

Sigma notation

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Summary

Sigma notation is used to express many additions at once. Understanding what this notation is, how it works, and how to manipulate them is a valuable skill to learn for use in almost any area of mathematics.

Before reading this guide, it is recommended that you read [GUIDE](#) and [GUIDE](#)

What is sigma notation?

If you want to add infinitely many things together, then it would be nice to have a quick way of writing this down! This is where **sigma notation** comes in.

Definition of sum and sigma notation

A **sum** is any addition of two or more real numbers. If a_k, a_{k+1}, \dots, a_n are real numbers (where k and n are some natural numbers with $k \leq n$), then you can use **sigma notation** to write their sum as

$$a_k + a_{k+1} + \dots + a_n = \sum_{i=k}^n a_i$$

where the right hand side reads 'the sum from $i = k$ to $i = n$ of the elements a_i '. The symbol i is known as the **index** of the sum; the index of a sum can notionally be any letter.

* Examples

Here's some examples of sigma notation.

(b) What is the value of $\sum_{n=2}^5 n^2$?

Before tackling a problem using sigma notation, it can be best to read it out loud. Here,

$$\sum_{n=2}^5 n^2 \text{ is 'the sum from } n = 2 \text{ to } n = 5 \text{ of } n^2\text{'}$$

This translates to

$$\sum_{n=2}^5 n^2 = 2^2 + 3^2 + 4^2 + 5^2$$

and $2^2 + 3^2 + 4^2 + 5^2 = 4 + 9 + 16 + 25 = 54$.

(c) What is the value of $\sum_{n=1}^N n = S$?

In this case, you're being asked to find $S = 1 + 2 + 3 + \dots + N$. The following method is due to [Gauss](#), who came up with this answer during a maths lesson at school when he was seven (hinting at the genius to follow).

First of all, you can reorder S to write that $S = N + (N - 1) + \dots + 2 + 1$. Adding two lots of S together gives the following:

$$\begin{array}{rcccccccc} S & = & 1 & + & 2 & + & 3 & + & \dots & + & N \\ + S & = & N & + & (N-1) & + & (N-2) & + & \dots & + & 1 \\ \hline 2S & = & (N+1) & + & (N+1) & + & (N+1) & + & \dots & + & (N+1) \end{array}$$

Therefore, $2S$ is N lots of $(N + 1)$; you can write this as $2S = N(N + 1)$. Dividing both sides by 2 gives $S = N(N + 1)/2$.

Writing sums using sigma notation

(a) Write $2 + 4 + 6 + 8 + 10 + 12$ using sigma notation.

You can tell that these are the first six multiples of 2; so you can list these elements as $2n$ for $n = 1$ up to $n = 6$. Therefore, you can write that

$$2 + 4 + 6 + 8 + 10 + 12 = \sum_{n=1}^6 2n.$$

Properties

Double sums

i Additional sums

A **sum** is any addition of two or more real numbers. If a_k, a_{k+1}, \dots, a_n are real numbers (where k and n are some natural numbers with $k \leq n$), then you can use **sigma notation** to write their sum as

$$a_k + a_{k+1} + \dots + a_n = \sum_{i=k}^n a_i$$

where the right hand side reads 'the sum from $i = k$ to $i = n$ of the elements a_i '. The symbol i is known as the **index** of the sum; the index of a sum can notionally be any letter.

Problems

Further reading