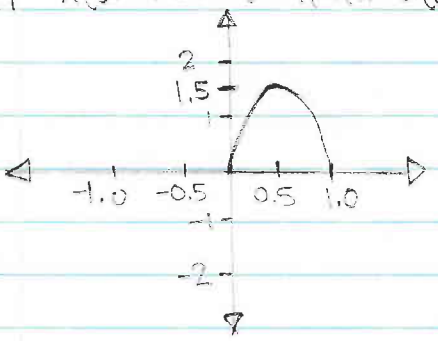
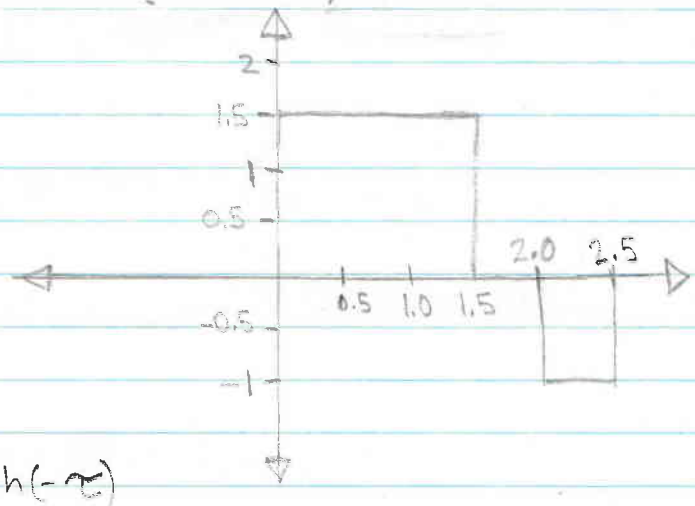
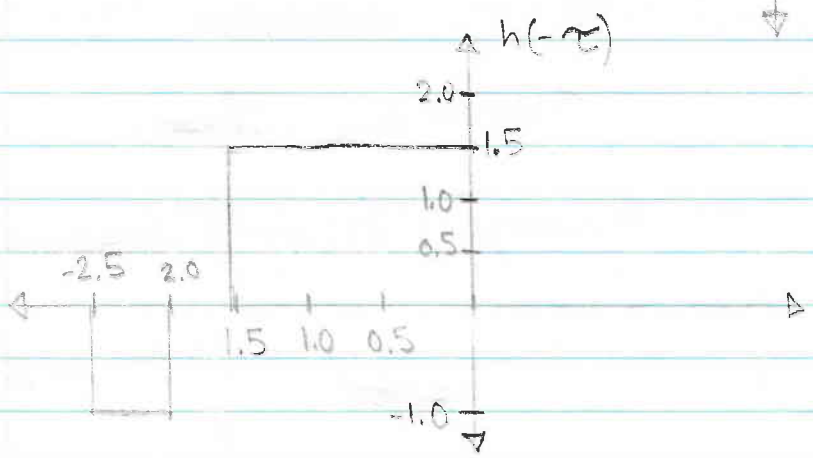
ELE 532 Lab 2 Part D.1

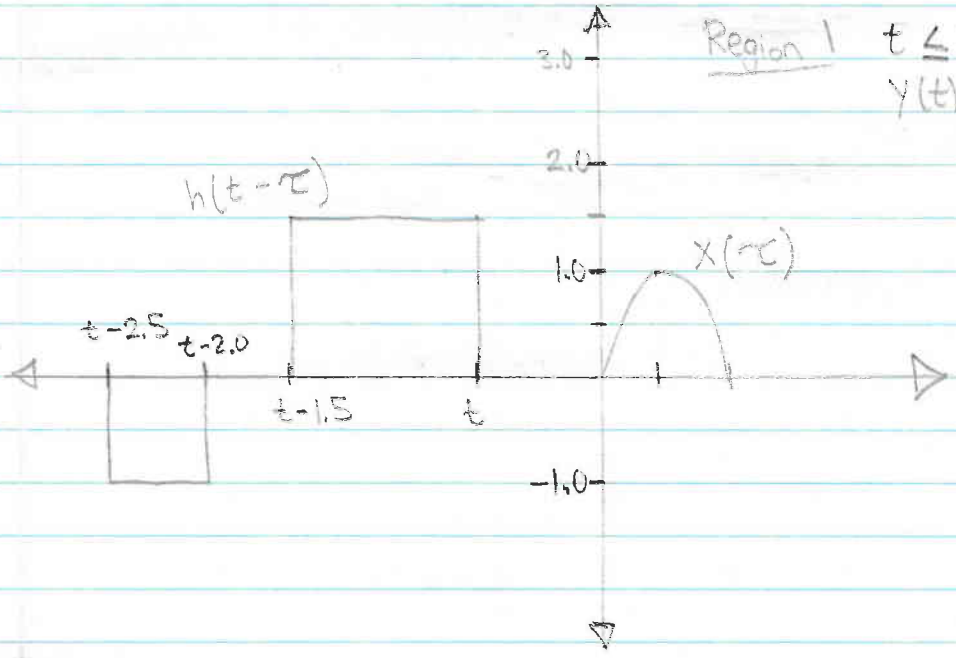
$h(t) = 1.5\big(u(t) - u(t-1.5)\big) - u(t-2) + u(t-2.5)$

B.1 $x(t) = 1.5\sin(\pi t)\big(u(t) - u(t-1)\big)$





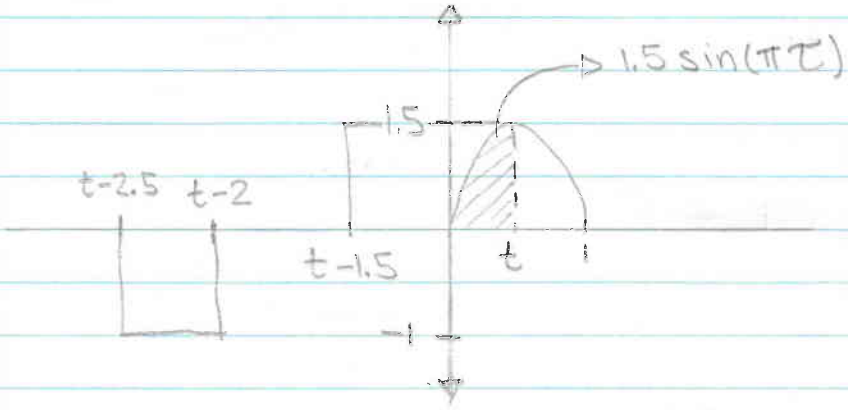




Region 1 $t \leq 0$

$y(t) = 0$

Region 2 $0 \le t \le 1$



$$y(t) = \int_0^t (1.5)(1.5\sin(\pi\tau))\,d\tau \qquad u = \pi\tau, \quad du = \pi\,d\tau$$

$$= \int_0^t 2.25\sin(\pi\tau)\,d\tau$$

$$= \int_0^t 2.25\sin(u)\frac{1}{\pi}\,du$$

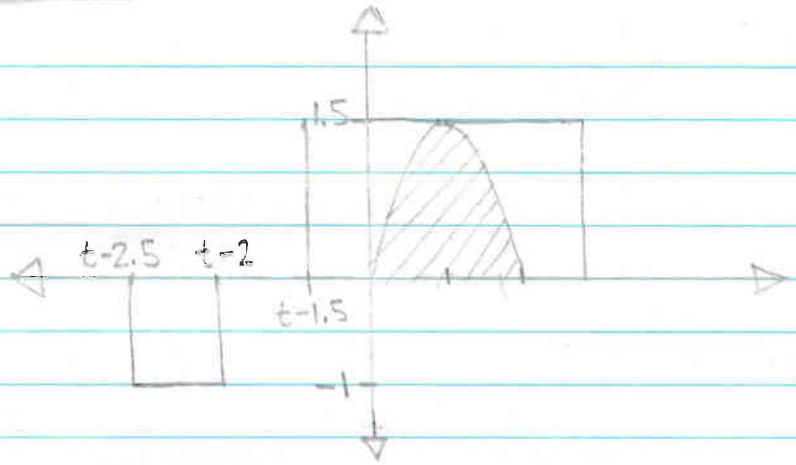
$$= \frac{2.25}{\pi}\int_0^t \sin(u)\,du = \frac{2.25}{\pi}\left(-\cos(u)\right)\Big|_0^t$$

$$= \frac{2.25}{\pi}\left(-\cos(\pi\tau)\right)\Big|_0^t$$

$$= -\frac{2.25}{\pi}\cos(\pi t) + \frac{2.25}{\pi}\cos(0)$$

$$\boxed{y(t) = \frac{2.25}{\pi}\left(1 - \cos(\pi t)\right)}$$

Region 3 , $t-1.5 < 0$, $t \geq 1$



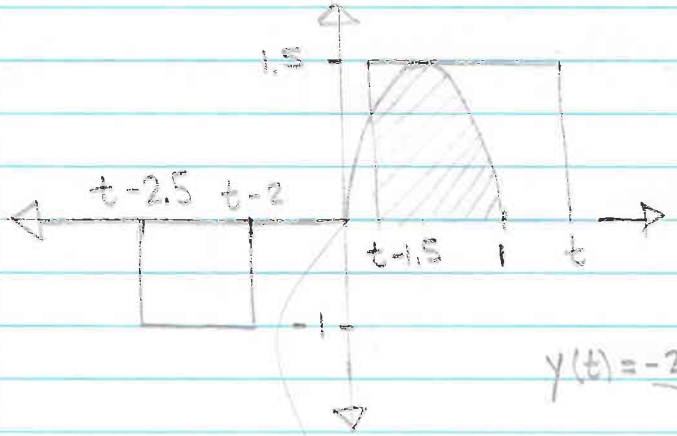
$$y(t) = \int_0^1 (1.5)(1.5\sin(\pi\tau))\,d\tau$$

$$= -\frac{2.25}{\pi}\cos(\pi\tau)\Big|_0^1 = -\frac{2.25}{\pi}\cos(\pi) + \frac{2.25}{\pi}\cos(0)$$

$$= \frac{2.25}{\pi} + \frac{2.25}{\pi}$$

$$\boxed{y(t) = \frac{4.5}{\pi} \doteq 1.432}$$

Region 4 $\quad 0 \leq t-1.5 \leq 0.5 \rightarrow 1.5 \leq t \leq 2$



$$y(t) = \int_{t-1.5}^1 (1.5)\,1.5\sin(\pi\tau)\,d\tau$$

$$= -\frac{2.25}{\pi}\cos(\pi\tau)\Big|_{t-1.5}^1$$

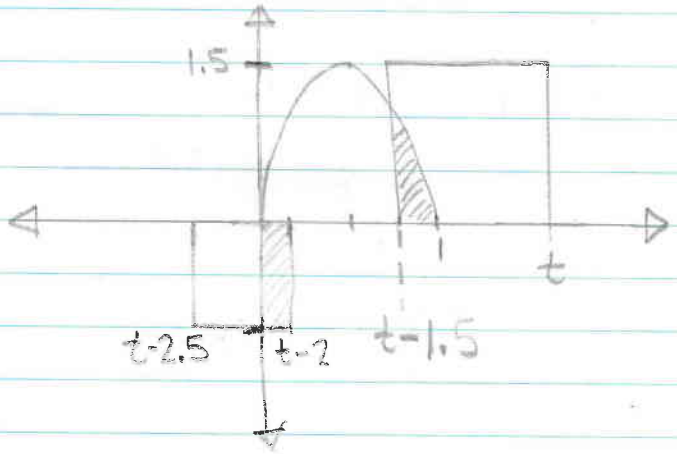
$$y(t) = -\frac{2.25\cos(\pi)}{\pi} + \frac{2.25\cos(\pi(t-1.5))}{\pi}$$

$$\boxed{y(t) = \frac{2.25}{\pi}\left[1 + \cos(\pi(t-1.5))\right]}$$

Region 5 $\quad 0 \le t-2 \quad t-1.5 \le 1 \rightarrow t \le 2.5$

$\hookrightarrow 2 \le t$

$2 \le t \le 2.5$



$$y(t) = \int_0^{t-2} (-1)(1.5\sin(\pi\tau))\,d\tau + \int_{t-1.5}^{1} (1.5)(1.5\sin(\pi\tau))\,d\tau$$

$$y(t) = \int_0^{t-2} -1.5\sin(\pi\tau)\,d\tau + \int_{t-1.5}^{1} 2.25\sin(\pi\tau)\,d\tau$$

$$y(t) = -1.5\int_0^{t-2} \sin(\pi\tau)\,d\tau + \text{integral from region 4}$$

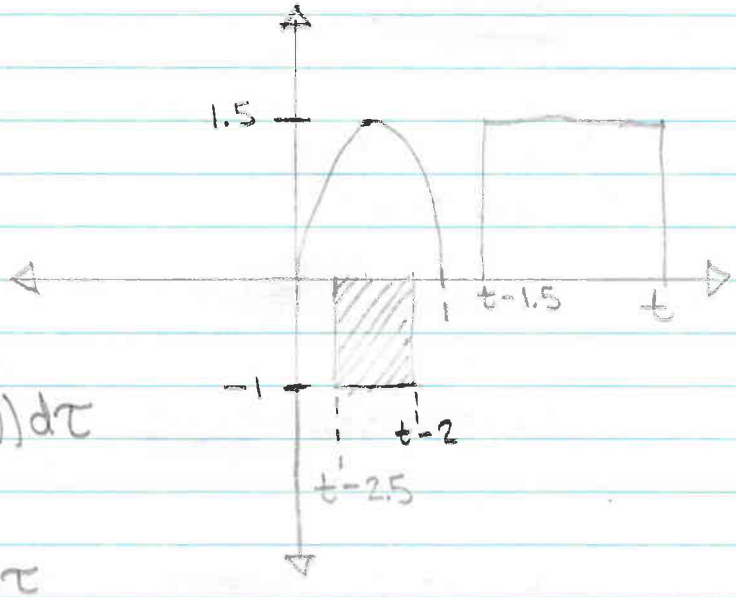
$$= \frac{1.5}{\pi}\cos(\pi\tau)\Big|_0^{t-2}$$

$$y(t) = \frac{1.5}{\pi}\left[\cos(\pi(t-2)) - 1\right] + \frac{2.25}{\pi}\left[1 + \cos(\pi(t-1.5))\right]$$

Region 6

$0 \le t-2.5 \qquad t-2 \le 1$

$2.5 \le t \le 3$



$$y(t) = \int_{t-2.5}^{t-2} (-1)(1.5\sin(\pi\tau))\,d\tau$$

$$= \int_{t-2.5}^{t-2} -1.5\sin(\pi\tau)\,d\tau$$

$$= \frac{1.5}{\pi}\cos(\pi\tau)\Big|_{t-2.5}^{t-2}$$

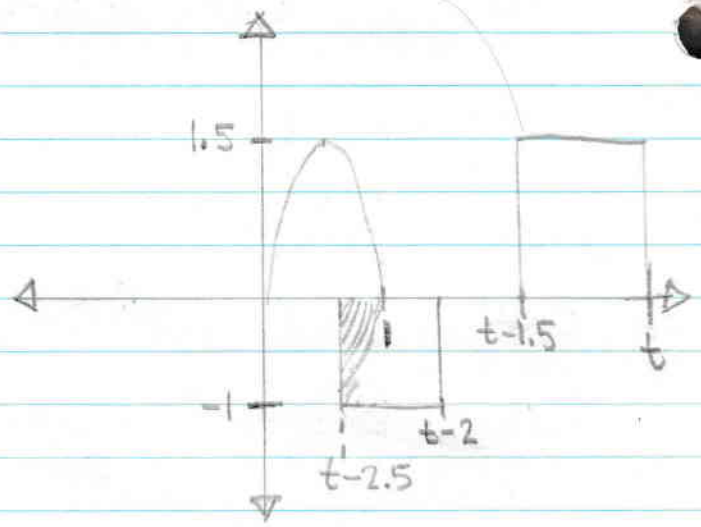
$$= \frac{1.5}{\pi}\cos(\pi(t-2)) - \frac{1.5}{\pi}\cos(\pi(t-2.5))$$

$$\boxed{y(t) = \frac{1.5}{\pi}\left[\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right]}$$

Region 7

$t-2.5 \le 1 \qquad 1 \le t-2$

$3 \le t \le 3.5$



$$y(t) = \int_{t-2.5}^{1} (-1)\,1.5\sin(\pi\tau)\,d\tau$$

$$= \int_{t-2.5}^{1} -1.5\sin(\pi\tau)$$

$$= \frac{1.5}{\pi}\cos(\pi\tau)\Big|_{t-2.5}^{1}$$

$$y(t) = \frac{1.5}{\pi}\left[-1 - \cos(\pi(t-2.5))\right]$$

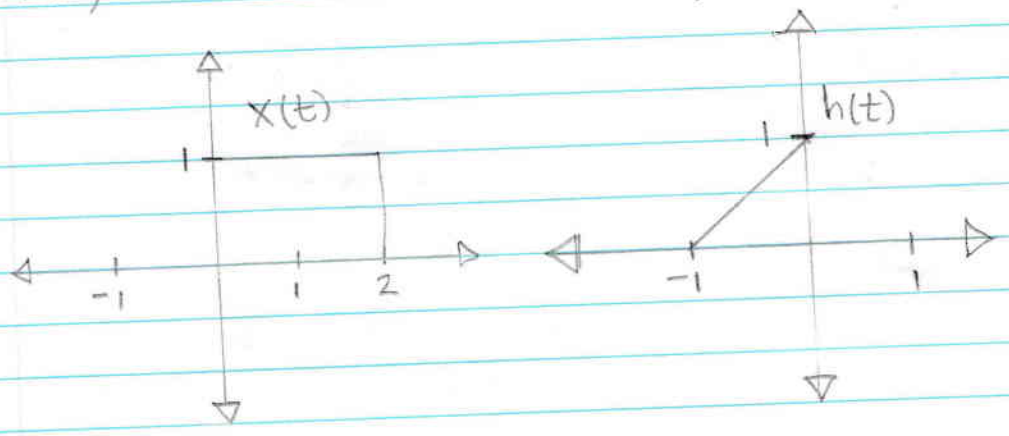
✱ Region 8 $3.5 \le t$

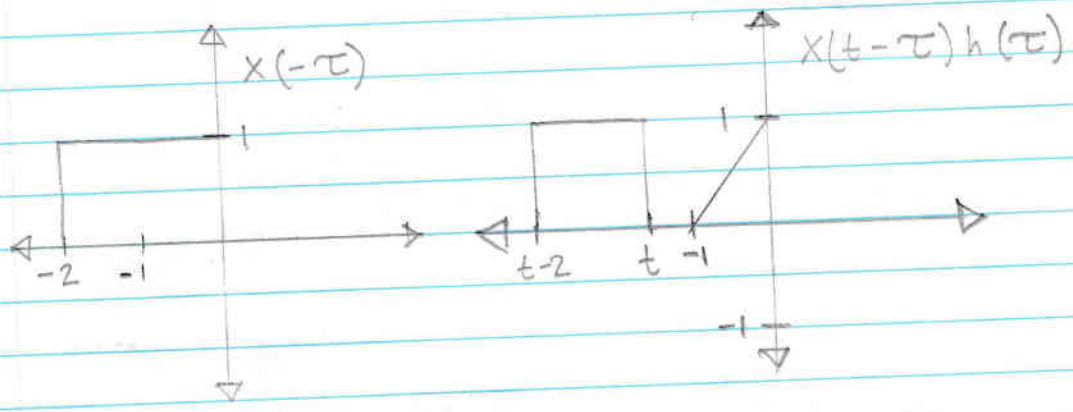
$y(t) = 0$

non-overlapping

$$y(t) = \begin{cases} 0 & t \le 0 \\ \frac{2.25}{\pi}(1-\cos(\pi t)) & 0 \le t \le 1 \\ \frac{4.5}{\pi} & 1 \le t \le 1.5 \\ \frac{2.25}{\pi}\left[1+\cos(\pi(t-1.5))\right] & 1.5 \le t \le 2 \\ \frac{1.5}{\pi}\left[\cos(\pi(t-2))-1\right] + \frac{2.25}{\pi}\left[1+\cos(\pi(t-1.5))\right] & 2 \le t \le 2.5 \\ \frac{1.5}{\pi}\left[\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right] & 2.5 \le t \le 3 \\ \frac{1.5}{\pi}\left[-1-\cos(\pi(t-2.5))\right] & 3 \le t \le 3.5 \\ 0 & 3.5 \le t \end{cases}$$

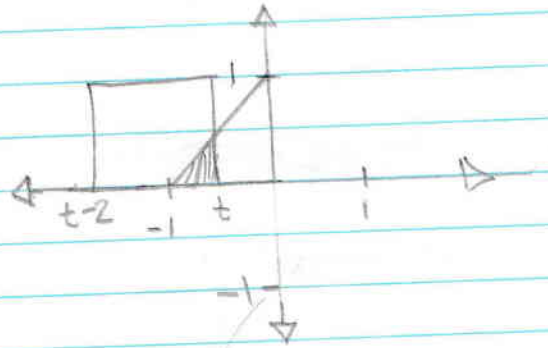
D.1) B.2 $x(t) = u(t) - u(t-2)$ $h(t) = (t+1)(u(t+1) - u(t))$





Region 1 $t \leq 1$, $y(t) = 0$, no overlap

Region 2 $-1 \leq t \leq 0$



$$y(t) = \int_{-1}^{t} (\tau + 1)\, d\tau$$

$$= \int_{-1}^{t} \tau\, d\tau + \int_{-1}^{t} d\tau$$

$$= \frac{\tau^2}{2}\Big|_{-1}^{t} + \tau\Big|_{-1}^{t}$$

$$= \frac{t^2}{2} - \frac{1}{2} + t - 1$$

$$\boxed{y(t) = \frac{t^2}{2} + t + \frac{1}{2}}$$

Library

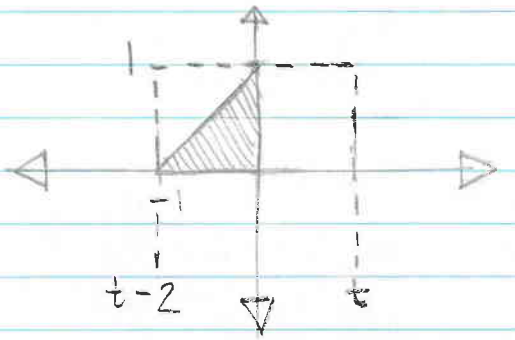
Region 3 $\quad t \le 1, \; t \ge 0, \; 0 \le t \le 1$

$$y(t) = \int_{-1}^{0} (\tau + 1)\, d\tau$$

$$= \frac{\tau^2}{2} + \tau \Big|_{-1}^{0}$$

$$= \frac{1}{2} - 1$$

$$\boxed{y(t) = -0.5}$$

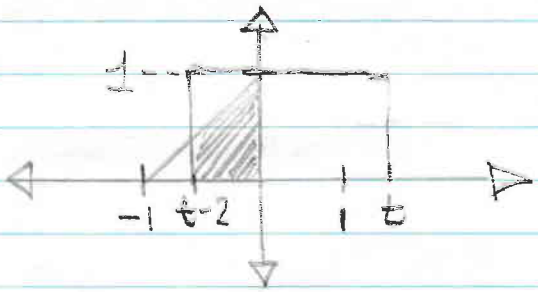


Region 4 $\quad t-2 \le 0, \; -1 \le t-2, \; 1 \le t \le 2$

$$y(t) = \int_{t-2}^{0} (\tau + 1)\, d\tau$$

$$= \frac{\tau^2}{2} + \tau \Big|_{t-2}^{0}$$

$$\boxed{y(t) = -\left[\frac{(t-2)^2}{2} + (t-2)\right]}$$



Region 5 $\quad t \ge 2$

$y(t) = 0$

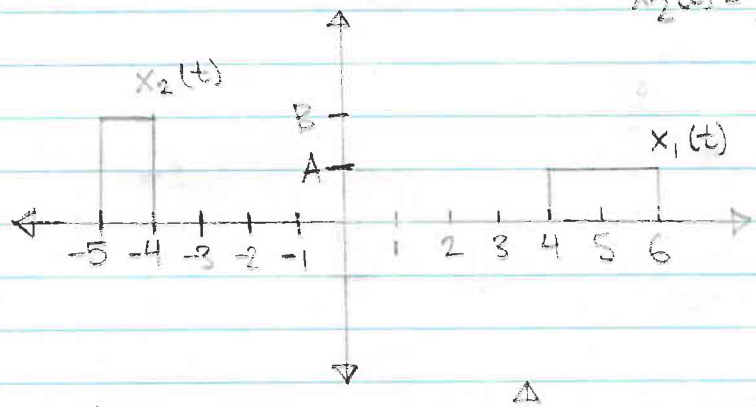
$$y(t) = \begin{cases} 0 & t \le -1 \\ \frac{t^2}{2} + t + \frac{1}{2} & -1 \le t \le 0 \\ -0.5 & 0 \le t \le 1 \\ -\left[\frac{(t-2)^2}{2} + (t-2)\right] & 1 \le t \le 2 \\ 0 & t \ge 2 \end{cases}$$

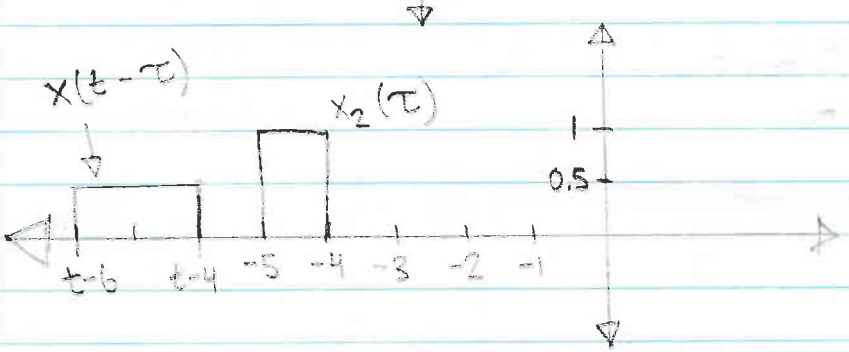
$x_1(t) = \frac{1}{2}[u(t-4) - u(t-5)]$

$x_2(t) = [u(t+5) - u(t+4)]$

D.1) B.3 Let A = 0.5, B = 1

a)





Region 1 $t-4 \leq -5$, $t \leq -1$

non-overlapping, $y(t) = 0$

Region 2 $-5 \leq t-4 \leq -4 \rightarrow -1 \leq t \leq 0$

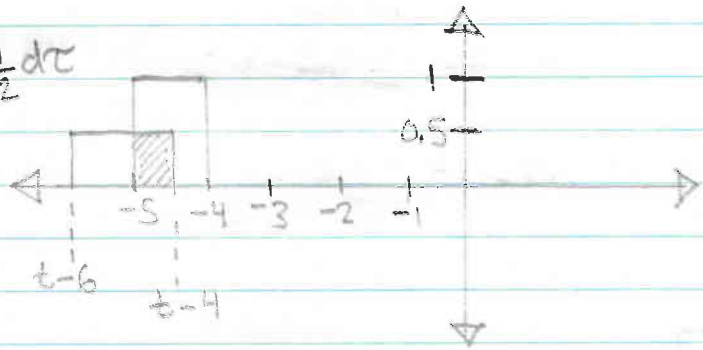
$$y(t) = \int_{-5}^{t-4} A(B)\,d\tau = \int_{-5}^{t-4} \frac{1}{2}\,d\tau$$

$$= \frac{1}{2}\tau \Big|_{-5}^{t-4}$$

$$= \frac{1}{2}(t-4) + \frac{1}{2}(5)$$

$$= \frac{1}{2}t - 2 + 2.5$$

$$\boxed{y(t) = \frac{1}{2}t + 0.5}$$



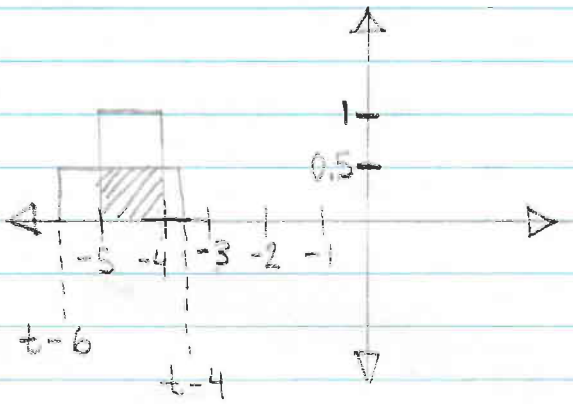
Region 3 $t-4 \geq -4$, $t-6 \leq -5$ $\rightarrow$ $0 \leq t \leq 1$

$$y(t) = \int_{-5}^{-4} \frac{1}{2}\, d\tau$$

$$= \frac{1}{2}\tau \Big|_{-5}^{-4}$$

$$= \frac{1}{2}(-4) - \frac{1}{2}(-5)$$

$$\boxed{y(t) = 0.5}$$



Region 4 $t-6 \leq -4$, $t-6 \geq -5$ $\rightarrow$ $1 \leq t \leq 2$

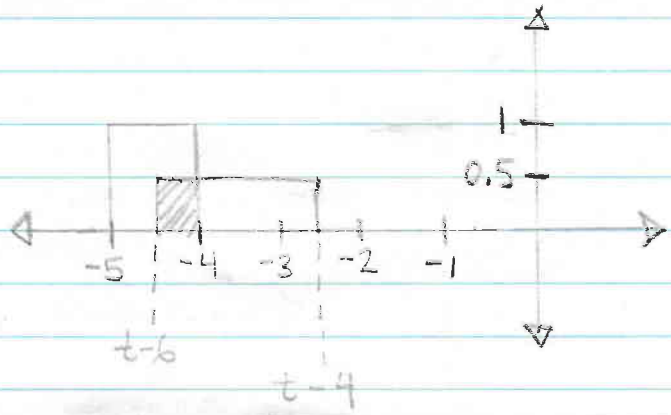
$$y(t) = \int_{t-6}^{-4} \frac{1}{2}\, d\tau$$

$$= \frac{1}{2}\tau \Big|_{t-6}^{-4}$$

$$= \frac{1}{2}(-4) - \frac{1}{2}(t-6)$$

$$= -2 - \frac{1}{2}t + 3$$

$$\boxed{y(t) = -\frac{1}{2}t + 1}$$



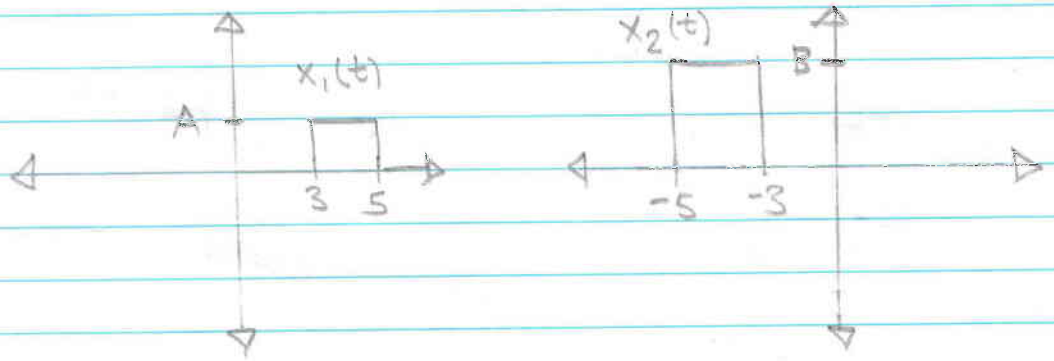
(6) Region 5 $t \geq 2$

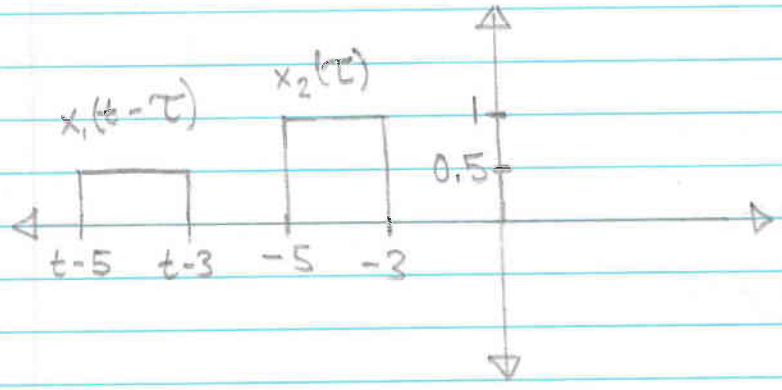
$y(t) = 0$, non-overlapping

$$y(t) = \begin{cases} 0 & t \leq 1 \\ \frac{1}{2}t + 0.5 & -1 \leq t \leq 0 \\ 0.5 & 0 \leq t \leq 1 \\ -\frac{1}{2}t + 1 & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

$x_1(t) = \frac{1}{2}[u(t-3) - u(t-5)]$

D.1) B.3 b) Let A=0.5, B=1 $\quad x_2(t) = [u(t+5) - u(t+3)]$





Region 1 $\quad t-3 \leq -5 \rightarrow t \leq -2$

$y(t) = 0$

Region 2 $\quad t-3 \leq -3,\ t-3 \geq -5 \rightarrow -5 \leq t-3 \leq -3$

$\rightarrow -2 \leq t \leq 0$

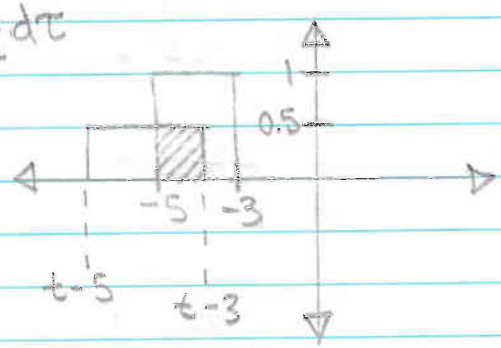
$$y(t) = \int_{-5}^{t-3} A(B)\,d\tau = \int_{-5}^{t-3} \frac{1}{2}\,d\tau$$

$$= \frac{1}{2}\tau \Big|_{-5}^{t-3}$$

$$= \frac{1}{2}(t-3) - \frac{1}{2}(-5)$$

$$= \frac{1}{2}t - 1.5 + 2.5$$

$$\boxed{y(t) = \frac{1}{2}t + 1}$$



Region 3 $\quad t-5 \geq -5, \; t-5 \leq -3$

$-5 \leq t-5 \leq -3$

$\hookrightarrow 0 \leq t \leq 2$

$$y(t) = \int_{t-5}^{-3} \frac{1}{2} d\tau$$

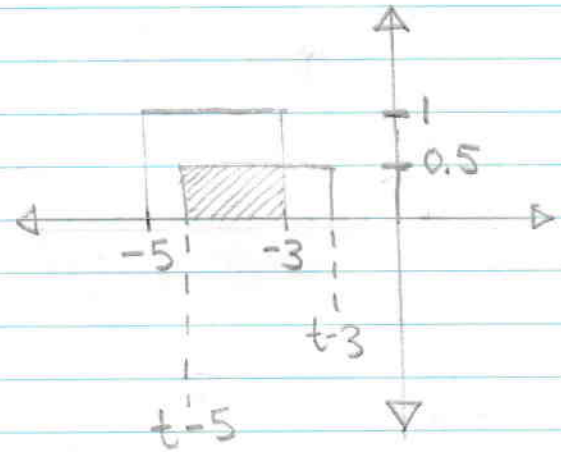
$$= \frac{1}{2}\tau \Big|_{t-5}^{-3}$$

$$= \frac{1}{2}(-3) - \frac{1}{2}(t-5)$$

$$= \frac{1}{2}(-3) - \frac{1}{2}t + \frac{1}{2}(5)$$

$$= -\frac{1}{2}t + 2.5 - 1.5$$

$$\boxed{y(t) = -\frac{1}{2}t + 1}$$



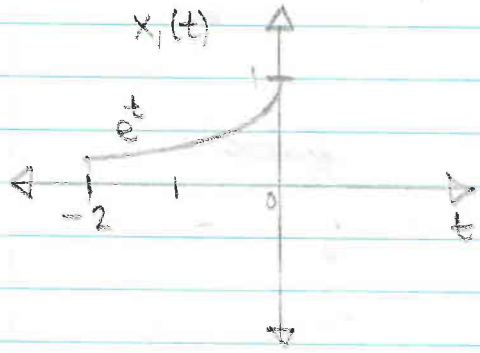
Region 4 $\quad t-5 \geq -3 \rightarrow t \geq 2$

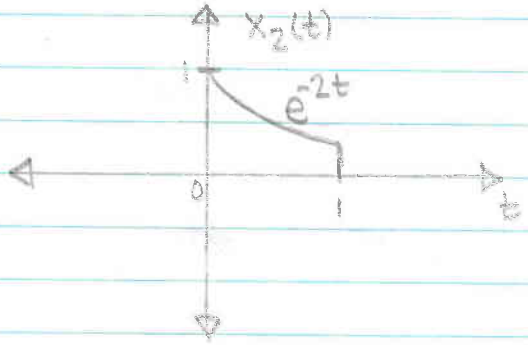
$y(t) = 0$

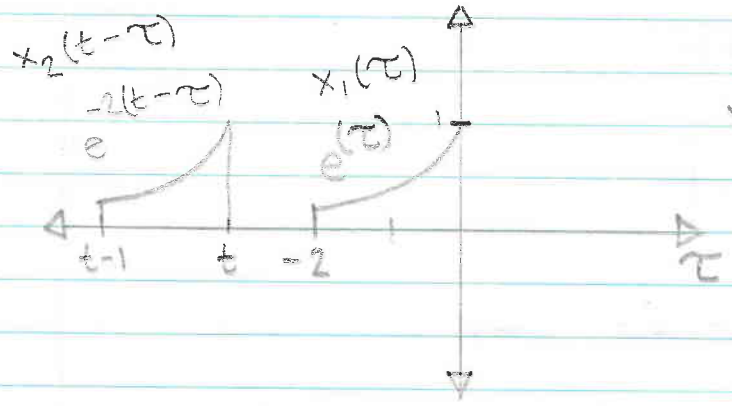
$$y(t) = \begin{cases} 0 & t \leq -2 \\ \frac{1}{2}t + 1 & -2 \leq t \leq 0 \\ -\frac{1}{2}t + 1 & 0 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

D.1) B.3 (h) $x_1(t) = e^{t}\left(u(t+2) - u(t)\right)$

$x_2(t) = e^{-2t}\left(u(t) - u(t-1)\right)$







$x_1(\tau) = e^{\tau}$

$x_2(t-\tau) = e^{-2(t-\tau)}$

Region 1 $\quad t \leq -2$

$y(t) = 0$, no overlap

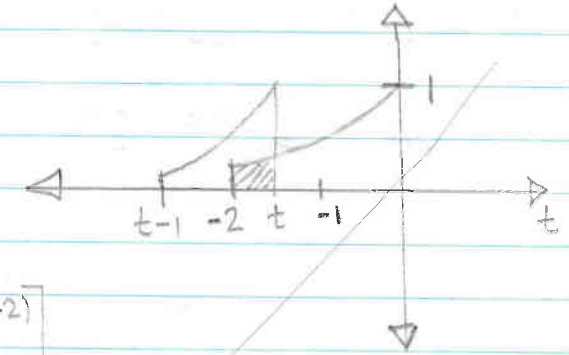
Region 2 $\quad t \geq -2$, $t-1 \leq -2 \rightarrow \; -2 \leq t \leq -1$

$$y(t) = \int_{-2}^{t} e^{\tau} e^{-2(t-\tau)}\, d\tau$$

$$= e^{-2t} \int_{-2}^{t} e^{\tau} e^{2\tau}\, d\tau$$

$$= e^{-2t}\left[\frac{1}{3} e^{3\tau}\right]_{-2}^{t} = \frac{1}{3} e^{-2t}\left[e^{3t} - e^{3(-2)}\right]$$

$$\boxed{y(t) = \frac{1}{3}\left[e^{t} - e^{-2(t+3)}\right]}$$



Region 3 $t-1 \geq -2$, $t \leq 0$ $\rightarrow$ $-1 \leq t \leq 0$

$$y(t) = \int_{t-1}^{t} e^{\tau} e^{-2(t-\tau)} d\tau$$

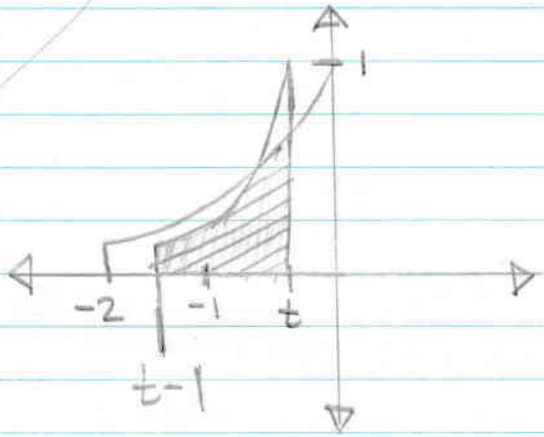
$$= \int_{t-1}^{t} e^{\tau} e^{-2t} e^{2\tau} d\tau$$

$$= e^{-2t} \int_{t-1}^{t} e^{3\tau} d\tau$$

$$= e^{-2t} \left[\frac{1}{3} e^{3\tau}\right]_{t-1}^{t}$$

$$= \frac{1}{3} e^{-2t} \left[e^{3t} - e^{3(t-1)}\right]$$

$$\boxed{y(t) = \frac{1}{3}\left[e^{t} - e^{t-3}\right]}$$



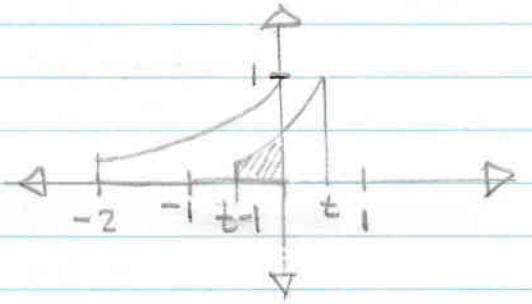
Region 4 $t-1 \leq 0$, $t \geq 0$ $\rightarrow$ $0 \leq t \leq 1$

$$y(t) = \int_{t-1}^{0} e^{\tau} e^{-2(t-\tau)} d\tau$$

$$= e^{-2t} \int_{t-1}^{0} e^{3\tau} d\tau$$

$$= e^{-2t} \left[\frac{1}{3} e^{3\tau}\right]_{t-1}^{0}$$

$$\boxed{y(t) = \frac{1}{3}\left[e^{-2t} - e^{t-3}\right]}$$



Region 5 $t-1 \geq 0 \rightarrow t \geq 1$

$y(t) = 0$, no overlapping

$$y(t) = \begin{cases} 0 & t \leq -2 \\ \frac{1}{3}\left[e^{t} - e^{-2(t+3)}\right] & -2 \leq t \leq -1 \\ \frac{1}{3}\left[e^{t} - e^{t-3}\right] & -1 \leq t \leq 0 \\ \frac{1}{3}\left[e^{-2t} - e^{t-3}\right] & 0 \leq t \leq 1 \\ 0 & t \geq 1 \end{cases}$$

ELE 532 Lab 2. A.2

A.2) $R_1 = R_2 = R_3 = 10k\Omega$ $\quad C_1 = C_2 = 1\mu F$

$$y''(t) + \frac{1}{C_2}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]y'(t) + \frac{1}{R_1R_2C_1C_2}y(t) = -\frac{1}{R_1R_3C_1C_2}x(t)$$

$$y''(t) + 300y'(t) + 10000\,y(t) = -10000\,x(t)$$

$$(D^2 + 300D + 10000)y(t) = -10000\,x(t)$$

$N = 2 \quad M = 0$ $\quad N \neq M,\ b_0 = 0$

↳ degree of input

↳ degree of output

* $y(t) \to h(t),\ x(t) \to \delta(t)$

$$\lambda^2 + 300\lambda + 10000 = 0$$

$$\frac{-b + \sqrt{b^2-4ac}}{2a} = -38.1966 \qquad \frac{-b - \sqrt{b^2-4ac}}{2a} = -261.8034$$

$$h(t) = \left(C_1 e^{-38.1966t} + C_2 e^{-261.8034t}\right)u(t)$$

$$h'(t) = \left(-38.1966\,C_1 e^{-38.1966t} - 261.8034\,C_2 e^{-261.8034t}\right)u(t) + (C_1 + C_2)\delta(t)$$

$$h''(t) = \left(1458.98\,C_1 e^{-38.1966t} + 68541.10\,C_2 e^{-261.8034t}\right)u(t)$$

$$+ (-38.1966\,C_1 - 261.8034\,C_2)\delta(t) + (C_1 + C_2)\delta'(t)$$

$$h''(t) + 300h'(t) + 10000\,h(t) = -10000\,\delta(t)$$

$$\hookrightarrow = 0 \times u(t) + (261.8034\,C_1 + 38.1966\,C_2)\delta(t) + (C_1 + C_2)\delta'(t)$$

$$\begin{cases} 261.8034\,C_1 + 38.1966\,C_2 = -10000 \\ C_1 + C_2 = 0 \end{cases}$$

Hilroy

① $261.8034\,C_1 + 38.1966\,C_2 = -10000$

② $C_1 + C_2 = 0$

③ $C_1 = -C_2$ sub. ③ into ①

$261.8034(-C_2) + 38.1966\,C_2 = -10000$

$-223.6068\,C_2 = -10000$

$\boxed{C_2 = 44.7214}$

$\boxed{C_1 = -C_2 = -44.7214}$

$h(t) = \left(C_1 e^{-38.1966t} + C_2 e^{-261.8034t}\right) u(t)$

$\boxed{h(t) = \left(-44.7214\, e^{-38.1966t} + 44.7214\, e^{-261.8034t}\right) u(t)}$