Linearize Dynamic Equations - Cart Pole

$$\ddot{x} = \frac{-mL\sin\theta\dot{\theta}^2 + mg\sin\theta\cos\theta + F_x}{[M+m](1-\cos^2\theta)}$$

$$\ddot{\theta} = \frac{-mL\sin\theta\cos\theta\cdot\dot{\theta}^2 + (M+m)g\sin\theta - \cos\theta F_x}{([M+m]\sin^2\theta)L}$$

Establish state-space: $\underline{x} = [x\ \theta\ \dot{x}\ \dot{\theta}]^T$

EQB: $\underline{x}_{eq} = [0\ 0\ 0\ 0]^T$ ASSUME $\theta = 0$ ↑ (upright)

LINEARIZED FORM: $\dot{\underline{x}} = A\underline{x} + B\underline{u}$

$\dot{\underline{x}} = [\dot{x}\ \dot{\theta}\ \ddot{x}\ \ddot{\theta}]^T$; $A = \left.\frac{\partial \dot{\underline{x}}}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_{eq}}$

$B = \left.\frac{\partial \dot{\underline{x}}}{\partial F_x}\right|_{\underline{x}=\underline{x}_{eq}}$

$$A = \begin{bmatrix} \frac{\partial}{\partial x}(\dot{x}) & \frac{\partial}{\partial\theta}(\dot{x}) & \frac{\partial}{\partial\dot{x}}(\dot{x}) & \frac{\partial}{\partial\dot{\theta}}(\dot{x}) \\ \frac{\partial}{\partial x}(\dot{\theta}) & \frac{\partial}{\partial\theta}(\dot{\theta}) & \frac{\partial}{\partial\dot{x}}(\dot{\theta}) & \frac{\partial}{\partial\dot{\theta}}(\dot{\theta}) \\ \frac{\partial}{\partial x}(\ddot{x}) & \frac{\partial}{\partial\theta}(\ddot{x}) & \frac{\partial}{\partial\dot{x}}(\ddot{x}) & \frac{\partial}{\partial\dot{\theta}}(\ddot{x}) \\ \frac{\partial}{\partial x}(\ddot{\theta}) & \frac{\partial}{\partial\theta}(\ddot{\theta}) & \frac{\partial}{\partial\dot{x}}(\ddot{\theta}) & \frac{\partial}{\partial\dot{\theta}}(\ddot{\theta}) \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(M+m)g}{ML} & 0 & 0 \end{bmatrix}$$

CA $A(4,2) = \frac{(M+m)g}{ML}$

$$B = \frac{d\dot{x}}{dF_x} = \begin{bmatrix} \frac{d\dot{x}}{dF_x} = 0 \\ \frac{d\dot{\theta}}{dF_x} = 0 \\ \frac{d}{dF_x}(\ddot{x}) = 1/m \\ d/dF_x(\ddot{\theta}) = +1/mL \end{bmatrix}$$

$$B = [0 \quad 0 \quad 1/m \quad 1/mL]^T$$