# HW5

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Exercise Set 5

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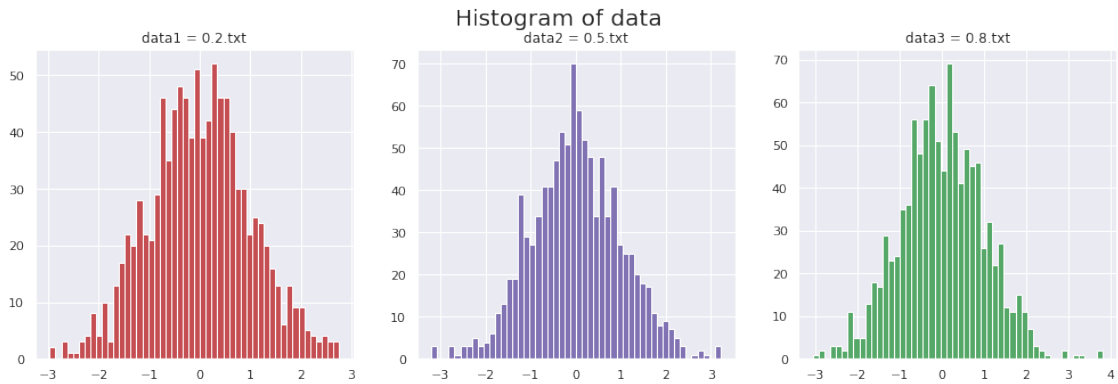
[ ]:
```
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import seaborn as sns
from math import comb
from numba import njit
sns.set()
```

[ ]:
```
#load data in data_5
data1 = np.loadtxt("Data_5/0.2.txt")
data2 = np.loadtxt("Data_5/0.5.txt")
data3 = np.loadtxt("Data_5/0.8.txt")
```

[ ]:
```
#plot histogram of the data
fig, axs = plt.subplots(1,3, figsize=(17,5))

axs[0].hist(data1, bins=50, color="r")
axs[0].set_title("data1 = 0.2.txt")

axs[1].hist(data2, bins=50, color="m")
axs[1].set_title("data2 = 0.5.txt")

axs[2].hist(data3, bins=50, color="g")
axs[2].set_title("data3 = 0.8.txt")

fig.suptitle("Histogram of data", fontsize=20)
plt.show()
```




# 1 Q1:

[ ]:
```
#number of moment and cumulant that want to calculate
n_m = [1, 2, 3, 4, 5, 10]
n_k = [1, 2, 3, 4, 5, 6, 10]
```

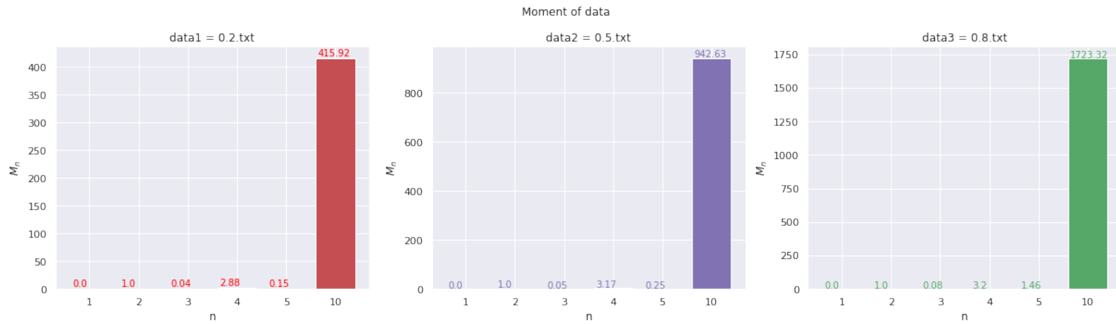
[ ]:
```
def moment(x,k):
    """calculate kth central moment of x data

    Args:
        x (array_like): data
        k (int): order of central moment

    Returns:
        float: k-th central moment
    """
    return np.mean((x)**k)
```

[ ]:
```
def cumulant(x, k, M):
    """calculate kth cumulant of x data

    Args:
        x (array_like): data
        k (int): order of cumulant

    Returns:
        float: k-th cumulant
    """
    K = np.zeros((k,k))
    for n in range(1,k+1):
        K[n-1, 0] = M(x,n)
        for i in range(1, n):
```

```
            K[n-1, i] = comb(n-1, n-i) * M(x,n-i)
        if n != k:
            K[n-1, n] = 1

    return  ((-1)**(k-1)) * np.linalg.det(K)
```

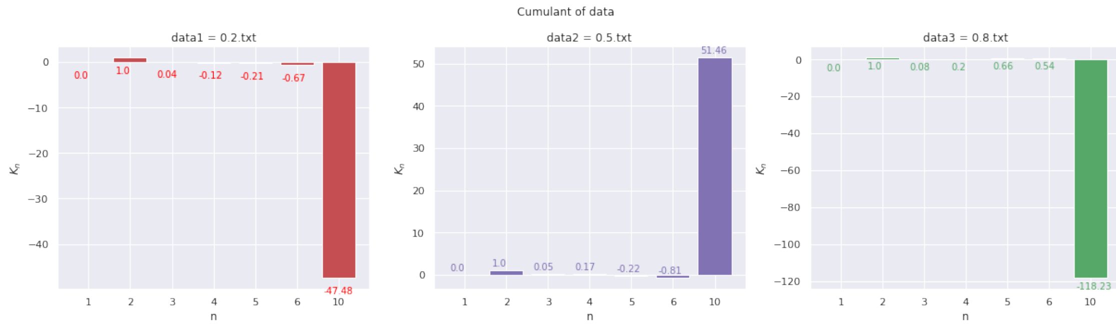
[ ]:
```
#find moment for 3 data
Moment1, Moment2, Moment3 = [], [], []
for n in n_m:
    Moment1.append(moment(data1, n))
    Moment2.append(moment(data2, n))
    Moment3.append(moment(data3, n))
```

[ ]:
```
#plot the values of moment for datas
fig, axs = plt.subplots(1,3, figsize=(17,5))

axs[0].bar([str(i) for i in n_m], Moment1, color="r")
axs[0].set_title("data1 = 0.2.txt")
axs[0].set_xlabel("n")
axs[0].set_ylabel(r"$M_n$")
for i, v in enumerate(Moment1):
    axs[0].text(i-.35,v+4, str(round(v,2)), color='red')

axs[1].bar([str(i) for i in n_m], Moment2, color="m")
axs[1].set_title("data2 = 0.5.txt")
axs[1].set_xlabel("n")
axs[1].set_ylabel(r"$M_n$")
for i, v in enumerate(Moment2):
    axs[1].text(i-.35,v+4, str(round(v,2)), color='m')

axs[2].bar([str(i) for i in n_m], Moment3, color="g")
axs[2].set_title("data3 = 0.8.txt")
axs[2].set_xlabel("n")
axs[2].set_ylabel(r"$M_n$")
for i, v in enumerate(Moment3):
    axs[2].text(i-.35,v+4, str(round(v,2)), color='g')

fig.suptitle("Moment of data")
plt.tight_layout()
plt.show()
```




[ ]:
```
#calculate cumulant for datas
Cumulant1, Cumulant2, Cumulant3 = [], [], []
for n in n_k:
    Cumulant1.append(cumulant(data1, n, moment))
    Cumulant2.append(cumulant(data2, n, moment))
    Cumulant3.append(cumulant(data3, n, moment))
```

[ ]:
```
#plot the cumulants of datas
fig, axs = plt.subplots(1,3, figsize=(17,5))

axs[0].bar([str(i) for i in n_k], Cumulant1, color="r")
axs[0].set_title("data1 = 0.2.txt")
axs[0].set_xlabel("n")
axs[0].set_ylabel(r"$K_n$")

for i, v in enumerate(Cumulant1):
    axs[0].text(i-.35,v-3.5, str(round(v,2)), color='red')

axs[1].bar([str(i) for i in n_k], Cumulant2, color="m")
axs[1].set_title("data2 = 0.5.txt")
axs[1].set_xlabel("n")
axs[1].set_ylabel(r"$K_n$")
for i, v in enumerate(Cumulant2):
    axs[1].text(i-.35,v+1, str(round(v,2)), color='m')

axs[2].bar([str(i) for i in n_k], Cumulant3, color="g")
axs[2].set_title("data3 = 0.8.txt")
axs[2].set_xlabel("n")
axs[2].set_ylabel(r"$K_n$")
for i, v in enumerate(Cumulant3):
    axs[2].text(i-.35,v-6, str(round(v,2)), color='g')

fig.suptitle("Cumulant of data")
plt.tight_layout()
plt.show()
```




## 2 Q2:

[ ]:
```python
def skewness(x):
    """calculate skewness of data

    Args:
        x (array_like): data

    Returns:
        float: skewness of data
    """
    return np.mean((x-x.mean())**3)
```

[ ]:
```python
def kurtosis(x):
    """calculate kurtosis of data

    Args:
        x (array_like): data

    Returns:
        float: kurtosis of data
    """
    m2 = np.mean(x**2)
    m4 = np.mean(x**4)
    return m4 / (m2**2)
```

[ ]:
```python
print("The skewness of data 1 is", skewness(data1))
print("The skewness of data 2 is", skewness(data2))
print("The skewness of data 3 is", skewness(data3))
```

```
The skewness of data 1 is 0.035624108266680136
The skewness of data 2 is 0.04754127376432882
The skewness of data 3 is 0.08001875525507664
```

[ ]:
```
print("The kurtosis of data 1 is", kurtosis(data1))
print("The kurtosis of data 2 is", kurtosis(data2))
print("The kurtosis of data 3 is", kurtosis(data3))
```

```
The kurtosis of data 1 is 2.880808301142841
The kurtosis of data 2 is 3.1674377255966975
The kurtosis of data 3 is 3.204739567661185
```

# 3 Q3:

## 3.1 A:

[ ]:
```
data = np.loadtxt("0.800")
data
```

[ ]:
```
array([[ 1.00000000e+00,  1.02577223e-01],
       [ 2.00000000e+00, -8.85575575e-01],
       [ 3.00000000e+00, -9.97431011e-01],
       …,
       [ 6.55340000e+04, -6.02874431e-01],
       [ 6.55350000e+04,  5.36658614e-02],
       [ 6.55360000e+04,  9.21246424e-01]])
```

[ ]:
```
x = data[:,0]
y = data[:,1]
```

[ ]:
```
def _int(x):
    """calculate integer part of the number
    example:
        _float(-0.5) = -1

    Args:
        x (float):

    Returns:
        int:
    """
    if x>= 0:
        return int(x)
    else:
        return int(x) -1
```

[ ]:
```
def PDF(X,dx):
  """probability of data

  Args:
      X (1d_array): data
```

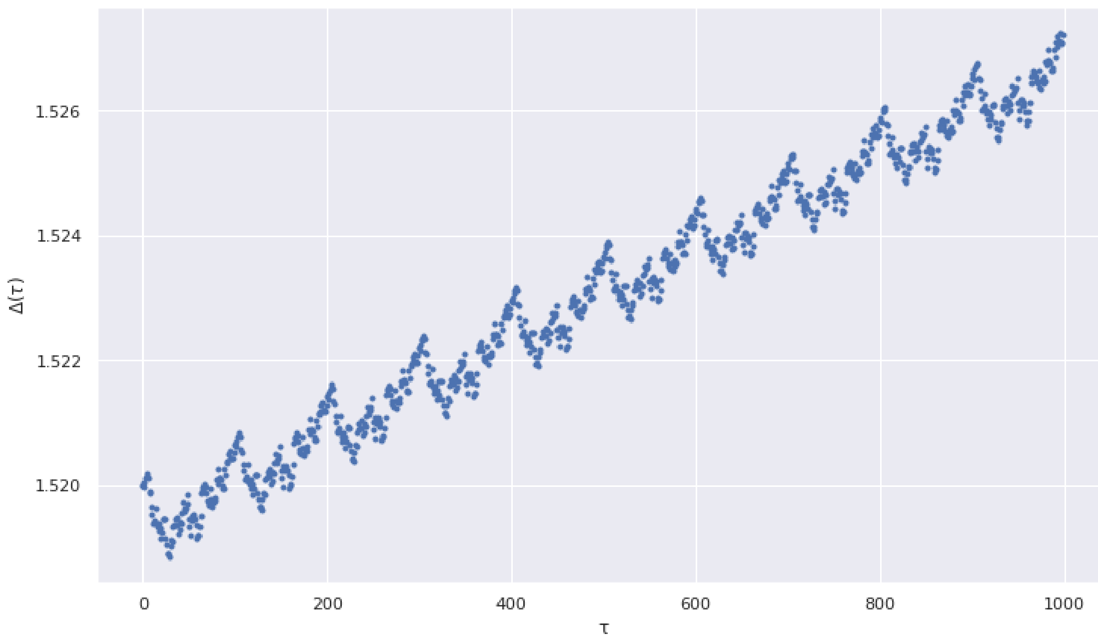
```
        dx (float): size of steps

    Returns:
        tuple: axis of probability and probability --> x, p(x)
    """
    n = int((X.max()-X.min())/dx) + 1

    axis = np.linspace(X.min(), X.max(), n)

    pdf = np.zeros(n)

    X -= X.min()

    for i in range(len(X)):
      k = _int(X[i]/dx)
      pdf[k] += 1

    pdf /= (np.sum(pdf)*dx)

    return axis,pdf
```

[ ]:
```
def p_joint(x, y, dx, dy, nx=0, ny=1, tau=0):
    """calculate joint probablity --> p(x(t+nx*tau),y(t+ny*tau))

    Args:
        x (1d_array): first data
        y (1d-array): second data
        dx (float): size of steps for x data
        dy (float): size of steps for y data
        nx (int, optional): coefficient of tau for x. Defaults to 0.
        ny (int, optional): coefficient of tau for x. Defaults to 1.
        tau (int, optional): delay time. Defaults to 0.
    Returns:
        2d_array: joint probability of x,y
    """

    numx = int((x.max()-x.min())/dx)+1
    numy = int((y.max()-y.min())/dy)+1

    pdf = np.zeros((numx, numy))

    x -= x.min()
    y -= y.min()

    for i in range(len(x)- np.max((nx,ny))*tau):
        k1 = _int(x[i+(nx*tau)]/dx)
        k2 = _int(y[i+(ny*tau)]/dy)
```

```
            pdf[k1,k2] += 1

    return pdf/(np.sum(pdf)*dx*dy)
```

[ ]:
```
delta_tau = []
dx = 100
dy = 0.01
x_p = PDF(x,dx)[1]
y_p = PDF(y,dy)[1]
for t in range(1000):
    d = 0
    joint_p = p_joint(x,y,dx,dy,tau=t)
    for i in range(len(x_p)):
        for j in range(len(y_p)):
            d += abs(joint_p[i,j] - (x_p[i] * y_p[j]))
    delta_tau.append(d)
```

[ ]:
```
plt.figure(figsize=(12,7))
plt.plot(delta_tau, ls="", marker=".")
plt.xlabel(' ')
plt.ylabel(r'$\Delta( )$')
plt.show()
```



### 3.2 B:

```
[ ]: dy = 0.01
     joint_p = p_joint(y,y,dy,dy,2,0,tau=t)
     y_p = PDF(y,dy)[1]
```

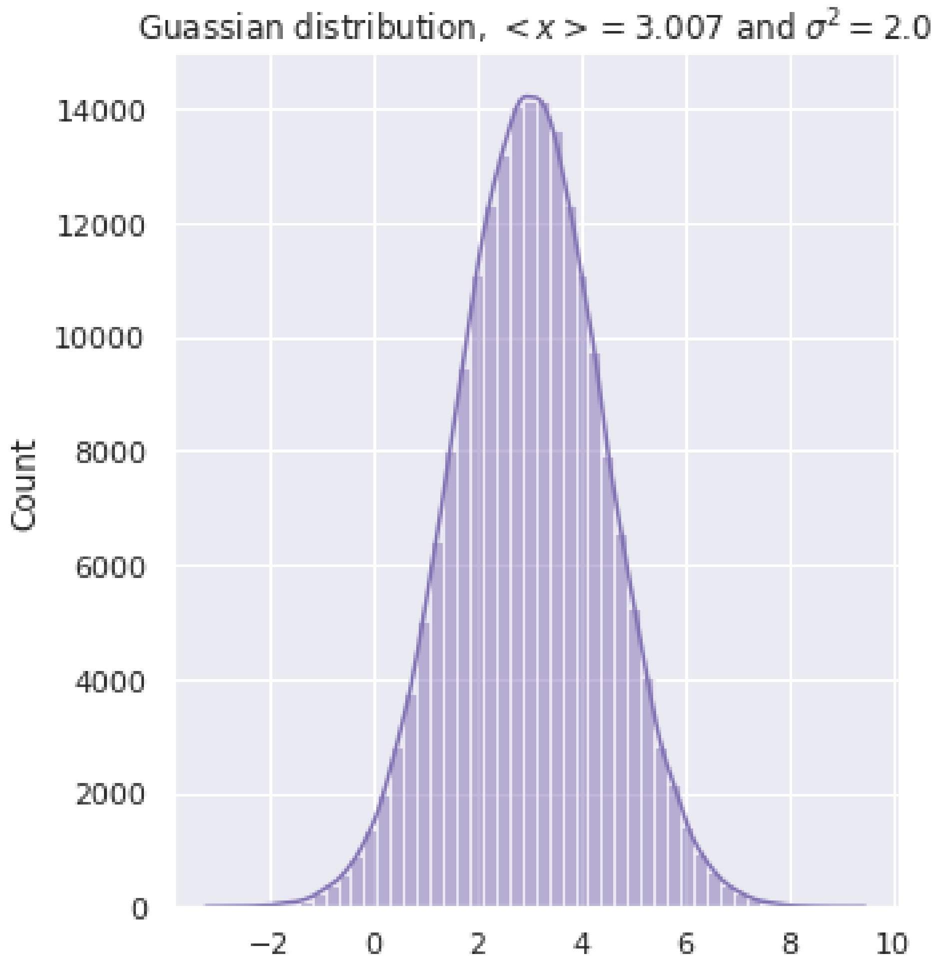
# 4 Q4:

```
[ ]: def GenerateNormal():
       """generate normal distribution from uniform dist. with BoX Muller algorithm

       Returns:
           float: random variable with normal distiribution
       """
       u1, u2 = np.random.uniform(0, 1, 2)
       z1 = ((-2*np.log(u1))**(0.5))* np.cos(2*np.pi*u2)
       z2 = ((-2*np.log(u2))**(0.5))* np.sin(2*np.pi*u1)
       return z1,z2
```

```
[ ]: normal = []
     for i in range(100000):
       z1,z2 = GenerateNormal()
       normal.append(z1)
       normal.append(z2)
```

```
[ ]: #tranfor normal dist, to guassian dist. with the mean is 3 and the variance is 2
     guassian = np.array(normal)* np.sqrt(2) + 3
```

```
[ ]: sns.displot(guassian, kde=True, bins= 50, color="m", )
     plt.title(rf'Guassian distribution, $<x> = ${round(guassian.mean(),3)} and $␣
       ↪\sigma^2 = {round(guassian.var(),3)}$')
     plt.show()
```

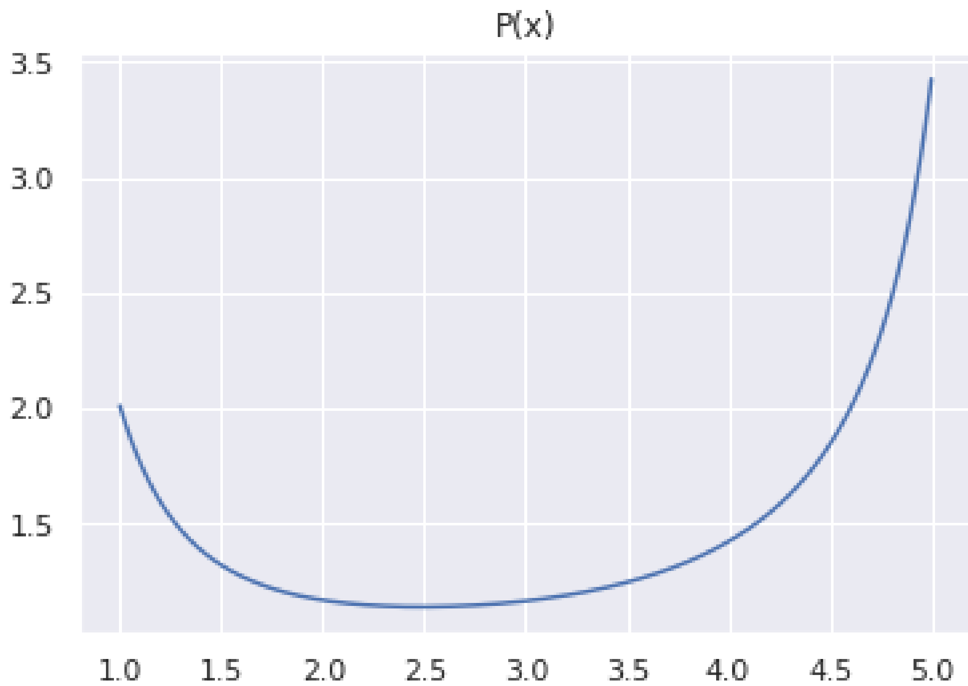



## 5 Q5:

```
[ ]: def p(x):
         """probability

         Args:
             x (float):
         """
         return np.sin(x**2/100) + 1/(np.cos(x**3/100)) + x**(-3)
     p = np.vectorize(p)
```

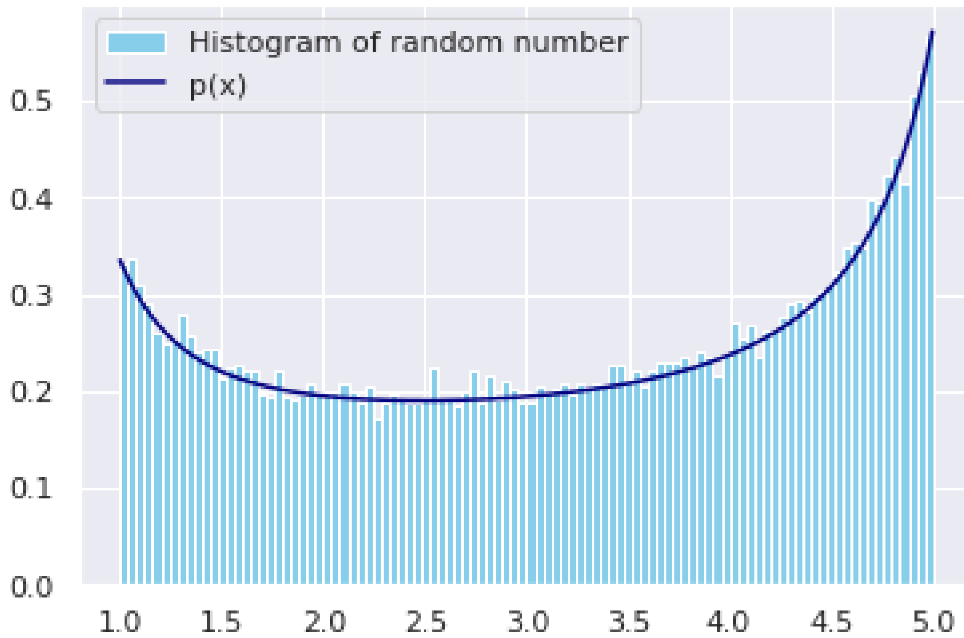
```
[ ]: X = np.linspace(1,5,1000)
     Prob = p(X)
     w = Prob.max()
     plt.plot(X, Prob)
```

```
plt.title("P(x)")
plt.show()
```




[ ]:
```
#generate random number from p(x) distrinution according to Von-Neumann␣
↪alghorithm
XX = []
for i in range(10**5):
    x = np.random.uniform(1,5)
    y = np.random.uniform(0,w)
    P = p(x)
    if y <= P:
        XX.append(x)
```

[ ]:
```
plt.hist(XX, bins=100, density=True, color="skyblue", label="Histogram of␣
↪random number")
plt.plot(X, Prob/6, color="navy", label="p(x)")
plt.legend()
plt.show()
```




# 6 Q6:

$$6 \rightarrow \rho(t) \rightarrow x = A \sin \omega t = g(t) \qquad \frac{dg}{dt} = A\omega \cos \omega t \,, \quad t = \frac{1}{\omega} \sin^{-1}\left(\frac{x}{A}\right)$$

$$\rho(x) = \rho\left(\frac{1}{\omega} \sin^{-1}\left(\frac{x}{A}\right)\right) \frac{1}{\underbrace{A\omega \cos \omega \left[\frac{1}{\omega} \sin^{-1}\left(\frac{x}{A}\right)\right]}_{\sqrt{1-\left(\frac{x}{A}\right)^2}}} = \rho\left(\frac{1}{\omega} \sin^{-1}\left(\frac{x}{A}\right)\right) \frac{1}{A\omega \left(1-\left(\frac{x}{A}\right)^2\right)^{1/2}}$$

# 7 Q7:

7 → $p(x) = \frac{a}{x^{a+1}}$ for $1 \le x < \infty$

A: $y = x^2$, $p(y) = ?$ $\quad x = \pm\sqrt{y} \quad \frac{dg}{dx} = 2x$

$g(x) = x^2$

$$p(y) = p(\sqrt{y})\frac{1}{2\sqrt{y}} + p(-\sqrt{y})\frac{1}{-2\sqrt{y}} = \frac{a}{y^{\frac{a+1}{2}}}\frac{1}{2y^{1/2}} + \frac{a}{(-1)^{a+1}\, y^{\frac{a+1}{2}}}\frac{1}{(-2)y^{1/2}}$$

$$= \frac{a/2}{y^{1+a/2}} + \frac{a/2}{(-1)^{a}\, y^{a/2+1}}$$

B: $z = \frac{1}{x}$, $p(z) = ?$ $\quad x = \frac{1}{z}, \quad \frac{dg}{dx} = \frac{-1}{x^2} = -z^2$

$g(x) = 1/x$

$$p(z) = p\left(\frac{1}{z}\right)(-z^2) = \frac{a}{(1/z)^{a+1}}(-1)z^2 = -a z^{a+1} z^2 = -a z^{a+3}$$

C: $T = \ln(x) \to g(x) = \ln x \to x = e^T \quad \frac{dg}{dx} = \frac{1}{x} = e^{-T}$

$$p(T) = p(e^T)\, e^{-T} = \frac{a}{(e^T)^{a+1}} e^{-T} = \frac{a}{e^{Ta+T} e^T} = \frac{a}{e^{T(a+2)}}$$

# 8 Q8:

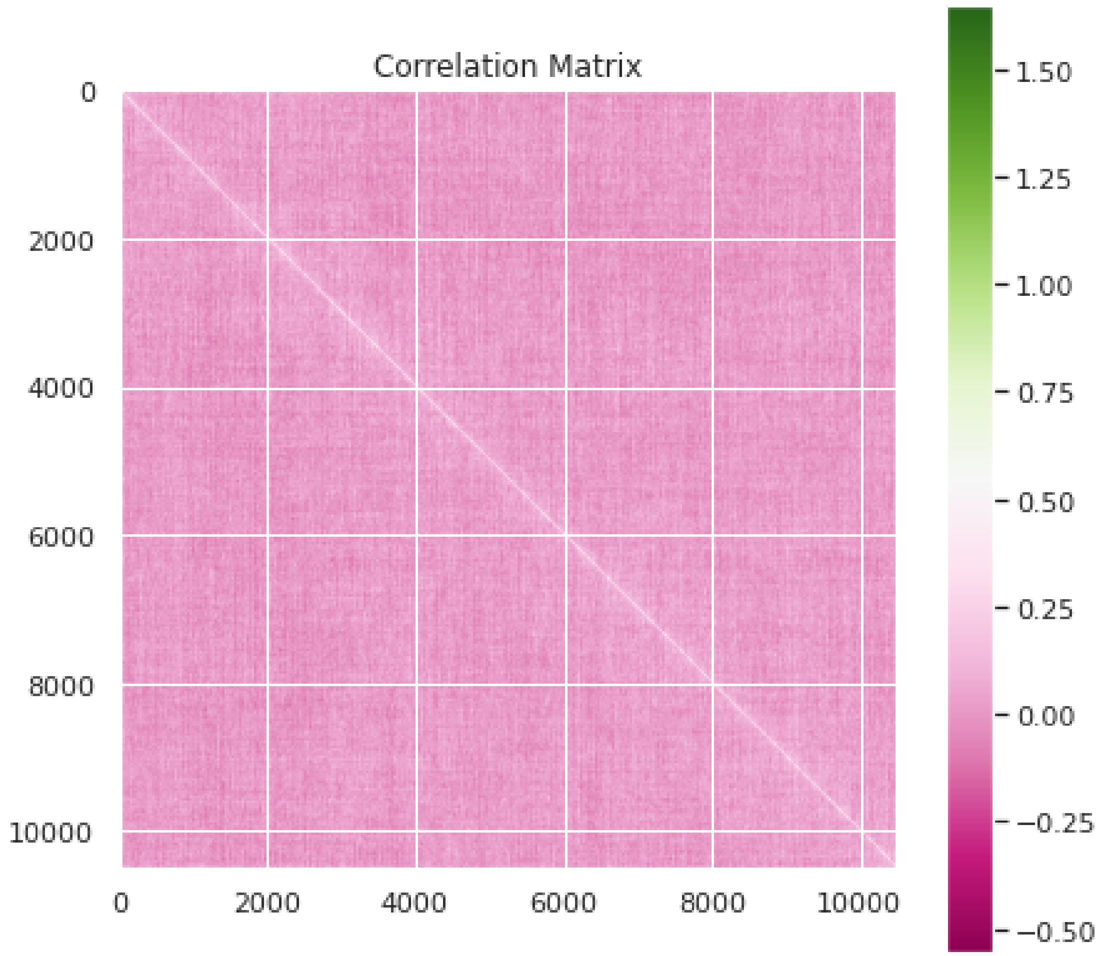
[ ]:
```python
data = np.loadtxt("data.txt")
r = len(data)%100
splited_data = np.array(np.array_split(data[r:], 100))  #for equal length
```

[ ]:
```python
@njit
def corr(data):
    """compute correlation matrix

    Args:
        data (2d_array): data

    Returns:
        2d_array: correlation
    """
    n = data.shape[1]
    C = np.zeros((n,n))
    for i in range(n):
```

```
        for j in range(i+1):
            C[i,j] = np.mean(data[:,i]*data[:,j])
            C[j,i] = C[i,j]
    return C
```

[ ]:
```
C = corr(splited_data)
```

[ ]:
```
plt.figure(figsize=(7,7))
plt.imshow(C, cmap="PiYG")
plt.colorbar()
plt.title("Correlation Matrix")
plt.show()
```




[ ]:
```
c1, c2, c3, c4, c5 = np.zeros(n), np.zeros(n), np.zeros(n), np.zeros(n), np.
 ↪zeros(n)
#5 sets of data
x1 = splited_data[93]
```

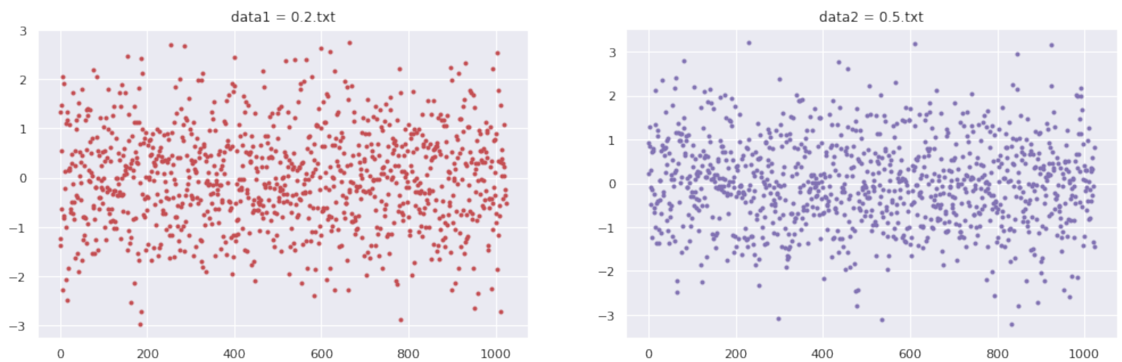
```
x2 = splited_data[34]
x3 = splited_data[18]
x4 = splited_data[40]
x5 = splited_data[88]

for tau in range(0,n):
    for i in range(n-tau):
        c1[tau] +=  x1[i]*x1[i+tau]
        c2[tau] +=  x2[i]*x2[i+tau]
        c3[tau] +=  x3[i]*x3[i+tau]
        c4[tau] +=  x4[i]*x4[i+tau]
        c5[tau] +=  x5[i]*x5[i+tau]
    c1[tau] /= n-tau
    c2[tau] /= n-tau
    c3[tau] /= n-tau
    c4[tau] /= n-tau
    c5[tau] /= n-tau
```

[ ]:

```
plt.figure(figsize=(12,7))
plt.plot(c1, label='Set 1', ls="", marker=".")
plt.plot(c2, label='Set 2', ls="", marker=".")
plt.plot(c3, label='Set 3', ls="", marker=".")
plt.plot(c4, label='Set 4', ls="", marker=".")
plt.plot(c5, label='Set 5', ls="", marker=".")
plt.xlabel(' ')
plt.ylabel('C( )')
plt.legend()
plt.show()
```

# 9 Q9:

[ ]:
```
#plot the data
fig, axs = plt.subplots(1,2, figsize=(17,5))

axs[0].plot(data1, ".r", )
axs[0].set_title("data1 = 0.2.txt")

axs[1].plot(data2, ".m")
axs[1].set_title("data2 = 0.5.txt")

plt.show()
```




[ ]:
```
def pearson_coef(x,y):
    """calculate pearson coefficent

    Args:
        x (array_like): data
        y (array_like): data

    Returns:
        float: pearson coefficent
    """
    return np.mean((x-np.mean(x))* (y-np.mean(y))) / np.sqrt(np.var(x)*np.
 ↪var(y))
```

[ ]:
```
print("The degree of correlation between 0.2.txt and 0.5.txt is",␣
 ↪pearson_coef(data1,data2))
print("The degree of correlation between 0.2.txt and itself is",␣
 ↪pearson_coef(data1,data1))
```

```
print("The degree of correlation between 0.5.txt and itself is",␣
 ↪pearson_coef(data2,data2))
```

```
The degree of correlation between 0.2.txt and 0.5.txt is 0.0804956891229922
The degree of correlation between 0.2.txt and itself is 1.0
The degree of correlation between 0.5.txt and itself is 1.0
```

## 10 Q10:

[ ]:
```
n = len(data1)
C1, C2, C3 = np.zeros(n), np.zeros(n), np.zeros(n)
for tau in range(0,n):
    for i in range(n-tau):
        C1[tau] +=  data1[i]*data1[i+tau]
        C2[tau] +=  data2[i]*data2[i+tau]
        C3[tau] +=  data3[i]*data3[i+tau]

    C1[tau] /= n-tau
    C2[tau] /= n-tau
    C3[tau] /= n-tau
```

[ ]:
```
plt.figure(figsize=(15,7))
plt.plot(C1, label='Set 1', ls="", marker=".")
plt.plot(C2, label='Set 2', ls="", marker=".")
plt.plot(C3, label='Set 3', ls="", marker=".")

plt.xlabel(' ')
plt.ylabel('C( )')
plt.legend()
plt.show()
```

For all three datasets, $C ( ) $ is almost equal to zero. So they are uncorrelated.

# 11 Q11:

$$I(X;Y) = H(X) + H(Y) - H(X,Y), \tag{1}$$

$$H(X) = -\Sigma_{i=1}^{L} p_i \ln p_i, \tag{2}$$

$$H(X,Y) = -\Sigma_{i=1}^{L} \Sigma_{j=1}^{M} p_{ij} \ln p_{ij}. \tag{3}$$

[ ]:
```
#set step size
dx = 0.01
```

[ ]:
```
#calculate probability of datasets according to PDF function in Q3
axis_data1, p_data1 = PDF(data1, dx)
axis_data2, p_data2 = PDF(data2, dx)
axis_data3, p_data3 = PDF(data3, dx)
```

[ ]:
```
#calculate joint probability of datasets according to p_joint function in Q3
p_12 = p_joint(data1, data2, dx, dx, ny=0)
p_13 = p_joint(data1, data3, dx, dx, ny=0)
p_23 = p_joint(data2, data3, dx, dx, ny=0)
```

[ ]:
```
def H(prob):
    """entropy

    Args:
        prob (nd_array): probability

    Returns:
        float: entropy
    """
    h = 0
    for p in prob.flatten():
        if p != 0:
            h += p*np.log(p)
    return -h
```

[ ]:
```
#compute entropy
h1 = H(p_data1)
h2 = H(p_data2)
h3 = H(p_data3)
h12 = H(p_12)
```

```
h13 = H(p_13)
h23 = H(p_23)
```

[ ]:
```
#compute mutual information
I12 = h1 + h2 - h12
I13 = h1 + h3 - h13
I23 = h2 + h3 - h23
```

[ ]:
```
print("The mutual information between 0.2.txt and 0.5.txt is", I12)
print("The mutual information between 0.2.txt and 0.8.txt is", I13)
print("The mutual information between 0.5.txt and 0.8.txt is", I23)
```

```
The mutual information between 0.2.txt and 0.5.txt is 23040.28183708713
The mutual information between 0.2.txt and 0.8.txt is 23104.788649055798
The mutual information between 0.5.txt and 0.8.txt is 23067.721903333804
```

# 12 Q12:

[ ]:
```
#use pearson_coef function of Q9
pc12 = pearson_coef(data1, data2)
pc13 = pearson_coef(data1, data3)
pc23 = pearson_coef(data2, data3)
```

[ ]:
```
print("The pearson's coeficent between 0.2.txt and 0.5.txt is", pc12)
print("The pearson's coeficent between 0.2.txt and 0.8.txt is", pc13)
print("The pearson's coeficent between 0.5.txt and 0.8.txt is", pc23)
```

```
The pearson's coeficent between 0.2.txt and 0.5.txt is 0.08049568912299221
The pearson's coeficent between 0.2.txt and 0.8.txt is -0.020389936658351003
The pearson's coeficent between 0.5.txt and 0.8.txt is -0.0156085973117271
```

[ ]:
```
def spearman_coef(x, y):
    rank_x = (-x).argsort()+1
    rank_y = (-y).argsort()+1
    d = rank_x - rank_y
    N = len(x)
    return 1 - ((6* np.sum(d**2)) / (N*(N**2-1)))
```

[ ]:
```
sc12 = spearman_coef(data1,data2)
sc13 = spearman_coef(data1,data3)
sc23 = spearman_coef(data2,data3)
```

[ ]:
```
print("The Spearman's coeficent between 0.2.txt and 0.5.txt is", sc12)
print("The Spearman's coeficent between 0.2.txt and 0.8.txt is", sc13)
print("The Spearman's coeficent between 0.5.txt and 0.8.txt is", sc23)
```

```
The Spearman's coeficent between 0.2.txt and 0.5.txt is -0.04522486991273866
The Spearman's coeficent between 0.2.txt and 0.8.txt is 0.04098728855232103
The Spearman's coeficent between 0.5.txt and 0.8.txt is 0.010495605643373151
```

We know that a positive value for either coefficient indicates a positive relationship between the two variables, while a negative value indicates a negative relationship. The magnitude of the coefficient indicates the strength of the relationship; a coefficient of 1 indicates a perfect positive relationship, while a coefficient of -1 indicates a perfect negative relationship. In this question, all Spearman's and Pearson's coefficients are near zero, so they have no correlation with each other.