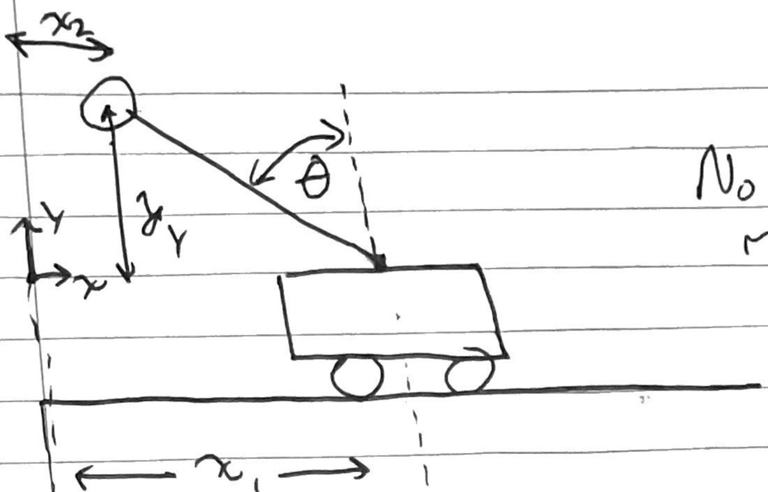

No friction,
Mass-less rod

(1). Kinetic Energy

$$K_{sys} = K_p + K_c$$

$$K_c = \frac{1}{2} m_c v_c^2 = \frac{1}{2} m_c v_c^2$$

$$K_p = \frac{1}{2} m_p v_p^2 = \frac{1}{2} m_p (\dot{x}_2^2 + \dot{y}_2^2)$$

Compute $\dot{x}_2, \dot{y}_2$

$$\dot{x}_2 = \frac{d}{dt}[x_1 - l\sin\theta] = \dot{x}_1 - l\dot{\theta}\cos\theta$$

$$\dot{y}_2 = d/dt[l\cos\theta] = -l\dot{\theta}\sin\theta$$

$$K_p = \frac{1}{2} m_2 (\dot{x}_1^2 - 2l\dot{x}_1\dot{\theta}\cos\theta + l^2\dot{\theta}^2)$$

$$K_p + K_c = K_{sys} = \frac{1}{2}(m_1 + m_2)\dot{x}_1^2 - m_2 l\dot{x}_1\dot{\theta}\cos\theta + \frac{1}{2} m l^2\dot{\theta}^2$$

(2). Potenial Energy

$$PE = m_c g l\cos\theta$$

(3) Lagrange

$$L = KE_{sys} - PE$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \partial L/\partial x_1 = F \quad (1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \partial L/\partial\theta = 0 \quad (2) \rightarrow$$

$$L = \frac{1}{2}(m_c + m_p)\dot{x}_1^2 - m_p l \dot{x}_1 \dot{\theta}\cos\theta + \frac{1}{2} m_p l^2 \dot{\theta}^2 - m_p g l \cos\theta$$

$$\frac{\partial L}{\partial \dot{x}_1} = (m_c + m_p)\dot{x}_1 - m_p l \dot{\theta}\cos\theta + 0 + 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m_c + m_p)\ddot{x}_1 - m_p l \ddot{\theta}\cos\theta + m_p l \dot{\theta}^2 \sin\theta$$

$$\partial L / \partial x_1 = 0$$

$$F = (m_c + m_p)\ddot{x}_1 - m_p l \ddot{\theta}\cos\theta + m_p l \dot{\theta}^2 \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = -m_p l \dot{x}_1 \cos\theta + m_p l^2 \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -m_p l \ddot{x}_1 \cos\theta + m_p l \dot{x}_1 \dot{\theta} \sin\theta + m_p \ddot{\theta} l^2$$

$$\frac{\partial L}{\partial \theta} = -m_p l \dot{x} \sin\theta + m_p g l \sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \cancel{l \ddot{x}_1 \cos\theta - g \sin\theta = 0}$$

$$= \cancel{m_p l \ddot{x}_1 \cos\theta - m_p l \ddot{\theta} \cos\theta + m_p l}$$

$$-m_p l \ddot{x}_1 \cos\theta + m_p l^2 \ddot{\theta} - m_p g l \sin\theta = 0$$

$$F_x = (M_c + m_p)\ddot{x} + m_p l \sin\theta \cdot \dot{\theta}^2 - m_p l \cos\theta\, \ddot{\theta} \quad (1)$$

$$0 = m_p l^2 \ddot{\theta} - m_p l \cos\theta\, \ddot{x} - m_p g l \sin\theta \quad (2)$$

Solve $\ddot{\theta}$ w/ (2)

$$\cancel{m_p l} \cos\theta \cdot \ddot{x} + \cancel{m_p} g \cancel{l} \sin\theta = \cancel{m_p l^2} \ddot{\theta}$$

$$\cancel{\frac{\ddot{x}\cos\theta + g\sin\theta}{l} = \ddot{\theta}} \qquad \ddot{\theta} = \frac{\ddot{x}\cos\theta + g\sin\theta}{L}$$

Solve for $\ddot{x}$

$$F_x = (M_c + M_p)\ddot{x} + M_p l \sin\theta \cdot \dot{\theta}^2 - M_p l \cos\theta \cdot \left(\frac{\ddot{x}\cos\theta + g\sin\theta}{l} \right)$$

$$F_x = (M_c + M_p)\ddot{x} + M_p l \sin\theta \cdot \dot{\theta}^2 - \frac{M_p l \cos\theta \cdot \ddot{x}\cos\theta - M_p l \cos\theta \cdot g \sin\theta}{l}$$

$$F_x = (M_c + M_p)\ddot{x} + M_p l \sin\theta \cdot \dot{\theta}^2 - \ddot{x} M_p \cos^2\theta - M_p \cos\theta \sin\theta\, g$$

$$F_x + M_p g \sin\theta\cos\theta - M_p l \sin\theta \cdot \dot{\theta}^2 = \ddot{x} M_c + \ddot{x} M_p - \ddot{x} M_p \cos^2\theta$$
$$= \ddot{x}[M_c + M_p - M_p \cos^2\theta]$$

$$\boxed{\frac{F_x + M_p g \sin\theta\cos\theta - M_p l \sin\theta \cdot \dot{\theta}^2}{[M_c + M_p(1 - \cos^2\theta)]} = \ddot{x}}$$

~~Now, plug $\ddot{x}$ back into $\ddot{\theta}$~~

~~$\ddot{\theta}$:~~

$$\cancel{\frac{F_x + M_p g \sin\theta\cos\theta - M_p l \sin\theta \cdot \dot{\theta}^2}{[M_c + M_p(1 - \cos^2\theta)]}} \quad \cancel{\cos\theta + g\sin\theta}$$

Solve for $\ddot{\theta}$

$$\ddot{\theta} = \frac{F_x + M_p g \sin\theta\cos\theta - M_p l \sin\theta \cdot \dot{\theta}^2}{[M_c + M_p[1-\cos^2\theta]]} \cdot \frac{\cos\theta}{L} + \frac{g\sin\theta}{L}$$

$$\frac{F_x\cos\theta + \cancel{m_p g \sin\theta\cos^2\theta} - m_p l \sin\theta \cdot \dot{\theta}^2 \cos\theta + M_c g \sin\theta + M_p g \sin\theta - \cancel{M_p \cos^2\theta\, g\sin\theta}}{L[M_c + m_p[1-\cos^2\theta]]}$$

$$\boxed{\ddot{\theta} = \frac{F_x\cos\theta - M_p l \sin\theta \cdot \dot{\theta}^2 \cos\theta + (M_c + M_p) g \sin\theta}{L(M_c + M_p[1-\cos^2\theta])}}$$