Powers and nth roots: Answers

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Summary

Answers to questions relating to the guide on powers and nth roots.

These are the answers to [Questions: Powers and nth roots] **Please attempt the questions before reading these answers!**

Q1

Simplify the following expressions, leaving your answer as a singular power:

1.1. $(a)^2 \cdot (b)^2 = (ab)^2$

1.2. $(a)^{-4} \cdot (b)^{-4} = (ab)^{-4} = \frac{1}{(ab)^4}$

1.3. $2^2 \cdot 3^2 = (2 \cdot 3)^2 = 6^2$

1.4. $8^5 \cdot 6^5 = (8 \cdot 6)^5 = 48^5$

1.5. $12^{11} \cdot 3^{11} = (12 \cdot 3)^{11} = 36^{11}$

1.6. $(\frac{x^5}{y^5}) = (\frac{x}{y})^5$

1.7. $(\frac{4^5}{2^5}) = (\frac{4}{2})^5 = 2^5$

1.8. $(\frac{2^{-2}}{13^{-2}}) = (\frac{2}{13})^{-2} = (\frac{13}{2})^2$

1.9. $(\frac{a}{b})^n \cdot (\frac{c}{d})^n = ((\frac{a}{b}) \cdot (\frac{c}{d}))^n = (\frac{ac}{bd})^n$

1.10. $(\frac{2}{3})^{14} \cdot (\frac{9}{12})^{14} = ((\frac{2}{3}) \cdot (\frac{9}{12}))^{14} = ((\frac{18}{36}))^{14} = ((\frac{1}{2}))^{14}$

1.11. $(a^{\frac{1}{2}}) \cdot (b^{\frac{1}{2}}) = (ab)^{\frac{1}{2}} = \sqrt{ab}$

Q2

Evaluate the following:

2.1. $(\frac{4^3 \cdot 3^3}{6^3}) = (\frac{4 \cdot 3}{6})^3 = (\frac{12}{6})^3 = 2^3 = 8$

2.2. $(\frac{4^2 \cdot 8^2}{2^2}) \cdot (\frac{1}{2})^2 = (\frac{4 \cdot 8}{2})^2 \cdot (\frac{1}{2})^2 = (\frac{4 \cdot 8 \cdot 1}{2 \cdot 2})^2 = (\frac{32}{4})^2 = 8^2 = 64$

2.3. $(\frac{a}{b})^4 \cdot (\frac{c}{d})^4 \cdot (\frac{e}{f})^4 = (\frac{ace}{bdf})^4$

2.4. $\frac{(\frac{-2}{3})^{-3} \cdot (\frac{-3}{5})^{-3}}{(\frac{2}{3})^{-3}} = \frac{(\frac{6}{15})^{-3}}{(\frac{2}{3})^{-3}} = \frac{(\frac{15}{6}^3)}{(\frac{3}{2})^3} = (\frac{15 \cdot 2}{6 \cdot 3})^3 = (\frac{5}{3})^3 = \frac{125}{27}$

2.5. $\frac{5^{x+1} \cdot 6^{x+1}}{3^{x+1}} = (\frac{5 \cdot 6}{3})^{x+1} = 10^{x+1}$

2.6. $\frac{\frac{(\frac{1}{2})^4}{(\frac{3}{5})^4}}{(\frac{8}{3})^4} = \frac{(\frac{5}{6})^4}{(\frac{8}{3})^4} = (\frac{15}{48})^4$

Q3

For the following, find the value of x:

3.1. $(4^x) \cdot (2^x) = 64$ then $(4 \cdot 2)^x = 64$ and hence $8^x = 64$ and so $x = 2$

3.2. $\frac{5^{x+1} \cdot 6^{x+1}}{3^{x+1}} = (\frac{5 \cdot 6}{3})^{x+1} = 100$ hence, $10^{x=1} = 100$ and so $x = 1$

3.3. $\frac{(\frac{1}{2})^x \cdot (\frac{-1}{4})^x}{(\frac{2}{3})^x} = \frac{-27}{4096}$, combining the first two fractions, $\frac{(\frac{1}{8})^x}{(\frac{2}{3})^x} = (\frac{-1}{8})^x \cdot (\frac{3}{2})^x = (\frac{-3}{16})^x = \frac{-27}{4096}$ and so $x = 3$

Q4

Simplify the following expressions:

4.1. $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

4.2. $\sqrt{3} \cdot \sqrt{7} = \sqrt{3 \cdot 7} = \sqrt{21}$

4.3. $(\frac{\sqrt{24}}{\sqrt{6}}) = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$

4.4. $(\sqrt{5})^2 = \sqrt{25} = 5$

4.5. $(\sqrt{2})^4 = (\sqrt{2})^2 \cdot (\sqrt{2})^2 = 2 \cdot 2 = 4$

4.6. $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

4.7. $\sqrt{4^3} \cdot \sqrt[3]{8}$
$\sqrt{4^3} = (\sqrt{4})^3 = 2^3$
$\sqrt[3]{8} = 2$
$\sqrt{4^3} \cdot \sqrt[3]{8} = 2^3 \cdot 2 = 2^4 = 16$

4.8. $\sqrt{6} \cdot \sqrt{15} = \sqrt{3 \cdot 3 \cdot 2 \cdot 5} = 3\sqrt{10}$

4.9. $\sqrt{75} - \sqrt{27} = 5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$

4.10. $(8 + \sqrt{2}) \cdot (3 - \sqrt{2}) = 24 - 2 = 3\sqrt{2} - 8\sqrt{8} = 22 - 5\sqrt{2}$

4.11. $(3\sqrt{7})^2(8\sqrt{3})^2 = (3\sqrt{7} \cdot 8\sqrt{3})^2 = (24\sqrt{21})^2 = 24^2 \cdot 21 = 12096$

4.12. $(3 - \sqrt{8})^2 = 9 + 8 - 2(3\sqrt{8}) = 17 - 6\sqrt{8}$

4.13. $(7 + \sqrt{5}) \cdot (1 + \sqrt{5}) = 7 + 5 + \sqrt{5} + 7\sqrt{5} = 12 + 8\sqrt{5}$

4.14. $\sqrt{45} + \sqrt{125} = \sqrt{9} \cdot \sqrt{5} + \sqrt{25} \cdot \sqrt{5} = 3\sqrt{5} + 5\sqrt{5} = 8\sqrt{5}$

4.15. $\sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$

Q5

Simplify the following:

5.1. $\frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$

5.2. $\frac{7+\sqrt{5}}{3+\sqrt{5}} = \frac{7+\sqrt{5}}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{16-4\sqrt{5}}{9-5} = \frac{16-4\sqrt{5}}{4} = 4 - \sqrt{5}$

5.3. $(\frac{2-\sqrt{3}}{3+\sqrt{3}}) = (\frac{(2-\sqrt{3})\cdot(3-\sqrt{3})}{(3+\sqrt{3})\cdot(3-\sqrt{3})}) = (\frac{6+3-3\sqrt{3}-2\sqrt{3}}{9-3}) = (\frac{9-5\sqrt{3}}{6})$

5.4. $\frac{21}{2+\sqrt{3}} = \frac{42-21\sqrt{3}}{4-3} = 42 - 21\sqrt{3}$

5.5. $\frac{1}{4-\sqrt{8}} = \frac{4+\sqrt{8}}{(4-\sqrt{8})\cdot(4+\sqrt{8})} = \frac{4+\sqrt{8}}{16-8} = \frac{4+\sqrt{8}}{8} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$
