In the name of God

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COMPUTATIONAL PHYSICS

Exercise Set 5

(Due Date: 1402/01/25)

1. Moment and Cumulant: For data package, compute M_n for $n = 1, 2, 3, 4, 5, 10$ and plot M_n as a function of n for mentioned n. Also plot $\mathcal{K}_n$ for $n = 1, 2, 3, 4, 5, 6, 10$.

2. Skewness and Kurtosis: Compute α_3 and α_4 for data package.

3. Joint PDF:
A : For the input data set, compute $\Delta(\tau) \equiv \int dxdy|p(x,t;y,t+\tau) - p(x,t)p(y,t+\tau)|$ as a function of τ. Explain your results.
B : For the input data set, compute $\Delta(\tau) \equiv \int dx_1dx_3|p(x_3,t+2\tau;x_1,t) - \int dx_2 p(x_3,t+2\tau|x_2,t+\tau)p(x_2,t+\tau|x_1,t)p(x_1,t)|$ as a function of τ. Explain your results.

4. According to Box-Muller algorithm, generate Gaussian random field with $\sigma_0^2 = 2$ and $\langle x\rangle = 3$. Check your results by fitting a Gaussian function on the computed PDF of your generated data.

5. According to Von-Neumann method, generate a set of random data set in the range $x \in [1-5]$ with PDF as: $p(x) = \sin(x^2/100) + \frac{1}{\cos(x^3/100)} + x^{-3}$.

6. PDF transformation: Suppose that in a black box a harmonic oscillator is oscillating and you made a series of snapshots randomly through time. Determine the PDF of the location of the oscillator in the stationary case.

7. Suppose that x has the Pareto distribution, $p(x) = \frac{a}{x^{a+1}}$ for $1 \leq x < \infty$. Find the probability density function of each of the following random variables:

A : $y = x^2$.
B : $z = \frac{1}{x}$.
C : $T = \ln(x)$.

8. Using the input file (data1), write a proper program to do the following tasks. Remember that you must split data into 100 part with equal size.

A : Compute $C(i,j) = \langle x(t_i)x(t_j)\rangle$. To this end, you must do the averaging on the 100 data sets. Make a matrix and plot it as a density plot.
B : Compute $C_i(\tau) = \langle x(t+\tau)x(t)\rangle$ for series and plot it for 5 sets of you data.

9. According to Pearson correlation coefficient, compute the degree of correlation between 0.2.txt and 0.5.txt as well as with themselves.

10. Compute $C(\tau) = \langle x(t+\tau)x(t)\rangle$ for 0.2.txt and 0.5.txt and 0.8.txt data sets. Interpret your results.

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11. Non-linear correlation. There are many methods to compute non-linear correlation coefficient. According to Wang, Qiang, Yi Shen, and Jian Qiu Zhang. "A nonlinear correlation measure for multivariable data set." Physica D: Nonlinear Phenomena 200.3-4 (2005): 287-295, and use the Eqs. (1), (2) and (3) of mentioned paper, compute the mutual information between all pairs of 0.2.txt, 0.5.txt and 0.8.txt.

12. Linear and non-linear correlation coefficients. Pearson's coefficient is a familiar measure to quantify the linear-correlation, while for assessing non-linear relation and even to determine the degree of correlation in the presence of outliers the Spearman's correlation coefficient is used. For all available pairs of 0.2.txt, 0.5.txt and 0.8.txt data sets, compute Spearman's and Pearson's correlation coefficient compare your results. Where:

$$\rho_p \equiv \frac{\langle [x - \langle x \rangle][y - \langle y \rangle] \rangle}{\sigma_x \sigma_y}$$

$$\rho_s \equiv 1 - 6 \frac{\sum_i d_i^2}{N(N^2 - 1)}$$

and $d_i \equiv [Rank(x_i) - Rank(y_i)]$ and $Rank$ means the order of value of variable in a set. Suppose that for $\{x\} : \{20, 100, 30, 50, 160, 10\}$. Then the $Rank(x) : \{5, 2, 4, 3, 1, 6\}$.

Good luck, Movahed