Rationalizing the denominator

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Summary

Rationalizing the denominator is a technique for simplifying fractions involving square roots in the denominator. This study guide also covers the topic of quadratic conjugates which are sometimes used to rationalize the denominator of a fraction.

*Before reading this guide, it is recommended that you read* [*Guide: Laws of indices*](lawsofindices.qmd) *and* [*Guide: Expanding Brackets*](expandingthebrackets.qmd)*. The only irrational numbers you will see in this guide are are square roots, if you want to learn more about irraional numbers, have a look at the* [*Guide: Number Theory*](numbertheory.qmd)*.*

# What is rationalizing the denominator?

When you rationalize the denominator, you rewrite a fraction so that the denominator contains no square roots or other irrational numbers.

For example, in the fraction , the denominator contains a square root, which is irrational. You want to rewrite the fraction so that it looks like this: where the numerator can be irrational, but the denominator is free of square roots and a rational number, in this case 2.

This process is helpful for doing operations like addition or subtraction on the fraction and can also be useful when trying to approximate such a fraction.

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| Note |
| When you rewrite fractions you might multiply the numerator and denominator by the same value where is any number. When you are doing this, you are multiplying the fraction by which is the same as multiplying the fraction by .  You are therefore not changing the value of the fraction, you are rewriting it. |

# Expressions of the form

For fractions like , where a is any number and b and c are integers, you rationalize the denominator by multiplying both the numerator and denominator by the square root in the denominator: . This gives you:

Simplifying the denominator gives you

The denominator is now rational and free of square roots as and are integers so is also an integer. Example 1 shows you how to rationalize the fraction you saw above:

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|  | **Example 1**  Simplify by rationalizing the denominator:  To rationalize this, you multiply both the numerator and denominator by , giving you:  Simplifying gives you: |

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|  | **Example 2**  Simplify by rationalizing the denominator:  Multiply both the numerator and denominator by , giving you:  Simplifying gives you: |

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|  | **Example 3**  Simplify by rationalizing the denominator:  Multiply both the numerator and denominator by , giving you:  [Expanding the brackets](expandingthebrackets.qmd) and simplifying then gives you: |

# What is a quadratic conjugate?

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| Definition of a quadratic conjugate |
| Given an expression of the form , where , and are integers, the quadratic conjugate is the expression with the same terms but with the opposite sign in front of the term with a square root.  The quadratic conjugate of the expression is therefore . |

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|  | **Example 4**  What is the quadratic conjugate of : |

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|  | **Example 5**  What is the quadratic conjugate of : |

These conjugates are useful for eliminating square roots when rationalizing denominators of a particular form. Multiplying an expression by its quadratic conjugate eliminates the square root. You can see this when you multiply by its quadratic conjugate to get

Simplifying this gives you

This result is rational, free of square roots.

# Expressions of the form

Here a is any number and b, c and d are integers. In this case, rationalizing the denominator involves multiplying the numerator and the denominator by the quadratic conjugate of , which would be . As you saw above, the denominator will become . The entire fraction therefore becomes:

As you can see, the denominator is now and has been successfully rationalized.

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|  | **Example 6**  Simplify by rationalizing the denominator. Provide your answers in their simplest form and with a positive denominator:  The quadratic conjugate of the denominator is:  Multiplying the numerator and denominator by the quadratic conjugate gives you:  Expanding the brackets and simplifying the denominator gives you:  Further simplifying the denominator gives you: |

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|  | **Example 7**  Simplify by rationalizing the denominator. Provide your answers in their simplest form and with a positive denominator:  The quadratic conjugate of the denominator is:  Multiplying the numerator and denominator by the quadratic conjugate gives you:  Expanding the brackets and simplifying the denominator gives you:  Further simplifying the denominator gives you:  Multiplying both the numerator and the denominator by to get a positive denominator gives you: |

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|  | **Example 8**  Simplify by rationalizing the denominator. Provide your answers in their simplest form and with a positive denominator:  The quadratic conjugate of the denominator is:  Multiplying the numerator and denominator by the quadratic conjugate gives you:  Expanding the brackets gives you:  This simplifies to:  Simplifying the denominator gives you: |

# Quick check problems

Rationalize the denominator for each of the following expressions. Provide your answers in their simplest form and with a positive denominator.

# Further reading

[For more questions on the subject, please go to Questions: Rationalizing the denominator.](../questions/qs-rationalizingthedenominator.qmd)

## Version history

v0.1: Draft version created 9/24 by Maximilian Volmar.