Full Car Vibrating Model

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Background

The present automotive industry is witnessing a neck-to-neck fight among automotive companies to produce highly developed models for better performance. One of the performance requirements is advanced suspension systems to give better vehicle handling for smooth driving leading to passenger comfort. Most of the research activities during the last decades have been directed to vehicle vibration control, which is influenced by the harmful effects of vibrations caused by road irregularities on drivers' comfort. The present work aims at developing a details analytical formation of governing equations for a full car model. At first, a mathematical full car model considering seven degrees of freedom is developed using the passive suspension. Then through a state space matrix, an analytical solution for the displacement of the vehicle body has obtained. This report also discusses the 7-DOF full-car model and the validation of that model with an analytical solution.

Literature Review

Griffin et al [1], has shown that a vehicle's interior vibration significantly affects comfort and road-holding capability. To reduce this vibration, the manufacturer's efforts have led to a suspension system installed between road excitation and the vehicle body. Gundogdu [2] presented an optimization of a four-degree-of-freedom quarter car seat and suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver's seat. Wong [3], through his elaborative research, has established the role of road surface irregularities, ranging from potholes to random variations of the surface elevation profile, acts as a significant source that excites the vibration of the vehicle body through the tire/wheel assembly and the suspension system. Agharkakli [4] has obtained passive and active suspension systems mathematical model for the quarter car model and offered a compromise between two conflicting criteria, good road handling and improved passenger comfort. It mainly focuses on the effect of road irregularities on ride comfort and road holding of quarter-car and half-car model. However, there remains ample scope for further studies, such as validating the full car model with analytical models.

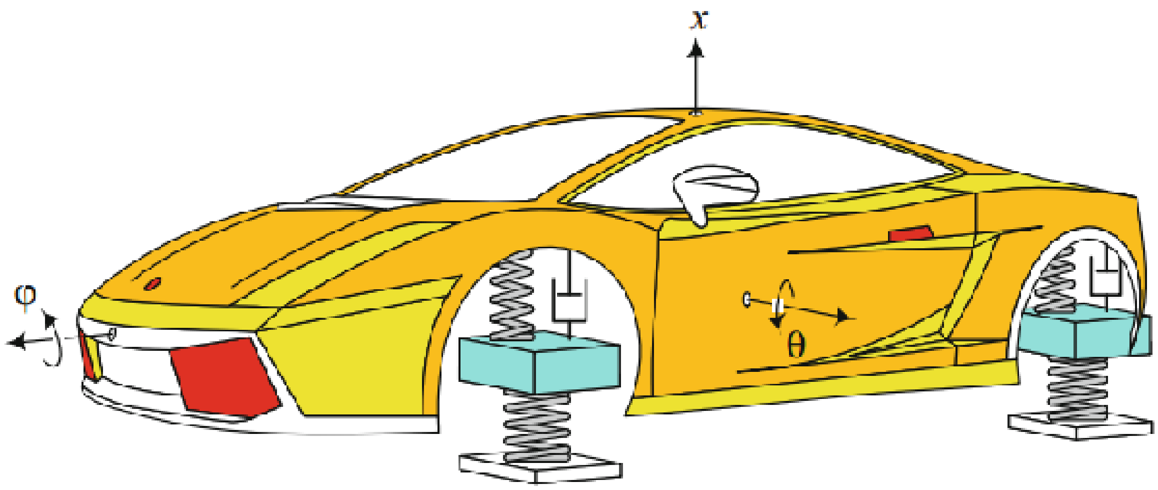
Full Car Vibrating Model-Approach to the problem

Degree of Freedoms-

Our model has seven DOF i.e., body bounce x, body roll φ, body pitch θ, wheels hop x_1, x_2, x_3, and x_4

Road Excitations-

The model is based on four independent road excitations y_1, y_2, y_3, and y_4.



[5] Full Car Vibrating Model

Assumptions: -

The body or chassis of the vehicle is modelled as a rigid slab which has a mass m, (the total body mass), I_x (longitudinal mass moment of inertia), and I_y(lateral mass moment of inertia).

The moments of inertia are only the body mass moments of inertia not the vehicle's mass moments of inertia. The wheels have a mass m_1, m_2, m_3, and m_4 respectively. We have assumed without loss of generality that both front wheels and rear wheels (and associated systems) are identical to each other, i.e., $m_1 = m_2 = m_f$ and $m_3 = m_4 = m_r$.

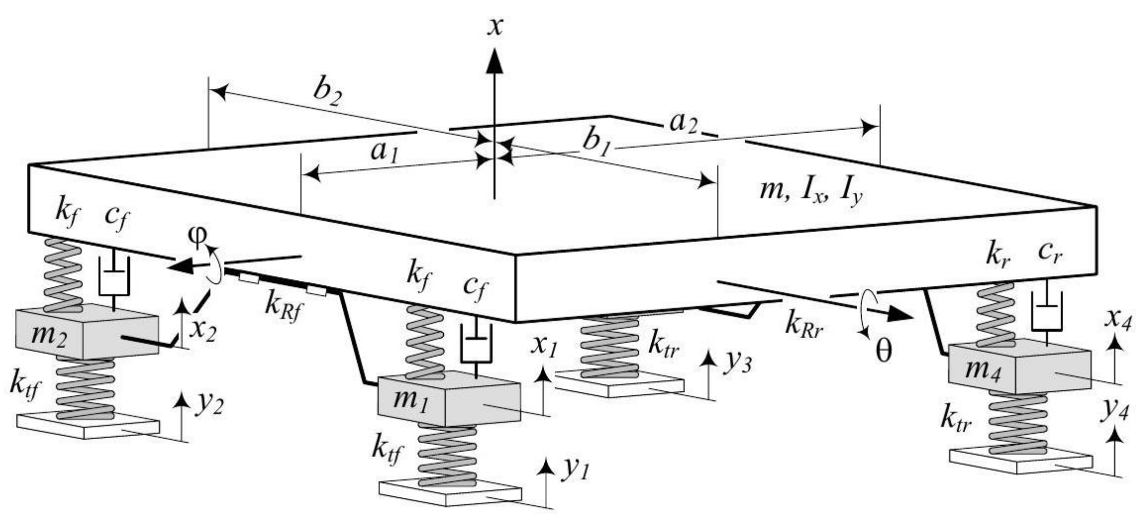
The front and rear tires stiffness is indicated by k_{t_f} and k_{t_r} respectively.

Because the damping of tires is much smaller than the damping of shock absorbers, we decided to ignore the tires' damping for simpler calculation.

The suspension of the car has stiffness k_f and damping c_f in the front and stiffness k_r and damping c_r in the rear. This is to make the suspension of the left and right wheels mirror. So, their stiffness and damping are equal.

Since most cars only have an antiroll bar in front, the moment of the antiroll bar can be referred to

$$M_R = -k_R\left(\varphi - \frac{x_1 - x_2}{b_1 + b_2}\right)$$



[5] Full Car Vibrating Model

Calculations: -

To find the equations of motion for the full car vibrating model, we used the Lagrange method. The kinetic and potential energies of the system and dissipation function are

- $K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m_f(\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2}m_r(\dot{x}_3^2 + \dot{x}_4^2) + \frac{1}{2}I_x\dot{\varphi}^2 + \frac{1}{2}I_y\dot{\theta}^2$

- $V = \frac{1}{2}k_f(x - x_1 + b_1\varphi - a_1\theta)^2 + \frac{1}{2}k_f(x - x_2 - b_2\varphi - a_1\theta)^2 + \frac{1}{2}k_r(x - x_3 - b_1\varphi + a_2\theta)^2 + \frac{1}{2}k_r(x - x_4 + b_2\varphi + a_2\theta)^2 + \frac{1}{2}k_R\left(\varphi - \frac{x_1 - x_2}{b_1 + b_2}\right)^2 + \frac{1}{2}k_{t_f}(x_1 - y_1)^2 + \frac{1}{2}k_{t_f}(x_2 - y_2)^2 + \frac{1}{2}k_{t_r}(x_3 - y_3)^2 + \frac{1}{2}k_{t_r}(x_4 - y_4)^2$
- $D = \frac{1}{2}c_f\left(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} - a_1\dot{\theta}\right)^2 + \frac{1}{2}c_f\left(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} - a_1\dot{\theta}\right)^2 + \frac{1}{2}c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right)^2 + \frac{1}{2}c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right)^2$

Using Lagrange method

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial D}{\partial \dot{q}_r} + \frac{\partial V}{\partial q_r} = 0 \quad r = 1,2,\cdots 7$$

provides the following 7 equations of motion.

1. $m\ddot{x} + c_f\left(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} - a_1\dot{\theta}\right) + c_f\left(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} - a_1\dot{\theta}\right) + c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right) + c_r\left(\dot{x} - \dot{x}_4 + b_2\dot{\varphi} + a_2\dot{\theta}\right) + k_f(x - x_1 + b_1\varphi - a_1\theta) + k_f(x - x_2 - b_2\varphi - a_1\theta) + k_r(x - x_3 - b_1\varphi + a_2\theta) + k_r(x - x_4 + b_2\varphi + a_2\theta) = 0$

2. $I_x\ddot{\varphi} + b_1c_f\left(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} - a_1\dot{\theta}\right) - b_2c_f\left(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} - a_1\dot{\theta}\right) - b_1c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right) + b_2c_r\left(\dot{x} - \dot{x}_4 + b_2\dot{\varphi} + a_2\dot{\theta}\right) + b_1k_f(x - x_1 + b_1\varphi - a_1\theta) - b_2k_f(x - x_2 - b_2\varphi - a_1\theta) - b_1k_r(x - x_3 - b_1\varphi + a_2\theta) + b_2k_r(x - x_4 + b_2\varphi + a_2\theta) + k_R\left(\varphi - \frac{x_1 - x_2}{b_1 + b_2}\right) = 0$

3. $I_y\ddot{\theta} - a_1c_f\left(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} - a_1\dot{\theta}\right) - a_1c_f\left(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} - a_1\dot{\theta}\right) + a_2c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right) + a_2c_r\left(\dot{x} - \dot{x}_4 + b_2\dot{\varphi} + a_2\dot{\theta}\right) - a_1k_f(x - x_1 + b_1\varphi - a_1\theta) - a_1k_f(x - x_2 - b_2\varphi - a_1\theta) + a_2k_r(x - x_3 - b_1\varphi + a_2\theta) + a_2k_r(x - x_4 + b_2\varphi + a_2\theta) = 0$

4. $m_f\ddot{x}_1 - c_f\left(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} - a_1\dot{\theta}\right) - k_f(x - x_1 + b_1\varphi - a_1\theta) - \frac{k_R}{b_1 + b_2}\left(\varphi - \frac{x_1 - x_2}{b_1 + b_2}\right) + k_{t_f}(x_1 - y_1) = 0$

5. $m_f\ddot{x}_2 - c_f\left(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} - a_1\dot{\theta}\right) - k_f(x - x_2 - b_2\varphi - a_1\theta) + \frac{k_R}{b_1+b_2}\left(\varphi - \frac{x_1-x_2}{b_1+b_2}\right) +$
$$k_{t_f}(x_2 - y_2) = 0$$

6. $m_r\ddot{x}_3 - c_r\left(\dot{x} - \dot{x}_3 - b_1\dot{\varphi} + a_2\dot{\theta}\right) - k_r(x - x_3 - b_1\varphi + a_2\theta) + k_{t_r}(x_3 - y_3) = 0$

7. $m_r\ddot{x}_4 - c_r\left(\dot{x} - \dot{x}_4 + b_2\dot{\varphi} + a_2\dot{\theta}\right) - k_r(x - x_4 + b_2\varphi + a_2\theta) + k_{t_r}(x_4 - y_4) = 0$

The above equations have been verified from research paper link given: -
https://www.researchgate.net/publication/248398906_Road_Profile_Estimation_Using_Wavelet_Neural_Network_and_7-DOF_Vehicle_Dynamic_Systems

The set of equations of motion can be rearranged in a matrix form

$$[m]\ddot{\mathbf{x}} + [c]\dot{\mathbf{x}} + [k]\mathbf{x} = \mathbf{f}$$

where **x, f**, [m], [c] and [k] are

$$\mathbf{x} = \begin{bmatrix} x \\ \varphi \\ \theta \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad [m] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_r \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ y_1k_{t_f} \\ y_2k_{t_f} \\ y_3k_{t_r} \\ y_4k_{t_r} \end{bmatrix}$$

$[c] =$

$$\begin{bmatrix} 2c_f + 2c_r & b_1c_f - b_2c_f - b_1c_r + b_2c_r & 2a_2c_r - 2a_1c_f & -c_f & -c_f & -c_r & -c_r \\ b_1c_f - b_2c_f - b_1c_r + b_2c_r & b_1^2c_f + b_2^2c_f + b_1^2c_r + b_2^2c_r & a_1b_2c_f - a_1b_1c_f - a_2b_1c_r + a_2b_2c_r & -b_1c_f & b_2c_f & b_1c_r & -b_2c_r \\ 2a_2c_r - 2a_1c_f & a_1b_2c_f - a_1b_1c_f - a_2b_1c_r + a_2b_2c_r & 2c_fa_1^2 + 2c_ra_2^2 & a_1c_f & a_1c_f & -a_2c_r & -a_2c_r \\ -c_f & -b_1c_f & a_1c_f & c_f & 0 & 0 & 0 \\ -c_f & b_2c_f & a_1c_f & 0 & c_f & 0 & 0 \\ -c_r & b_1c_r & -a_2c_r & 0 & 0 & c_r & 0 \\ -c_r & -b_2c_r & -a_2c_r & 0 & 0 & 0 & c_r \end{bmatrix}$$

$[k] =$

$$\begin{bmatrix} 2k_f + 2k_r & b_1k_f - b_2k_f - b_1k_r + b_2k_r & 2a_2k_r - 2a_1k_f & -k_f & -k_f & -k_r & -k_r \\ b_1k_f - b_2k_f - b_1k_r + b_2k_r & k_R + b_1^2k_f + b_2^2k_f + b_1^2k_r + b_2^2k_r & a_1b_2k_f - a_1b_1k_f - a_2b_1k_r + a_2b_2k_r & -b_1k_f - \frac{k_R}{b_1+b_2} & b_2k_f + \frac{k_R}{b_1+b_2} & b_1k_r & -b_2k_r \\ 2a_2k_r - 2a_1k_f & a_1b_2k_f - a_1b_1k_f - a_2b_1k_r + a_2b_2k_r & 2k_fa_1^2 + 2k_ra_2^2 & a_1k_f & a_1k_f & -a_2k_r & -a_2k_r \\ -k_f & -b_1k_f - \frac{k_R}{b_1+b_2} & a_1k_f & k_f + k_{t_f} + \frac{k_R}{(b_1+b_2)^2} & -\frac{k_R}{(b_1+b_2)^2} & 0 & 0 \\ -k_f & b_2k_f + \frac{k_R}{b_1+b_2} & a_1k_f & -\frac{k_R}{(b_1+b_2)^2} & k_f + k_{t_f} + \frac{k_R}{(b_1+b_2)^2} & 0 & 0 \\ -k_r & b_1k_r & -a_2k_r & 0 & 0 & k_r + k_{t_r} & 0 \\ -k_r & -b_2k_r & -a_2k_r & 0 & 0 & 0 & k_r + k_{t_r} \end{bmatrix}$$

<u>Full vehicle model parameter.</u>

Parameter	Value
Front suspension stiffness	k_r = 16088 (N/m)
Rear suspension average stiffness	k_r = 15401 (N/m)
Front tire vertical stiffness	k_{tf} = 16088 (N/m)
Rear tire vertical stiffness	k_{tr} = 15401 (N/m)
Anti-roll bar stiffness	$k_R = k_{Rf} = k_{Rr}$ = 20000 N m/rad
Front vertical damping	c_f = 2305 (N.s/m)
Rear vertical damping	c_r = 1226 (N.s/m)
Body vehicle mass	m = 930 kg
Front wheel mass	$m_f = m_1 = m_2$ = 31.5 kg
Rear wheel mass	$m_r = m_3 = m_4$ = 29 kg
Pitch moment of inertia	1243 (kg.m^2)
Roll moment of inertia	298 (kg.m^2)
Wheelbase	3.45 m

Parameter	Value
Body vertical motion coordinate	x (m)
Front right wheel vertical motion coordinate	x_1 (m)
Front left wheel vertical motion coordinate	x_2 (m)
Rear right wheel vertical motion coordinate	x_3 (m)
Rear left wheel vertical motion coordinate	x_4 (m)
Body pitch motion coordinate	θ (rad/s)
Body roll motion coordinate	φ (rad/s)
Road excitation at the front right wheel	y_1 (m)
Road excitation at the front left wheel	y_2 (m)
Road excitation at the rear right wheel	y_3 (m)
Road excitation at the rear left wheel	y_4 (m)
Body longitudinal mass moment of inertia	I_x = 298 (kg.m^2)
Body lateral mass moment of inertia	I_y = 1243 (kg.m^2)
Distance of C from front axle	a_1 = 1.7 (m)
Distance of C from rear axle	a_2 = 1.75 (m)
Distance of C from right side	b_1 = 0.7 (m)
Distance of C from left side	b_2 = 0.705 (m)

Solving 2nd order differential equation: -

$$[m]\ddot{\boldsymbol{x}} + [c]\dot{\boldsymbol{x}} + [k]\boldsymbol{x} = \boldsymbol{f}$$

If we define x as row vector, one can obtain new version of above equation as: -

$$\ddot{\boldsymbol{x}}[m] + \dot{\boldsymbol{x}}[c] + \boldsymbol{x}[k] = \boldsymbol{f}'$$

Multiplying equation with $[m]^{-1}$, we get new equation in form: -

$$\ddot{\boldsymbol{x}} + \dot{\boldsymbol{x}}[C] + \boldsymbol{x}[K] = \boldsymbol{F}$$

$where\ [C] = [c][m]^{-1}\ \ ,\ [K] = [k][m]^{-1}\ \ ,\boldsymbol{F} = \boldsymbol{f}'[m]^{-1}$

Now we can define $\boldsymbol{v} = \dot{\boldsymbol{x}}$ *to convert above into 2 first order equations.*

$$\dot{\boldsymbol{v}} + \boldsymbol{v}[C] + \boldsymbol{x}[K] = \boldsymbol{F} \text{ and } \boldsymbol{v} = \dot{\boldsymbol{x}}$$

If we define variable ***Y = [x, v]****, we get equation: -*

$$\boldsymbol{Y} = \boldsymbol{Y}A + B$$

Where $A = \begin{bmatrix} O & I \\ -K & -C \end{bmatrix}$ *and* $B = [o, \boldsymbol{F}]$ *; O and I are zero and identity matrix of size 7 and o is zero row vector of size 7.*

Using MATLAB function ode45 or using numerical techniques like Euler's method, Runge-Kutta method, we can get our solutions defining time span and initial state of system.

Also, we can get eigen values of A using eig() function in MATLAB which can help us get 14 modes of vibration **–** *>2 for each degree of freedom.*

Result-

We get 14 eigen values: - lambda =

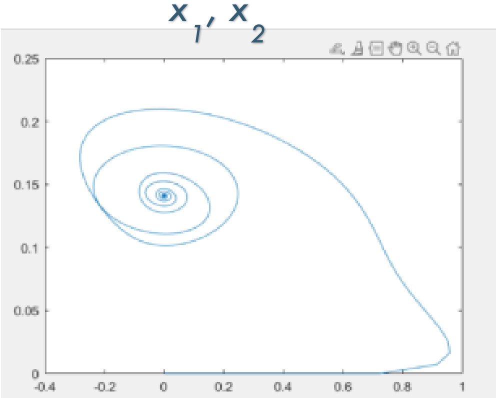
{-75.6135 + 0.0000i, -40.3310 + 6.8642i , -40.3310 - 6.8642i,-24.4211 +20.8686i ,-24.4211 -20.8686i ,-22.5924 +23.6151i ,-22.5924 -23.6151i ,-11.1494 + 0.0000i , -0.7690 + 6.0002i ,-0.7690 - 6.0002i ,-1.3302 + 9.2293i ,-1.3302 - 9.2293i ,-0.6493 + 7.8319i ,-0.6493 - 7.8319i}

From negative real components of eigen value and non-zero value of complex component, we can predict that phase diagram would be inward spiral.

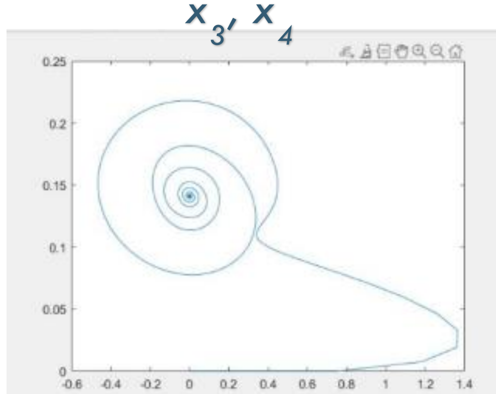
We can observe phase plots (x vs v) for $y_1 = y_2 = y_3 = y_4$

$y_i = Sin(0.01t)$

x_1, x_2



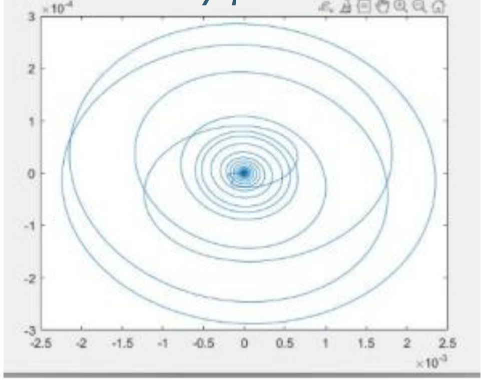
x_3, x_4



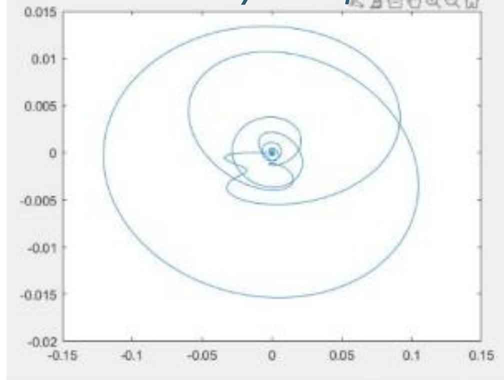
As it can be seen, phase plot of front and rear wheels are identical to each other.

Overall nature is spiral inward as expected

Body pitch ϑ



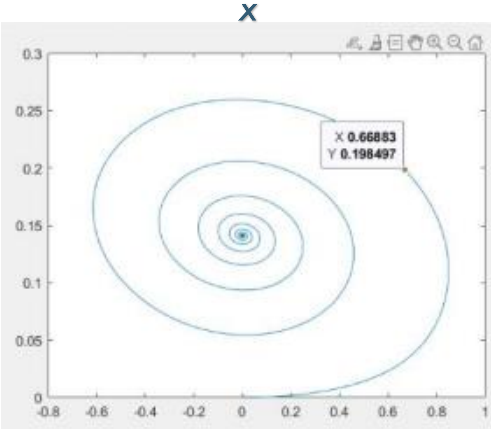
Body roll φ



The body pitch and roll is relatively more elliptical in nature about centre 0 which is equilibrium points also

Overall nature is spiral inward as expected

x

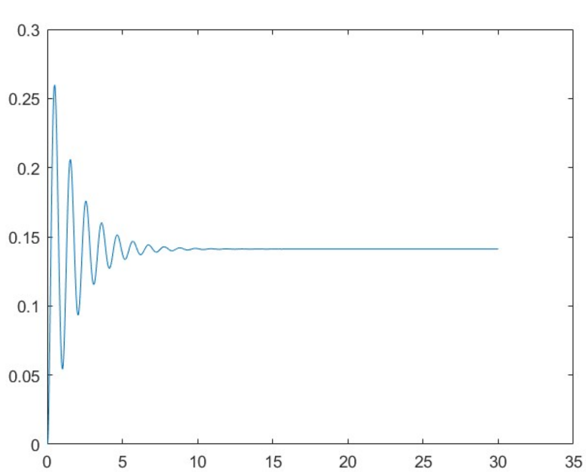


Overall nature is spiral inward as expected but centre of spiral is at non-zero x ~ 0.141 and v = 0 which is obvious as our excitations are non-zero and varies with time.

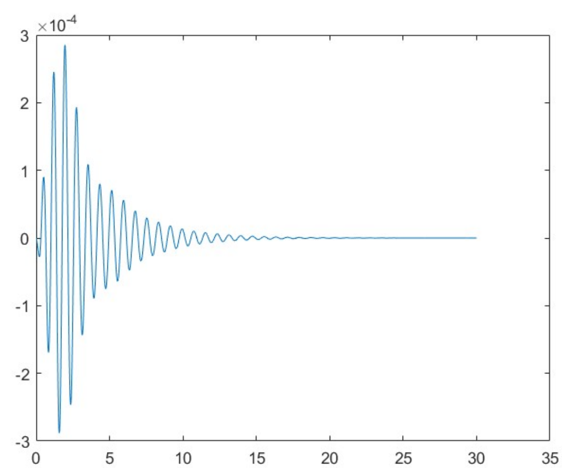
Note that our graphs are phase plots of q_i *vs* $\dot{q}_i$ *for 7 degree of freedoms- body bounce* x, *body roll* φ, *body pitch* θ, *wheels hop* x_1, x_2, x_3, *and* x_4

Moreover, front wheels and back wheels have similar behavior in all as I should be by symmetry. Following are q_i *vs t graphs* $y_i = Sin(0.01t)$

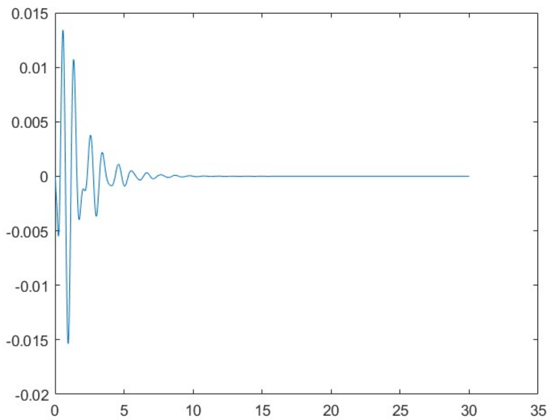
$x\ v/s\ t$



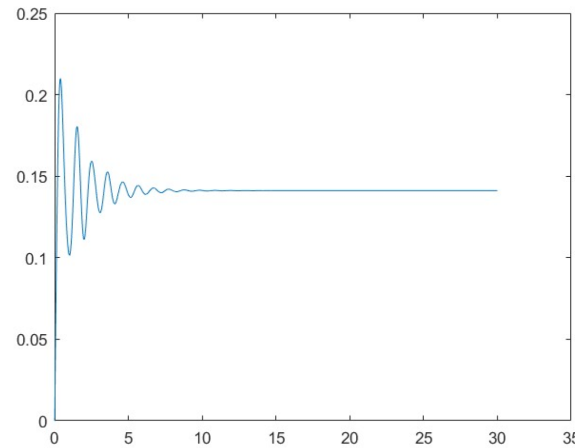
$\varphi\ v/s\ t$



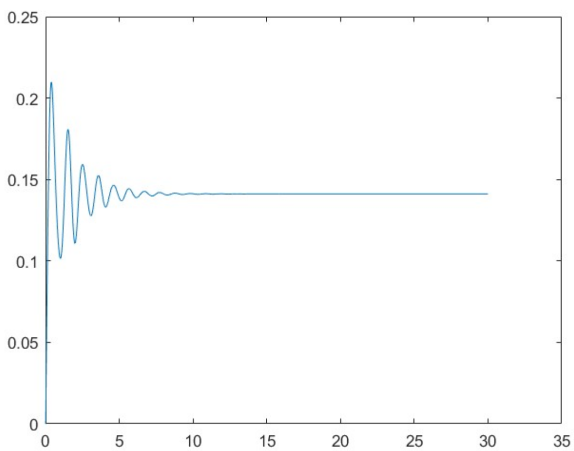
$\theta\ v/s\ t$



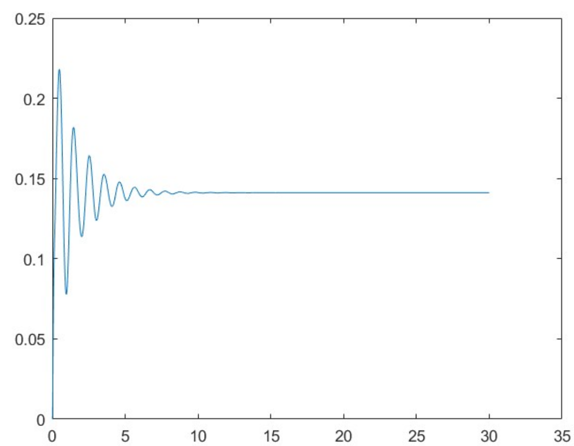
$x_1\ v/s\ t$



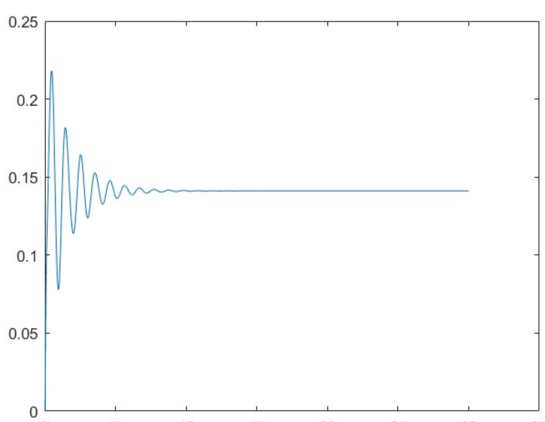
$x_2\ v/s\ t$



$x_3\ v/s\ t$



$x_4\ v/s\ t$

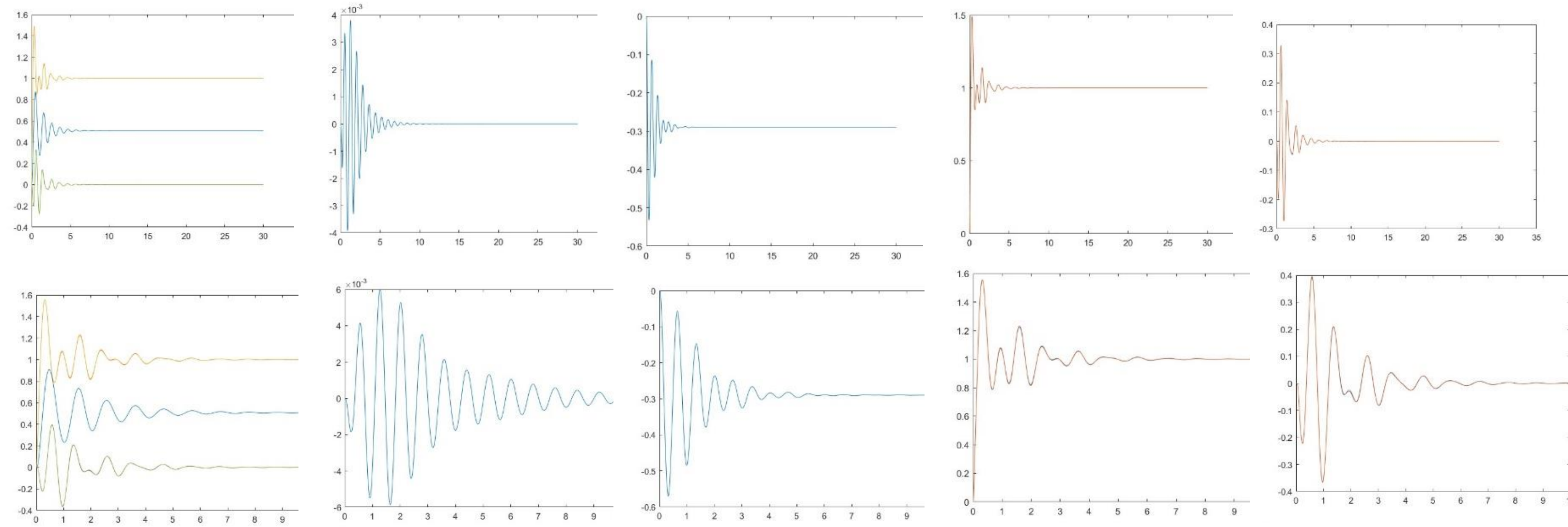


*The example was solved using Euler's method with initial condition all 0. The variation with time was damped sinusoidal. The vertical displacements (x, x1, x2, x3, x4) tend to 0 in case 1 tends to 0.141 in case 2. The angular displacements (*body roll φ and body pitch θ) tend to 0 in both cases which is obviously expected as system would tend to stabilize itself along the symmetry plane.

Moving on to better numerical methods and comparing results with Euler. (y1 = y2 = 1 and y3 = y4 = 0)

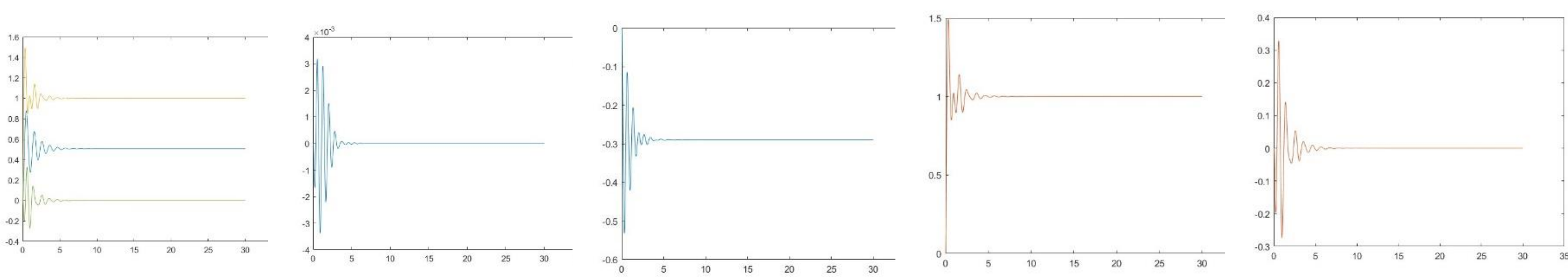
We used self-implemented Runge-Kutta method and euler method for constant slope path and compared results.

Vertical displacements vs t | ***Body roll φ vs t*** | ***Body pitch ϑ vs t*** | x_1, x_2 vs t | x_3, x_4 vs t



As it can be observed, behavior estimated by Runge-Kutta is similar as compared to Euler's method and one can see that body roll ultimately subsidize while body pitch stabilizes at non zero value due to slope path. The vertical displacements stabilize s.t. front wheel tends to y_front and back wheel tends to y_rear and ***x*** *stabilize midway b/w front and rear wheel at 0.5.*

Effect of Anti-roll spring

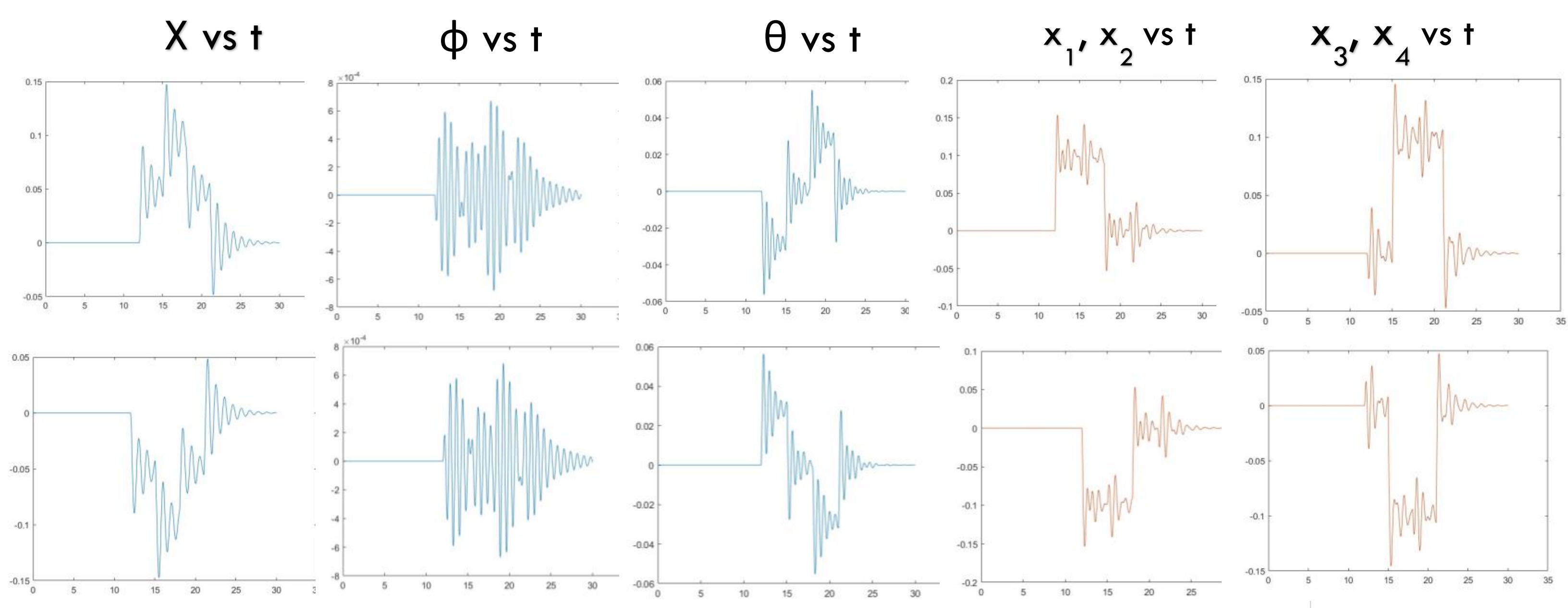


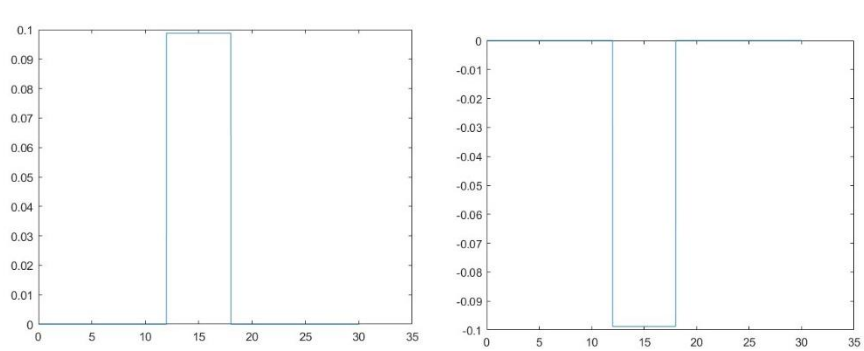
*As we can see that for KR=0,plots of vertical displacement is not visible in general graph while one can see that body roll stabilizes fast than before from graph of **φ**.*

Note that we have not involved y(t) till now. Next up is simulation where y varies with t.

Simulation of bumps (positive and negative amplitude) on horizontal path

Below are attached simulations of Simulation results of sinusoidal bump in straight path with negative and positive amplitudes respectively. The bump height was -0.1m in case 1 and 0.1 m in case 2 which was encountered by front tire during 12 to 18 s and back tire during 15 to 21 s in simulation of 30s journey. The car speed is assumed to be constant = 1.4/3 =0.467 m/s during simulation.





Path for negative and positive amplitudes.

This is with respect to front tire while it will be delayed by 3 seconds for rear tires

Glossary

The vehicle can have an antiroll bar in front and in the back, with a torsional stiffness k_{R_f} and k_{R_r}.

Using a simple model, the antiroll bar provides a torque M_R proportional to the roll angle φ.

$$\begin{aligned} M_R &= -\left(k_{R_f} + k_{R_r}\right)\varphi \\ &= -k_R\varphi \end{aligned}$$

However, a better model of the antiroll bar reaction is

$$M_R = -k_{R_f}\left(\varphi - \frac{x_1 - x_2}{w_f}\right) - k_{R_f}\left(\varphi - \frac{x_4 - x_3}{w_r}\right).$$

Most cars only have an antiroll bar in front. For these cars, the moment of the antiroll bar simplifies to

$$M_R = -k_R\left(\varphi - \frac{x_1 - x_2}{w}\right)$$

if we use (as per model standards)

$$\begin{aligned} w_f &\equiv w = b_1 + b_2 \\ k_{R_f} &\equiv k_R. \end{aligned}$$

MATLAB code for reference: -

```
m=930;
mf=31.5;
mr=29;
Ix=298;
Iy=1243;
ktf=160880;
ktr=154010;
kf=16088;
kr=15401;
kR=20000;
cf=2305;
cr=1226;
a1=1.7;
a2=1.75;
b1=0.7;
b2=0.705;
y1=0;
y2=0;
y3=0;
y4=0;
a2=1.75;
b1=0.7;
b2=0.705;
y1=0;
y2=0;
y3=0;
y4=0;
```

```
C = c*minv;
K = k*minv;
F = f*minv;



O=zeros(7,7);
I=eye(7);
o = zeros(1,7);

A = [O I;-1*K -
1*C];
lambda=eig(A);

B = [o;F];
```

```
%This is ode45 method
vo = ones(1,14);

g = @(t,x,Ka,Ca,Fa)[(x(8:14)) ; -
Ka*x(1:7)-Ca*x(8:14)+Fa'];

[t,xa] = ode45(@(t,x)
g(t,x,K,C,F),[0 10],vo);
```

```
mass = [m 0 0 0 0 0 0;
    0 Ix 0 0 0 0 0;
    0 0 Iy 0 0 0 0;
    0 0 0 mf 0 0 0;
    0 0 0 0 mf 0 0;
    0 0 0 0 0 mr 0;
    0 0 0 0 0 0 mr];

minv = [1/m 0 0 0 0 0 0;
    0 1/Ix 0 0 0 0 0;
    0 0 1/Iy 0 0 0 0;
    0 0 0 1/mf 0 0 0;
    0 0 0 0 1/mf 0 0;
    0 0 0 0 0 1/mr 0;
    0 0 0 0 0 0 1/mr];

k11=2*kf+2*kr;
k12=b1*kf-b2*kf-b1*kr+b2*kr;
k13=2*a2*kr-2*a1*kf;
k22=kR+b1*b1*kf+b2*b2*kf+b1*b1*kr+b2*b2*kr;
k23=a1*b2*kf-a1*b1*kf-a2*b1*kr+a2*b2*kr;
k24=-b1*kf-(kR/(b1+b2));
k25=b2*kf+(kR/(b1+b2));
k33=2*kf*a1*a1+2*kr*a2*a2;
k44=kf+ktf+(kR/((b1+b2)*(b1+b2)));
k55=kf+ktf+kR/((b1+b2)*(b1+b2));

k = [k11 k12 k13 -kf -kf -kr -kr;
    k12 k22 k23 k24 k25 b1*kr -b2*kr;
    k13 k23 k33 a1*kf a1*kf -a2*kr -a2*kr;
    -kf k24 a1*kf k44 -kR/((b1+b2)*(b1+b2))
0 0;
    -kf  k25 a1*kf -kR/((b1+b2)*(b1+b2)) k55
0 0;
    -kr b1*kr -a2*kr 0 0 kr+ktr 0;
    -kr -b2*kr -a2*kr 0 0 0 kr+ktr];

c11=2*cf+2*cr;
c12=b1*cf-b2*cf-b1*cr+b2*cr;
c13=2*a2*cr-2*a1*cf;
c22=b1*b1*cf+b2*b2*cf+b1*b1*cr+b2*b2*cr;
c23= a1*b2*cf-a1*b1*cf-a2*b1*cr+a2*b2*cr;
c33=2*cf*a1*a1 + 2*cr*a2*a2;

c = [ c11 c12 c13 -cf -cf -cr -cr;
    c12 c22  c23 -b1*cf b2*cf b1*cr -b2*cr;
    c13 c23 c33 a1*cf a1*cf -a2*cr -a2*cr;
    -cf -b1*cf a1*cf cf 0 0 0;
    -cf b2*cf a1*cf 0 cf 0 0;
    -cr b1*cr -a2*cr 0 0 cr 0;
    -cr -b2*cr -a2*cr 0 0 0 cr];

f = [0,0,0,y1*ktf,y2*ktf,y3*ktr,y4*ktr];
```

```
%This is Euler’s Method
u=3000;
vo = zeros(u,14);
t = zeros(u,1);
w=0.004;
for i = 1:u
    y1=sin(w*u);
    y2=sin(w*u);
    y3=sin(w*u);
    y4=sin(w*u);
    f = 
[0,0,0,y1*ktf,y2*ktf,y3*ktr,y4*ktr];
    F = f*minv;
    an = zeros(1,14);
    an(1:7) = vo(i,8:14);
    an(8:14)=-vo(i,1:7)*K-
vo(i,8:14)*C+F;
%     o=[(vo(i,8:14))' ;(-vo(i,1:7)*K-
vo(i,8:14)*C+F)']
    vo(i+1,:) = vo(i,:) + 0.01.*an;
    t(i+1)=t(i)+0.01;
end
```

```
figure()
plot(t,ij)
figure()
plot(t,vo(:,1))
figure()
plot(t,vo(:,2))
figure()
plot(t,vo(:,3))
figure()
plot(t,vo(:,4),t,vo(:,5))
figure()
plot(t,vo(:,6),t,vo(:,7))
```

```
%This is Runge Kutta Method
u=3000;
vo = zeros(u,14);
t = zeros(u,1);
w=0.01;

for i = 1:u
y1=1;
y2=1;
y3=0;
y4=0;
f = 
[0,0,0,y1*ktf,y2*ktf,y3*ktr,y4*ktr];
F = f*minv;
    an = zeros(1,14);
    an(1:7) = vo(i,8:14);
    an(8:14)=-vo(i,1:7)*K-
vo(i,8:14)*C+F;
    temp = vo(i,:)+w.*an/2;
    an2(1:7) = temp(8:14);
    an2(8:14)=-temp(1:7)*K-
temp(8:14)*C+F;
    temp = vo(i,:)+w.*an2/2;
    an3(1:7) = temp(8:14);
    an3(8:14)=-temp(1:7)*K-
temp(8:14)*C+F;
    temp = vo(i,:)+w.*an3;
    an4(1:7) = temp(8:14);
    an4(8:14)=-temp(1:7)*K-
temp(8:14)*C+F;
    vo(i+1,:) = vo(i,:) + 
(w/6).*(an+2*an2+2*an3+an4);
    t(i+1)=t(i)+0.01;
end
```

References: -

[1] M J Griffin, Handbook of Human Vibration

[2] O. Gundogdu, "Optimal seat and suspension design for a quarter car with driver model using genetic algorithms

[3] J. Y. Wong, Theory of Ground Vehicles

[4] A. Agharkakli, G. S. Sabet, A. Barouz, Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile

[5] A. Solhmirzaei, Sh. Azadi, R. Kazemi Road Profile Estimation Using Wavelet Neural Network and 7-DOF Vehicle Dynamic Systems