Cart Pole Dynamic Equations

$$\ddot{x} = \frac{-mL\sin\theta\dot{\theta}^2 + mg\sin\theta\cos\theta + F_x}{[M+m]\sin^2\theta}$$

$$\ddot{\theta} = \frac{-mL\sin\theta\cos\theta\cdot\dot{\theta}^2 + (M+m)g\sin\theta - \cos\theta F_x}{([M+m]\sin^2\theta)L}$$

TO CONTROL → LINEARIZE EQUATIONS

$$\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \underline{x} \; ; \; \underline{x}_{eq} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \; \dot{\underline{x}} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix}$$

LINEARIZED EQUATIONS

$$\dot{\underline{x}} = A\underline{x} + B\nu$$

$$A = \frac{d\dot{\underline{x}}}{d\underline{x}}\Big|_{\underline{x}=x_{eq}}, \quad B = \frac{d\dot{\underline{x}}}{d\nu}\Big|_{\underline{x}=x_{eq}}$$

$$A = \begin{bmatrix} \partial/\partial x(\dot{x}) & \frac{\partial}{\partial\theta}(\dot{x}) & \frac{\partial}{\partial\dot{x}}(\dot{x}) & \partial/\partial\dot{\theta}(\dot{x}) \\ \partial/\partial x(\dot{\theta}) & \frac{\partial}{\partial\theta}(\dot{\theta}) & \partial/\partial\dot{x}(\dot{\theta}) & \partial/\partial\dot{\theta}(\dot{\theta}) \\ \partial/\partial x(\ddot{x}) & \frac{\partial}{\partial\theta}(\ddot{x}) & \partial/\partial\dot{x}(\ddot{x}) & \partial/\partial\dot{\theta}(\ddot{x}) \\ \partial/\partial x(\ddot{\theta}) & \partial/\partial\theta(\ddot{\theta}) & \partial/\partial\dot{x}(\ddot{\theta}) & \frac{\partial}{\partial\dot{\theta}}(\ddot{\theta}) \end{bmatrix}$$

SCRATCH COMPUTATIONS

$\frac{d}{dx}(\dot{x}) = 0$, $\frac{d}{d\theta}(\dot{x}) = 0$, $\frac{d}{d\dot{x}}(\dot{x}) = 1$, $\frac{d}{d\dot{\theta}}(\dot{x}) = 0$

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$\frac{d}{dx}(\ddot{x}) = 0$, ~~$\frac{d}{d\theta}(\ddot{x}) = \ldots = 0$~~ $-mg/M \Big|_{x = x_{eq}}$, $\frac{d}{d\dot{x}}(\ddot{x}) = 0$, $\frac{d}{d\dot{\theta}}(\ddot{x})\Big|_{x = x_{eq}} = 2\sin(\theta) = 0$

$$\frac{d}{d\theta}(\ddot{x})\Big|_{x = x_{eq}} = -mg/L$$

$d/dx(\ddot{\theta}) = 0$, $d/d\theta(\ddot{\theta})\Big|_{x = x_e} = \frac{(M+m)g}{LM}$, $\frac{d}{d\dot{x}}(\ddot{\theta}) = 0$, $\frac{d}{d\dot{\theta}}[\ddot{\theta}] = 0$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -mg/L & 0 & 0 \\ 0 & \frac{(M+m)g}{LM} & 0 & 0 \end{bmatrix}$$

NOTE : $A(3,2) = -mg/L$

$$B = \frac{d\dot{x}}{dF_x} = \begin{bmatrix} 0 = d\dot{x}/dF_x \\ 0 = d\dot{\theta}/dF_x \\ d/dF_x(\ddot{x})\Big|_{x = x_{eq}} = \frac{1}{(M)+(m)(\sin^2\theta)} = 1/M \\ d/dF_x(\ddot{\theta})\Big|_{x = x_{eq}} = 1/[M + m\sin^2\theta]L = 1/-ML \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 1/ML \end{bmatrix}$$

$\therefore$ A & B matrix have been defined.

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$