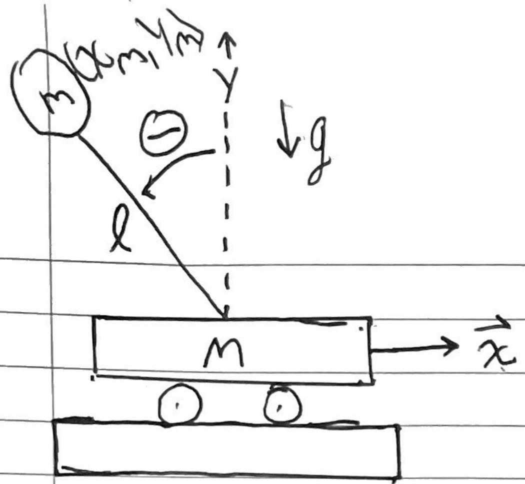

Kinematics

$X_m = X - l\sin\theta$ ①

$Y_m = l\cos\theta$ ②

$X_m = \dot{X} - l\dot{\theta}\cos\theta$ ③

$\dot{Y}_m = -l\dot{\theta}\sin\theta$ ④

TO APPLY LAGRANGE'S EQUATIONS,
DERIVE POTENTIAL & KINETIC ENERGY

POTENTIAL ENERGY

$V = mgY_m = mgl\cos\theta$ ⑤

KINETIC ENERGY

$$T = \tfrac{1}{2}M\dot{X}^2 + \tfrac{1}{2}m(\dot{x}_m^2 + \dot{Y}_m^2)$$

$$= \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}m[\dot{x}^2 - 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2\overbrace{(\cos^2\theta + \sin^2\theta)}^{1}]$$

$$T = \tfrac{1}{2}(M+m)\dot{x}^2 + \tfrac{1}{2}ml^2\dot{\theta}^2 - ml\dot{\theta}\dot{x}\cos\theta \quad ⑥$$

LAGRANGE'S EQUATIONS

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad ⑦$$

$$L = T - V \quad ⑧$$

Substitute ⑤, ⑥ into ⑧

$$L = \underbrace{\tfrac{1}{2}(M+m)\dot{x}^2 + \tfrac{1}{2}ml^2\dot{\theta}^2 - ml\dot{\theta}\dot{x}\cos\theta}_{T} - \underbrace{mgl\cos\theta}_{V} \quad ⑨$$

EQUATIONS OF MOTION

x/ Substitute (9) → (7) where q is x

$$x: (M+m)\ddot{x} - (ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta) = F(t)$$

$$\boxed{F(t) = (M+m)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta \quad (10)}$$

θ/ substitute (9) → (7) where q is θ

$$\theta: ml^2\ddot{\theta} - ml\ddot{x} + ml\dot{x}\cancel{\phi}\sin\theta - ml\dot{\theta}\dot{x}\sin\theta - mg\cancel{l}\sin\theta = 0$$

Divide by ml:

$$\boxed{l\ddot{\theta} - \ddot{x}\cos\theta - g\sin\theta = 0 \quad (11)}$$

NOTE, θ'S EQN EQUALS ZERO B/C THERE IS NO EXTERNAL LOAD