# The Scalar Product: Solutions

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**Summary**

Answers to questions relating to the guide on the Scalar Product.

*These are the answers to The Scalar Product: Questions.* **Please attempt the questions before reading these answers!**

## Answers

### Q1

Find the scalar product of $\mathbf{u}$ and $\mathbf{v}$.

1.1. For $\mathbf{u} = \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$, the scalar product is 26.

1.2. For $\mathbf{u} = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ -5 \\ 13 \end{pmatrix}$, the scalar product is 117.

1.3. For $\mathbf{u} = \begin{pmatrix} -4.4 \\ -1.2 \\ 0.3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 6.1 \\ -2.5 \\ 9.3 \end{pmatrix}$, the scalar product is -22.37.

1.4. For $\mathbf{u} = \begin{pmatrix} 54 \\ 38 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 32 \\ -55 \\ 13 \end{pmatrix}$, the scalar product is -362.

1.5. For $\mathbf{u} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$, the scalar product is 48.

1.6. For $\mathbf{u} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$, the scalar product is -195.

1.7. For $\mathbf{u} = 17\mathbf{j} + 23\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$, the scalar product is -575.

1.8. For $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = \mathbf{j}$, the scalar product is 0.

As the scalar product of $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = \mathbf{j}$ is 0, they are perpendicular to each other. This is true for any combination of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

## Q2

Find the value(s) of $\lambda$ for which **u** and **v** are perpendicular.

2.1. For $\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix}$, $\lambda$= 3.

2.2. For $\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\lambda$= -$\frac{2}{3}$.

2.3. For $\mathbf{u} = \begin{pmatrix} 9 \\ -2 \\ 11 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ -\lambda \\ 3 \end{pmatrix}$, $\lambda$= -3.

2.4. For $\mathbf{u} = \begin{pmatrix} \lambda \\ 6 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ \lambda \\ 8 \end{pmatrix}$, $\lambda$= -2 or $\lambda$= -4.

2.5. For $\mathbf{u} = \begin{pmatrix} -2\lambda^2 \\ 4 \\ 14 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 2\lambda \\ 1 \end{pmatrix}$, $\lambda$= $\frac{7}{3}$ or $\lambda$= -1.

2.6. For $\mathbf{u} = \begin{pmatrix} -5 \\ 9 \\ 2\lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ -2 \\ \lambda \end{pmatrix}$, $\lambda$= $\frac{9}{2}$ or $\lambda$= -2.

2.7. For $\mathbf{u} = \begin{pmatrix} -7 \\ 4 \\ 2\lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2\lambda \\ 1 \\ 6\lambda \end{pmatrix}$, $\lambda$= $\frac{2}{3}$ or $\lambda$= $\frac{1}{2}$.

2.8. For $\mathbf{u} = \begin{pmatrix} -25 \\ -1\lambda^2 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3\lambda \\ -11 \\ 7 \end{pmatrix}$, $\lambda$= 7 or $\lambda$= -$\frac{2}{11}$.

## Q3

Find the angle $\theta$ in between **u** and **v** .

3.1. For $\mathbf{u} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix}$, $\theta = 132.2°$.

3.2. For $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\theta = 70.5°$.

3.3. For $\mathbf{u} = \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$, $\theta = 108.7°$.

3.4. For $\mathbf{u} = \begin{pmatrix} 1.2 \\ -1.4 \\ -3.1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -5.4 \\ 9.7 \\ -7.5 \end{pmatrix}$, $\theta = 86.2°$.

3.5. For $\mathbf{u} = \begin{pmatrix} 45 \\ 65 \\ 54 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -19 \\ -58 \\ 71 \end{pmatrix}$, $\theta = 95.1°$.

3.6. For $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\theta = 90.0°$.

3.7. For $\mathbf{u} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$, $\theta = 43.0°$.

3.8. For $\mathbf{u} = \begin{pmatrix} -17 \\ 3 \\ 8 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 12 \\ -19 \\ -16 \end{pmatrix}$, $\theta = 137.8°$.