Introduction to vectors: answers

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Summary

Answers to questions relating to the guide on introduction to vectors

These are the answers to Questions: Introduction to vectors. **Please attempt the questions before reading these answers!**

Q1

Are these vectors parallel?

1.1. $\overrightarrow{CD} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ so $\overrightarrow{CD}$ is a multiple of $\overrightarrow{AB}$ so they are parallel to each other.

1.2. You can use factorisation to get $\overrightarrow{EF} = \begin{pmatrix} 6x \\ -3y \end{pmatrix} = 3 \begin{pmatrix} 2x \\ -y \end{pmatrix}$ thus showing that it is a multiple and therefore parallel.

1.3. You can show that $\overrightarrow{BC} = \begin{pmatrix} 8 \\ -4 \end{pmatrix} = 4 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ therefore being a multiple, so they are parallel.

1.4. Since $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ is not a multiple of $\begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix}$, so $\overrightarrow{DE}$ is not parallel to $\overrightarrow{BC}$.

1.5. $\overrightarrow{MO} = \begin{pmatrix} 6 \\ 8 \\ 10 \end{pmatrix} = -2 \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix}$ so they are parallel.

Q2

Find the magnitude of the following vectors

2.1. $|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

2.2. $|\mathbf{b}| = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$

2.3. $|\mathbf{c}| = \sqrt{1^2 + (-1)^2 + 4^2} = \sqrt{1 + 1 + 16} = \sqrt{18} = 3\sqrt{2}$

2.4. $|\mathbf{d}| = \sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{25 + 4 + 1} = \sqrt{30}$

2.5. $|\mathbf{e}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$

2.6. $|\mathbf{f}| = \sqrt{(-3)^2 + 6^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

2.7. $|\mathbf{g}| = \sqrt{5^2 + 1^2 + (\sqrt{2})^2} = \sqrt{25 + 1 + 2} = \sqrt{28} = 2\sqrt{7}$

2.8. $|\mathbf{h}| = \sqrt{6^2 + 2^2 + 2^2} = \sqrt{36 + 4 + 4} = \sqrt{44} = 2\sqrt{11}$

2.9. $|\mathbf{m}| = \sqrt{(-3)^2 + 3^2 + (-3)^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$

2.10. $|\mathbf{n}| = \sqrt{2^2 + 4^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

2.11. $|\mathbf{p}| = \sqrt{8^2 + (-2)^2 + 16^2} = \sqrt{64 + 4 + 256} = \sqrt{324} = 18$

2.12. $|\mathbf{q}| = \sqrt{5^2 + (-2)^2 + 14^2} = \sqrt{25 + 4 + 196} = \sqrt{225} = 15$

2.13. $|\mathbf{u}| = \sqrt{7^2 + 2^2 + (-1)^2} = \sqrt{49 + 4 + 1} = \sqrt{54} = 3\sqrt{6}$

2.14. $|\mathbf{v}| = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$

Q3

Find the unit vectors for the following vectors

3.1. Find the magnitude of the vector first, $|\mathbf{a}| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$. Then $\hat{\mathbf{a}} = \frac{-2\mathbf{i}+3\mathbf{j}}{\sqrt{13}} = \frac{-2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$

3.2. Find the magnitude of the vector first, $|\mathbf{b}| = \sqrt{(-2)^2 + 4^2 + (-6)^2} = \sqrt{56} = 2\sqrt{14}$. Then$\hat{\mathbf{b}} = \frac{-2\mathbf{i}+4\mathbf{j}-6\mathbf{k}}{2\sqrt{14}} = \frac{-1}{\sqrt{14}}\mathbf{i} + \frac{2}{\sqrt{14}}\mathbf{j} - \frac{3}{\sqrt{14}}\mathbf{k}$

3.3. Find the magnitude of the vector first, $|\mathbf{c}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$. Then $\hat{\mathbf{c}} = \frac{\mathbf{i}+2\mathbf{j}+4\mathbf{k}}{\sqrt{21}} = \frac{1}{\sqrt{21}}\mathbf{i} + \frac{2}{\sqrt{21}}\mathbf{j} + \frac{4}{\sqrt{21}}\mathbf{k}$

3.4. Find the magnitude of the vector first, $|\mathbf{d}| = \sqrt{4^2 + (-2)^2 + 3^2} = \sqrt{29}$. Then $\hat{\mathbf{d}} = \frac{4\mathbf{i}-2\mathbf{j}+3\mathbf{k}}{\sqrt{29}} = \frac{4}{\sqrt{29}}\mathbf{i} - \frac{2}{\sqrt{29}}\mathbf{j} + \frac{3}{\sqrt{29}}\mathbf{k}$

3.5. Find the magnitude of the vector first, $|\mathbf{e}| = \sqrt{3^2 + 2^2} = \sqrt{13}$. Then $\hat{\mathbf{e}} = \frac{3\mathbf{i}+2\mathbf{k}}{\sqrt{13}} = \frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{k}$

3.6. Find the magnitude of the vector first, $|\mathbf{f}| = \sqrt{(-3)^2 + 1^2 + 7^2} = \sqrt{59}$. Then $\hat{\mathbf{f}} = \frac{-3\mathbf{i}+\mathbf{j}+7\mathbf{k}}{\sqrt{59}} = -\frac{3}{\sqrt{59}}\mathbf{i} + \frac{1}{\sqrt{59}}\mathbf{j} + \frac{7}{\sqrt{59}}\mathbf{k}$

3.7. Find the magnitude of the vector first, $|\mathbf{g}| = \sqrt{(-5)^2 + (\sqrt{2})^2} = \sqrt{27} = 3\sqrt{3}$. Then $\hat{\mathbf{g}} = \frac{-5\mathbf{i}+\sqrt{2}\mathbf{k}}{3\sqrt{3}} = -\frac{5}{3\sqrt{3}}\mathbf{i} + \frac{\sqrt{2}}{3\sqrt{3}}\mathbf{k}$

3.8. Find the magnitude of the vector first, $|\mathbf{h}| = \sqrt{(-3)^2 + 1^2 + 1^2} = \sqrt{11}$. Then

$\hat{\mathbf{h}} = \frac{-3\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{11}} = \frac{-3}{\sqrt{11}}\mathbf{i} + \frac{1}{\sqrt{11}}\mathbf{j} + \frac{1}{\sqrt{11}}\mathbf{k}$

3.9. Find the magnitude of the vector first, $|\mathbf{m}| = \sqrt{(-3)^2 + 3^2 + (-3)^2} = \sqrt{27} = 3\sqrt{3}$. Then $\hat{\mathbf{m}} = \frac{-3\mathbf{i}+3\mathbf{j}-3\mathbf{k}}{3\sqrt{3}} = -\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}$

3.10. Find the magnitude of the vector first, $|\mathbf{n}| = \sqrt{3^2 + 6^2 + 9^2} = \sqrt{126} = 3\sqrt{14}$. Then $\hat{\mathbf{n}} = \frac{3\mathbf{i}+6\mathbf{j}+9\mathbf{k}}{3\sqrt{14}} = \frac{1}{\sqrt{14}}\mathbf{i} + \frac{2}{\sqrt{14}}\mathbf{j} + \frac{3}{\sqrt{14}}\mathbf{k}$

3.11. Find the magnitude of the vector first, $|\mathbf{p}| = \sqrt{3^2 + (-4)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$. Then $\hat{\mathbf{p}} = \frac{3\mathbf{i}-4\mathbf{j}-5\mathbf{k}}{5\sqrt{2}} = \frac{3}{5\sqrt{2}}\mathbf{i} - \frac{4}{5\sqrt{2}}\mathbf{j} - \frac{1}{\sqrt{2}}\mathbf{k}$

3.12. Find the magnitude of the vector first, $|\mathbf{q}| = \sqrt{4^2 + (-3)^2 + 12^2} = \sqrt{169} = 13$. Then $\hat{\mathbf{q}} = \frac{4\mathbf{i}-3\mathbf{j}+12\mathbf{k}}{13} = \frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}$

3.13. Find the magnitude of the vector first, $|\mathbf{u}| = \sqrt{6^2 + 5^2 + 4^2} = \sqrt{77} =$. Then $\hat{\mathbf{u}} = \frac{6\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{\sqrt{77}} = \frac{6}{\sqrt{77}}\mathbf{i} + \frac{5}{\sqrt{77}}\mathbf{j} + \frac{4}{\sqrt{77}}\mathbf{k}$

3.14. Find the magnitude of the vector first, $|\mathbf{v}| = \sqrt{2^2 + 4^2 + 8^2} = \sqrt{84} = 2\sqrt{21}$. Then $\hat{\mathbf{v}} = \frac{2\mathbf{i}+4\mathbf{j}+8\mathbf{k}}{2\sqrt{21}} = \frac{1}{\sqrt{21}}\mathbf{i} + \frac{2}{\sqrt{21}}\mathbf{j} + \frac{4}{\sqrt{21}}\mathbf{k}$