Answers: Introduction to vectors

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Summary

Answers to questions relating to the guide on introduction to vectors.

These are the answers to Questions: Introduction to vectors.

Please attempt the questions before reading these answers!

Q1

1.1. $|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

1.2. $|\mathbf{b}| = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$

1.3. $|\mathbf{c}| = \sqrt{1^2 + (-1)^2 + 4^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

1.4. $|\mathbf{d}| = \sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{25+4+1} = \sqrt{30}$

1.5. $|\mathbf{e}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4+1+16} = \sqrt{21}$

1.6. $|\mathbf{f}| = \sqrt{(-3)^2 + 6^2 + 2^2} = \sqrt{9+36+4} = \sqrt{49} = 7$

1.7. $|\mathbf{g}| = \sqrt{5^2 + 1^2 + (\sqrt{2})^2} = \sqrt{25+1+2} = \sqrt{28} = 2\sqrt{7}$

1.8. $|\mathbf{h}| = \sqrt{6^2 + 2^2 + 2^2} = \sqrt{36+4+4} = \sqrt{44} = 2\sqrt{11}$

1.9. $|\mathbf{m}| = \sqrt{(-3)^2 + 3^2 + (-3)^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$

1.10. $|\mathbf{n}| = \sqrt{2^2 + 4^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

1.11. $|\mathbf{p}| = \sqrt{8^2 + (-2)^2 + 16^2} = \sqrt{64+4+256} = \sqrt{324} = 18$

1.12. $|\mathbf{q}| = \sqrt{5^2 + (-2)^2 + 14^2} = \sqrt{25+4+196} = \sqrt{225} = 15$

1.13. $|\mathbf{u}| = \sqrt{7^2 + 2^2 + (-1)^2} = \sqrt{49+4+1} = \sqrt{54} = 3\sqrt{6}$

1.14. $|\mathbf{v}| = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144+81+64} = \sqrt{289} = 17$

Q2

2.1. Find the magnitude of the vector first, so $|\mathbf{a}| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$.

Then

$$\hat{\mathbf{a}} = \frac{-2\mathbf{i} + 3\mathbf{j}}{\sqrt{13}} = \frac{-2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

2.2. Find the magnitude of the vector first, so $|\mathbf{b}| = \sqrt{(-2)^2 + 4^2 + (-6)^2} = \sqrt{56} = 2\sqrt{14}$.

Then

$$\hat{\mathbf{b}} = \frac{-2\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{2\sqrt{14}} = \frac{-1}{\sqrt{14}}\mathbf{i} + \frac{2}{\sqrt{14}}\mathbf{j} - \frac{3}{\sqrt{14}}\mathbf{k}$$

2.3. Find the magnitude of the vector first, so $|\mathbf{c}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$.

Then

$$\hat{\mathbf{c}} = \frac{\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}}{\sqrt{21}} = \frac{1}{\sqrt{21}}\mathbf{i} + \frac{2}{\sqrt{21}}\mathbf{j} + \frac{4}{\sqrt{21}}\mathbf{k}$$

2.4. Find the magnitude of the vector first, so $|\mathbf{d}| = \sqrt{4^2 + (-2)^2 + 3^2} = \sqrt{29}$.

Then

$$\hat{\mathbf{d}} = \frac{4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{29}} = \frac{4}{\sqrt{29}}\mathbf{i} - \frac{2}{\sqrt{29}}\mathbf{j} + \frac{3}{\sqrt{29}}\mathbf{k}$$

2.5. Find the magnitude of the vector first, so $|\mathbf{e}| = \sqrt{3^2 + 2^2} = \sqrt{13}$.

Then

$$\hat{\mathbf{e}} = \frac{3\mathbf{i} + 2\mathbf{k}}{\sqrt{13}} = \frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{k}$$

2.6. Find the magnitude of the vector first, so $|\mathbf{f}| = \sqrt{(-3)^2 + 1^2 + 7^2} = \sqrt{59}$.

Then

$$\hat{\mathbf{f}} = \frac{-3\mathbf{i} + \mathbf{j} + 7\mathbf{k}}{\sqrt{59}} = -\frac{3}{\sqrt{59}}\mathbf{i} + \frac{1}{\sqrt{59}}\mathbf{j} + \frac{7}{\sqrt{59}}\mathbf{k}$$

2.7. Find the magnitude of the vector first, so $|\mathbf{g}| = \sqrt{(-5)^2 + (\sqrt{2})^2} = \sqrt{27} = 3\sqrt{3}$.

Then

$$\hat{\mathbf{g}} = \frac{-5\mathbf{i} + \sqrt{2}\mathbf{k}}{3\sqrt{3}} = -\frac{5}{3\sqrt{3}}\mathbf{i} + \frac{\sqrt{2}}{3\sqrt{3}}\mathbf{k}$$

2.8. Find the magnitude of the vector first, so $|\mathbf{h}| = \sqrt{(-3)^2 + 1^2 + 1^2} = \sqrt{11}$.

Then

$$\hat{\mathbf{h}} = \frac{-3\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{11}} = \frac{-3}{\sqrt{11}}\mathbf{i} + \frac{1}{\sqrt{11}}\mathbf{j} + \frac{1}{\sqrt{11}}\mathbf{k}$$

2.9. Find the magnitude of the vector first, so $|\mathbf{m}| = \sqrt{(-3)^2 + 3^2 + (-3)^2} = \sqrt{27} = 3\sqrt{3}$.

Then

$$\hat{\mathbf{m}} = \frac{-3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}{3\sqrt{3}} = -\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}$$

2.10. Find the magnitude of the vector first, so $|\mathbf{n}| = \sqrt{3^2 + 6^2 + 9^2} = \sqrt{126} = 3\sqrt{14}$.

Then

$$\hat{\mathbf{n}} = \frac{3\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}}{3\sqrt{14}} = \frac{1}{\sqrt{14}}\mathbf{i} + \frac{2}{\sqrt{14}}\mathbf{j} + \frac{3}{\sqrt{14}}\mathbf{k}$$

2.11. Find the magnitude of the vector first, so $|\mathbf{p}| = \sqrt{3^2 + (-4)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$.

Then

$$\hat{\mathbf{p}} = \frac{3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}}{5\sqrt{2}} = \frac{3}{5\sqrt{2}}\mathbf{i} - \frac{4}{5\sqrt{2}}\mathbf{j} - \frac{1}{\sqrt{2}}\mathbf{k}$$

2.12. Find the magnitude of the vector first, so $|\mathbf{q}| = \sqrt{4^2 + (-3)^2 + 12^2} = \sqrt{169} = 13$.

Then

$$\hat{\mathbf{q}} = \frac{4\mathbf{i} - 3\mathbf{j} + 12\mathbf{k}}{13} = \frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}$$

2.13. Find the magnitude of the vector first, so $|\mathbf{u}| = \sqrt{6^2 + 5^2 + 4^2} = \sqrt{77} =$.

Then

$$\hat{\mathbf{u}} = \frac{6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}}{\sqrt{77}} = \frac{6}{\sqrt{77}}\mathbf{i} + \frac{5}{\sqrt{77}}\mathbf{j} + \frac{4}{\sqrt{77}}\mathbf{k}$$

2.14. Find the magnitude of the vector first, so $|\mathbf{v}| = \sqrt{2^2 + 4^2 + 8^2} = \sqrt{84} = 2\sqrt{21}$.

Then

$$\hat{\mathbf{v}} = \frac{2\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{2\sqrt{21}} = \frac{1}{\sqrt{21}}\mathbf{i} + \frac{2}{\sqrt{21}}\mathbf{j} + \frac{4}{\sqrt{21}}\mathbf{k}$$

Version history and licensing

v1.0: initial version created 08/23 by Zheng Chen as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.