

# Answers: The scalar product

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## Summary

Answers to questions relating to the guide on the scalar product.

*These are the answers to [Questions: The scalar product](#).*

**Please attempt the questions before reading these answers!**

## Q1

1.1. For  $\mathbf{a} = \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = 26$ .

1.2. For  $\mathbf{a} = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3 \\ -5 \\ 13 \end{pmatrix}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = 117$ .

1.3. For  $\mathbf{a} = \begin{pmatrix} -44 \\ -12 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 61 \\ -25 \\ 93 \end{pmatrix}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = -2237$ .

1.4. For  $\mathbf{a} = \begin{pmatrix} 54 \\ 38 \\ 0 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 32 \\ -55 \\ 13 \end{pmatrix}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = -362$ .

1.5. For  $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = 48$ .

1.6. For  $\mathbf{a} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$  and  $\mathbf{b} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = -195$ .

1.7. For  $\mathbf{a} = 17\mathbf{j} + 23\mathbf{k}$  and  $\mathbf{b} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = -575$ .

1.8. For  $\mathbf{a} = \mathbf{i}$  and  $\mathbf{b} = \mathbf{j}$ , the scalar product is  $\mathbf{a} \cdot \mathbf{b} = 0$ .

As the scalar product of  $\mathbf{a} = \mathbf{i}$  and  $\mathbf{b} = \mathbf{j}$  is 0, they are perpendicular to each other. This is true for any combination of any *distinct* pair of  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ . However, since any vector is parallel to itself, it follows that  $\mathbf{i} \cdot \mathbf{i} = |\mathbf{i}||\mathbf{i}| = |1||1| = 1$ ; similar results hold for  $\mathbf{j} \cdot \mathbf{j}$  and  $\mathbf{k} \cdot \mathbf{k}$ .

## Q2

2.1. For  $\mathbf{a} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix}$ , the angle  $\theta$  is  $132.2^\circ$ .

2.2. For  $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ , the angle  $\theta$  is  $70.5^\circ$ .

2.3. For  $\mathbf{a} = \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$ , the angle  $\theta$  is  $108.7^\circ$ .

2.4. For  $\mathbf{a} = \begin{pmatrix} 1.2 \\ -1.4 \\ -3.1 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -5.4 \\ 9.7 \\ -7.5 \end{pmatrix}$ , the angle  $\theta$  is  $86.2^\circ$ .

2.5. For  $\mathbf{a} = \begin{pmatrix} 45 \\ 65 \\ 54 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -19 \\ -58 \\ 71 \end{pmatrix}$ , the angle  $\theta$  is  $95.1^\circ$ .

2.6. For  $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ , the angle  $\theta$  is  $90^\circ$ .

2.7. For  $\mathbf{a} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$ , the angle  $\theta$  is  $43.0^\circ$ .

2.8. For  $\mathbf{a} = \begin{pmatrix} -17 \\ 3 \\ 8 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 12 \\ -19 \\ -16 \end{pmatrix}$ , the angle  $\theta$  is  $137.8^\circ$ .

### Q3

3.1. For  $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix}$  to be perpendicular, then  $\lambda = 3$ .

3.2. For  $\mathbf{a} = \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  to be perpendicular, then  $\lambda = -\frac{2}{3}$ .

3.3. For  $\mathbf{a} = \begin{pmatrix} 9 \\ -2 \\ 11 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ -\lambda \\ 3 \end{pmatrix}$  to be perpendicular, then  $\lambda = -3$ .

3.4. For  $\mathbf{a} = \begin{pmatrix} \lambda \\ 6 \\ 1 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ \lambda \\ 8 \end{pmatrix}$  to be perpendicular, then  $\lambda = -2$  or  $\lambda = -4$ .

3.5. For  $\mathbf{a} = \begin{pmatrix} -2\lambda^2 \\ 4 \\ 14 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3 \\ 2\lambda \\ 1 \end{pmatrix}$  to be perpendicular, then  $\lambda = \frac{7}{3}$  or  $\lambda = -1$ .

3.6. For  $\mathbf{a} = \begin{pmatrix} -5 \\ 9 \\ 2\lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} \lambda \\ -2 \\ \lambda \end{pmatrix}$  to be perpendicular, then  $\lambda = \frac{9}{2}$  or  $\lambda = -2$ .

3.7. For  $\mathbf{a} = \begin{pmatrix} -7 \\ 4 \\ 2\lambda \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 2\lambda \\ 1 \\ 6\lambda \end{pmatrix}$  to be perpendicular, then  $\lambda = \frac{2}{3}$  or  $\lambda = \frac{1}{2}$ .

3.8. For  $\mathbf{a} = \begin{pmatrix} -25 \\ -1\lambda^2 \\ -2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 3\lambda \\ -11 \\ 7 \end{pmatrix}$  to be perpendicular, then  $\lambda = 7$  or  $\lambda = -\frac{2}{11}$ .

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## **Version history and licensing**

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.

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