Introduction to Logarithms: Answers

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Summary

Answers to questions relating to the study guide on Logarithms.

These are answers to: Questions: Introduction to Logarithms. **Please attempt the questions before reading these answers!**

Q1

For the following, find the value of x if x is a rational number:

1.1. $\log_7(x) = 1$ rearranged gives $7^1 = x$ so $x = 7$

1.2. $\log_8(x) = 3$ rearranged gives $8^3 = x$ so $x = 512$

1.3. $\log_{12}(x) = 0$ rearranged gives $12^0 = x$ so $x = 1$

1.4. $\log_{10}(100) = x$ rearranged gives $10^x = 100$ so $x = 2$

1.5. $\log_2(17) = x$ rearranged gives $2^x = 17$ but 17 is odd so this would not result in a rational number for x

1.6. $\log_4(2) = x$ rearranged gives $4^x = 2$ so $x = \frac{1}{2}$

1.7. $\log_3(27) = x$ rearranged gives $3^x = 27$ so $x = 3$

1.8. $\log_{10}(1) = x$ rearranged gives $10^x = 1$ so $x = 0$

1.9. $\log_x(16) = 4$ rearranged gives $x^4 = 16$ so $x = \sqrt[4]{16} = 2$

1.10. $\log_x(49) = 2$ rearranged gives $x^2 = 49$ so $x = \sqrt{49} = 7$

1.11. $\log_x(13) = 4$ rearranged gives $x^4 = 13$ so $x = \sqrt[4]{13}$

1.12. $\log_{2x}(12) = -1$ rearranged gives $(2x)^{-1} = 12$ so $x = \dfrac{1}{24}$

Q2

Before attempting this question, write out the 5 laws next to their names:

1. The Product Rule: $\log_a(M \cdot N) = \log_a(M) + \log_a(N)$
2. The Quotient Rule: $\log_a\left(\dfrac{M}{N}\right) = \log_a(M) - \log_a(N)$
3. The Power Rule: $\log_a(M^k) = k \cdot \log_a(M)$
4. The Zero Rule: $\log_a(1) = 0$
5. The Identity Rule: $\log_a(a) = 1$

For the following, find the value of x :

2.1. $\log_3(\dfrac{1}{27}) = x$

$$\begin{aligned} x &= \log_3(1) - \log_3(27) && \text{(Using Law 2)} \\ &= 0 - \log_3(27) && \text{(Using Law 4)} \\ -x &= \log_3(27) \end{aligned}$$

Let $y = -x$, so Rearranging gives: $3^y = 27$, so $y = 3 = -x$ implying that $x = -3$

2.2. $\log_4(16) = x$

$$
\begin{aligned}
x &= \log_4(4^2) \\
&= 2 \cdot \log_4(4) && \text{(Using Law 3)} \\
&= 2 \cdot 1 && \text{(Using Law 5)} \\
&= 2
\end{aligned}
$$

2.3. $\log_7\left(\frac{2}{49}\right) = x$

$$
\begin{aligned}
x &= \log_7(2) - \log_7(49) && \text{(Using Law 2)} \\
&= \log_7(2) - 2 \\
x + 2 &= \log_3(27)
\end{aligned}
$$

Rearranging gives: $3^{x+2} = 27$, so $x + 2 = 3$ implying that $x = 1$

2.4. $\log_x(YZ) = M$
From question 1, $\log_x(YZ) = M$ rearranged gives $x^M = YZ$ so $x = \sqrt[M]{YZ}$

2.5. $\log_6\left(\frac{36}{x}\right) = 1$

$$
\begin{aligned}
1 &= \log_6(36) - \log_6(x) && \text{(Using Law 2)} \\
&= 2 - \log_6(x) \\
\log_6(x) &= 1 \\
6^1 &= x
\end{aligned}
$$

2.6. $\log_5(25) = x$

$$
\begin{aligned}
x &= \log_5(5 \cdot 5) && \text{(Using Law 1)} \\
&= \log_5(5) + \log_5(5) \\
&= 1 + 1 && \text{(Using Law 5)} \\
&= 2
\end{aligned}
$$

Q3

Convert the following logs to the given base and evaluate

3.1. $\log_3(25)$ to base 5

$$\log_3(25) = \frac{\log_5(25)}{\log_5(3)}$$

3.2. $\log_4(64)$ to base 16

$$\log_4(64) = \frac{\log_{16}(64)}{\log_{16}(4)}$$

3.3. $\log_e(100)$ to base 10

$$\log_e(100) = \frac{2}{\log_{10}(e)}$$

3.4. $\ln(27)$ to base 3

$$\ln(27) = \log_3(27)$$

3.5. $\log_4(8)$ to base 2

$$\log_4(8) = \frac{\log_2(8)}{\log_2(4)} = \frac{3}{2}$$

Q4

For the following, find the value of x:

4.1. $3^{x+1} = 7^x$

$$x = -\frac{\log(3)}{\log\left(\frac{7}{3}\right)}$$

4.2. $17^{2x} = 4^{x-1}$

$$x = \frac{\log(2)}{\log\left(\frac{2}{17}\right)}$$

4.3. $5^{x+1} + 5^x = 12$

$$x = \frac{\log(2)}{\log(5)}$$

4.4. $2^{3x-1} = 10^x$

$$x = -\frac{\log(2)}{\log\left(\frac{5}{4}\right)}$$

4.5. $11^x = 122^{(x-1)}$

$$x = \frac{\log(122)}{\log\left(\frac{122}{11}\right)}$$

4.6. $2^{2x} - 8 \cdot 2^x - 16 = 0$

$$x = 2 + \frac{log(1+\sqrt{2})}{log(2)}$$
