Proof: properties of sigma notation

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Summary

An explanation as to why some of the properties of sigma notation are true.

Before reading this proof sheet, it is recommended that you read Guide: Introduction to sigma notation and [Guide: Proof by induction].

Proof of properties of sigma notation

Distributivity

Distributivity

Hello

Proof of distributivity

You can see this is true by writing the entire sum out, like this:

$$\begin{aligned} \sum_{i=k}^{n} Ca_i &= Ca_k + Ca_{k+1} + Ca_{k+2} + ... + Ca_n \\ &= C(a_k + a_{k+1} + a_{k+2} + ... + a_n) \\ &= C\sum_{i=k}^{n} a_k \end{aligned}$$

Combining and decomposing sums

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Hello

Proof of combining and decomposing sums

Similar to the distributive property, you can show this is true by writing the entire sum out:

$$\begin{aligned}\sum_{i=k}^{n} a_i + \sum_{i=k}^{n} b_i &= (a_k + a_{k+1} + ... + a_n) + (b_k + b_{k+1} + ... + b_n) \\ &= (a_k + b_k) + (a_{k+1} + b_{k+1}) + ... + (a_n + b_n) \\ &= \sum_{i=k}^{n} (a_k + b_k).\end{aligned}$$

In a similar way, you can show that $\sum_{i=k}^{n} a_i - \sum_{i=k}^{n} b_i = \sum_{i=k}^{n} (a_i - b_i)$ is also true.

Further reading

Guide: Using the quadratic formula

Questions: Using the quadratic formula

Version history

v1.0: created in 04/24 by tdhc.