

# Questions: The scalar product

Ritwik Anand

## Summary

A selection of questions for the study guide on the scalar product

*Before attempting these questions, it is highly recommended that you read [Guide: The scalar product](#), as well as [Guide: Introduction to quadratic equations](#).*

## Q1

Find the scalar product of  $\mathbf{a}$  and  $\mathbf{b}$ .

1.1.  $\mathbf{a} = \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

1.2.  $\mathbf{a} = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3 \\ -5 \\ 13 \end{pmatrix}$

1.3.  $\mathbf{a} = \begin{pmatrix} -44 \\ -12 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 61 \\ -25 \\ 93 \end{pmatrix}$

1.4.  $\mathbf{a} = \begin{pmatrix} 54 \\ 38 \\ 0 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 32 \\ -55 \\ 13 \end{pmatrix}$

1.5.  $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$

1.6.  $\mathbf{a} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$  and  $\mathbf{b} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$

1.7.  $\mathbf{a} = 17\mathbf{j} + 23\mathbf{k}$  and  $\mathbf{b} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$

1.8.  $\mathbf{a} = \mathbf{i}$  and  $\mathbf{b} = \mathbf{j}$ .

What can you say about the result of 1.8.? Can you deduce similar conclusions for the scalar product of different combinations of the vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ ?

## Q2

Using the geometric definition of the scalar products, find the smallest angle  $\theta$  in between  $\mathbf{a}$  and  $\mathbf{b}$  in degrees. If your answer is not a whole number, give your answer to an accuracy of one decimal place.

$$2.1. \mathbf{a} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix}$$

$$2.2. \mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$2.3. \mathbf{a} = \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$$

$$2.4. \mathbf{a} = \begin{pmatrix} 1.2 \\ -1.4 \\ -3.1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -5.4 \\ 9.7 \\ -7.5 \end{pmatrix}$$

$$2.5. \mathbf{a} = \begin{pmatrix} 45 \\ 65 \\ 54 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -19 \\ -58 \\ 71 \end{pmatrix}$$

$$2.6. \mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$2.7. \mathbf{a} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$$

$$2.8. \mathbf{a} = \begin{pmatrix} -17 \\ 3 \\ 8 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 12 \\ -19 \\ -16 \end{pmatrix}$$

### Q3

Find the value(s) of  $\lambda$  for which  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular.

3.1.  $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix}$

3.2.  $\mathbf{a} = \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

3.3.  $\mathbf{a} = \begin{pmatrix} 9 \\ -2 \\ 11 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ -\lambda \\ 3 \end{pmatrix}$

3.4.  $\mathbf{a} = \begin{pmatrix} \lambda \\ 6 \\ 1 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ \lambda \\ 8 \end{pmatrix}$

3.5.  $\mathbf{a} = \begin{pmatrix} -2\lambda^2 \\ 4 \\ 14 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3 \\ 2\lambda \\ 1 \end{pmatrix}$

3.6.  $\mathbf{a} = \begin{pmatrix} -5 \\ 9 \\ 2\lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ -2 \\ \lambda \end{pmatrix}$

3.7.  $\mathbf{a} = \begin{pmatrix} -7 \\ 4 \\ 2\lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 2\lambda \\ 1 \\ 6\lambda \end{pmatrix}$

3.8.  $\mathbf{a} = \begin{pmatrix} -25 \\ -\lambda^2 \\ -2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3\lambda \\ -11 \\ 7 \end{pmatrix}$

---

After attempting the questions above, please click [this link](#) to find the answers.

---

## **Version history and licensing**

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.

This work is licensed under [CC BY-NC-SA 4.0](#).