final

June 21, 2023

1 Final

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[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
```

2 Q5:

N	N-point stencil Central Differences
3	$\dfrac{f_1 - f_{-1}}{2h}$
5	$\dfrac{f_{-2} - 8f_{-1} + 8f_1 - f_2}{12h}$
7	$\dfrac{-f_{-3} + 9f_{-2} - 45f_{-1} + 45f_1 - 9f_2 + f_3}{60h}$
9	$\dfrac{3f_{-4} - 32f_{-3} + 168f_{-2} - 672f_{-1} + 672f_1 - 168f_2 + 32f_3 - 3f_4}{840h}$

[ ]:
```
def derivative_5p(f):
    """ompute the derivative of signal with 5-point neighbors in central␣
 ↪difference formula

    Args:
        f (list or array): data

    Returns:
```

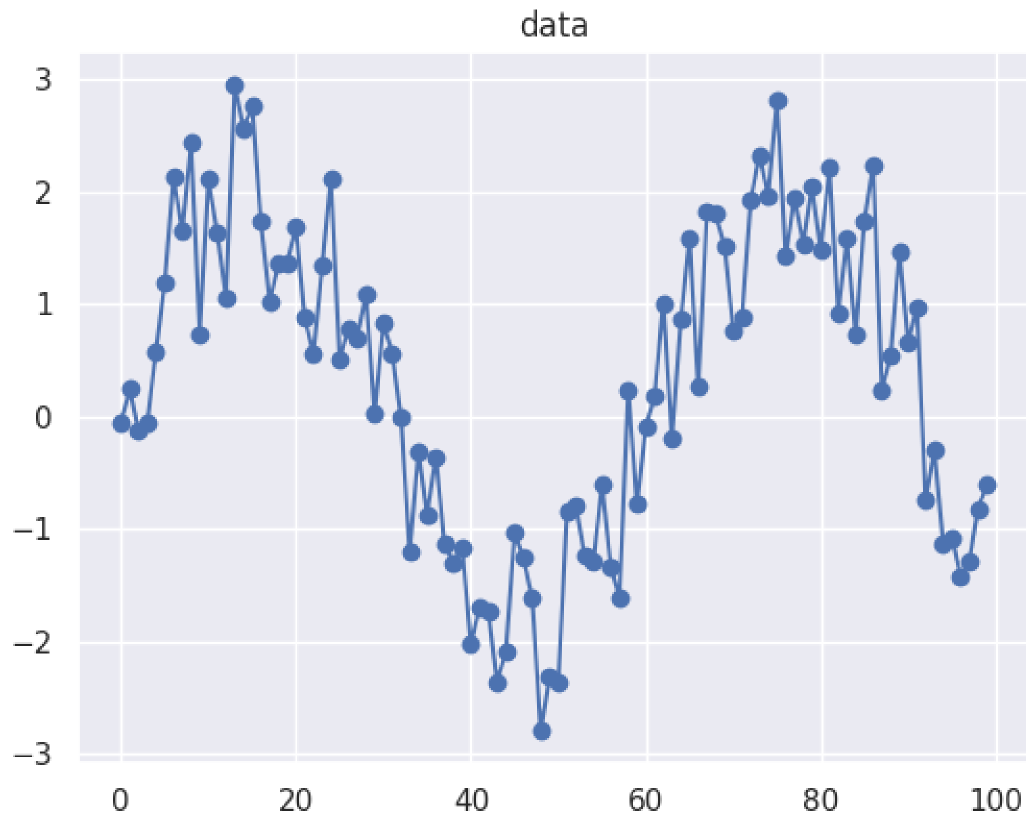
```
        1d_array: derivative of data
    """
    h = 1
    N = len(f)
    dr = []
    dr.append((f[1]-f[0])/(2*h))
    dr.append((f[2]-f[1])/(2*h))
    for i in range(2,N-2):
        dr.append((f[i-2]-8*f[i-1]+8*f[i+1]-f[i+2])/(12*h))
    dr.append((f[N-1]-f[N-2])/(2*h))
    return np.array(dr)
```

[ ]:
```
def derivative_7p(f):
    """ompute the derivative of signal with 7-point neighbors in central␣
 ↪difference formula

    Args:
        f (list or array): data

    Returns:
        1d_array: derivative of data
    """
    h = 1
    N = len(f)
    dr = []
    dr.append((f[1]-f[0])/(2*h))
    dr.append((f[2]-f[1])/(2*h))
    dr.append((f[3]-f[2])/(2*h))
    for i in range(3,N-3):
        dr.append((-f[i-3]+9*f[i-2]-45*f[i-1]+45*f[i+1]-9*f[i+2]+f[i+3])/(60*h))
    dr.append((f[N-2]-f[N-3])/(2*h))
    dr.append((f[N-1]-f[N-2])/(2*h))
    return np.array(dr)
```

2.1 Load data

[ ]:
```
data = np.loadtxt("data_obs2.txt")
data = data[:,1]
```

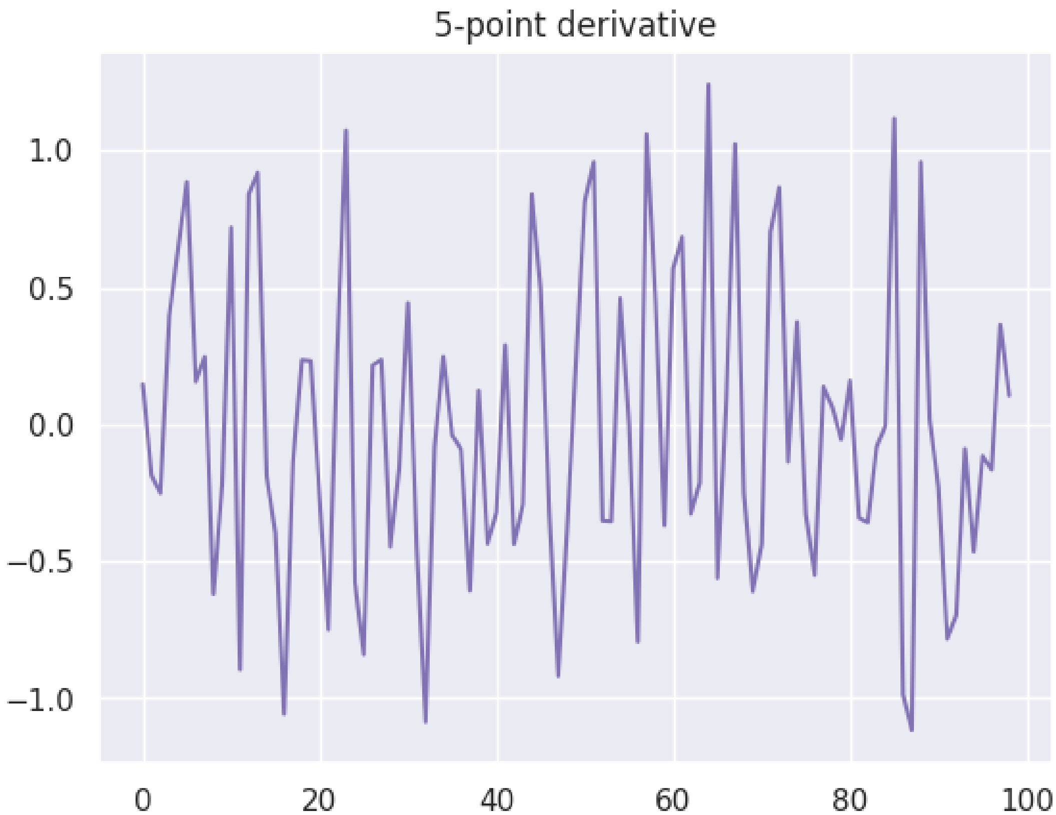
[ ]:
```
plt.plot(data,"-o")
plt.title("data")
plt.show()
```



2.2 Four neighbours:

[ ]:
```
d5 = derivative_5p(data)
```

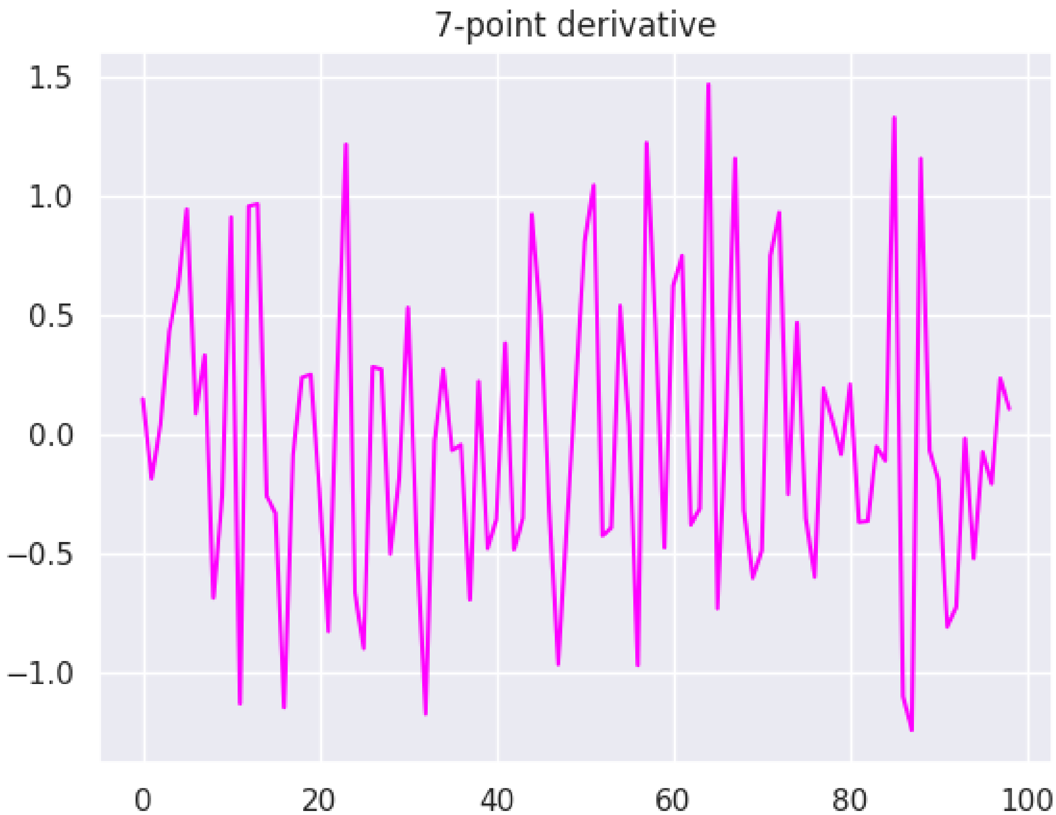
[ ]:
```
plt.plot(d5, color="m")
plt.title("5-point derivative")
plt.show()
```



2.3 Six neighbours:

[ ]:
```
d7 = derivative_7p(data)
```

[ ]:
```
plt.plot(d7, color="magenta")
plt.title("7-point derivative")
plt.show()
```



3 Q6:

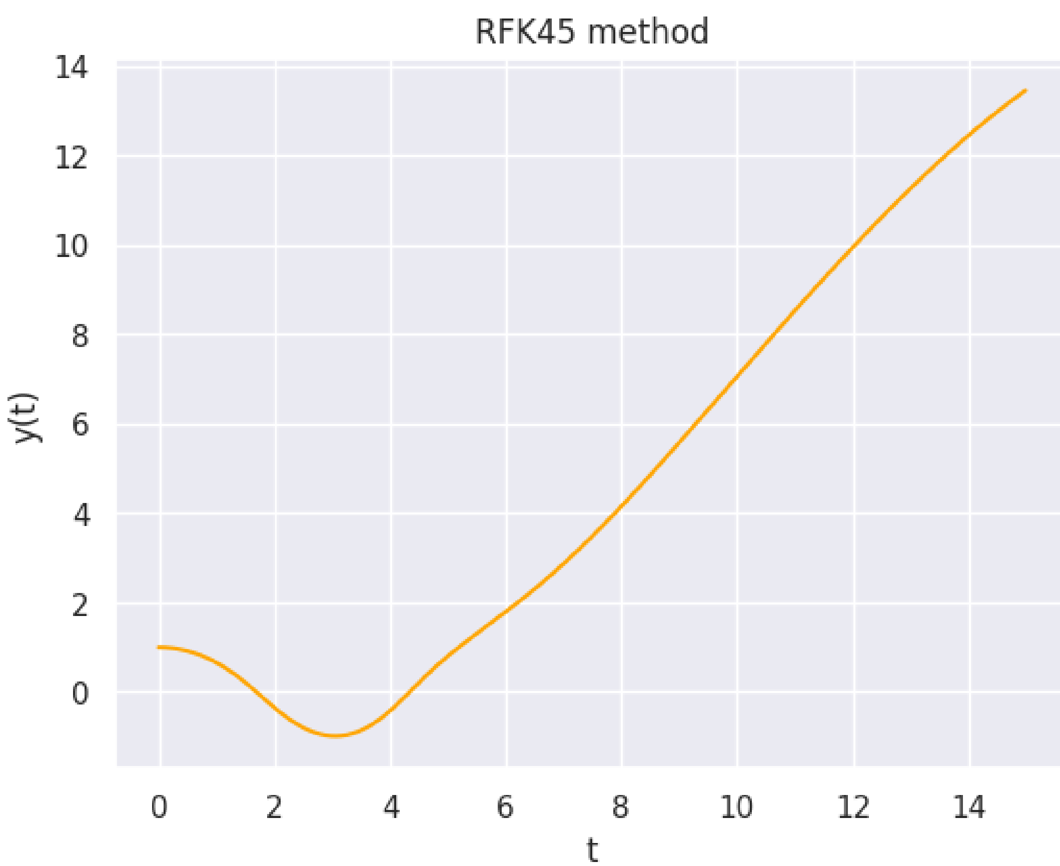
3.1 A)

```
[ ]: dt = 0.01
     T = np.arange(0, 15, dt)
     N = len(T)
     Y = np.zeros(N)
     Y[0] =1
     V = np.zeros(N)
```

```
[ ]: for t in range(N-1):
         f1 = V[t]
         k1 = np.sin(0.2*T[t]) - ((2*Y[t])/(Y[t]**2 +1)**(3/2)) - (0.4 * f1**2)

         f2 = V[t] + (dt/2)*k1
         k2 = np.sin(0.2*(T[t]+(dt/2))) -((2*(Y[t]+(dt/2)*f1))/((Y[t]+(dt/2)*f1)**2␣
      ↪+1)**(3/2)) - (0.4 * f2**2)

         f3 = V[t] + (dt/2)*k2
```

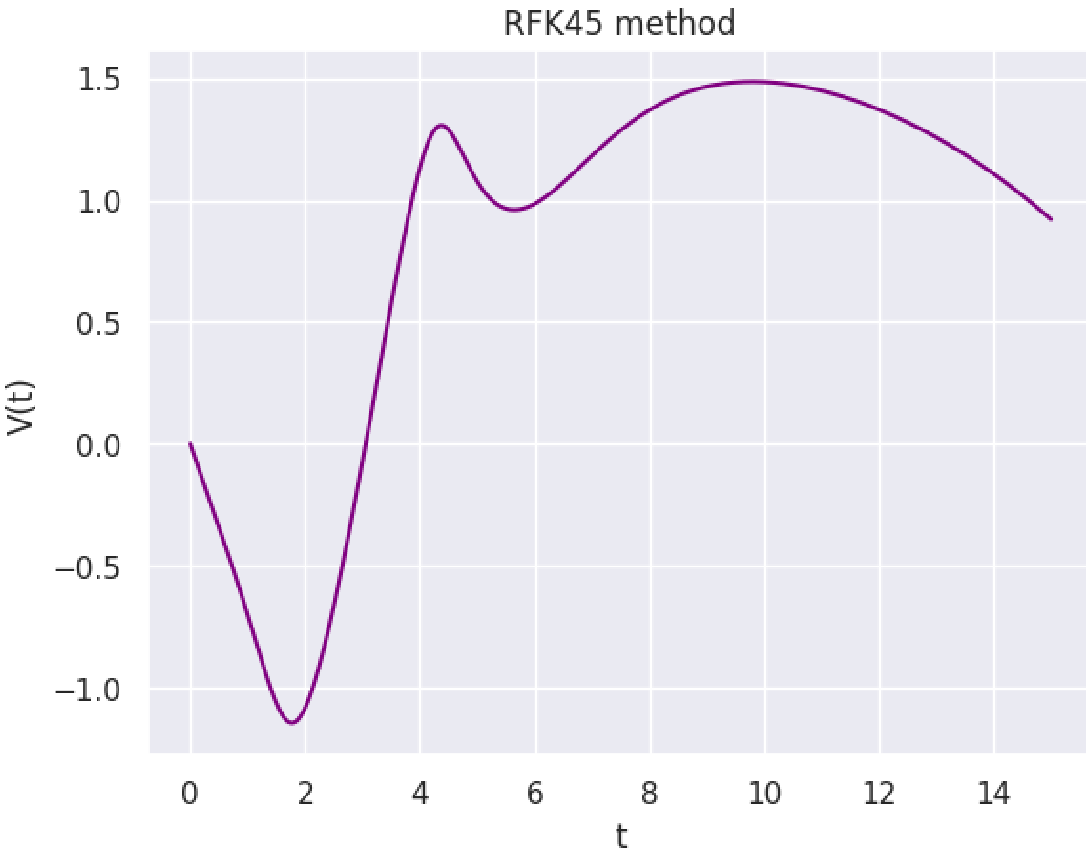
```
    k3 = np.sin(0.2*(T[t]+(dt/2))) -((2*(Y[t]+(dt/2)*f2))/((Y[t]+(dt/2)*f2)**2␣
↪+1)**(3/2))  - (0.4 * f3**2)

    f4 = V[t] + (dt)*k3
    k4 = np.sin(0.2*(T[t]+(dt))) - ((2*(Y[t]+dt*f3))/((Y[t]+dt*f3)**2 +1)**(3/
↪2)) - (0.4 * f4**2)

    V[t+1] = V[t] + (dt/6) * (k1+ 2*k2 + 2*k3 + k4)
    Y[t+1] = Y[t] + (dt/6) * (f1+ 2*f2 + 2*f3 + f4)
```

[ ]:
```
plt.plot(T,Y, color="orange")
plt.title("RFK45 method")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.show()
```

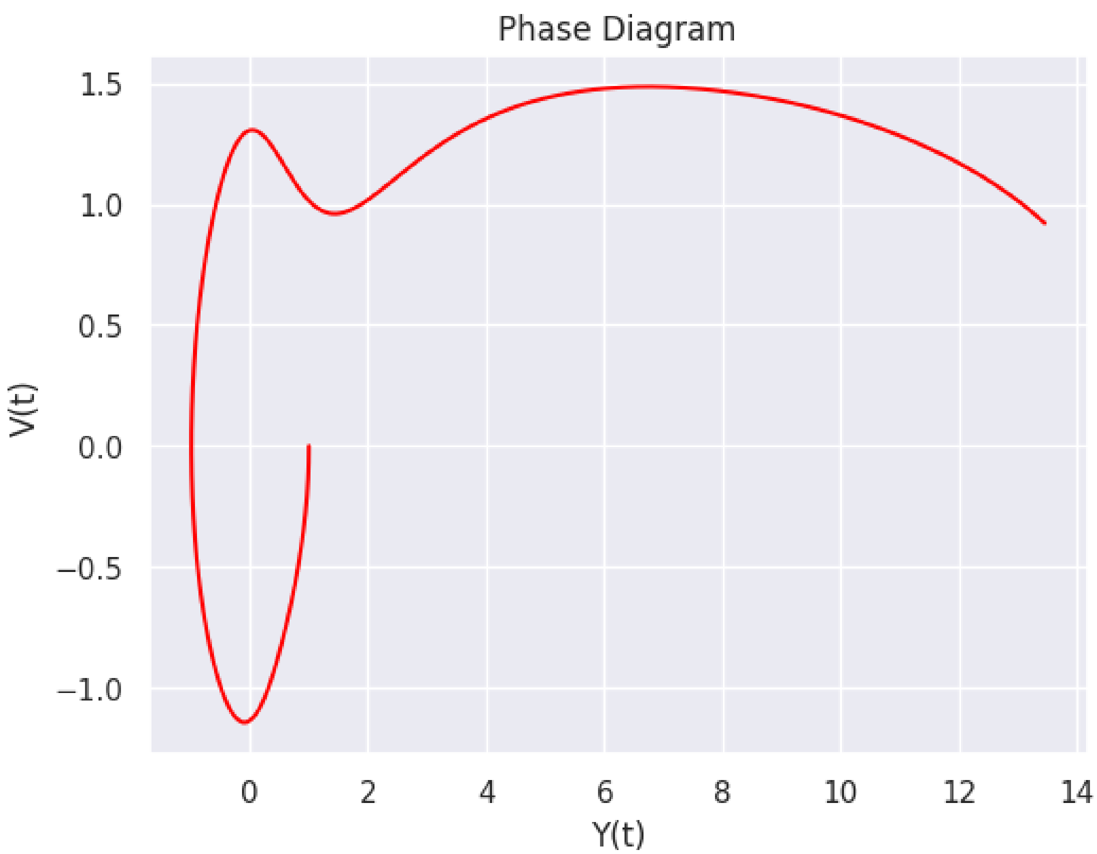


[ ]:
```
plt.plot(T,V, color="purple")
plt.title("RFK45 method")
plt.xlabel("t")
```

```
plt.ylabel("V(t)")
plt.show()
```



[ ]:
```
plt.plot(Y,V, color="red")
plt.title("Phase Diagram")
plt.xlabel("Y(t)")
plt.ylabel("V(t)")
plt.show()
```



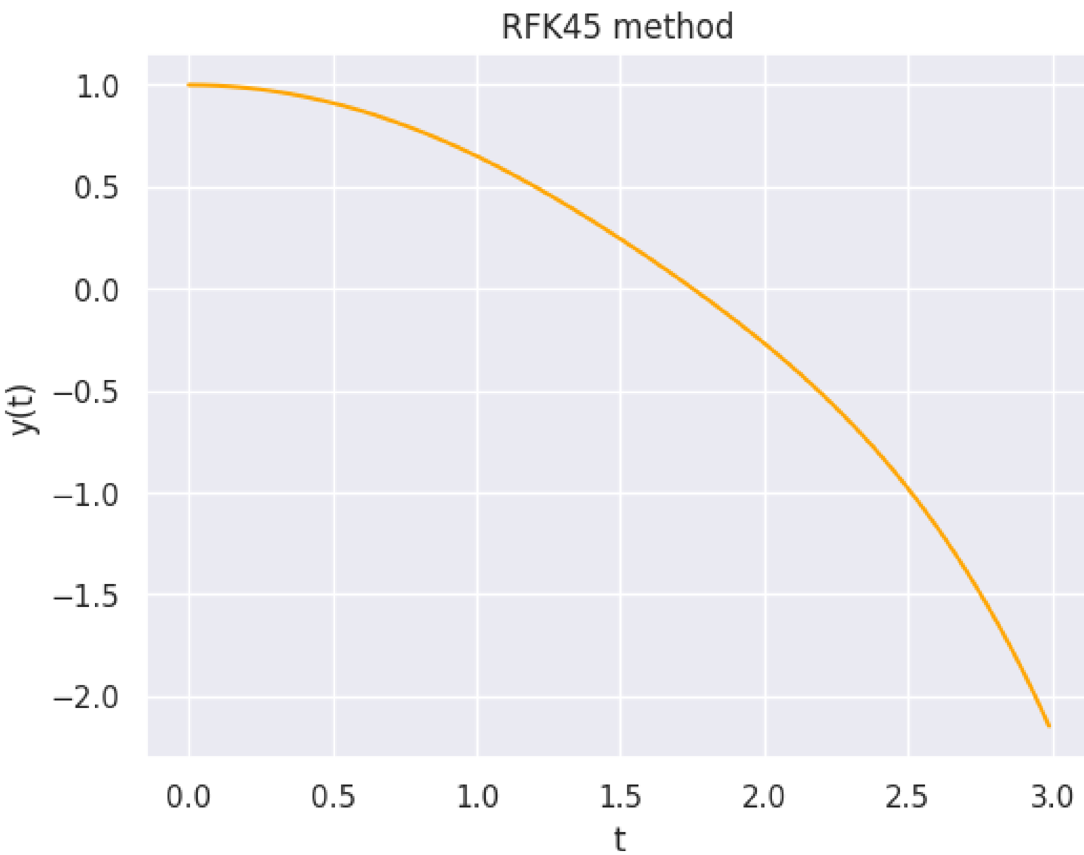
3.2 B)

```
[ ]: dt = 0.01
     T = np.arange(0, 3, dt)
     N = len(T)
     Y = np.zeros(N)
     Y[0] = 1
     V = np.zeros(N)
```

```
[ ]: for t in range(N-1):
         f1 = V[t]
         k1 = - ((2*np.cos(T[t])*Y[t])/(Y[t]**2 +1)**(3/2)) - (0.4 * f1**2)

         f2 = V[t] + (dt/2)*k1
         k2 = -((2*np.cos(T[t]+(dt/2))*(Y[t]+(dt/2)*f1))/((Y[t]+(dt/2)*f1)**2␣
      ↪+1)**(3/2)) - (0.4 * f2**2)

         f3 = V[t] + (dt/2)*k2
```

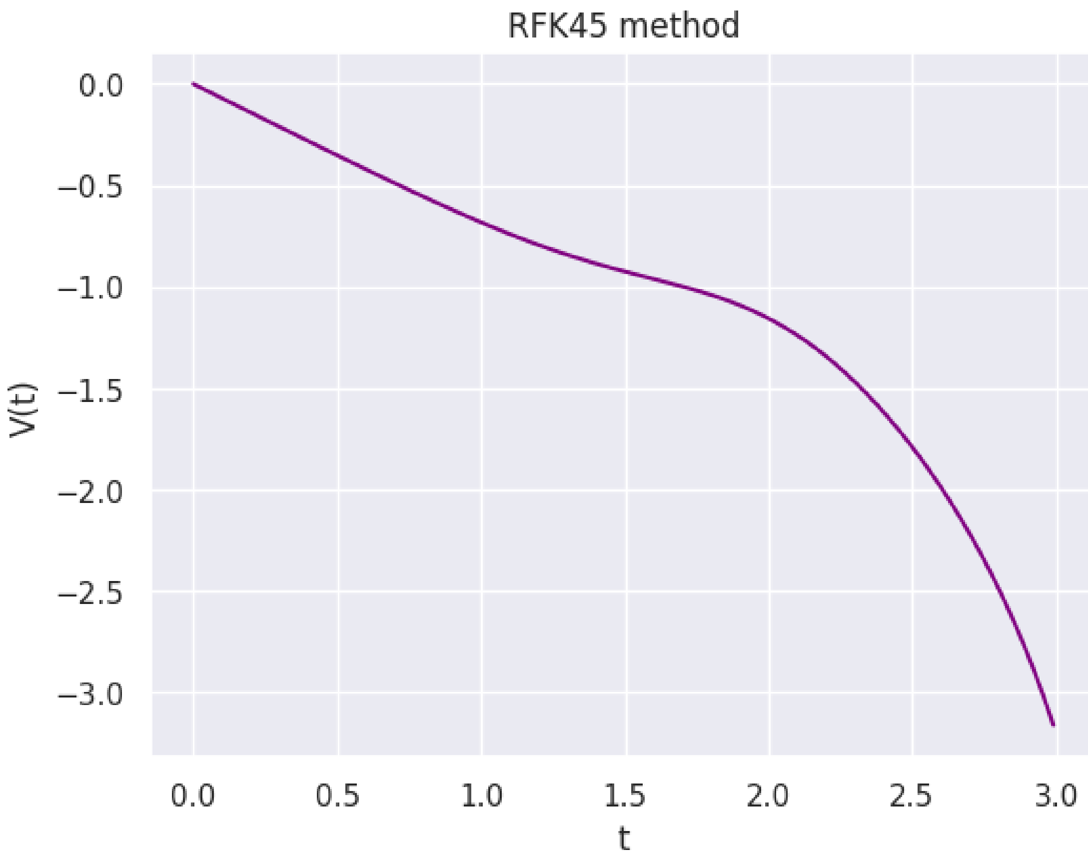
```
    k3 = -((2*np.cos(T[t]+(dt/2))*(Y[t]+(dt/2)*f2))/((Y[t]+(dt/2)*f2)**2␣
↪+1)**(3/2)) - (0.4 * f3**2)

    f4 = V[t] + (dt)*k3
    k4 = - ((2*np.cos(T[t]+dt)*(Y[t]+dt*f3))/((Y[t]+dt*f3)**2 +1)**(3/2)) - (0.
↪4 * f4**2)

    V[t+1] = V[t] + (dt/6) * (k1+ 2*k2 + 2*k3 + k4)
    Y[t+1] = Y[t] + (dt/6) * (f1+ 2*f2 + 2*f3 + f4)
```

[ ]:
```
plt.plot(T,Y, color="orange")
plt.title("RFK45 method")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.show()
```

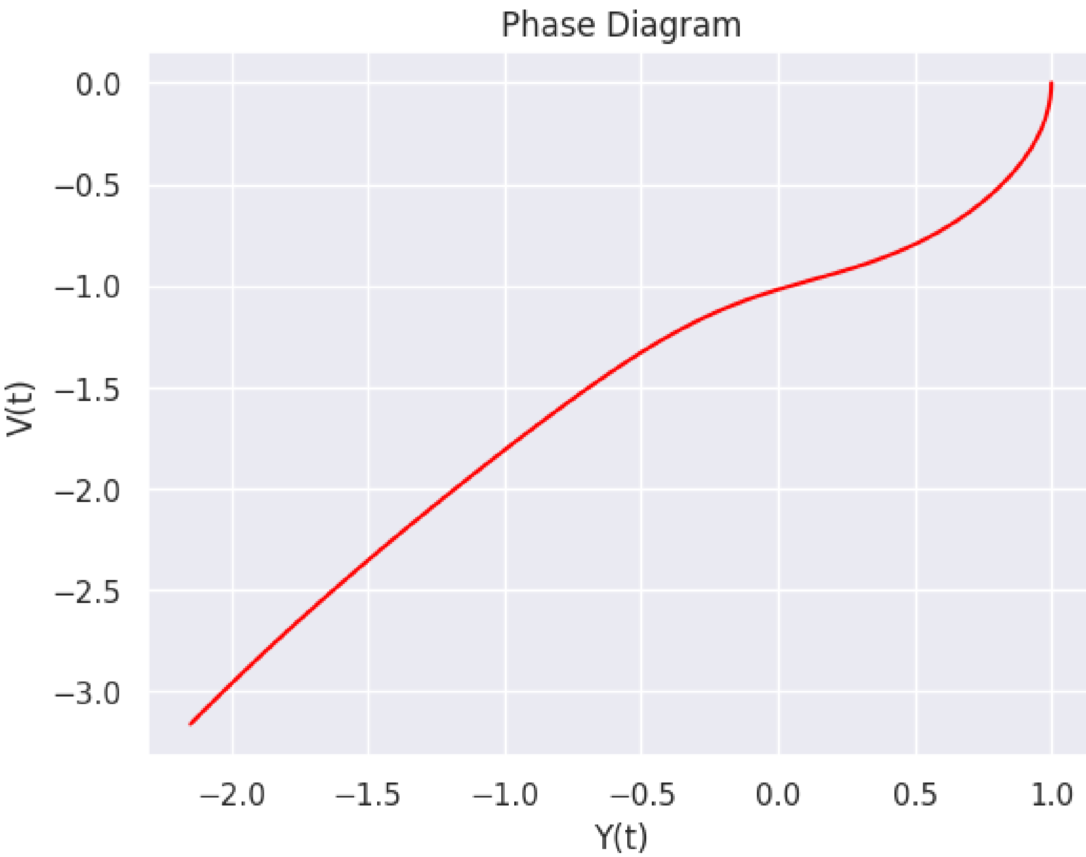


[ ]:
```
plt.plot(T,V, color="purple")
plt.title("RFK45 method")
plt.xlabel("t")
```

```
plt.ylabel("V(t)")
plt.show()
```



[ ]:
```
plt.plot(Y,V, color="red")
plt.title("Phase Diagram")
plt.xlabel("Y(t)")
plt.ylabel("V(t)")
plt.show()
```



3.2.1 Check Chaotic

```
[ ]: def eq1(V, Y):
         for t in range(N-1):
             f1 = V[t]
             k1 = np.sin(0.2*T[t]) - ((2*Y[t])/(Y[t]**2 +1)**(3/2)) - (0.4 * f1**2)

             f2 = V[t] + (dt/2)*k1
             k2 = np.sin(0.2*(T[t]+(dt/2))) -((2*(Y[t]+(dt/2)*f1))/((Y[t]+(dt/
      ↪2)*f1)**2 +1)**(3/2)) - (0.4 * f2**2)

             f3 = V[t] + (dt/2)*k2
             k3 = np.sin(0.2*(T[t]+(dt/2))) -((2*(Y[t]+(dt/2)*f2))/((Y[t]+(dt/
      ↪2)*f2)**2 +1)**(3/2))  - (0.4 * f3**2)

             f4 = V[t] + (dt)*k3
             k4 = np.sin(0.2*(T[t]+(dt))) - ((2*(Y[t]+dt*f3))/((Y[t]+dt*f3)**2␣
      ↪+1)**(3/2)) - (0.4 * f4**2)
```

```
        V[t+1] = V[t] + (dt/6) * (k1+ 2*k2 + 2*k3 + k4)
        Y[t+1] = Y[t] + (dt/6) * (f1+ 2*f2 + 2*f3 + f4)
    return V , Y
```

[ ]:
```
Y1 = np.zeros(N)
Y1[0] =1.01
V1 = np.zeros(N)
```

[ ]:
```
Y2 = np.zeros(N)
Y2[0] =1.02
V2 = np.zeros(N)
```

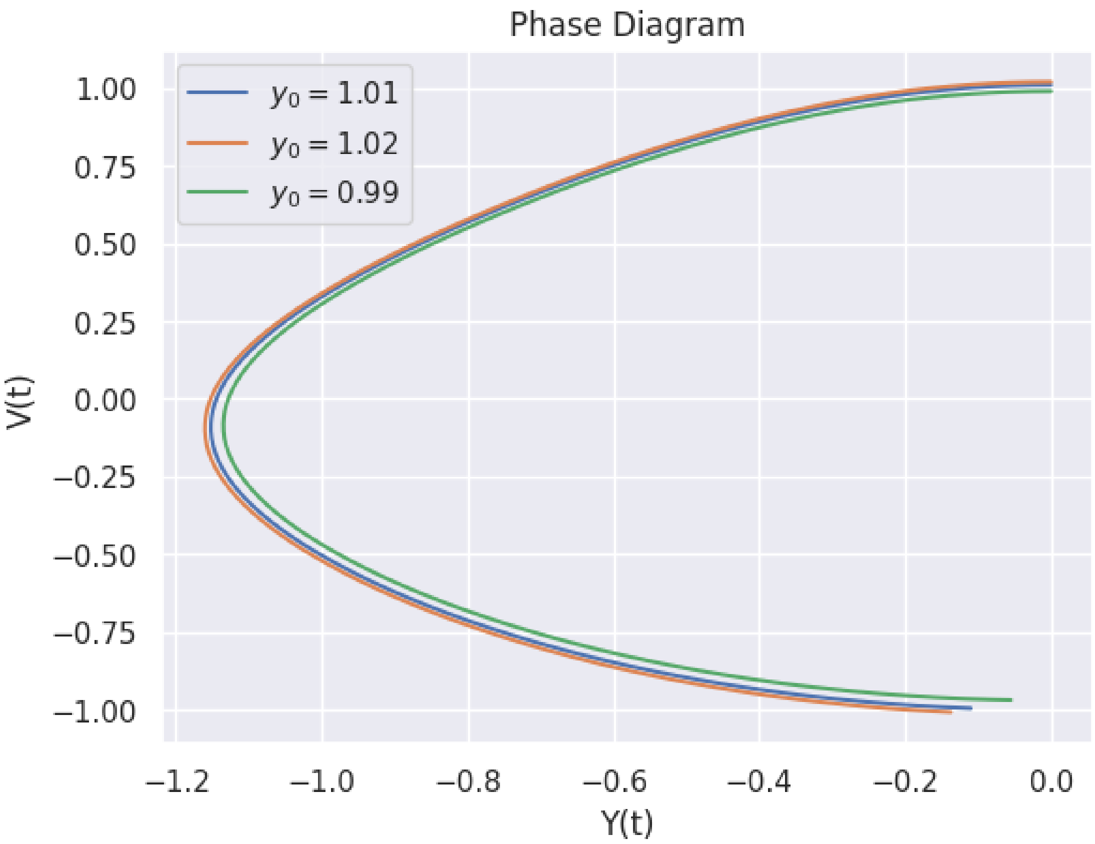
[ ]:
```
Y3 = np.zeros(N)
Y3[0] =0.99
V3 = np.zeros(N)
```

[ ]:
```
V1, Y1 = eq1(V1, Y1)
```

[ ]:
```
V2, Y2 = eq1(V2, Y2)
```

[ ]:
```
V3, Y3 = eq1(V3, Y3)
```

[ ]:
```
plt.plot(V1, Y1, label=r"$y_0 = 1.01$")
plt.plot(V2, Y2, label=r"$y_0 = 1.02$")
plt.plot(V3, Y3, label=r"$y_0 = 0.99$")
plt.title("Phase Diagram")
plt.xlabel("Y(t)")
plt.ylabel("V(t)")
plt.legend()
plt.show()
```



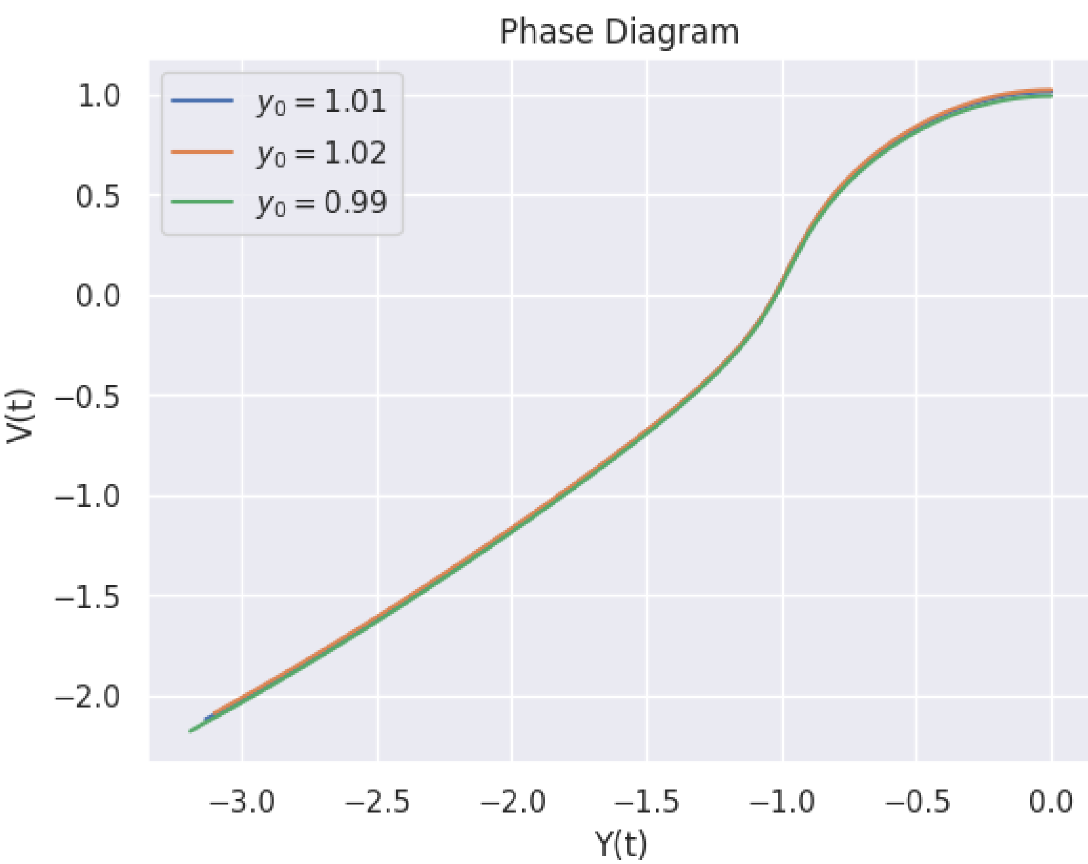
There is no chaotic behavior in the system, as we can see.

[ ]:
```
def eq2(V, Y):
    for t in range(N-1):
        f1 = V[t]
        k1 = - ((2*np.cos(T[t])*Y[t])/(Y[t]**2 +1)**(3/2)) - (0.4 * f1**2)

        f2 = V[t] + (dt/2)*k1
        k2 = -((2*np.cos(T[t]+(dt/2))*(Y[t]+(dt/2)*f1))/((Y[t]+(dt/2)*f1)**2
 ↪+1)**(3/2)) - (0.4 * f2**2)

        f3 = V[t] + (dt/2)*k2
        k3 = -((2*np.cos(T[t]+(dt/2))*(Y[t]+(dt/2)*f2))/((Y[t]+(dt/2)*f2)**2
 ↪+1)**(3/2)) - (0.4 * f3**2)

        f4 = V[t] + (dt)*k3
        k4 = - ((2*np.cos(T[t]+dt)*(Y[t]+dt*f3))/((Y[t]+dt*f3)**2 +1)**(3/2)) -
 ↪(0.4 * f4**2)

        V[t+1] = V[t] + (dt/6) * (k1+ 2*k2 + 2*k3 + k4)
```

```
        Y[t+1] = Y[t] + (dt/6) * (f1+ 2*f2 + 2*f3 + f4)
    return V , Y
```

[ ]:
```
V1, Y1 = eq2(V1, Y1)
V2, Y2 = eq2(V2, Y2)
V3, Y3 = eq2(V3, Y3)
```

[ ]:
```
plt.plot(V1, Y1, label=r"$y_0 = 1.01$")
plt.plot(V2, Y2, label=r"$y_0 = 1.02$")
plt.plot(V3, Y3, label=r"$y_0 = 0.99$")
plt.title("Phase Diagram")
plt.xlabel("Y(t)")
plt.ylabel("V(t)")
plt.legend()
plt.show()
```



There is no chaotic behavior in the system, as we can see.