

# Answers: Introduction to complex numbers

Tom Coleman

## Summary

Answers to questions relating to the guide on introduction to complex numbers.

*These are the answers to [Questions: Introduction to complex numbers](#).*

**Please attempt the questions before reading these answers!**

## Q1

For each of the quadratic equations below, identify the variable and the coefficients  $a, b, c$ .

- 1.1. For  $x^2 - 7x + 6 = 0$ , the variable is  $x$ , and the coefficients are  $a = 1, b = -7, c = 6$ .
- 1.2. For  $y^2 + 14y + 49 = 0$ , the variable is  $y$ , and the coefficients are  $a = 1, b = 14, c = 49$ .
- 1.3. For  $h^2 - h - 56 = 0$ , the variable is  $h$ , and the coefficients are  $a = 1, b = -1, c = -56$ .
- 1.4. For  $7y^4 - y^2 = 0$ , the variable is  $y^2$ , and the coefficients are  $a = 7, b = -1, c = 0$ .
- 1.5. For  $5n^2 - 14n + 100 = 0$ , the variable is  $n$ , and the coefficients are  $a = 5, b = -14, c = 100$ .
- 1.6. For  $A^2 - 144 = 0$ , the variable is  $A$ , and the coefficients are  $a = 1, b = 0, c = -144$ .
- 1.7. For  $25M^2 = 0$ , the variable is  $M$ , and the coefficients are  $a = 25, b = 0, c = 0$ .
- 1.8. For  $e^{2x} - 4e^x + 4 = 0$ , the variable is  $e^x$ , and the coefficients are  $a = 1, b = -4, c = 4$ .
- 1.9. For  $-9s^4 + 3s^2 - 1 = 0$ , the variable is  $s^2$ , and the coefficients are  $a = -9, b = 3, c = -1$ .
- 1.10. For  $2e^{6x} + e^{3x} + 1 = 0$ , the variable is  $e^{3x}$ , and the coefficients are  $a = 2, b = 1, c = 1$ .
- 1.11. For  $\cos^2(x) + 4\cos(x) - 4 = 0$ , the variable is  $\cos(x)$ , and the coefficients are  $a = 1, b = 4, c = -4$ .
- 1.12. For  $8x^8 - 4x^4 - 1 = 0$ , the variable is  $x^4$ , and the coefficients are  $a = 8, b = -4, c = -1$ .

## Q2

2.1. The discriminant of the equation  $x^2 - 7x + 6 = 0$  is  $D = 25$ , and therefore the equation has two distinct real roots.

2.2. The discriminant of the equation  $y^2 + 14y + 49 = 0$  is  $D = 0$ , and therefore the equation has one distinct real root.

2.3. The discriminant of the equation  $h^2 - h - 56 = 0$  is  $D = 217$ , and therefore the equation has two distinct real roots.

2.4. The discriminant of the equation  $7y^4 - y^2 = 0$  is  $D = 1$ , and therefore the equation has two distinct real roots.

2.5. The discriminant of the equation  $5n^2 - 14n + 100 = 0$  is  $D = -1804$ , and therefore the equation has no real roots (two distinct complex roots).

2.6. The discriminant of the equation  $A^2 - 144 = 0$  is  $D = 576$ , and therefore the equation has two distinct real roots.

2.7. The discriminant of the equation  $25M^2 = 0$  is  $D = 0$ , and therefore the equation has one distinct real root.

2.8. The discriminant of the equation  $e^{2x} - 4e^x + 4 = 0$  is  $D = 0$ , and therefore the equation has one distinct real root  $r$  in  $e^x$ . Whether or not it has a real root in  $x$  depends on whether or not  $r$  is positive. If  $r$  is positive, there is exactly one real root  $x = \ln(r)$ ; if  $r$  is negative, then there are no real roots.

2.9. The discriminant of the equation  $-9s^4 + 3s^2 - 1 = 0$  is  $D = -27$ , and therefore the equation has no real roots. This is true even with  $s^2$  as the variable, as if  $s^2$  is complex then  $s$  must also be complex.

2.10. The discriminant of the equation  $2e^{6x} + e^{3x} + 1 = 0$  is  $D = -7$ , and therefore the equation has no real roots. This is true even with  $e^{3x}$  as the variable, as if  $e^{3x}$  is complex then  $x$  must also be complex.

2.11. The discriminant of the equation  $\cos^2(x) + 4\cos(x) - 4 = 0$  is  $D = 32$ , and therefore the equation has two distinct real roots  $r_1$  and  $r_2$  in  $\cos(x)$ . Whether or not it has a real root in  $x$  depends on whether or not either of the roots is between  $-1$  and  $1$ . If both  $r_1$  and  $r_2$  are outside this range, then there are no real roots. If one of  $r_1$  or  $r_2$  is between  $-1$  and  $1$ , then there are infinitely many solutions.

2.12. The discriminant of the equation  $8x^8 - 4x^4 - 1 = 0$  is  $D = 48$ , and therefore the equation has two distinct real roots  $r_1$  and  $r_2$  in  $x^4$ . The amount of real roots depend on the signs of  $r_1$  and  $r_2$ .

- If  $r_1$  and  $r_2$  are both positive, then there are four real roots in  $x$ . This is because

$x^2 = \pm\sqrt{r_1}$  or  $x^2 = \pm\sqrt{r_2}$ ; square rooting the positive terms gives the roots in  $x$  as  $\pm\sqrt{(\sqrt{r_1})} = \pm\sqrt[4]{r_1}$  and  $\pm\sqrt{(\sqrt{r_2})} = \pm\sqrt[4]{r_2}$ . Any other roots must be complex, since you are taking square roots of the negative numbers  $-\sqrt{r_1}$  and  $-\sqrt{r_2}$ .

- If exactly one of  $r_1$  and  $r_2$  is positive (say  $r_i$ ), then there are two real roots in  $x$  given by  $\pm\sqrt[4]{r_i}$ . All other roots are complex.
- If both  $r_1$  and  $r_2$  are negative, then there are no real roots in  $x$ .

### Q3

3.1. Rearranging gives  $x^2 + x - 1 = 0$ . The discriminant of this is  $D = 5$ , and therefore the equation has two distinct real roots.

3.2. Rearranging gives  $y^2 + 10 = 0$ . The discriminant of this is  $D = -40$ , and therefore the equation has no real roots (two distinct complex roots).

3.3. Rearranging gives  $4m^2 + 4m + 1 = 0$ . The discriminant of this is  $D = 0$ , and therefore the equation has one distinct real root.

3.4. Rearranging gives  $t^4 + 1 = 0$ . The discriminant of this is  $D = -4$ , and therefore the equation has no real roots. This is true even with  $t^2$  as the variable, as if  $t^2$  is complex then  $t$  must also be complex.

3.5. Rearranging gives  $5x^2 - 11x - 1 = 0$ . The discriminant of this is  $D = 101$ , and therefore the equation has two distinct real roots.

3.6. Rearranging gives  $e^{2x} - 2e^x + 1 = 0$ . The discriminant of this is  $D = 0$ , and therefore the equation has one distinct real root  $r$  in  $e^x$ . Whether or not it has a real root in  $x$  depends on whether or not  $r$  is positive. If  $r$  is positive, there is exactly one real root  $x = \ln(r)$ ; if  $r$  is negative, then there are no real roots.

---



---

### Version history and licensing

v1.0: initial version created 09/24 by tdhc.

[This work is licensed under CC BY-NC-SA 4.0.](#)