Comparison of Implied Risk-neutral Distribution with the Observed Stock Prices' Distribution

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Abstract

We propose to work on an exploratory project for this course where we plan to compare the option-market implied risk-neutral distributions of stock prices vs. the realized distribution of the same. We expect that the risk-neutral distribution will differ somewhat from the observed distribution because of risk-premia as well as the evolution of stock prices turning out differently than expected.

We plan to analyze and explore this across regimes (crisis periods vs. *the Great Moderation*), sectors (high volatility sectors such as Technology vs. stable sectors like Utilities) and market capitalization (large, medium and small-caps if option data is available).

Below, we briefly describe how we plan to implement this project and some literature from which our techniques have been inspired.

1 Introduction

Most of the existing literature on implied risk-neutral distributions uses some variant of the Breeden Litzenberger (1978) [1] method and several implementations, such as Figlewski (2012) [2] use its results to generate the implied probability density functions. However this method uses European option prices and most of the options on single stocks with data in the Ivy Database have the American exercise feature.

We did a literature survey to compute implied distributions from American options prices and evaluated the method as described in Tian (2010) [3] as a possible candidate. This method, though promising, is complicated in implementation and suffers from scalability issues.

Our team discussed this and other potential methods with Professor Emanuel Derman and he suggested the following approach for this problem.

1. Extract implied volatilities from the American option prices using any appropriate model
2. Calculate European option prices using this implied volatility surface
3. Use the Breeden Litzenberger method to generate the implied risk-neutral distribution

While calculating the option prices, we would use the interpolated risk-free rate r from the IVY database interest rate table and for the appropriate dividends, we would use the implied dividends using put-call parity as described by Professor Lipkin during the session on hard-to-borrow stocks.

2 Implementation

We intend to first begin with a subset of the liquid options universe with data available for

at least 8-10 years on the IVY database. On this data after cleaning for outliers and aggregating MBBO values for monthly-expiry options, we would implement the method described in the previous section to generate European option prices and the corresponding implied distribution curve.

A curve for each expiration will be generated for each trading date for all available expirations traded on that date. This will help us get a rich data set of implied distributions.

Now, while the implied distribution curves for each expiry as seen from several trading days are available, the price realized on each of those expiries is a *single instance of a random variable*. And hence, the inputs will be distributions whereas the outputs would be single-point realizations.

To map this set of outcomes to a realized distribution, we would use a VAR like methodology, essentially comparing how frequently a move expected to occur $< p\%$ times based on the implied distribution actually occurs.

We would then compare this real distribution with the implied distribution and see if we can identify any interesting behavior when clustered by sectors, market-capitalization or across different regimes.

3 Conclusion

Using the methodology described above, we would like to explore how and if the shape of the implied distribution of prices matches with the eventual outcome and if we can create a simple functional form for transforming one to the other with bounded errors.

Also, for specific cases surrounding events such as earnings and other market-moving news, we would like to see if there are interesting phenomena observed with the behavior of these curves.

References

[1] Breeden, D., & Litzenberger, R. (1978). Prices of State-Contingent Claims Implicit in Option Prices. Journal of Business. Retrieved from http://www.jstor.org/stable/2352653

[2] Figlewski, S. (2012). What is Risk Neutral Volatility? Available at SSRN 2183969, 1-45

[3] Tian, Y. (2011). Extracting Risk-Neutral Density and its Moments from American Option Prices. Journal of Derivatives.