Sigma notation

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Sigma notation is used to express many additions at once. Understanding what this notation is, how it works, and how to manipulate them is a valuable skill to learn for use in almost any area of mathematics.

*Before reading this guide, it is recommended that you read GUIDE and GUIDE*

## What is sigma notation?

If you want to add infinitely many things together, then it would be nice to have a quick way of writing this down! This is where **sigma notation** comes in.

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| Definition of sum and sigma notation |
| A **sum** is any addition of two or more real numbers. If are real numbers (where and are some natural numbers with ), then you can use **sigma notation** to write their sum as  where the right hand side reads ‘the sum from to of the elements ’. The symbol is known as the **index** of the sum; the index of a sum can notionally be any letter. |

#### Examples

Here’s some examples of sigma notation.

1. What is the value of ?

* Before tackling a problem using sigma notation, it can be best to read it out loud. Here,
* This translates to
* and .

1. What is the value of ?

In this case, you’re being asked to find . The following method is due to [Gauss](https://mathshistory.st-andrews.ac.uk/Biographies/Gauss/), who came up with this answer during a maths lesson at school when he was seven (hinting at the genius to follow).

First of all, you can reorder to write that . Adding two lots of together gives the following:

$$
\begin{array}{cccccccccccc}
& S & = & 1 & + & 2 & + & 3 & + & \ldots & + & N \\
+ & S & = & N & + & (N-1) & + & (N-2) & + & \ldots & + & 1\\\hline
& 2S & = & (N+1) & + & (N+1) & + & (N+1) & + & \ldots & + & (N+1)
\end{array}
$$

Therefore, is lots of ; you can write this as . Dividing both sides by gives .

## Writing sums using sigma notation

1. Write using sigma notation.

* You can tell that these are the first six multiples of ; so you can list these elements as for up to . Therefore, you can write that

## Properties

## Double sums

|  |  |
| --- | --- |
|  | **Additional sums**  A **sum** is any addition of two or more real numbers. If are real numbers (where and are some natural numbers with ), then you can use **sigma notation** to write their sum as  where the right hand side reads ‘the sum from to of the elements ’. The symbol is known as the **index** of the sum; the index of a sum can notionally be any letter. |

## Problems

## Further reading