In [16]:
```
import numpy as np
def init_params(nx, nh, ny):
    """Initialise les poids et biais."""
    W1 = np.random.normal(0, 0.3, (nh, nx))
    b1 = np.zeros((nh, 1))
    W2 = np.random.normal(0, 0.3, (ny, nh))
    b2 = np.zeros((ny, 1))

    return {'W1': W1, 'b1': b1, 'W2': W2, 'b2': b2}
```

In [17]:
```
def forward(params, X):
    """Effectue une passe avant à travers le réseau."""
    W1, b1, W2, b2 = params['W1'], params['b1'], params['W2'], params['b2']

    Z1 = np.dot(X, W1.T) + b1.T  # Correction : b1.T → b1
    A1 = np.tanh(Z1)
    Z2 = np.dot(A1, W2.T) + b2.T
    A2 = np.exp(Z2) / np.sum(np.exp(Z2), axis=1, keepdims=True)  # Softmax

    return A2, Z1, A1
```

In [18]:
```
def loss_accuracy(Yhat, Y):
    """Calculates the cross-entropy loss and accuracy."""
    loss = -np.mean(np.sum(Y * np.log(Yhat + 1e-8), axis=1))  # Correction de tr
    accuracy = 100 * np.mean(np.argmax(Yhat, axis=1) == np.argmax(Y, axis=1))
    return loss, accuracy
```

Backpropagation Algorithm

The backpropagation algorithm is used to update the weights in a neural network. The weight update rule is given by:

W_ji <- W_ji + ΔW_ji

where the weight change ΔW_ji is calculated as:

ΔW_ji = -η * (dE_d / dW_ji)

Here, η is the learning rate, and E_d is the error for a given data point d.

Chain Rule Application

The weight W_ji influences the network through net_j, which is the weighted sum of inputs to neuron j:

net_j = sum_i (W_ji * X_ji)

Using the chain rule, we can express the partial derivative of the error with respect to the weight as:

dE_d / dW_ji = (dE_d / dnet_j) * (dnet_j / dW_ji)

Since $dnet_j / dW_{ji} = X_{ji}$, the equation simplifies to:

$dE_d / dW_{ji} = (dE_d / dnet_j) * X_{ji}$

Thus, the weight update rule becomes:

$\Delta W_{ji} = -\eta * (dE_d / dnet_j) * X_{ji}$

Remaining Task

The remaining task is to derive a convenient expression for $dE_d / dnet_j$, which depends on the activation function and the error propagation through the network.

Derivation of Back Propagation Algorithm

Case 1: Output Unit (Softmax Activation)

When unit j is an output unit of the network, the error can be directly computed based on the difference between the predicted output and the actual target.

Training Rule for Output Unit Weights

The weight update rule for output unit weights is given by:

$\Delta W_{ji} = -\eta * (dE_d / dW_{ji})$

Using the chain rule, we can express this as:

$\Delta W_{ji} = -\eta * (dE_d / dnet_j) * X_{ji}$

Where:

$dE_d / dnet_j = (o_j - t_j)$

Thus, the weight update rule becomes:

$\Delta W_{ji} = \eta * (o_j - t_j) * X_{ji}$

We can also define δ_j as:

$\delta_j = (o_j - t_j)$

So the weight update rule can be simplified to:

$\Delta W_{ji} = \eta * \delta_j * X_{ji}$

Case 2: Hidden Unit (Tanh Activation)

When unit j is an internal (hidden) unit of the network, the error is propagated backward from the output layers through the network.

Training Rule for Hidden Unit Weights

For hidden units, the error is propagated backward from the downstream layers. The derivative of the error with respect to the net input of a hidden unit net_j is given by:

dE_d / dnet_j = sum over downstream units k (dE_d / dnet_k * dnet_k / dnet_j)

Where:

- dnet_k / dnet_j = W_kj * do_j / dnet_j
- do_j / dnet_j is the derivative of the activation function (tanh).

For the **tanh** activation function:

do_j / dnet_j = 1 - o_j^2

Thus, the error term for the hidden unit j becomes:

δ_j = (sum over downstream units k (δ_k * W_kj)) * (1 - o_j^2)

The weight update rule for hidden unit weights is then:

ΔW_ji = η * δ_j * X_ji

In [19]:
```
def backward(params, outputs, Y, X):
    """Calcule les gradients de la perte par rapport aux paramètres."""
    Yhat, Z1, A1 = outputs
    m = Y.shape[0]

    δ2 = Yhat - Y  # Pas de transposition
    dW2 = np.dot(δ2.T, A1) / m  # Correction de dimension
    db2 = np.sum(δ2, axis=0, keepdims=True) / m
    db2 = db2.T

    δ1 = np.dot(δ2, params['W2']) * (1 - np.power(A1, 2))  # Derivée de tanh
    dW1 = np.dot(δ1.T, X) / m
    db1 = np.sum(δ1, axis=0, keepdims=True) / m
    db1 = db1.T

    grads = {'W1': dW1, 'b1': db1, 'W2': dW2, 'b2': db2}
    return grads
```

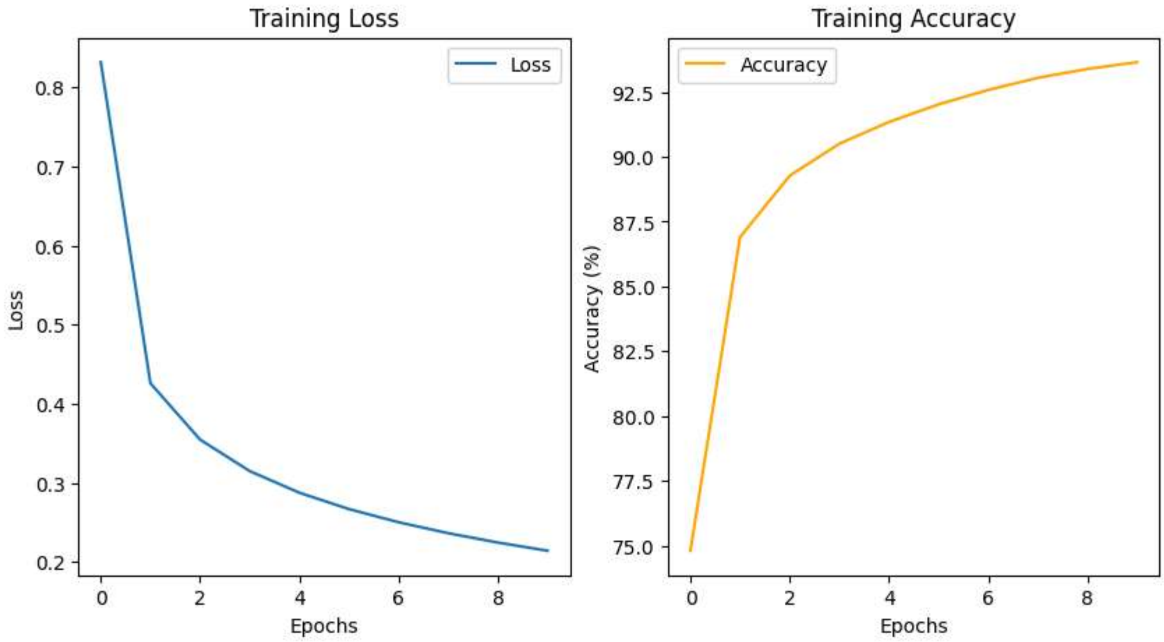
In [20]:
```
def sgd(params, grads, eta):
    """Met à jour les paramètres via la descente de gradient."""
    for key in params.keys():
        params[key] -= eta * grads[key]
```

In [21]:
```
import matplotlib.pyplot as plt
import pickle
from tensorflow.keras.datasets import mnist
```

```
# Load MNIST dataset
(X_train, Y_train), (X_test, Y_test) = mnist.load_data()

# Preprocess data
X_train = X_train.reshape(-1, 28 * 28) / 255.0
X_test = X_test.reshape(-1, 28 * 28) / 255.0
Y_train = np.eye(10)[Y_train]  # One-hot encode labels
Y_test = np.eye(10)[Y_test]

# Hyperparameters
nx = X_train.shape[1]
nh = 132
ny = 10
epochs = 10
mini_batch_size = 128
eta = 0.05

# Initialize parameters
params = init_params(nx, nh, ny)

# Training loop
loss_history = []
accuracy_history = []

for epoch in range(epochs):
    epoch_loss = []
    epoch_accuracy = []

    for i in range(0, X_train.shape[0], mini_batch_size):
        X_batch = X_train[i:i + mini_batch_size]
        Y_batch = Y_train[i:i + mini_batch_size]

        # Forward pass
        Yhat, Z1, A1 = forward(params, X_batch)

        # Backward pass
        grads = backward(params, (Yhat, Z1, A1), Y_batch, X_batch)

        # Update parameters
        sgd(params, grads, eta)

        # Calculate and store loss and accuracy
        loss, accuracy = loss_accuracy(Yhat, Y_batch)
        epoch_loss.append(loss)
        epoch_accuracy.append(accuracy)

    # Calcul de la moyenne de la loss et de l'accuracy pour l'epoch
    avg_loss = np.mean(epoch_loss)
    avg_accuracy = np.mean(epoch_accuracy)
    loss_history.append(avg_loss)
    accuracy_history.append(avg_accuracy)

    # Affichage des métriques pour l'epoch en cours
    print(f"Epoch {epoch+1}/{epochs} - Loss: {avg_loss:.4f} - Accuracy: {avg_acc

    # Condition d'arrêt si précision > 99%
    if avg_accuracy > 99:
        break
```

```python
# Sauvegarde des paramètres du modèle
with open("model_mnist.pkl", "wb") as file:
    pickle.dump(params, file)

print("✅ Modèle sauvegardé avec succès !")


# Plot loss and accuracy
plt.figure(figsize=(10, 5))

plt.subplot(1, 2, 1)
plt.plot(loss_history, label="Loss")
plt.title("Training Loss")
plt.ylabel("Loss")
plt.xlabel("Epochs")
plt.legend()

plt.subplot(1, 2, 2)
plt.plot(accuracy_history, label="Accuracy", color='orange')
plt.title("Training Accuracy")
plt.ylabel("Accuracy (%)")
plt.xlabel("Epochs")
plt.legend()

plt.show()

# Evaluate on test set
Yhat_test, _, _ = forward(params, X_test)
test_loss, test_accuracy = loss_accuracy(Yhat_test, Y_test)
print(f"Test Loss: {test_loss:.4f}, Test Accuracy: {test_accuracy:.2f}%")
```

```
Epoch 1/10 - Loss: 0.8317 - Accuracy: 74.80%
Epoch 2/10 - Loss: 0.4260 - Accuracy: 86.91%
Epoch 3/10 - Loss: 0.3547 - Accuracy: 89.27%
Epoch 4/10 - Loss: 0.3149 - Accuracy: 90.51%
Epoch 5/10 - Loss: 0.2876 - Accuracy: 91.35%
Epoch 6/10 - Loss: 0.2669 - Accuracy: 92.03%
Epoch 7/10 - Loss: 0.2502 - Accuracy: 92.59%
Epoch 8/10 - Loss: 0.2364 - Accuracy: 93.05%
Epoch 9/10 - Loss: 0.2246 - Accuracy: 93.40%
Epoch 10/10 - Loss: 0.2143 - Accuracy: 93.66%
✅ Modèle sauvegardé avec succès !
```




```
Test Loss: 0.2249, Test Accuracy: 93.04%
```

In [22]:
```
# Chargement du modèle sauvegardé
with open("model_mnist.pkl", "rb") as file:
    loaded_params = pickle.load(file)

print("✅ Modèle chargé avec succès !")

# Sélection d'une image aléatoire
index = np.random.randint(0, X_test.shape[0])
X_sample = X_test[index].reshape(1, -1)  # Reshape pour correspondre au format a
Y_sample_true = np.argmax(Y_test[index])  # Classe réelle

# Prédiction avec le modèle chargé
Yhat_sample, _, _ = forward(loaded_params, X_sample)
Y_sample_pred = np.argmax(Yhat_sample)  # Classe prédite

# Affichage de l'image et du résultat
plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
plt.title(f"Vrai: {Y_sample_true} | Prédit: {Y_sample_pred}")
plt.axis("off")
plt.show()
```

```
✅ Modèle chargé avec succès !
```

Vrai: 5 | Prédit: 5