Sigma notation

Tom Coleman

Summary

Sigma notation is used to express many additions at once. Understanding what this notation is, how it works, and how to manipulate them is a valuable skill to learn for use in almost any area of mathematics.

Before reading this guide, it is recommended that you read GUIDE and GUIDE

What is sigma notation?

If you want to add infinitely many things together, then it would be nice to have a quick way of writing this down! This is where **sigma notation** comes in.

Definition of sum and sigma notation

A **sum** is any addition of two or more real numbers. If $a_k, a_{k+1}, \dots, a_n$ are real numbers (where k and n are some natural numbers with $k \leq N$), then you can use **sigma notation** to write their sum as

$$a_k + a_{k+1} + \dots + a_N = \sum_{i=k}^{N} a_i$$

where the right hand side reads 'the sum from $i = k$ to $i = n$ of the elements a_i'. The symbol i is known as the **index** of the sum; the index of a sum can notionally be any letter.

* Examples

Here's some examples of sigma notation.

(b) What is the value of $\sum_{n=2}^{5} n^2$?

Before tackling a problem using sigma notation, it can be best to read it out loud. Here,

$$\sum_{n=2}^{5} n^2 \text{ is 'the sum from } n = 2 \text{ to } n = 5 \text{ of } n^2\text{'.}$$

This translates to

$$\sum_{n=2}^{5} n^2 = 2^2 + 3^2 + 4^2 + 5^2$$

and $2^2 + 3^2 + 4^2 + 5^2 = 4 + 9 + 16 + 25 = 54$.

(c) What is the value of $\sum_{n=1}^{N} n = S$?

In this case, you're being asked to find $S = 1 + 2 + 3 + ... + N$. The following method is due to Gauss, who came up with this answer during a maths lesson at school when he was seven (hinting at the genius to follow).

First of all, you can reorder S to write that $S = N + (N-1) + ... + 2 + 1$. Adding two lots of S together gives the following:

$$\begin{array}{cccccccccccc} & S & = & 1 & + & 2 & + & 3 & + & ... & + & N \\ + & S & = & N & + & (N-1) & + & (N-2) & + & ... & + & 1 \\ \hline & 2S & = & (N+1) & + & (N+1) & + & (N+1) & + & ... & + & (N+1) \end{array}$$

Therefore, $2S$ is N lots of $(N+1)$; you can write this as $2S = N(N+1)$. Dividing both sides by 2 gives $S = N(N+1)/2$.

Writing sums using sigma notation

(a) Write $2 + 4 + 6 + 8 + 10 + 12$ using sigma notation.

You can tell that these are the first six multiples of 2; so you can list these elements as $2n$ for $n = 1$ up to $n = 6$. Therefore, you can write that

$$2 + 4 + 6 + 8 + 10 + 12 = \sum_{n=1}^{6} 2n.$$

Properties

Double sums

Additional sums

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where the right hand side reads 'the sum from $i = k$ to $i = n$ of the elements a_i'. The symbol i is known as the **index** of the sum; the index of a sum can notionally be any letter.

Problems

Further reading