Dealing With Powers and nth roots

Zoe Gemmell

Simplifying expressions involving powers and th roots is a valuable skill to learn. In this guide you will learn a few manipulation techniques that will hopefully be useful going further into your maths career.

*Before reading this guide, it is recommended that you read (Guide: Laws of indices). This guide uses for multiplication. Power is another word for indice/index.*

This guide will focus on expressions with bases and powers. You can recognise them from combinations of expressions in the following format:

## What is a base?

A base is a number that is raised to a power. It can be any number you can think of; single digit, decimal numbers, a million. This is the number that gets multiplied by itself.

What is a power you might then ask? A power is any number that you can raise a base to, it dictates how many times the base is multiplied by itself.

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|  | **Example 1**  You would say this out loud as ‘three multiplied by itself five times’, ‘three to the fifth power’ or ‘three to the five’. |

## Working with powers - The Definitions

Before you have a look at the laws here are some definitions:

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| Definition of expression |
| A mathematical expression is a combination of symbols that has a finite length. |

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| Definition of commutativity |
| Changing the order of the numbers involved in your expression does not change the answer. |

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|  | **Example 2**  The order of the 2 and 3 can be swapped and the answer stays the same. |

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| Definition of terminoligy in a fraction |
| The numerator of a fraction is everything on top of the line, the denominator is everything underneath the line: |

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| Tip |
| Both addition and multiplication are commutative |

# Now onto the laws:

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| Law 7: Multiplication of variables with the same indices but different bases |
| When multiplying variables with the same powers you can expand and rewrite them into a nicer format:  Where the bases are multiplied together then raised to the rth power. |

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|  | **Example 3** |

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| Law 8: Division of variables with the same indices but different bases |
| When dividing variables with the same powers you can expand and rewrite them into a nicer format:  Where the numerator base (a) is divided by the denominator base (b) then raised to the rth power. |

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|  | **Example 4** |

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| Law 9: fractional power, the nth root |
| When multiplying by bases with fractional powers (some sort of nth root) :  Where the aim of the manipulation is to put everything inside the square root sign, then to continue with the calculation. |

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|  | **Example 5** |

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| Law 10: Multiplication under the root sign |
| When you have a term under a th root sign which you can factorise (split up into different factors) this can be done and the th root signs can be dealt with separately.  Where you are using the commutativity of multiplication to split up the th root and deal with terms separately. |

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|  | **Example 6** |

# Solving equations

Using the laws above you can solve equations

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| Tip |
| When dealing with equations, you may find yourself faced with the following form:  In which case you will want to use the **conjugate** of the denominator to *creatively* multiply the fraction by 1. The conjugate of is , to find this for any espression you change the sign in front of the root.  Now the fraction only has square roots in the numerator, this is better because it can often make expressions you are dealing with more concise. |

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|  | **Example 7**  From here you can count up powers of 6 to work out the value of x: , so |

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|  | **Example 8**  Here is a rather long example using multiple laws from this guide and the intro to indices guide.  to simplify the appearance of the calculation, swap for Finally, switch back to  So x=2 |

## Quick check problems

1. What is the singular of indices?
2. Solve for x
3. Determine whether the following calculations are correct:
4. is greater than for all values of

[For more questions on the subject, please go to Questions: Needs to be added](.qmd)