$e^{j\alpha t}$ $\alpha = n\omega_0$

Ele 532 Lab 3

A.1 $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$

Euler Formula: $= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{2}\left[\frac{1}{2}e^{j\frac{\pi}{10}t} + \frac{1}{2}e^{-j\frac{\pi}{10}t}\right]$

$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$

Fundamental Frequency: $\begin{cases} \cos\frac{3\pi}{10}t : \omega_{0_1} = \frac{3\pi}{10} \\ \cos\frac{\pi}{10}t : \omega_{02} = \frac{\pi}{10} \end{cases}$

$x(t + T_0) = x(t)$

$x_1(t + T_0) = \cos\frac{3\pi}{10}(t + T_0) + \frac{1}{2}\cos\frac{\pi}{10}(t + T_0)$

$\begin{cases} \frac{3\pi}{10}(t + T_0) = \frac{3\pi}{10}t + 2\pi k_1 \\ \frac{\pi}{10}(t + T_0) = \frac{\pi}{10}t + 2\pi k_2 \end{cases} \rightarrow \begin{cases} \frac{3\pi}{10}T_0 = 2\pi k_1 \\ \frac{\pi}{10}T_0 = 2\pi k_2 \end{cases} \rightarrow \begin{cases} T_0 = \frac{20}{3}k_1 \\ T_0 = 20k_2 \end{cases}$

T_0: Lowest common multiple (LCM) of $\frac{20}{3}$ and 20

$LCM(\frac{20}{3}, 20) = 20$, $T_0 = 20$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{20} = \boxed{\frac{\pi}{10}}$ $jn\omega_0 t = jn\frac{\pi}{10}t$

Powers: $\frac{3\pi}{10}$ $\frac{-3\pi}{10}$ $\frac{\pi}{10}$ $\frac{-\pi}{10}$

$(n)\omega_0 = n\frac{\pi}{10}$ $(3)\omega_0$ $(-3)\omega_0$ $(1)\omega_0$ $(-1)\omega_0$

$$x(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$$

$$x(t) = \frac{1}{2}e^{j3w_0t} + \frac{1}{2}e^{-j3w_0t} + \frac{1}{4}e^{jw_0t} + \frac{1}{4}e^{-jw_0t}$$

$$D_3 = \frac{1}{2} \qquad D_{-3} = \frac{1}{2} \qquad D_1 = \frac{1}{4} \qquad D_{-1} = \frac{1}{4}$$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jnw_0t} \qquad T_0 = 20 \qquad w_0 = \frac{\pi}{10}$$

$$D_n = \frac{1}{T_0}\int_{-\frac{1}{2}T_0}^{\frac{1}{2}T_0}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-jn\frac{\pi}{10}t}\,dt$$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j(3-n)\frac{\pi}{10}t} + \frac{1}{2}e^{-j(3+n)\frac{\pi}{10}t} + \frac{1}{4}e^{j(1-n)\frac{\pi}{10}t} + \frac{1}{4}e^{-j(1+n)\frac{\pi}{10}t}\right]dt$$

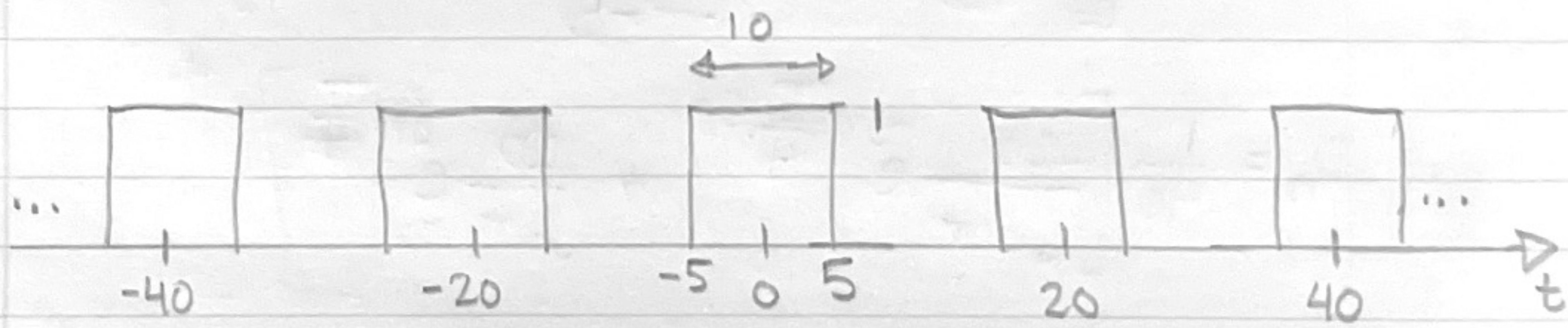
$$D_n = \frac{1}{20}\left[\frac{1}{2j(3-n)\left(\frac{\pi}{10}\right)}\left(e^{j(3-n)\frac{\pi}{10}(10)} - e^{j(3-n)\frac{\pi}{10}(-10)}\right)\right.$$
$$+ \frac{1}{2j(3+n)\left(\frac{\pi}{10}\right)}\left(e^{j(3+n)\frac{\pi}{10}(10)} - e^{j(3+n)\frac{\pi}{10}(-10)}\right)$$
$$+ \frac{1}{4j(1-n)\frac{\pi}{10}}\left(e^{j(1-n)\frac{\pi}{10}(10)} - e^{j(1-n)\frac{\pi}{10}(-10)}\right)$$
$$\left. + \frac{1}{4j(1+n)\frac{\pi}{10}}\left(e^{j(1+n)\frac{\pi}{10}(10)} - e^{j(1+n)\frac{\pi}{10}(-10)}\right)\right]$$

$$\frac{e^{j(\alpha t+\beta)} - e^{-j(\alpha t+\beta)}}{2j} = \sin(\alpha t+\beta) \qquad \alpha = nW_0$$

$$D_n = \frac{1}{2}\left[\frac{\sin[(3-n)\pi]}{(3-n)\pi}\right] + \frac{1}{2}\left[\frac{\sin[(3+n)\pi]}{(3+n)\pi}\right]$$

$$+ \frac{1}{4}\left[\frac{\sin[(1-n)\pi]}{(1-n)\pi}\right] + \frac{1}{4}\left[\frac{\sin[(1+n)\pi]}{(1+n)\pi}\right]$$

A.2 $x_2(t)$

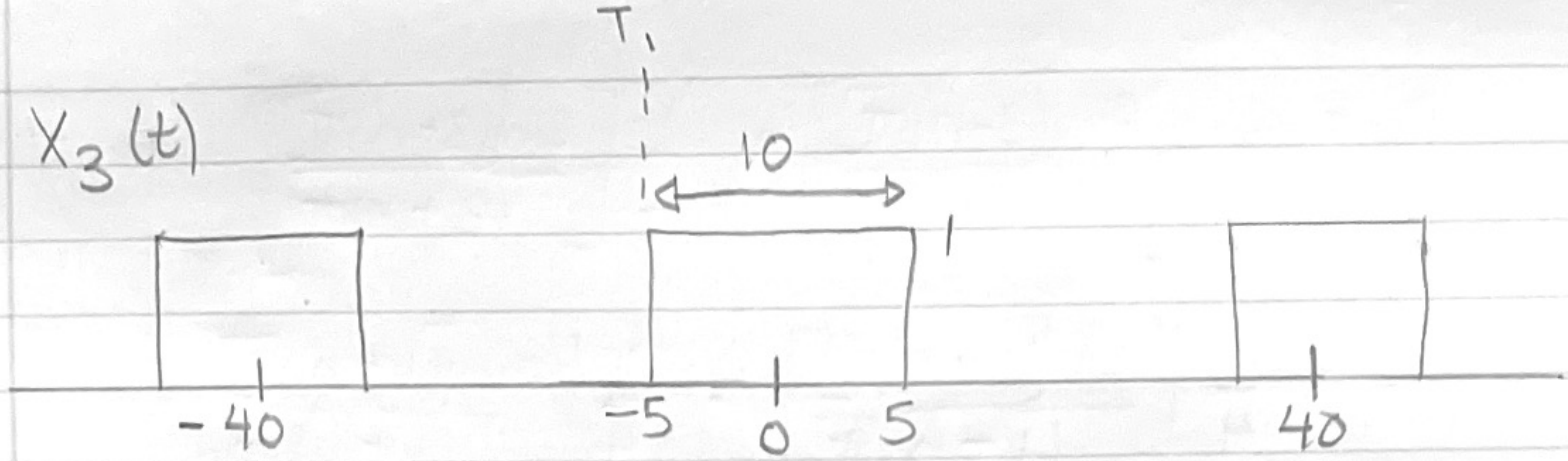


$$T_0 = 20 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{10} \qquad x_2(t) = 1$$

$$D_n = \frac{1}{20}\int_{-5}^{5} x_2(t)\, e^{-jnW_0 t}\, dt = \frac{1}{20}\int_{-5}^{5} (1)\, e^{-jn\frac{\pi}{10}t}\, dt$$

$$D_n = \frac{1}{20}\left[\frac{1}{-j\frac{\pi}{10}n} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5} = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi} e^{jn\frac{\pi}{2}}\right]$$

$$D_n = \frac{e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}}{2jn\pi} = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi} \qquad \boxed{D_n = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}}$$



$T_0 = 40 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{20}$

$$D_n = \frac{1}{T_0}\int_{-5}^{5} x_3(t)\, e^{-jn\omega_0 t} = \frac{1}{40}\int_{-5}^{5} (1)\, e^{-jn\frac{\pi}{20}t}$$

$$D_n = \frac{1}{40}\left[\frac{1}{-jn\frac{\pi}{20}}\, e^{-jn\frac{\pi}{20}t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{40}\left[\frac{-20}{jn\pi}\, e^{-jn\frac{\pi}{4}} + \frac{20}{jn\pi}\, e^{jn\frac{\pi}{4}}\right]$$

$$\boxed{D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right) = \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi}}$$

B.1) $x_1(t) = \cos\left(\frac{3\pi}{10}\right)t + \frac{1}{2}\cos\left(\frac{\pi}{10}\right)t$

$$w_{01} = \frac{3\pi}{10} \qquad w_{02} = \frac{\pi}{10}$$

$$w_0 = \frac{\text{GCF of numerator}}{\text{LCM of denominator}} = \frac{\pi}{10}$$

For $x_2(t)$: $T_0 = 20s$ $\qquad w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

For $x_3(t)$: $T_0 = 40s$ $\qquad w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

B.2) For $x_1(t)$ there are a fixed amount of fourier coefficients which are: $D_3 = 1/2$, $D_{-3} = 1/2$, $D_1 = 1/4$, and $D_{-1} = 1/4$. However, for $x_2(t)$ there are an infinite amount of fourier coefficients ~~as~~ expressed by the equation: $D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$.

B.3) $D_n = \frac{1}{n\pi}\sin\frac{n\pi}{2}$ for $x_2(t)$ and $D_n = \frac{1}{n\pi}\sin\frac{n\pi}{4}$ for $x_3(t)$. The affect of different periods can be seen in these equations. $x_3(t)$ has a smaller fundamental frequency than $x_2(t)$.

B.4) For $x_2(t)$: $D_0 = \frac{\sin(0)}{0}$

$$D_0 = \frac{\cancel{\pi}\cos(0)}{2\cancel{\pi}} \quad \text{l'hôpital rule}$$

$$D_0 = \frac{1}{2}$$

Since $x_4(t)$ is $x_2(t)$ shifted downward by $1/2$, D_0 for $x_4(t)$ is $D_0 = \frac{1}{2} - \frac{1}{2} = 0$. This is also proven by how the top and bottom portions of the graph cancel out.

B.5) For $x_1(t)$, the ~~reconstructed~~ reconstructed signal will not change because it has a finite number of fourier coefficients. For $x_2(t)$, as the number of fourier coefficients increases, the signal will be more and more accurate.

B.6) Since $x_1(t)$ has a finite number of D_n values, you would only need 4 fourier coefficients to perfectly reconstruct it. However, for $x_2(t)$ and $x_3(t)$ you would need an infinite number of D_n to perfectly reconstruct.

B.7) In periodic signals, there are an infinite number of fourier coefficients. Storing an infinite amount is not possible and so it is not viable. If there were a finite amount of fourier coefficients, then this strategy could work.