

# Question sheet

Sophie Chowgule

## Questions: PMFs, PDFs, and CDFs

A selection of questions to test your understanding of Probability Mass Functions (PMFs), Probability Density Functions (PDFs), and Cumulative Distribution Functions (CDFs).

**Before attempting these questions it is highly recommended that you read (Guide: PMFs, PDFs, and CDFs).**

### Q1

For each of the scenarios below, determine if the given distribution is a valid PMF and answer the following questions.

#### 1.1

Let  $X$  be the random variable representing the result of rolling a biased four sided-dice. The PMF of  $X$  is given by:

$x$	1	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$

- a. What is the probability of  $X = 4$ ?

#### 1.2

A random variable  $X$  has five possible outcomes 1, 2, 3, 4, 5, and the PMF is:

$x$	1	2	3	4	5
$P(X = x)$	0.25	0.35	0.05	0.2	0.1

- a. What is the probability of  $X = 3$  or  $X = 4$ ?

### 1.3

A coin is tossed, where the probability of tails is 70% and heads is 30%. Let  $X$  represent the result of the coin toss. Complete the table below:

$x$	Heads	Tails
$P(X = x)$		

### 1.4

A random variable  $X$  have the possible outcomes 1, 2, 3, 4, 5, 6, 7 with the following PMF:

$x$	1	2	3	4	5	6	7
$P(X = x)$	0.1	0.05	0.05	0.3	0.25	0.75	0.35

- a. Is this a valid PMF? Why or why not?

### 1.5

A bag contains 5 red, 3 blue, and 2 green sweets from a sweet shop. Let  $X$  represent the color of a randomly picked sweet:

- a. What is the probability of picking a blue sweet?
- b. Construct the PMF for this scenario by completing the table:

$x$	Red	Blue	Green
$P(X = x)$			

### 1.6

The PMF for a random variable  $X$  is given as:

$x$	1	2	3	4
$P(X = x)$	$p$	$2p$	$3p$	$4p$

- a. For what value of  $p$  is this a valid PMF?
- b. For this value of  $p$ , what is the probability of  $X = 3$ ?

## Q2

For each of the scenarios below, determine if the given distribution is a valid PDF and answer the following questions.

### 2.1

Let  $X$  be a continuous random variable on the interval  $[0, 2]$  with the PDF:

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- a. What is the probability that  $X$  lies between 1 and 2?

### 2.2

Let  $X$  be a continuous random variable with the PDF:

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- a. What is the probability that  $X$  lies between 0.5 and 1?  
b. What is  $P(0.25 > X > 0.75)$ ?

### 2.3

Let  $X$  be a continuous random variable uniformly distributed between 3 and 7. The PDF is:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

- a. What is the probability that  $X$  lies between 3 and 6?

### 2.4

The PDF of a random variable  $X$  is given by:

$$f(x) = \begin{cases} \frac{1}{9} & \text{if } 1 \leq x \leq 4 \\ \frac{1}{4} & \text{if } 5 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

- a. Is this a valid PDF? Why or why not?

## 2.5

Consider the PDF:

$$f(x) = \begin{cases} kx^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. For what value of  $k$  is this a valid PDF?
- b. For this value of  $k$ , what is the  $P(0.2 \leq x \leq 0.3)$ ?

## 2.6

The PDF of  $X$  is given by:

$$f(x) = \begin{cases} 4x & \text{if } 0 \leq x < 0.5, \\ 4 - 4x & \text{if } 0.5 \leq x < 0.75, \\ 0.5 & \text{if } 0.75 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- a. Is this a valid PDF? Why or why not?

---

## Q3

For each of the scenarios below, determine if the given distribution is a valid CDF and answer the following questions.

### 3.1

A scenario represented by a PMF has the following CDF:

$x$	1	2	3	4
$P(X = x)$	0.1	0.3	0.5	1

- a. What is  $F(3)$ ?

- b. What is  $P(X > 2)$ ?

### 3.2

For the random variable uniformly distributed on  $[0, 2]$  as seen in question 2.2:

- a. Write the CDF  $F(x)$  for the values 0.5, 1, and 2.  
b. What is  $F(1.5)$ ?  
c. What is  $F(3)$ ?

### 3.3

For the PDF given in question 2.3:

- a. Write the CDF  $F(x)$  at points 4, 5, and 6?  
b. What is  $P(X > 5)$ ?

### 3.4

The PDF of  $X$  for a scenario is given by:

$x$	1	2	3	4	5	6
$P(X = x)$	0.1	0.2	0.5	0.4	0.8	1

- a. Is this a valid CDF? Why or why not?