# HW8

May 12, 2023

# 1 Exercise Set 8

## 1.1 Mohaddeseh Mozaffari

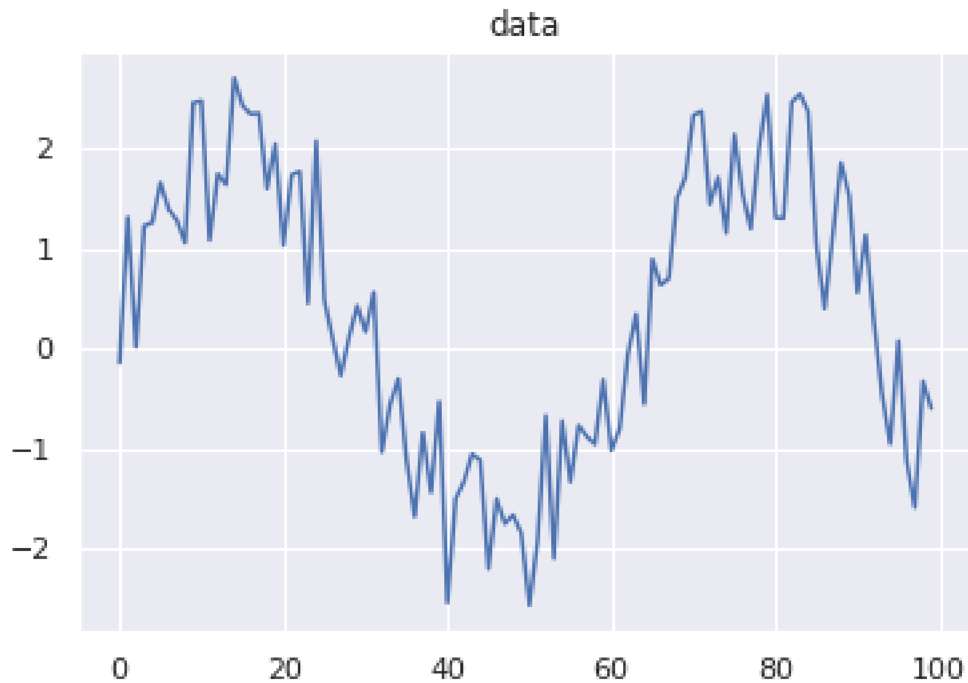
[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
```

# 2 Load data

[ ]:
```
data = np.loadtxt("/home/mohaddeseh/Documents/Programing/Computational/HW8/
    ↪dataprofile.txt")
```

[ ]:
```
data = data[:,1]
```

[ ]:
```
plt.plot(data)
plt.title("data")
plt.show()
```




## 3 Q1:

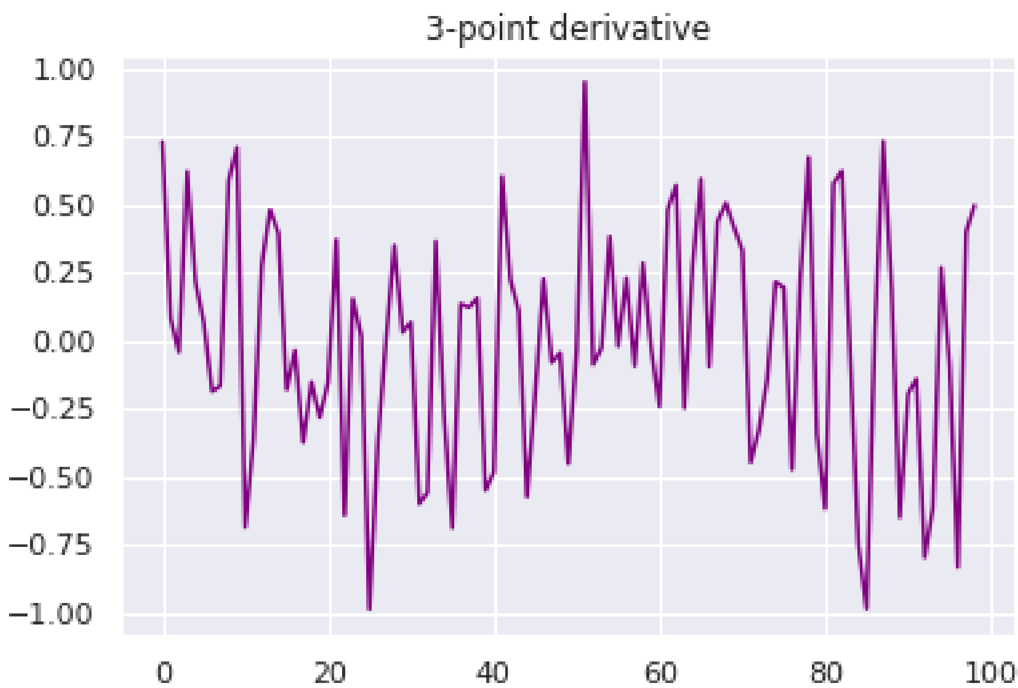
| $N$ | $N$-point stencil Central Differences |
|---|---|
| 3 | $\dfrac{f_1 - f_{-1}}{2h}$ |
| 5 | $\dfrac{f_{-2} - 8f_{-1} + 8f_1 - f_2}{12h}$ |
| 7 | $\dfrac{-f_{-3} + 9f_{-2} - 45f_{-1} + 45f_1 - 9f_2 + f_3}{60h}$ |
| 9 | $\dfrac{3f_{-4} - 32f_{-3} + 168f_{-2} - 672f_{-1} + 672f_1 - 168f_2 + 32f_3 - 3f_4}{840h}$ |

### 3.1 3-point:

[ ]:
```
def derivative_3p(f):
    """ompute the derivative of signal with 3-point neighbors in central␣
 ↪difference formula

    Args:
        f (list or array): data

    Returns:
        1d_array: derivative of data
    """
    h = 1
    N = len(f)
    dr = []
    dr.append((f[1]-f[0])/(2*h))
    for i in range(1,N-1):
        dr.append((f[i+1]-f[i-1])/(2*h))
    return np.array(dr)
```

[ ]:
```
d3 = derivative_3p(data)
```

[ ]:
```
plt.plot(d3, color="purple")
plt.title("3-point derivative")
plt.show()
```

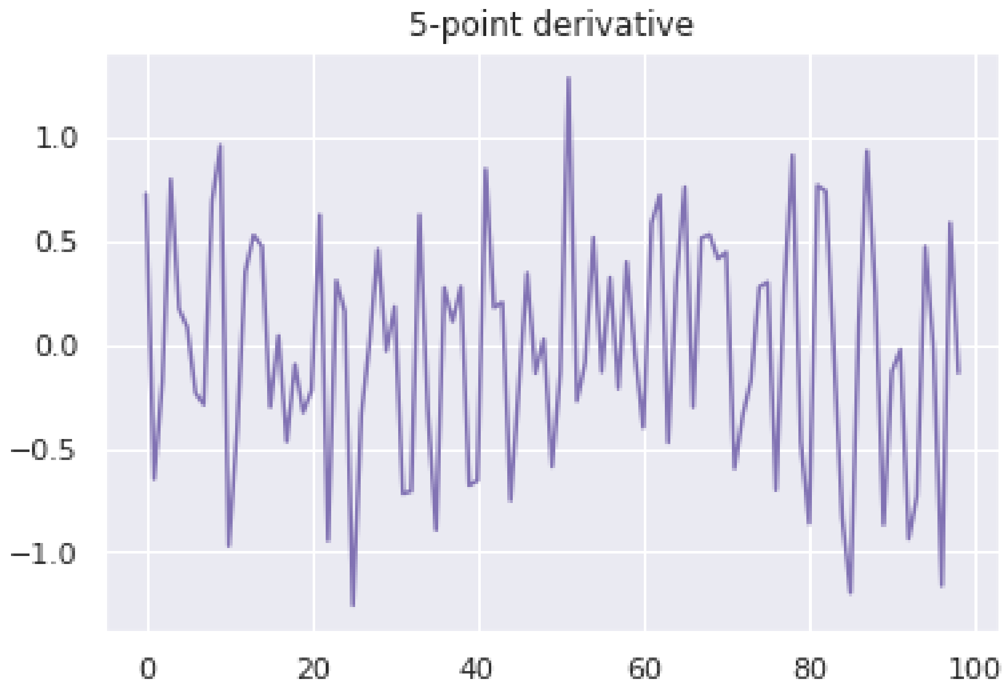


### 3.2 5-point:

[ ]:
```
def derivative_5p(f):
    """ompute the derivative of signal with 5-point neighbors in central␣
 ↪difference formula

    Args:
        f (list or array): data

    Returns:
        1d_array: derivative of data
    """
    h = 1
    N = len(f)
    dr = []
    dr.append((f[1]-f[0])/(2*h))
    dr.append((f[2]-f[1])/(2*h))
    for i in range(2,N-2):
        dr.append((f[i-2]-8*f[i-1]+8*f[i+1]-f[i+2])/(12*h))
    dr.append((f[N-1]-f[N-2])/(2*h))
    return np.array(dr)
```

[ ]:
```
d5 = derivative_5p(data)
```

[ ]:
```
plt.plot(d5, color="m")
plt.title("5-point derivative")
plt.show()
```

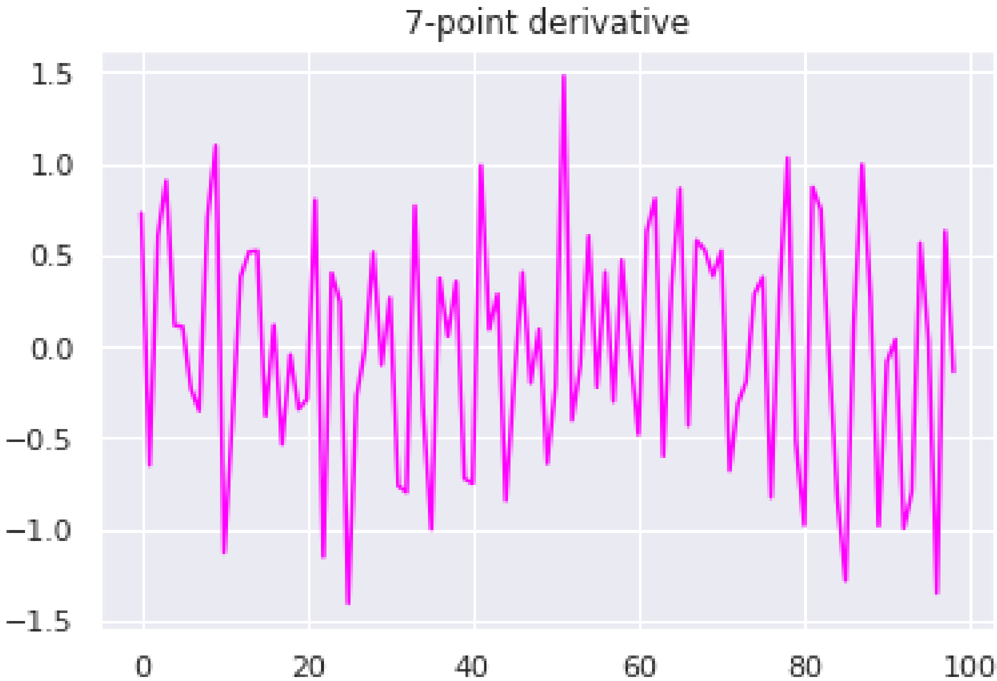



### 3.3 7-point:

```
[ ]: def derivative_7p(f):
         """ompute the derivative of signal with 7-point neighbors in central␣
      ↪difference formula

         Args:
             f (list or array): data

         Returns:
             1d_array: derivative of data
         """
         h = 1
         N = len(f)
         dr = []
         dr.append((f[1]-f[0])/(2*h))
         dr.append((f[2]-f[1])/(2*h))
         dr.append((f[3]-f[2])/(2*h))
         for i in range(3,N-3):
             dr.append((-f[i-3]+9*f[i-2]-45*f[i-1]+45*f[i+1]-9*f[i+2]+f[i+3])/(60*h))
         dr.append((f[N-2]-f[N-3])/(2*h))
         dr.append((f[N-1]-f[N-2])/(2*h))
         return np.array(dr)
```

[ ]:
```python
d7 = derivative_7p(data)
```

[ ]:
```python
plt.plot(d7, color="magenta")
plt.title("7-point derivative")
plt.show()
```



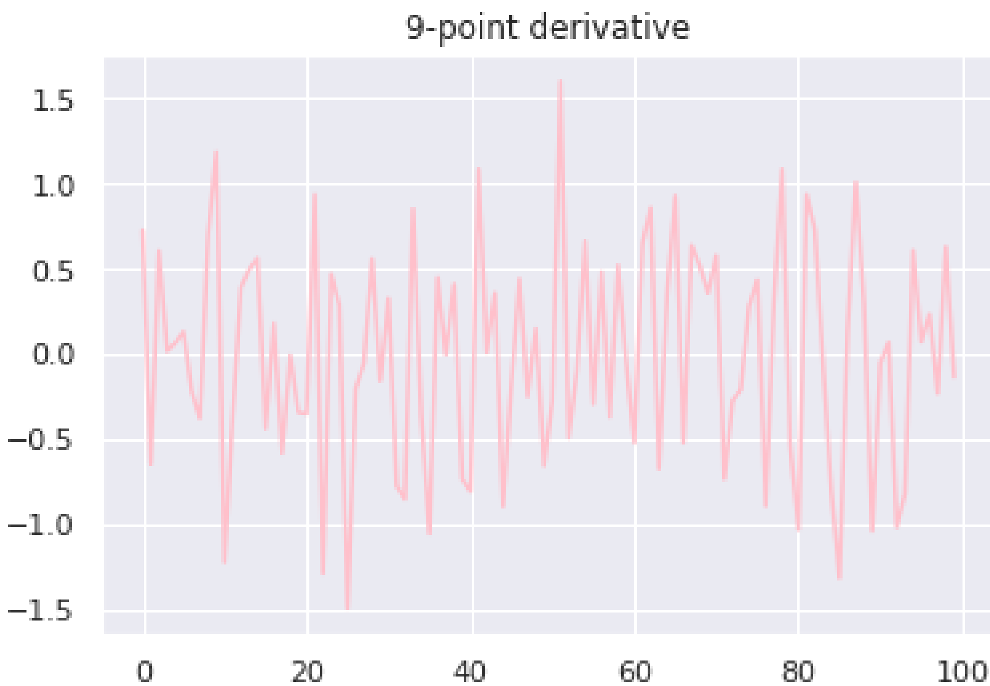

### 3.4 9-point:

[ ]:
```python
def derivative_9p(f):
    """ompute the derivative of signal with 9-point neighbors in central␣
 ↪difference formula

    Args:
        f (list or array): data

    Returns:
        1d_array: derivative of data
    """
    h = 1
    N = len(f)
    dr = []
    dr.append((f[1]-f[0])/(2*h))
    dr.append((f[2]-f[1])/(2*h))
    dr.append((f[3]-f[2])/(2*h))
```

```
    dr.append((f[4]-f[3])/(2*h))
    for i in range(4,N-4):
        dr.
↪append((3*f[i-4]-32*f[i-3]+168*f[i-2]-672*f[i-1]+672*f[i+1]-168*f[i+2]+32*f[i+3]-3*f[i+4])/
↪(840*h))
    dr.append((f[N-4]-f[N-3])/(2*h))
    dr.append((f[N-3]-f[N-4])/(2*h))
    dr.append((f[N-2]-f[N-3])/(2*h))
    dr.append((f[N-1]-f[N-2])/(2*h))
    return np.array(dr)
```

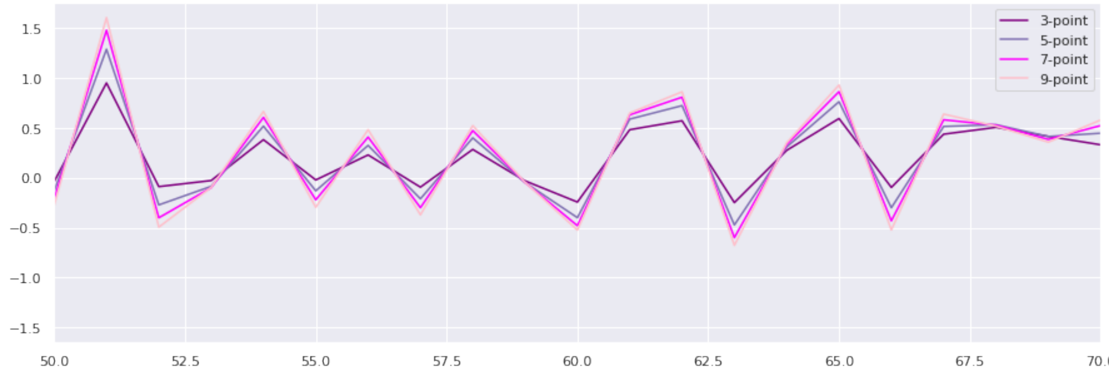
[ ]:
```
d9 = derivative_9p(data)
```

[ ]:
```
plt.plot(d9, color="pink")
plt.title("9-point derivative")
plt.show()
```




[ ]:
```
plt.figure(figsize=(15,5))
plt.plot(d3, color="purple", label="3-point")
plt.plot(d5, color="m", label="5-point")
plt.plot(d7, color="magenta", label="7-point")
plt.plot(d9, color="pink", label="9-point")
plt.legend()
plt.xlim(50,70)
```

```
plt.show()
```




# 4 Q2:

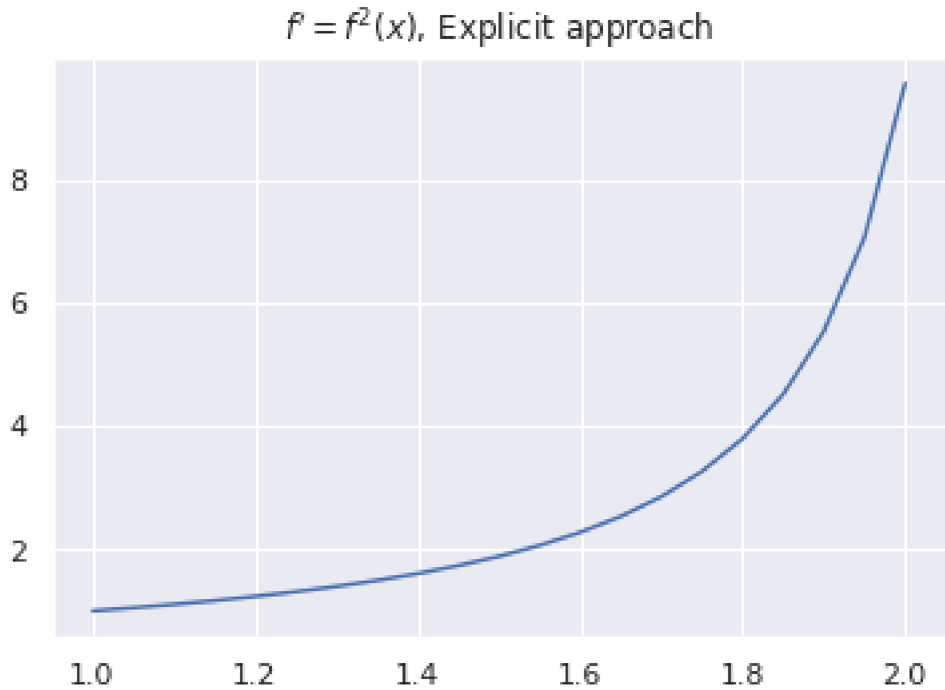
## 4.1 A)

[ ]:
```
#initial condition
dx = 0.05
f1 = 1
X = np.arange(1, 2.05, dx)
N = len(X)
```

### 4.1.1 Explicit approach

[ ]:
```
fe = np.zeros(N)
fe[0] = f1
for i in range(N-1):
    fe[i+1] = fe[i]+ dx*(fe[i]**2)
```

[ ]:
```
plt.plot(X, fe)
plt.title(r"$f'=f^2(x)$, Explicit approach")
plt.show()
```

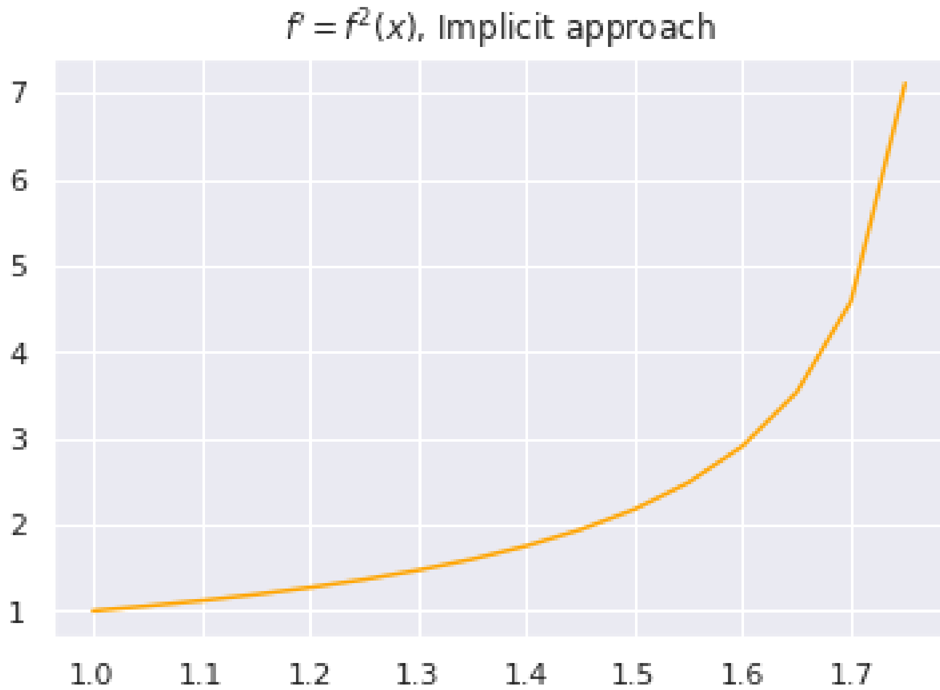



#### 4.1.2 Implicit approach

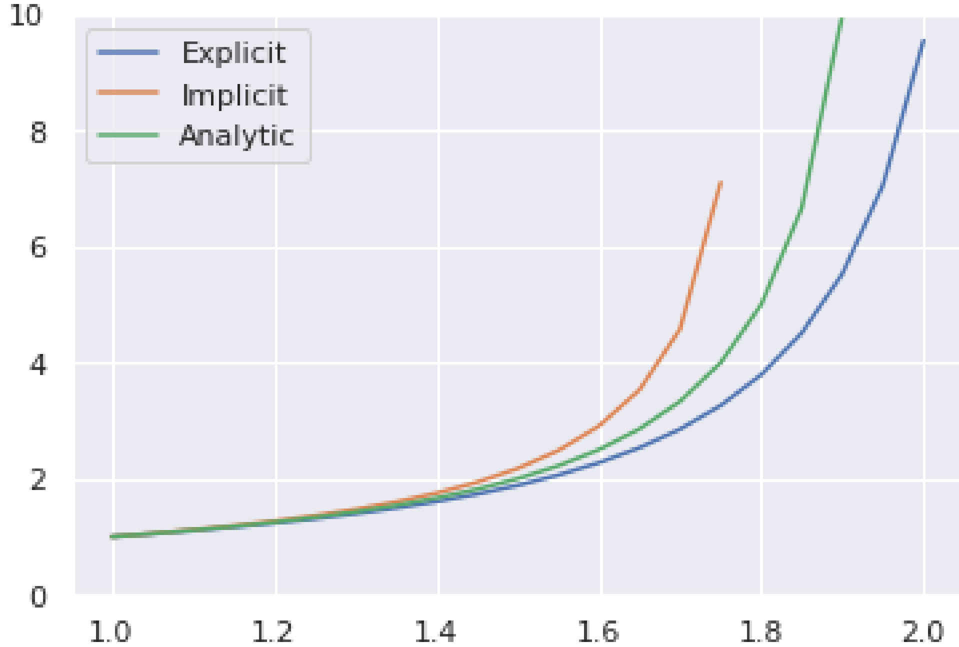
[ ]:
```
fi = np.zeros(N)
fi[0] = f1
for i in range(N-1):
    fi[i+1] = (1 - ((1 - (4*fi[i]*dx))**0.5))/(2*dx)
```

```
/tmp/ipykernel_27044/208837291.py:4: RuntimeWarning: invalid value encountered
in double_scalars
  fi[i+1] = (1 - ((1 - (4*fi[i]*dx))**0.5))/(2*dx)
```

[ ]:
```
plt.plot(X, fi, color="orange")
plt.title(r"$f'=f^2(x)$, Implicit approach")
plt.show()
```




[ ]:
```
plt.plot(X,fe, label="Explicit")
plt.plot(X,fi, label="Implicit")
plt.plot(X[:-1], (1/(2-X))[:-1], label="Analytic")
plt.ylim(0,10)
plt.legend()
plt.show()
```




As we can see, the explicit approach is closest to analytic approach.

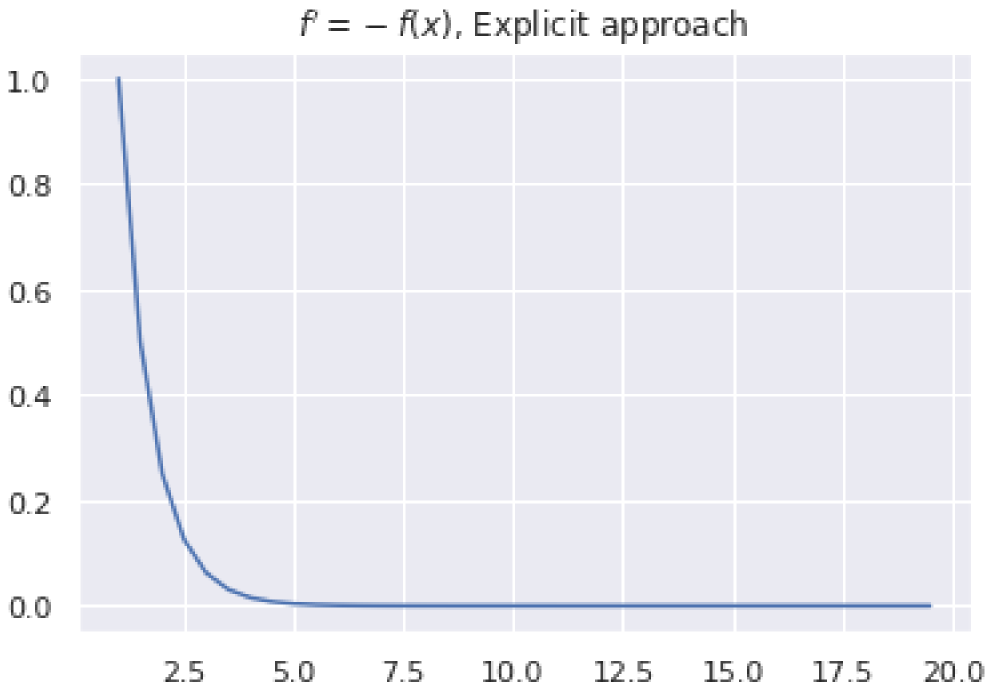
## 4.2 B)

[ ]:
```
#initial condition
dx = 0.5
f1 = 1
X = np.arange(1,20,dx)
N = len(X)
```

### 4.2.1 Explicit approach

[ ]:
```
fe = np.zeros(N)
fe[0] = f1
for i in range(N-1):
    fe[i+1] = (1-dx)*fe[i]
```

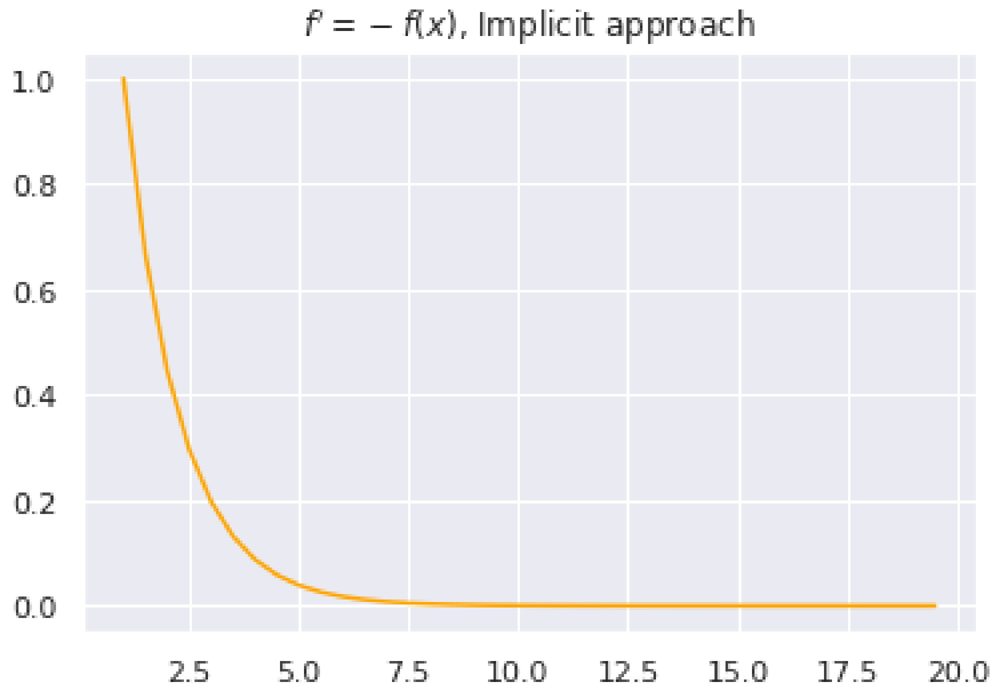
[ ]:
```
plt.plot(X,fe)
plt.title(r"$f'=-f(x)$, Explicit approach")
plt.show()
```



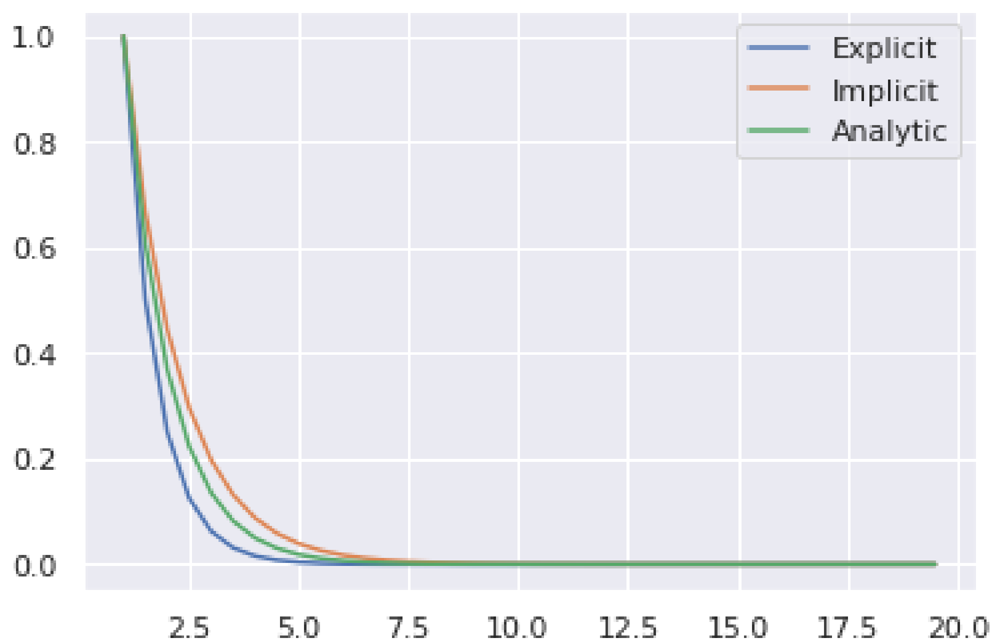

#### 4.2.2 Implicit approach

[ ]:
```
fi = np.zeros(N)
fi[0] = f1
for i in range(N-1):
    fi[i+1] = (fi[i])/(1+dx)
```

[ ]:
```
plt.plot(X,fi, color="orange")
plt.title(r"$f'=-f(x)$, Implicit approach")
plt.show()
```




[ ]:
```
plt.plot(X,fe, label="Explicit")
plt.plot(X,fi, label="Implicit")
plt.plot(X,np.exp(-X+1), label="Analytic")
plt.legend()
plt.show()
```



As we can see, the implicit approach is closest to analytic approach.

# 5 Q3:

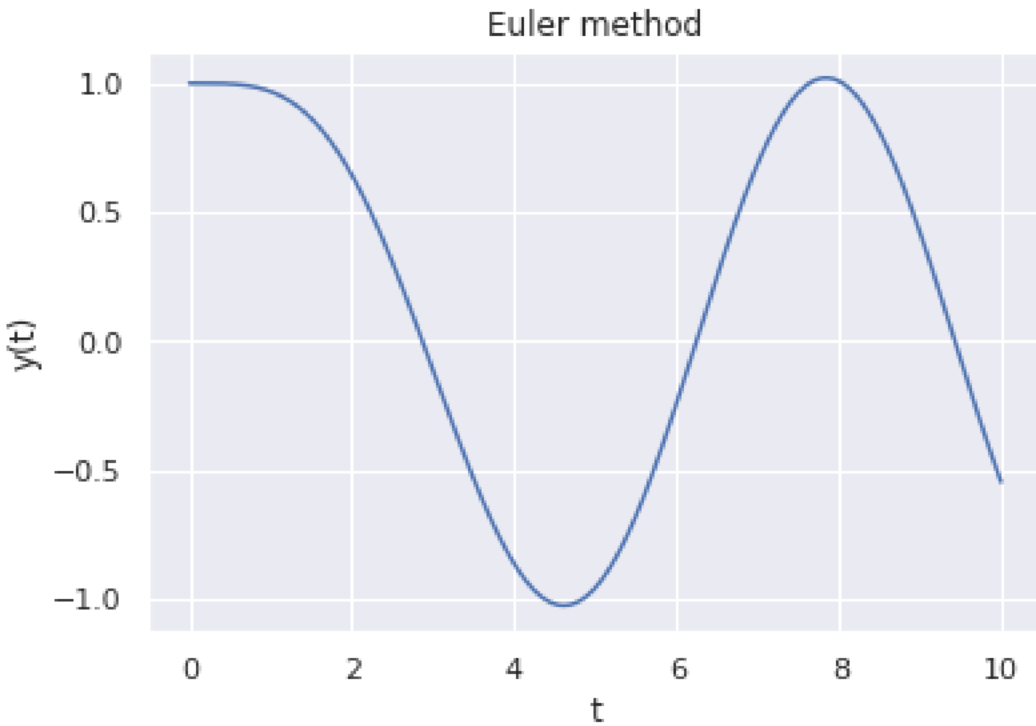
[ ]:
```
a = 1
w2 = 1
w1 = 1
dt = 0.01
T = np.arange(0, 10, dt)
N = len(T)
```

## 5.1 Euler method

[ ]:
```
y = np.zeros(N)
y[0] = 1
v = np.zeros(N)
```

[ ]:
```
for t in range(N-1):
    v[t+1] = v[t] + dt*(np.cos(w1*T[t]) - a*v[t] - w2 * y[t])
    y[t+1] = y[t] + dt*(v[t])
```

[ ]:
```
plt.plot(T,y)
plt.title("Euler method")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.show()
```

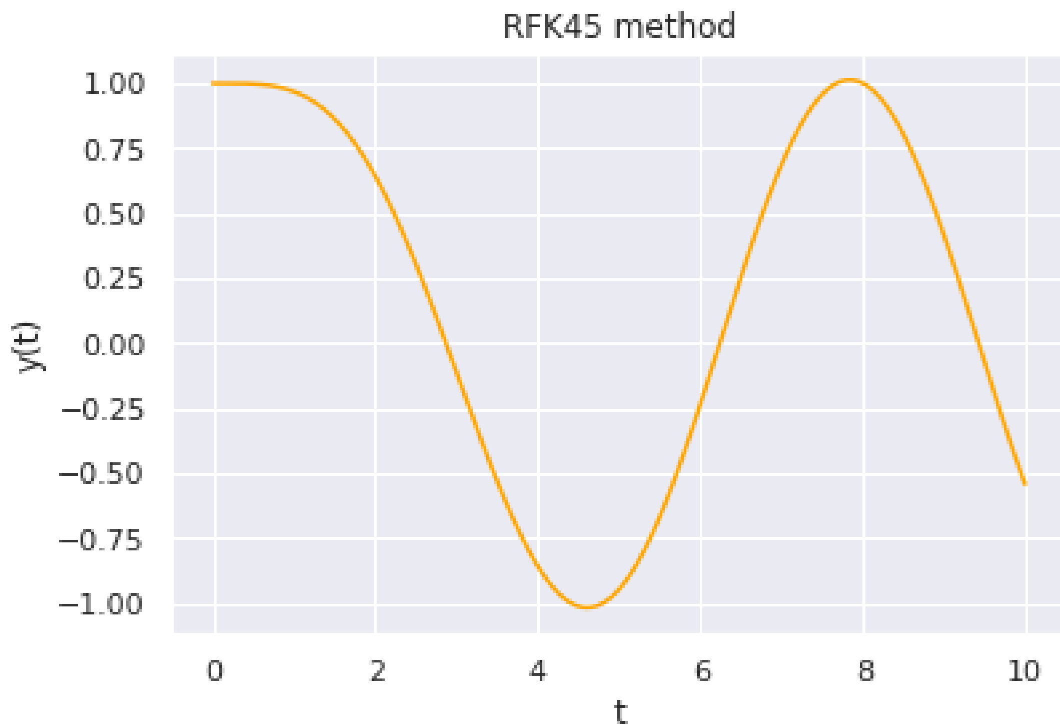



### 5.2 RKF45:

[ ]:
```
Y = np.zeros(N)
Y[0] = 1
V = np.zeros(N)
```

[ ]:
```
for t in range(N-1):
    f1 = V[t]
    k1 = np.cos(w1*T[t]) - w2*Y[t] - (a * f1)

    f2 = V[t] + (dt/2)*k1
    k2 = np.cos(w1*(T[t]+(dt/2))) - w2*(Y[t]+(dt/2)*f1) - f2

    f3 = V[t] + (dt/2)*k2
    k3 = np.cos(w1*(T[t]+(dt/2))) - w2*(Y[t]+(dt/2)*f2) - f3

    f4 = V[t] + (dt/2)*k3
    k4 = np.cos(w1*(T[t]+(dt/2))) - w2*(Y[t]+(dt/2)*f3) - f4

    V[t+1] = V[t] + (dt/6) * (k1+ 2*k2 + 2*k3 + k4)
    Y[t+1] = Y[t] + (dt/6) * (f1+ 2*f2 + 2*f3 + f4)
```

```
[ ]: plt.plot(T,Y, color="orange")
     plt.title("RFK45 method")
     plt.xlabel("t")
     plt.ylabel("y(t)")
     plt.show()
```




## 6 Q4:

$$y''(t) + a y'(t) + \omega^2 y(t) = \cos(\omega, t)$$

$$y''(t) = \frac{y(t+\Delta t) + y(t-\Delta t) - 2y(t)}{(\Delta t)^2}, \quad y'(t) = \frac{y(t+\Delta t) - y(t-\Delta t)}{2\Delta t}$$

$$\Rightarrow \frac{y(t+\Delta t) + y(t-\Delta t) - 2y(t)}{(\Delta t)^2} + \frac{a y(t+\Delta t) - a y(t-\Delta t)}{2\Delta t} + \omega^2 y(t) = \cos(\omega, t)$$

$$\Rightarrow 2y(t+\Delta t) + 2y(t-\Delta t) - 4y(t) + a\Delta t\, y(t+\Delta t) - a\Delta t\, y(t-\Delta t) + 2a\omega^2(\Delta t)^2 y(t) = 2(\Delta t)^2\cos(\omega, t)$$

$$\Rightarrow y(t)\left[-4 - 2a\omega^2(\Delta t)^2\right] = 2(\Delta t)^2\cos(\omega, t) - y(t+\Delta t)\left[2 + a\Delta t\right] - y(t-\Delta t)\left[2 - a\Delta t\right]$$

$$\Rightarrow y(t) = \frac{(2 + a\Delta t)\, y(t+\Delta t) + (2 - a\Delta t)\, y(t-\Delta t) - 2(\Delta t)^2\cos(\omega, t)}{4 + 2a\omega^2(\Delta t)^2}$$

[ ]:
```
a = 1
w2 = 1
w1 = 1
dt = 0.01
T = np.arange(0, 10, dt)
N = len(T)
f = np.random.uniform(-1,1,N)
f[0] = 1
f[-1] = (y[-1] + Y[-1]) /2
```

[ ]:
```
for _ in range(N**2):
    x = np.random.randint(1,N-1)
    f[x] = ((2+a*dt) * y[x+1] + (2-a*dt) * y[x-1] - (2*(dt**2)*np.
 ↪cos(w1*T[x])))/(4+(2*a*w2*dt**2))
```

[ ]:
```
plt.plot(T,f, color="green")
plt.title("Relaxation method")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.show()
```


As we can see, all four method are same result.

# 7 Q5:

[ ]:
```
a = 1
w2 = 1
w1 = 1
N = 1000
Tt = np.linspace(0, 10, N)
h = (Tt[-1] - Tt[0])/N
alpha = 1
beta = (y[-1] + Y[-1])/2
```

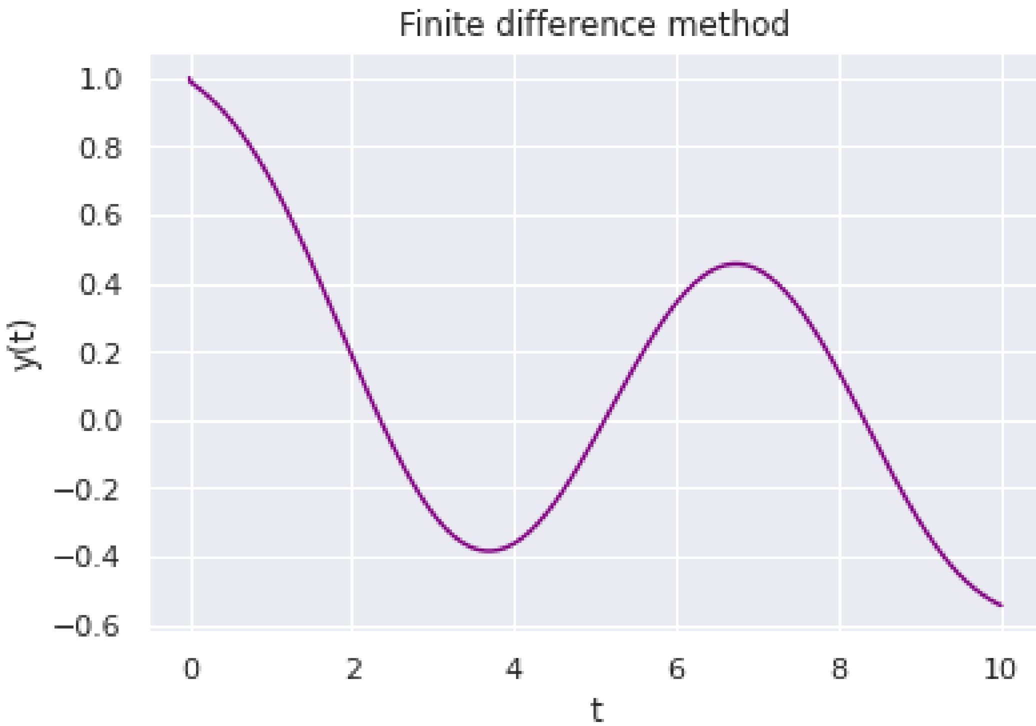
[ ]:
```
L = np.zeros((N-2, N-2))
L[0,0] = 2 + w2 * h**2
L[0,1] = (h/2)*a - 1
for i in range(1, N-3):
    L[i,i] = 2 + w2 * h**2
    L[i,i+1] = (h/2)*a - 1
    L[i,i-1] = (-h/2)*a - 1
L[N-3,N-3] = 2 + w2 * h**2
L[N-3,N-4] = (-h/2)*a - 1
```

[ ]:
```
A = (h**2)* np.cos(w1* Tt)
A = np.delete(A,0)
A = np.delete(A,-1)
A[0] += ((h/2)*(-a) + 1)* alpha
A[-1] += ((h/2)*(-a) + 1)* beta
```

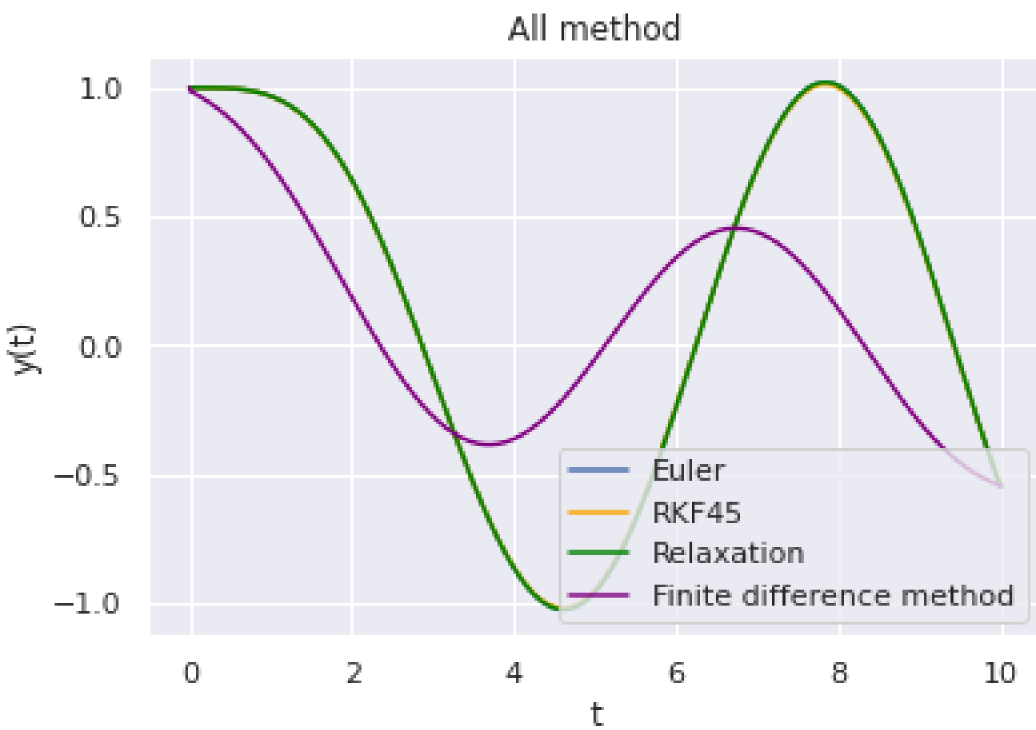
[ ]:
```
y3 = np.linalg.solve(L, A)
```

[ ]:
```
yy = np.zeros(N)
yy[0] = alpha
yy[1:N-1] = y3
yy[N-1] = beta
```

[ ]:
```
plt.plot(Tt, yy, color="purple")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.title("Finite difference method")
plt.show()
```




[ ]:
```
plt.plot(T,y, label="Euler")
plt.plot(T,Y, color="orange", label="RKF45")
plt.plot(T,f, color="green", label="Relaxation")
plt.plot(Tt, yy, color="purple", label="Finite difference method")
plt.title("All method")
plt.legend()
plt.xlabel("t")
plt.ylabel("y(t)")
plt.show()
```



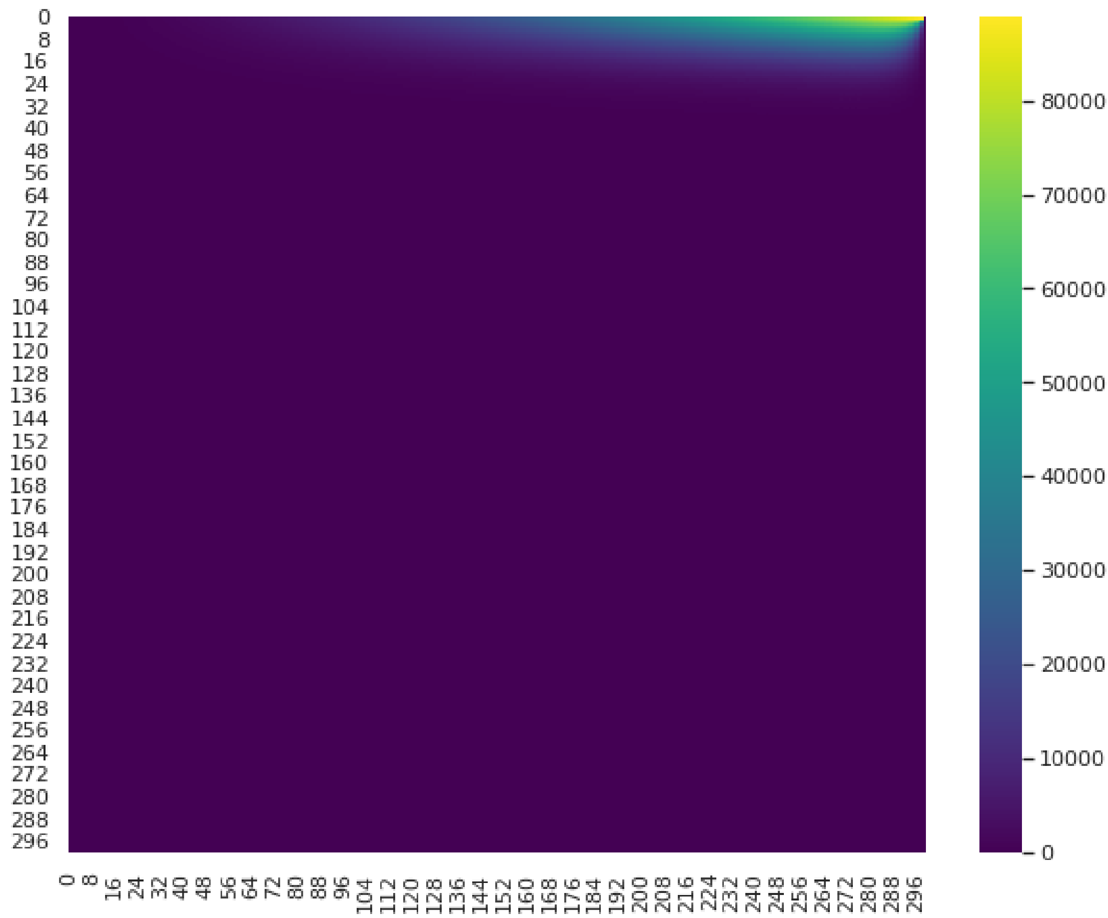

## 8 Q6:

```
[ ]: #initial random lattice
lattice = np.random.random((300,300))
```

```
[ ]: #set boundry value
lattice[:,0] = [x for x in range(300)]
lattice[0,:] = [y**2 for y in range(300)]
lattice[:,299] = 1
lattice[299,:] = 0
```

```
[ ]: #use relaxation method for solving laplace eq.
for _ in range(300**3):
    x,y= np.random.randint(1,299,2)
    lattice[x,y] = (lattice[x+1, y] + lattice[x-1, y] + lattice[x, y+1] +␣
 ↪lattice[x, y-1]) / 4
```

```
[ ]: plt.figure(figsize=(10,8))
sns.heatmap(lattice, cmap="viridis")
plt.show()
```



0
8
16
24
32
40
48
56
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152
160
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