

# Answers: Rearranging equations involving trigonometry and logarithms

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## Summary

This is an answer set relating to the questions based on Guide, Introduction to rearranging equations involving trigonometry and logarithms.

*These are the answers to [Questions: Introduction to rearranging equations using trigonometry and logarithms](#)*

**Please attempt the questions before reading these answers!**

## Q1

Solve the trigonometric equations in radians.

1.1 For  $\sin(x) = \frac{\sqrt{2}}{2}$ ,  $x$  is equal to  $\frac{\pi}{2}$  or 1.57.

1.2 For  $\cos(2x + 1) = \frac{1}{2}$ ,  $x$  is equal to  $\frac{\pi-3}{6}$  or 0.0234.

1.3 For  $\tan(5x - 1) = \frac{\sqrt{2}}{2}$ ,  $x$  is equal to 0.323.

1.4 For  $\cos(x^2 + 4x + 3) = 1$ ,  $x$  is equal to -1 or -3. To do this, you use that  $\cos^{-1}(1) = 0$  and so you need to solve the quadratic equation  $x^2 + 4x + 3 = 0$ .

## Q2

Rewrite  $\cot$  and  $\csc$  in terms of  $\sin$ ,  $\cos$ , and  $\tan$

$$1 + \frac{1}{\tan^2(x)} = \frac{1}{\sin^2(x)}$$

$$1 + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}$$

Then, multiply both sides of the equation by  $\sin^2(x)$

$$\sin^2(x) + \cos^2(x) = 1.$$

### Q3

Rewriting  $5 \cos(x) + 9 \sin(x)$  gives  $\sqrt{106} \sin(x + 0.507)$ . Setting this equal to 10 and solving gives  $x = 0.823$ . If you have a slightly different answer, this may be due to rounding at different points in the process.

### Q4

4.1  $a = 6, b = 36, c = 2$ .

4.2  $a = 3, b = 2187, c = 2187$ .

4.3  $a = e, b = y, c = x$ .

4.4  $a = 2, b = 9, c = 3.17\dots$

4.5  $a = 2, b = 4, c = 2$ .

### Q5

5.1 The solution to  $6 \log_3(x) + \log_3(5) = 9$  is  $x = \sqrt[6]{\frac{3^9}{5}}$ , or approximately 3.97.

5.2 The solution to  $\log_2(16x) = 6$  is  $x = 4$ .

5.3 If  $e^{\ln(3x)} = y$ , then  $y = 3x$ .

### Q6

Firstly, substitute  $y$  into the first equation. This gives  $2^{\log_2(x)} = 4x - 7$ . Via example 7, you can see that this means  $x = 4x - 7$ . Rearranging this gives  $x = \frac{7}{3}$  or approximately 2.33. Plugging this into the second equation gives  $y = \log_2(\frac{7}{3})$  or approximately 1.22.

### Q7

7.1 If  $e^{-x} + 3e^x = 12$ , then multiply everything by  $e^x$  and define  $y$  such that  $e^x = y$ . This makes  $1 + 3y^2 = 12y$  and solving this gives  $y = \frac{6 \pm \sqrt{33}}{3}$ . Then,  $\ln(y) = x = 1.36$  or  $-2.46$ .

7.2 Using the same method detailed above  $y = \frac{9 \pm \sqrt{65}}{8}$  and  $x = 0.757$  or  $-2.144$ .

### Q8

8.1 If  $\log_{16}(x) = \log_2(y)$ , then  $y = x^{\frac{1}{4}}$ .

8.2 If  $\log_3(x) = \log_{27}(y)$  , then  $y = x^3$ .

8.3 If  $\log_9(x) + \log_3(2x) = 6$ , then  $\log_9(x) = \log_3(x^{\frac{1}{2}})$ . Substituting gives  $\log_3(2x^{\frac{3}{2}}) = 6$ , thus  $3^6 = 2x^{\frac{3}{2}}$ . This means that  $x = 51.0$ .