Addition and scalar multiplication: answers

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Summary

Answers to questions relating to the guide on addition and scalar multiplication.

These are the answers to Questions: Addition and scalar multiplication. **Please attempt the questions before reading these answers!**

Q1

Solve the following questions.

1.1. For the $\mathbf{i}$ component, $4+8=12$.For the $\mathbf{j}$ component, $5+2=7$.For the $\mathbf{k}$ component, $7+4=11$. So the answer is $12\mathbf{i}+7\mathbf{j}+11\mathbf{k}$.

1.2. $\mathbf{a}+\mathbf{b}=2\mathbf{i}+3\mathbf{j}+9\mathbf{k}$.

1.3. $\mathbf{a}-\mathbf{b}=2\mathbf{i}-11\mathbf{j}+14\mathbf{k}$.

1.4. You can solve this by doing scalar addition component-wise. $\mathbf{i}$th component: $4-(3+11)=-10$, $\mathbf{j}$th component: $12-(-3-4)=19$, $\mathbf{k}$th component: $-7-(-2+9)=-14$. So the answer is $-10\mathbf{i}+19\mathbf{j}-14\mathbf{k}$.

Q2

Solve the following questions.

2.1. $\mathbf{a}+\mathbf{b}=\begin{pmatrix}4\alpha\\7\beta\end{pmatrix}$

2.2. $\mathbf{a}-\mathbf{b}=\begin{pmatrix}7\\3\beta-2\alpha\\-\gamma\end{pmatrix}$

2.3. $\mathbf{a}+\mathbf{b}-\mathbf{c}=\mathbf{0}$ or $\begin{pmatrix}0\\0\\0\end{pmatrix}$. This is different from the scalar 0.

2.4. This question is erroneous. You cannot add $\mathbf{a}$ to a scalar 0.

Q3

3.1. $3\mathbf{u} = (3 \times 5\mathbf{j}) + (3 \times 6\mathbf{k}) = 15\mathbf{j} + 18\mathbf{k}$.

3.2. $\begin{pmatrix} 0 \\ 18 \\ -42 \end{pmatrix}$.

3.3. $\begin{pmatrix} 0 \\ -27 \\ 10 \end{pmatrix}$

3.4. $\begin{pmatrix} -4 \\ -32 \\ -2 \end{pmatrix}$

Q4

4.1. By the laws of vector addition, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the respective coordinates of A and B written in vector form. We can solve for $\overrightarrow{AB}$ by solving the above equation. $\overrightarrow{AB} = \begin{pmatrix} -2-3 \\ 5-4 \\ 7-5 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix}$

4.2.$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -4 \\ -5 \end{pmatrix}$. $\overrightarrow{AB} - \overrightarrow{AC} = \begin{pmatrix} 6 \\ 10 \\ 5 \end{pmatrix}$. You can also calculate this by noticing $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$. Then $\overrightarrow{CB} = \begin{pmatrix} 6-0 \\ 11-1 \\ 7-2 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ 5 \end{pmatrix}$ as required

4.3. Let λ be a real scalar. $\overrightarrow{AB} = \lambda\overrightarrow{BC}$. $\overrightarrow{AB} = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 4k-12 \\ 3k-4 \end{pmatrix}$. This gives you the simultaneous equations $\begin{cases} 10 = \lambda(4k-12) \\ -5 = \lambda(3k-4) \end{cases}$. Solving this gives $k = 2$.

4.4. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. $\begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} - \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \\ -2 \end{pmatrix}$. Solving this gives $A = (-5, -2, 11)$.

4.5. Let λ and μ be a real scalar. $\lambda\mathbf{a} + \mu\mathbf{b} = 13\mathbf{i} - 9\mathbf{j}$. This gives you the simultaneous equations $\begin{cases} 2\lambda + 3\mu = 13 \\ 3\lambda - 5\mu = -9 \end{cases}$. Solving this gives $\mu = 3$, $\lambda = 2$. Which gives the answer $2\mathbf{a} + 3\mathbf{b}$.

4.6. $2\begin{pmatrix} 2 \\ 5 \\ \gamma \end{pmatrix} + 3\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ 0 \end{pmatrix}$. Solving this gives $\alpha = 3$,$\beta = 1$ and $\gamma = -6$.

4.7. Let λ be a real scalar. $\mathbf{a} = \lambda\mathbf{b}$. This gives the simultaneous equations $\begin{cases} k - 7 = -2\lambda \\ 5k + 1 = 8\lambda \end{cases}$. Solving this gives $k = 3$.

4.8. This gives the simultaneous equations $\begin{cases} 5\alpha + 5 = 2\alpha - 2 \\ 3 - \beta = 3\beta + 8 \\ 7 - 2 = \gamma + 12 \\ 1 + 5 = 2\delta + 2\delta \end{cases}$. Solving this gives $\alpha = -\frac{7}{3}$,$\beta = -\frac{5}{4}$,$\gamma = -7$ and $\delta = \frac{3}{2}$.