Introduction to quadratic equations: answers

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Summary

Answers to questions relating to the guide on introduction to quadratic equations.

These are the answers to Questions: Introduction to quadratic equations. **Please attempt the questions before reading these answers!**

Q1

For each of the quadratic equations below, identify the variable and the coefficients a, b, c.

1.1. For $x^2 - 7x + 6 = 0$, the variable is x, and the coefficients are $a = 1, b = -7, c = 6$.

1.2. For $y^2 + 14y + 49 = 0$, the variable is y, and the coefficients are $a = 1, b = 14, c = 49$.

1.3. For $h^2 - h - 56 = 0$, the variable is h, and the coefficients are $a = 1, b = -1, c = -56$.

1.4. For $7y^4 - y^2 = 0$, the variable is y^2, and the coefficients are $a = 7, b = -1, c = 0$.

1.5. For $5n^2 - 14n + 100 = 0$, the variable is n, and the coefficients are $a = 5, b = -14, c = 100$.

1.6. For $A^2 - 144 = 0$, the variable is A, and the coefficients are $a = 1, b = 0, c = -144$.

1.7. For $25M^2 = 0$, the variable is M, and the coefficients are $a = 25, b = 0, c = 0$.

1.8. For $e^{2x} - 4e^x + 4 = 0$, the variable is e^x, and the coefficients are $a = 1, b = -4, c = 4$.

1.9. For $-9s^4 + 3s^2 - 1 = 0$, the variable is s^2, and the coefficients are $a = -9, b = 3, c = -1$.

1.10. For $2e^{6x} + e^{3x} + 1 = 0$, the variable is e^{3x}, and the coefficients are $a = 2, b = 1, c = 1$.

1.11. For $\cos^2(x) + 4\cos(x) - 4 = 0$, the variable is $\cos(x)$, and the coefficients are $a = 1, b = 4, c = -4$.

1.12. For $8x^8 - 4x^4 - 1 = 0$, the variable is x^4, and the coefficients are $a = 8, b = -4, c = -1$.

Q2

2.1. The discriminant of the equation $x^2 - 7x + 6 = 0$ is $D = 25$, and therefore the equation has two distinct real roots.

2.2. The discriminant of the equation $y^2 + 14y + 49 = 0$ is $D = 0$, and therefore the equation has one distinct real root.

2.3. The discriminant of the equation $h^2 - h - 56 = 0$ is $D = 217$, and therefore the equation has two distinct real roots.

2.4. The discriminant of the equation $7y^4 - y^2 = 0$ is $D = 1$, and therefore the equation has two distinct real roots.

2.5. The discriminant of the equation $5n^2 - 14n + 100 = 0$ is $D = -1804$, and therefore the equation has no real roots (two distinct complex roots).

2.6. The discriminant of the equation $A^2 - 144 = 0$ is $D = 576$, and therefore the equation has two distinct real roots.

2.7. The discriminant of the equation $25M^2 = 0$ is $D = 0$, and therefore the equation has one distinct real root.

2.8. The discriminant of the equation $e^{2x} - 4e^x + 4 = 0$ is $D = 0$, and therefore the equation has one distinct real root r in e^x. Whether or not it has a real root in x depends on whether or not r is positive. If r is positive, there is exactly one real root $x = \ln(r)$; if r is negative, then there are no real roots.

2.9. The discriminant of the equation $-9s^4 + 3s^2 - 1 = 0$ is $D = -27$, and therefore the equation has no real roots. This is true even with s^2 as the variable, as if s^2 is complex then s must also be complex.

2.10. The discriminant of the equation $2e^{6x} + e^{3x} + 1 = 0$ is $D = -7$, and therefore the equation has no real roots. This is true even with e^{3x} as the variable, as if e^{3x} is complex then x must also be complex.

2.11. The discriminant of the equation $\cos^2(x) + 4\cos(x) - 4 = 0$ is $D = 32$, and therefore the equation has two distinct real roots r_1 and r_2 in $\cos(x)$. Whether or not it has a real root in x depends on whether or not either of the roots is between -1 and 1. If both r_1 and r_2 are outside this range, then there are no real roots. If one of r_1 or r_2 is between -1 and 1, then there are infinitely many solutions.

2.12. The discriminant of the equation $8x^8 - 4x^4 - 1 = 0$ is $D = 48$, and therefore the equation has two distinct real roots r_1 and r_2 in x^4. The amount of real roots depend on the signs of r_1 and r_2.

- If r_1 and r_2 are both positive, then there are four real roots in x. This is because

$x^2 = \pm\sqrt{r_1}$ or $x^2 = \pm\sqrt{r_2}$; square rooting the positive terms gives the roots in x as $\pm\sqrt{(\sqrt{r_1})} = \pm\sqrt[4]{r_1}$ and $\pm\sqrt{(\sqrt{r_2})} = \pm\sqrt[4]{r_2}$. Any other roots must be complex, since you are taking square roots of the negative numbers $-\sqrt{r_1}$ and $-\sqrt{r_2}$.

- If exactly one of r_1 and r_2 is positive (say r_i), then there are two real roots in x given by $\pm\sqrt[4]{r_i}$. All other roots are complex.
- If both r_1 and r_2 are negative, then then there are no real roots in x.

Q3

3.1. Rearranging gives $x^2 + x - 1 = 0$. The discriminant of this is $D = 5$, and therefore the equation has two distinct real roots.

3.2. Rearranging gives $y^2 + 10 = 0$. The discriminant of this is $D = -40$, and therefore the equation has no real roots (two distinct complex roots).

3.3. Rearranging gives $4m^2 + 4m + 1 = 0$. The discriminant of this is $D = 0$, and therefore the equation has one distinct real root.

3.4. Rearranging gives $t^4 + 1 = 0$. The discriminant of this is $D = -4$, and therefore the equation has no real roots. This is true even with t^2 as the variable, as if t^2 is complex then t must also be complex.

3.5. Rearranging gives $5x^2 - 11x - 1 = 0$. The discriminant of this is $D = 101$, and therefore the equation has two distinct real roots.

3.6. Rearranging gives $e^{2x} - 2e^x + 1 = 0$. The discriminant of this is $D = 0$, and therefore the equation has one distinct real root r in e^x. Whether or not it has a real root in x depends on whether or not r is positive. If r is positive, there is exactly one real root $x = \ln(r)$; if r is negative, then there are no real roots.