The Scalar Product: Questions

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Summary

A selection of questions for the study guide on the Scalar Product

Before attempting these questions, it is highly recommended that you read Guide: The Scalar Product, as well as the Guide: Introduction to Quadratic Equations.

Questions

Q1

Find the scalar product of **u** and **v**.

1.1. $\mathbf{u} = \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

1.2. $\mathbf{u} = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ -5 \\ 13 \end{pmatrix}$

1.3. $\mathbf{u} = \begin{pmatrix} -4.4 \\ -1.2 \\ 0.3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 6.1 \\ -2.5 \\ 9.3 \end{pmatrix}$

1.4. $\mathbf{u} = \begin{pmatrix} 54 \\ 38 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 32 \\ -55 \\ 13 \end{pmatrix}$

1.5. $\mathbf{u} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$

1.6. $\mathbf{u} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$

1.7. $\mathbf{u} = 17\mathbf{j} + 23\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$

1.8. $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = \mathbf{j}$.

What can you say about the result of (1.8.)? Can you deduce similar conclusions for the scalar product of different combinations of the base vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$?

Q2

Find the value(s) of λ for which $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

2.1. $\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix}$

2.2. $\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

2.3. $\mathbf{u} = \begin{pmatrix} 9 \\ -2 \\ 11 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ -\lambda \\ 3 \end{pmatrix}$

2.4. $\mathbf{u} = \begin{pmatrix} \lambda \\ 6 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ \lambda \\ 8 \end{pmatrix}$

2.5. $\mathbf{u} = \begin{pmatrix} -2\lambda^2 \\ 4 \\ 14 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 2\lambda \\ 1 \end{pmatrix}$

2.6. $\mathbf{u} = \begin{pmatrix} -5 \\ 9 \\ 2\lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \lambda \\ -2 \\ \lambda \end{pmatrix}$

2.7. $\mathbf{u} = \begin{pmatrix} -7 \\ 4 \\ 2\lambda \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2\lambda \\ 1 \\ 6\lambda \end{pmatrix}$

2.8. $\mathbf{u} = \begin{pmatrix} -25 \\ -\lambda^2 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3\lambda \\ -11 \\ 7 \end{pmatrix}$

Q3

Using the Geometric Definition of Scalar Products, find the angle θ in between $\mathbf{u}$ and $\mathbf{v}$ in degrees to 1 decimal point.

3.1. $\mathbf{u} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix}$

3.2. $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

3.3. $\mathbf{u} = \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$

3.4. $\mathbf{u} = \begin{pmatrix} 1.2 \\ -1.4 \\ -3.1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -5.4 \\ 9.7 \\ -7.5 \end{pmatrix}$

3.5. $\mathbf{u} = \begin{pmatrix} 45 \\ 65 \\ 54 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -19 \\ -58 \\ 71 \end{pmatrix}$

3.6. $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

3.7. $\mathbf{u} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$

3.8. $\mathbf{u} = \begin{pmatrix} -17 \\ 3 \\ 8 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 12 \\ -19 \\ -16 \end{pmatrix}$