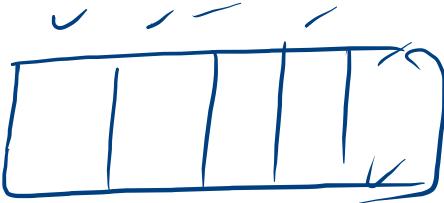
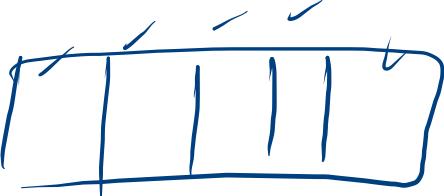


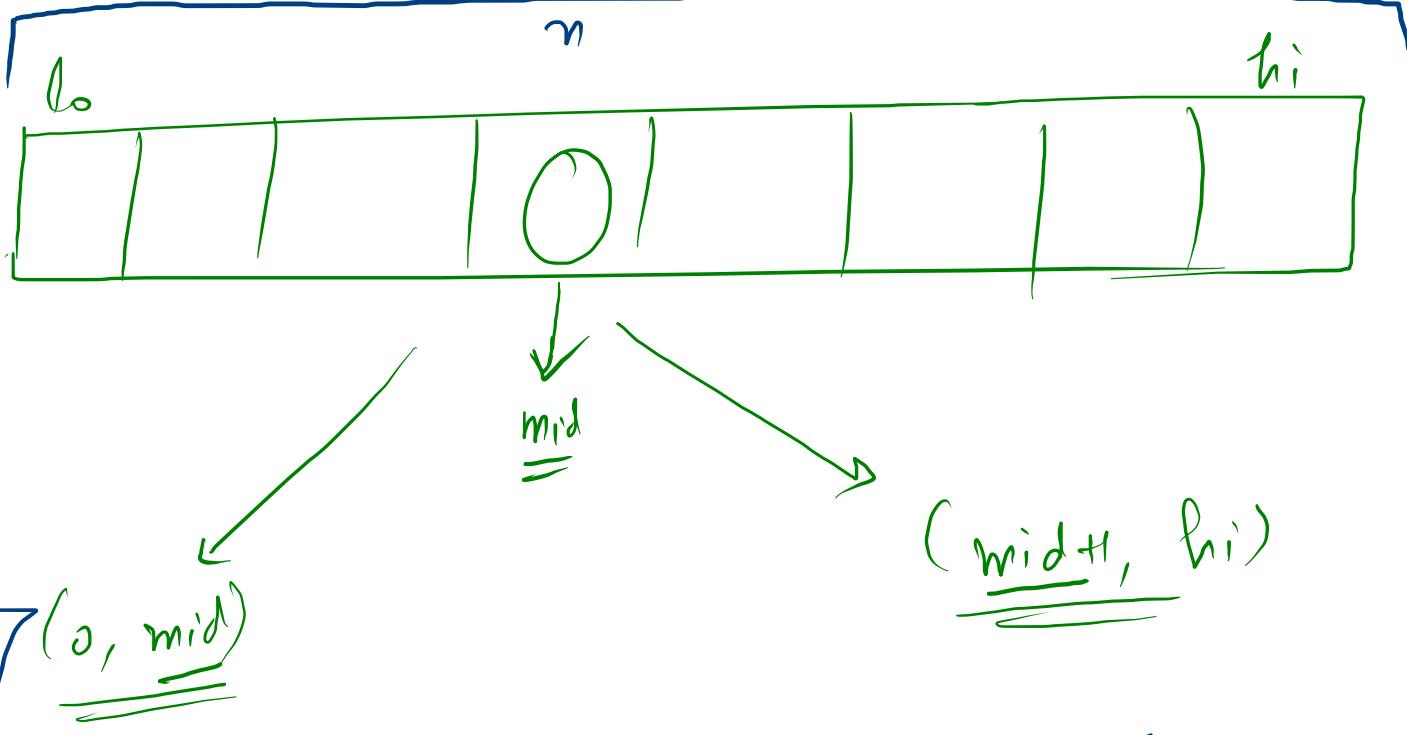
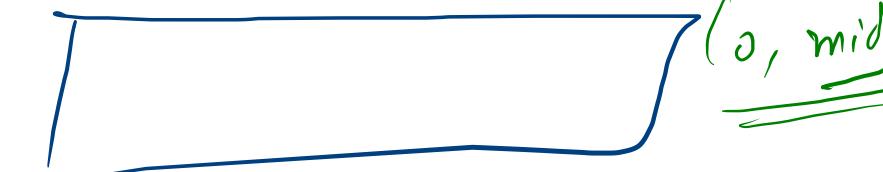
→ merge Sort



→ Quick Sort



→ Quick Select



```
public static int[] mergeSort(int[] arr, int lo, int hi) {  
    if(lo == hi){  
        int b[] = new int[1];  
        b[0] = arr[lo];  
        return b;  
    }  
    int mid = (lo+hi)/2;  
    int lp[] = mergeSort(arr,lo,mid);  
    int rp[] = mergeSort(arr,mid+1,hi);  
    return mergeTwoSortedArrays(lp,rp);  
}
```

Annotations on the code:

- The first block of code (base case) is annotated with  $O(1)$  and "constant time".
- The assignment of  $mid$  is annotated with  $O(1)$ .
- The assignments of  $lp[]$  and  $rp[]$  are annotated with  $\frac{n}{2}$  and checked marks.
- The final return statement is annotated with  $O(n)$  and checked marks.

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n$$

## MERGE SORT

↙

$n = \frac{n}{2^0}$

$n/2 = \frac{n}{2^1}$

$n/4 = \frac{n}{2^2}$

$n/8 = \frac{n}{2^3}$

⋮

$1 = \frac{n}{2^{K-1}}$

$\overbrace{\hspace{10em}}^{K \text{ times}}$

**Pow Smaller**

$$\frac{0 \rightarrow 10}{\overbrace{\hspace{10em}}^{5 \text{ times}}} \quad \frac{0 \rightarrow K}{\overbrace{\hspace{10em}}^{K+1}}$$

$$\frac{0 \rightarrow (K-1)}{\overbrace{\hspace{10em}}^{K \text{ times}}}$$

$T(n) = 2 \cdot T(\frac{n}{2}) + n$

$2 \cdot T(\frac{n}{2}) = 4 \cdot T(\frac{n}{4}) + n$

$4 \cdot T(\frac{n}{4}) = 8 \cdot T(\frac{n}{8}) + n$

⋮

$T(1) \Rightarrow$

$$T(n) = n + n + n + \dots$$

$\underbrace{\hspace{10em}}_{K \text{ times}}$

$$\boxed{T(n) = n \cdot K}$$

$$\cancel{n_1} = \cancel{n_2}$$

$n_1 \approx n_2$

$$T(n) = n[\log_2 n + 1]$$

$$T(n) = n \log_2 n + n$$

$$T.C. \Rightarrow O(n \log_2 n)$$

$$1 = \frac{n}{2^{K-1}}$$

$$2^{K-1} = n$$

taking

$$K-1 = \log_2 n$$

$K = \log_2 n + 1$



```
public static void quickSort(int[] arr, int lo, int hi) {  
    if(lo > hi){  
        return;  
    }  
    ✓ int pivot = arr[hi];  
  
    int pivotIdx = partition(arr, pivot, lo, hi);  
    quickSort(arr, lo, pivotIdx-1);  
    quickSort(arr, pivotIdx+1, hi);  
}
```

```

while(i < len)
    if(arr[i] < v)
        Swap
            i++
            j++
    } else {
        i++
    }
}

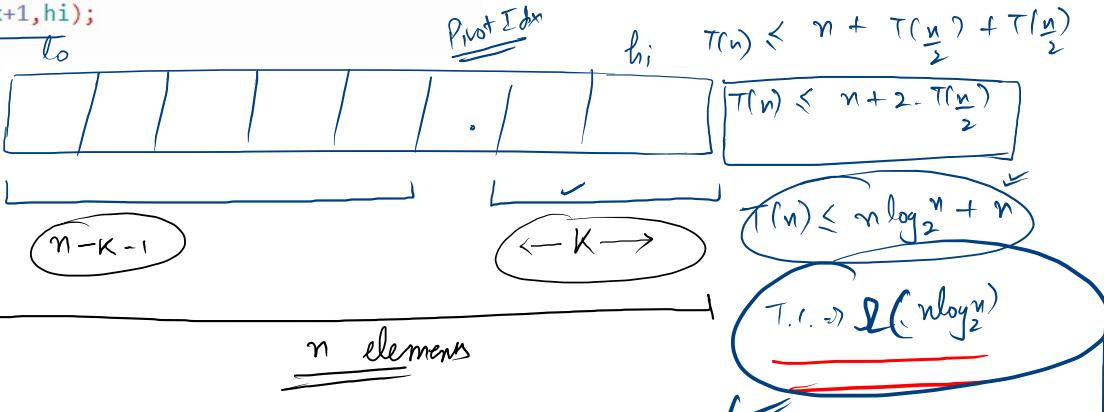
```

$$T(n) \Rightarrow n + T(n-k-1) + T(k)$$

$$\checkmark \underline{\underline{B.C.}} \rightarrow K = n/2$$

$$T(n) = n + T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right)$$

$$\text{Pivot } I_{dn} \quad f_i : T(n) \leq n + T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right)$$



W.C.

k = 0

$$T(n) = n + T(n-1)$$

$$\text{~~T(n-1)~~} = (n-1) + \text{~~T(n-2)~~}$$

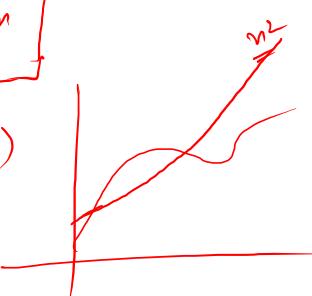
$$\cancel{T(n-2)} = (n-2) + \cancel{T(n-3)}$$

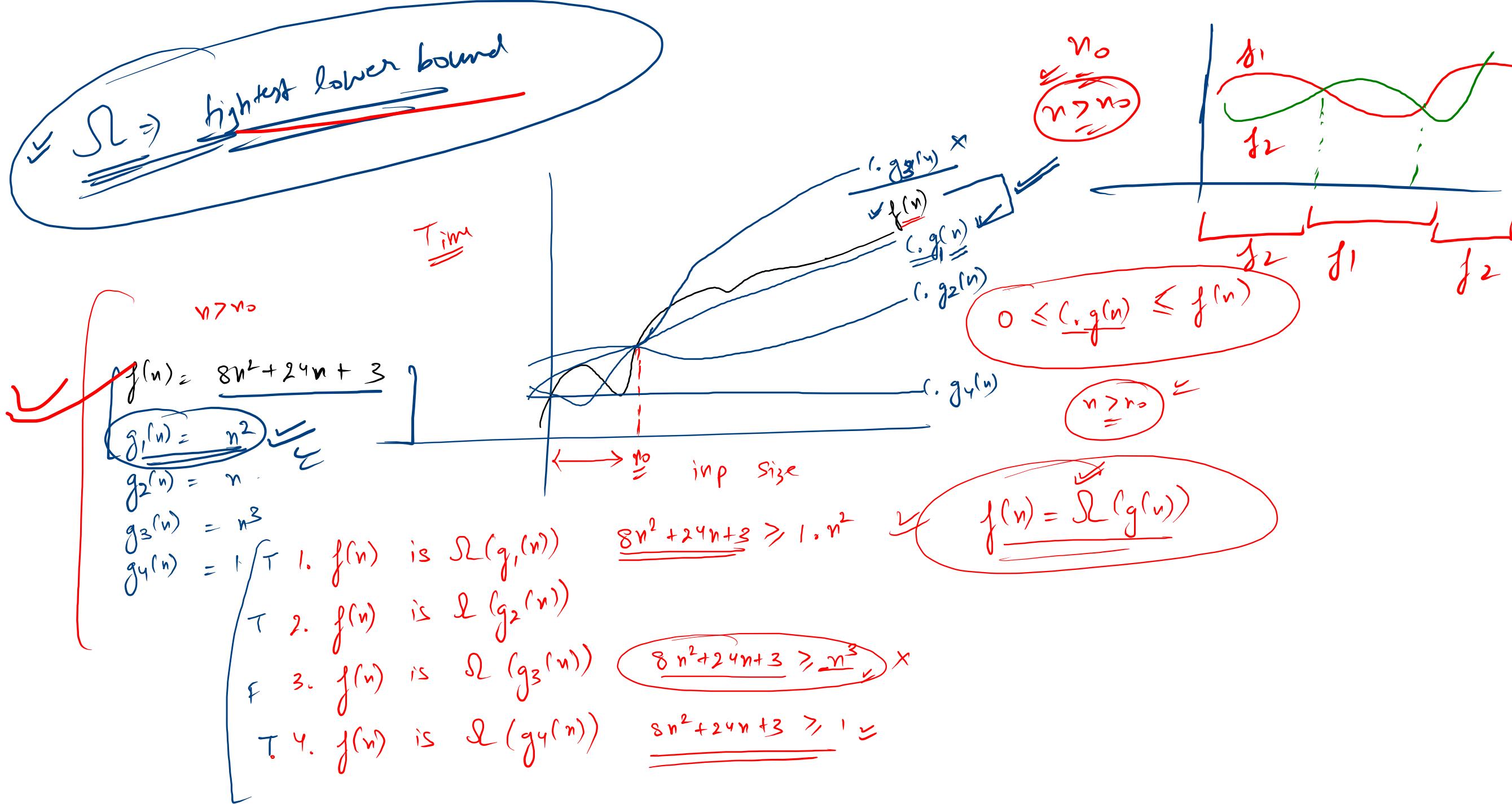
$$T(n) = \overbrace{n + (n-1) + (n-2) + \dots + 3 + 2 + 1 + 0}^{\text{Sum of first } n \text{ natural numbers}}$$

$$T(n) \geq \frac{n(n+1)}{2}$$

$$T(n) = \frac{n^2 + n}{2}$$

$$\frac{1}{T_0(\cdot, \varepsilon)} O(n^2)$$





```
public static int fib(int n){
```

```
    if(n==0 || n==1) { }
```

```
    return n;
```

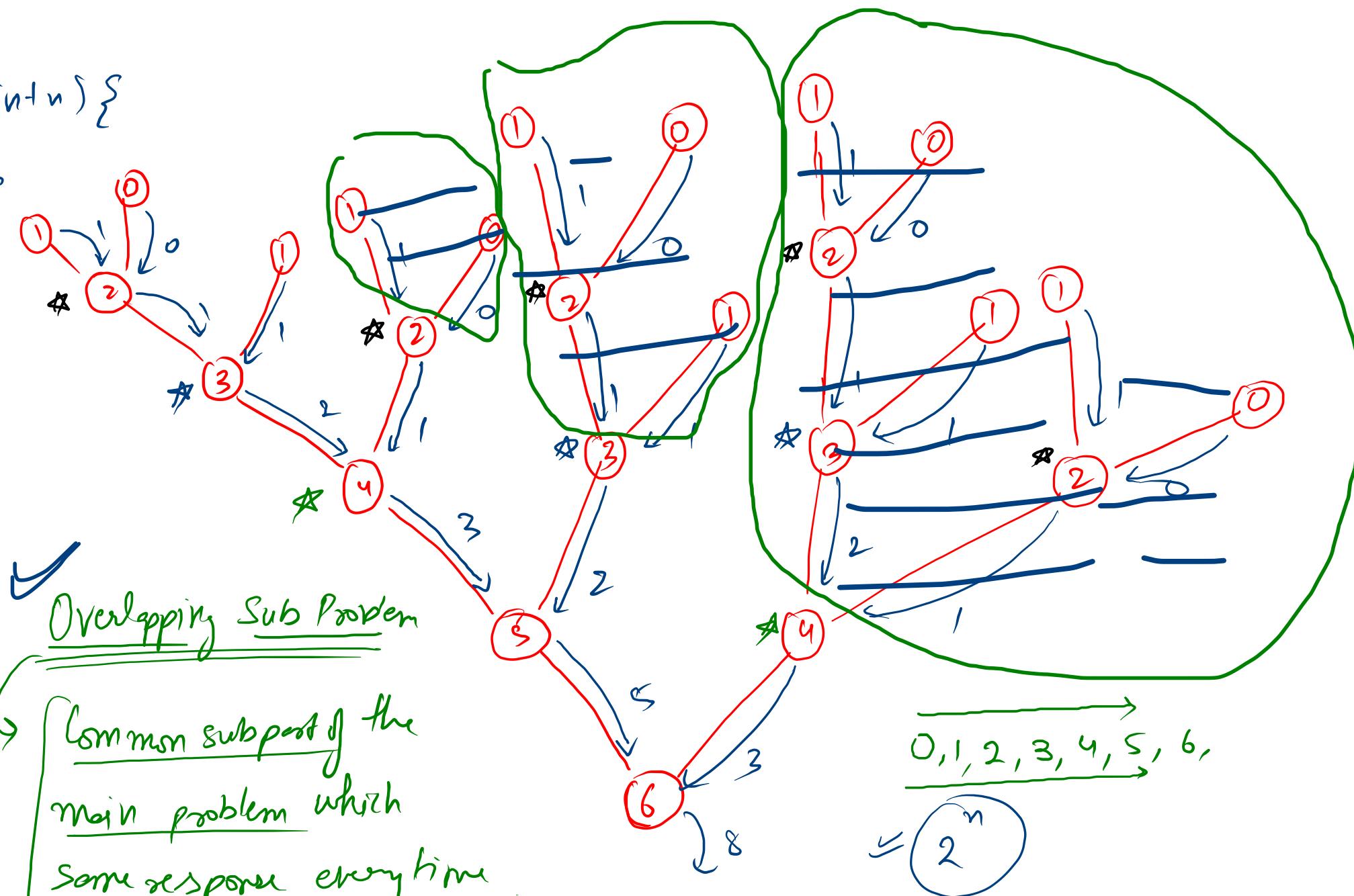
```
}
```

```
fibNm1 ← fib(n-1) ✓
```

```
fibNm2 ← fib(n-2) ✓
```

```
return fibNm1 + fibNm2 }
```

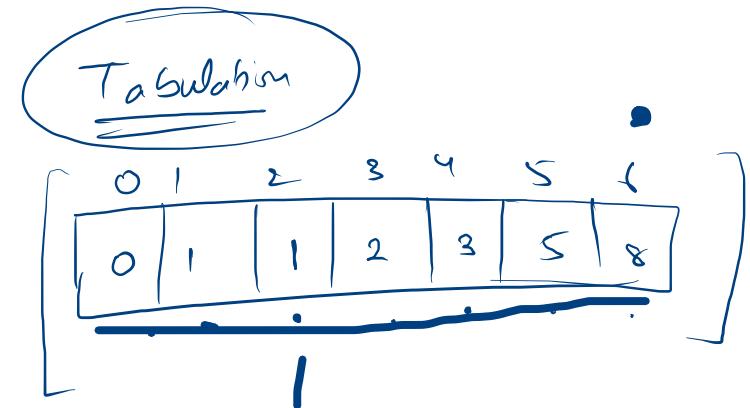
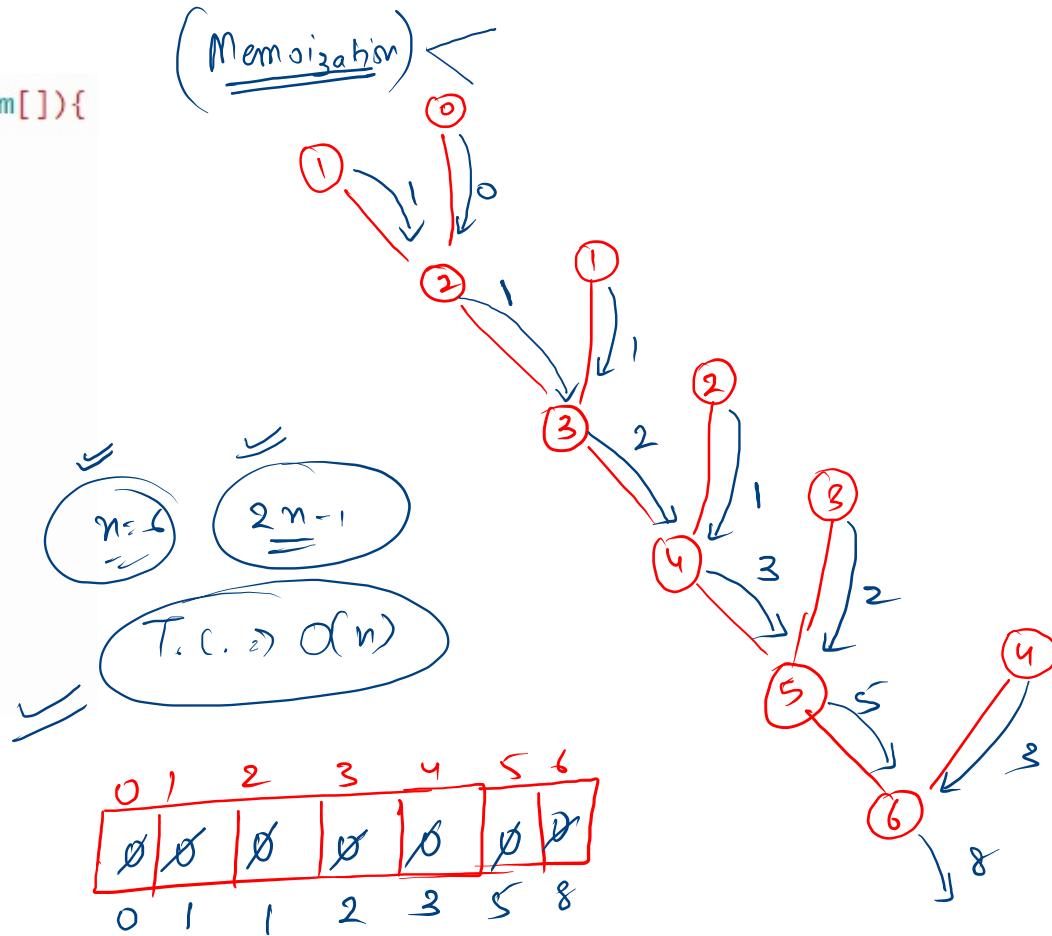
Overlapping Sub Problem  
Common subpart of the  
main problem which  
some response every time

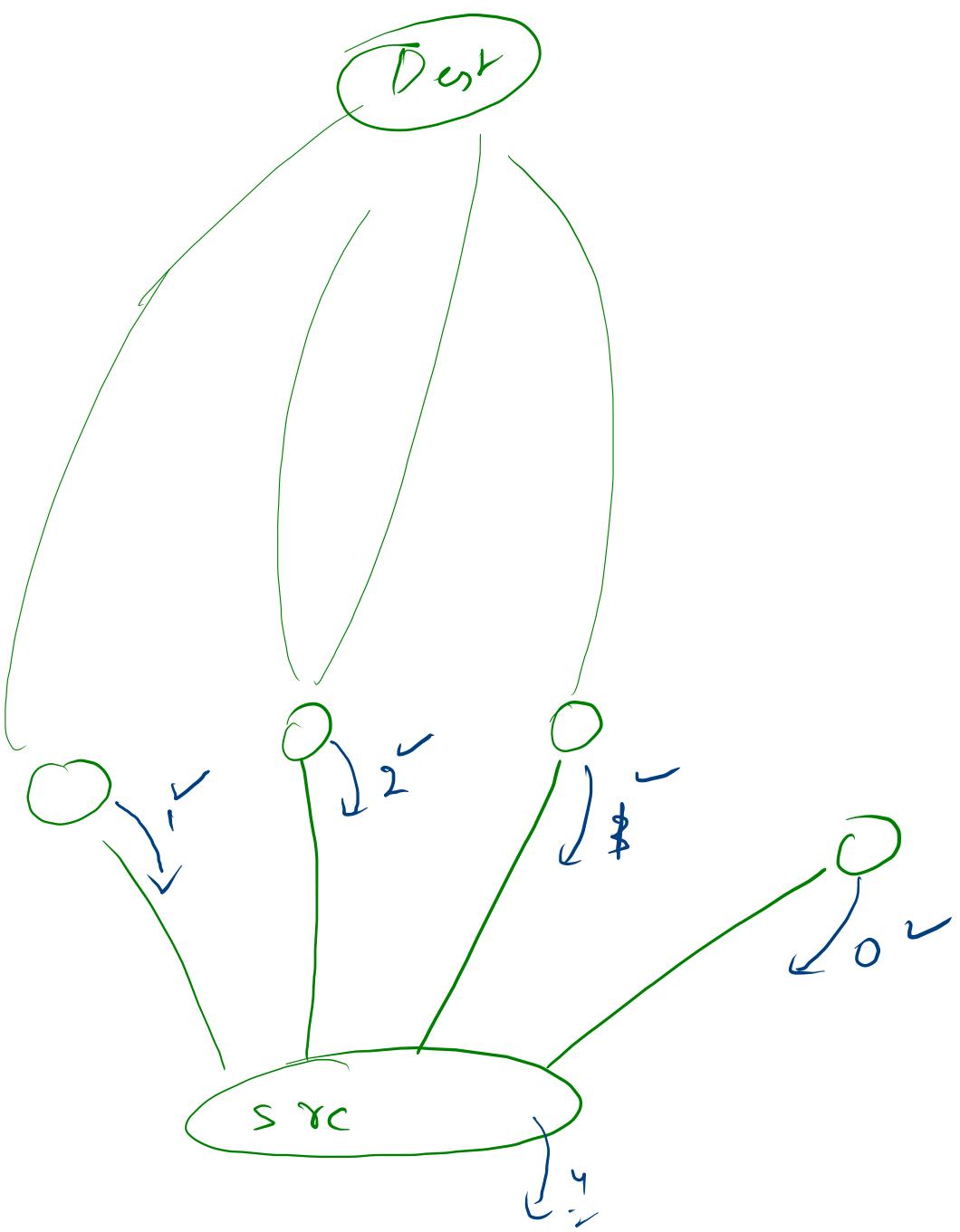


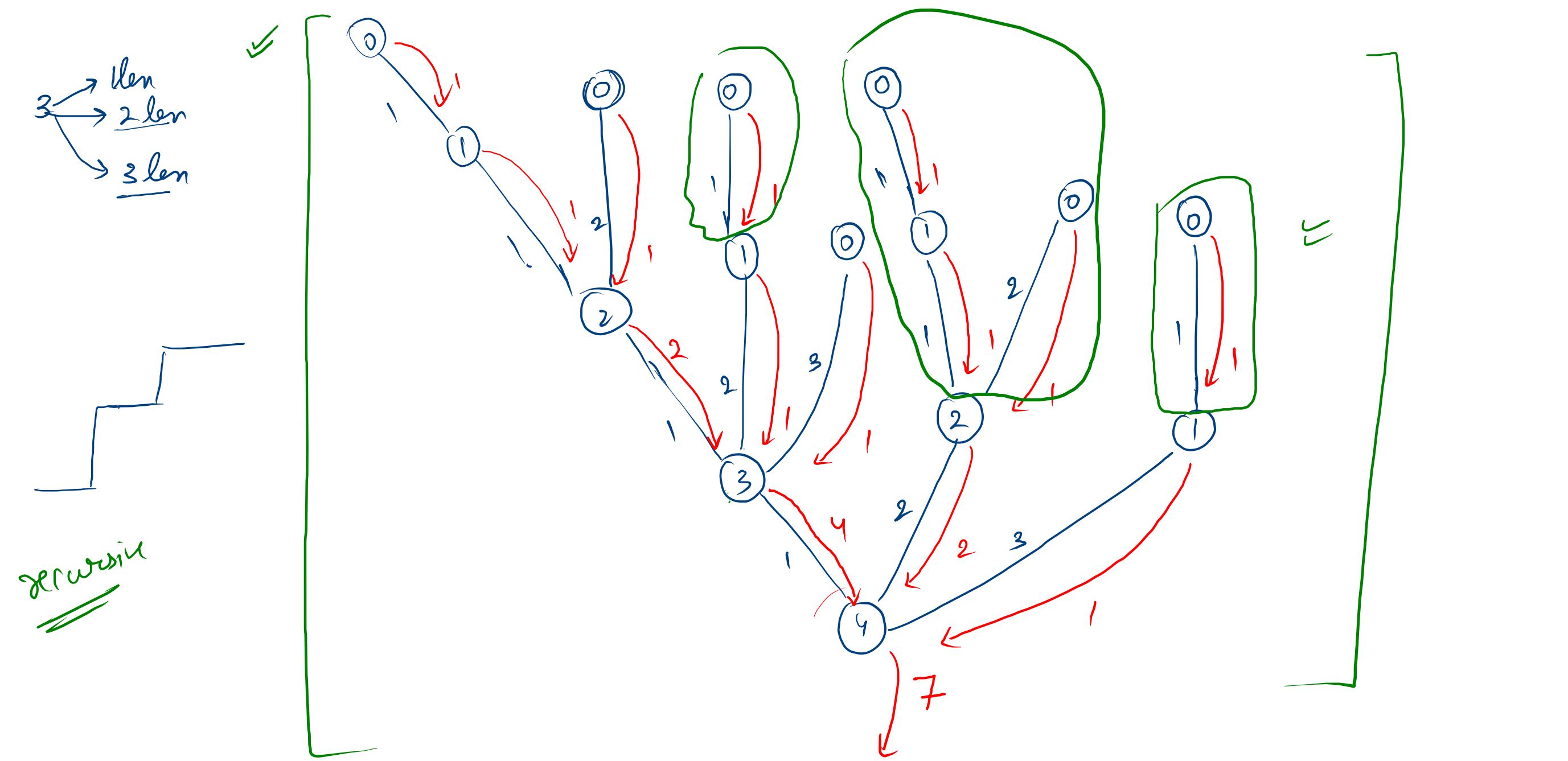
```

public static int fib1(int n, int mem[]){
    if(n == 1 || n == 0){
        return mem[n] = n;
    }
    if(mem[n] != 0){
        return mem[n];
    }
    int fibNm1 = fib1(n-1,mem);
    int fibNm2 = fib1(n-2,mem);
    int fibN = fibNm1+fibNm2;
    return mem[n] = fibN;
}

```







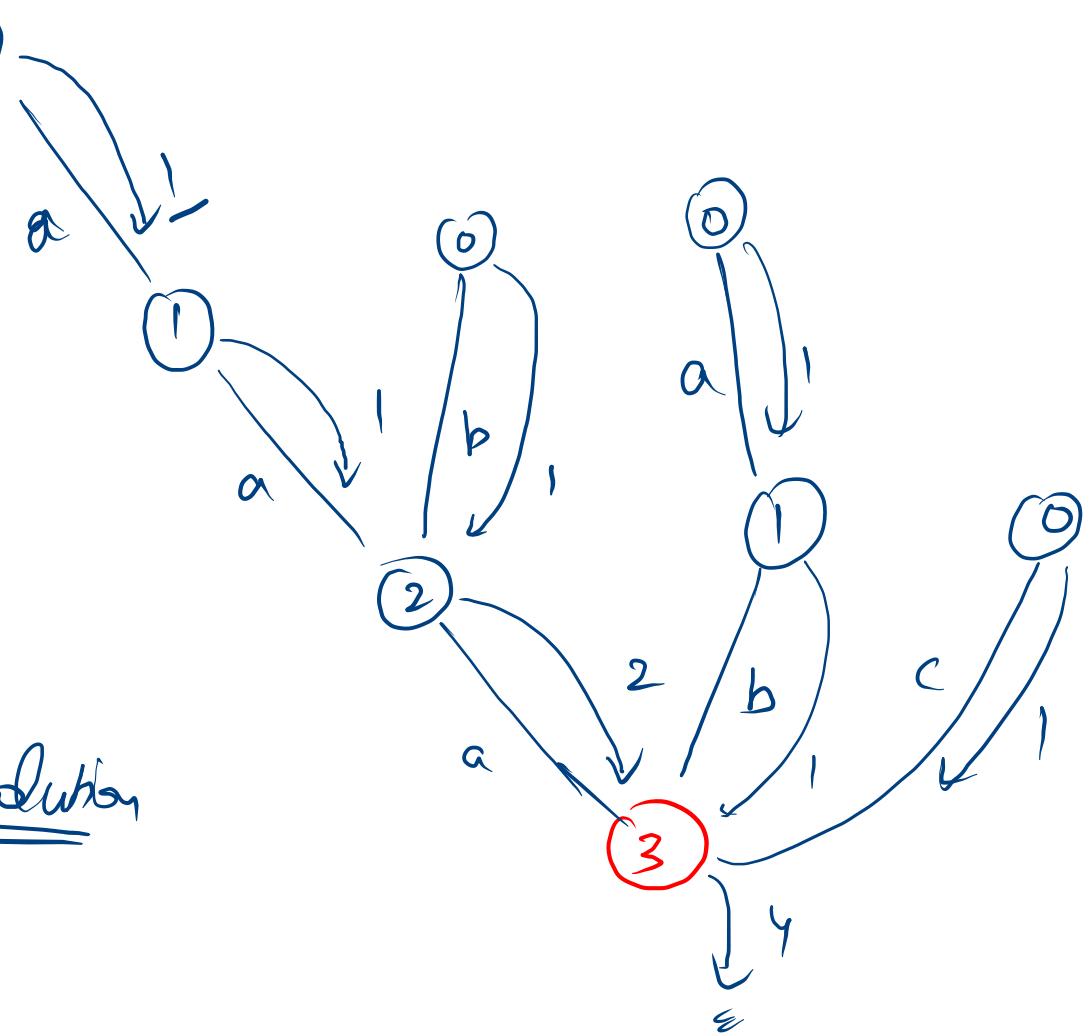
$\checkmark$   $n \leq 2$   $\checkmark$

```
public static int climbStairsRec(int n){  
    if(n == 0){  
        return 1;  
    }  
  
    int tways = 0;  
  
    if(n-1 >= 0){  
        tways += climbStairsRec(n-1);  $\rightarrow a$   
    }  
    if(n-2 >= 0){  
        tways += climbStairsRec(n-2);  $\rightarrow b$   
    }  
    if(n-3 >= 0){  
        tways += climbStairsRec(n-3);  $\rightarrow c$   
    }  
  
    return tways;  
}
```

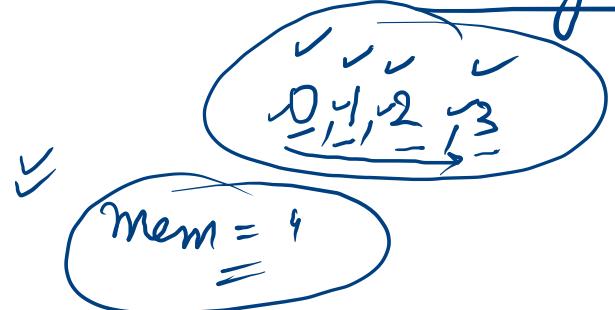
① Recursion

② Memoization

③ Tabulation



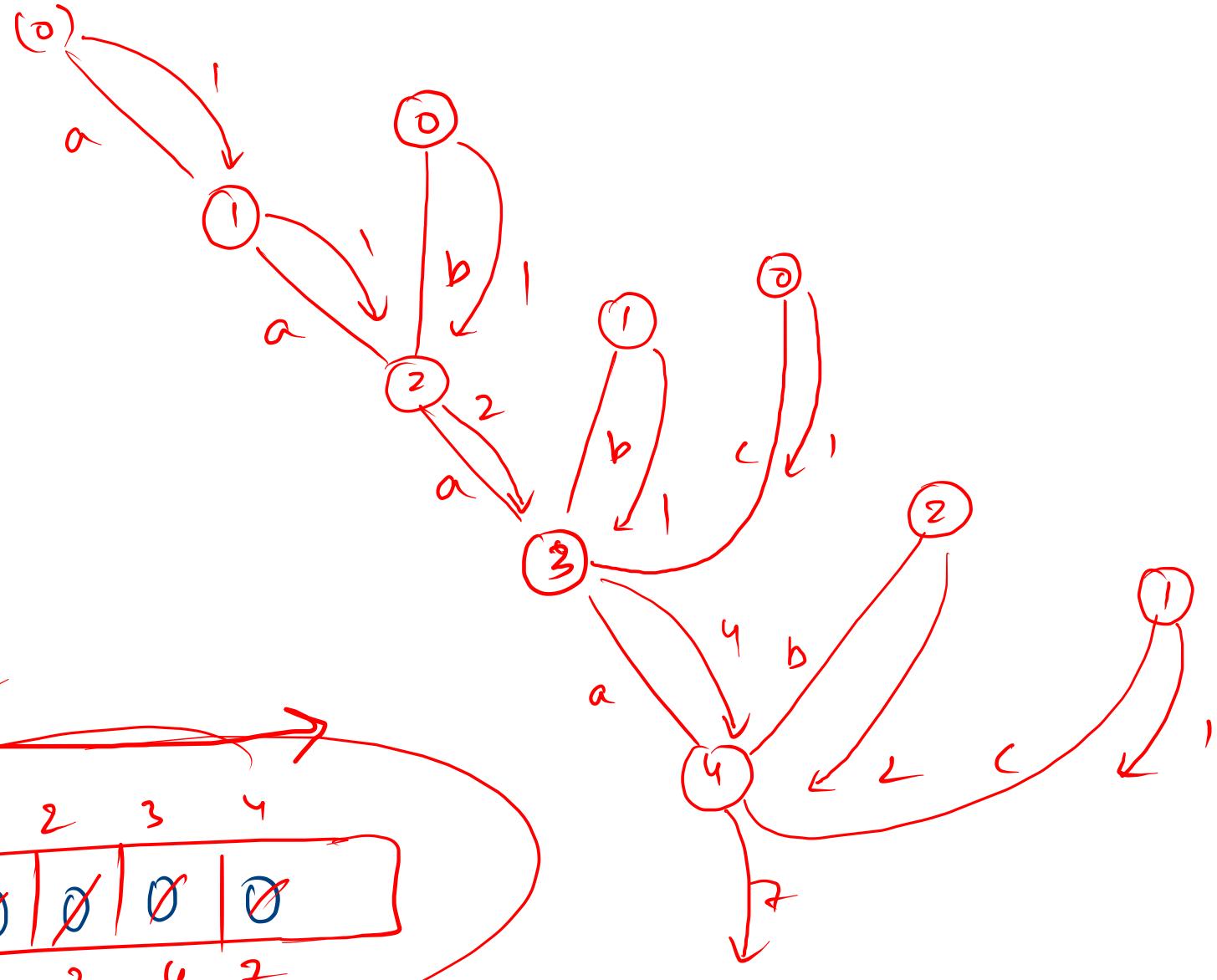
order of solution



```

public static int climbStairsM(int n, int mem[]){
    if(n == 0){
        return mem[n] = 1;
    }
    if(mem[n] != 0){
        return mem[n];
    }
    int tways = 0;
    if(n-1 >= 0){
        tways += climbStairsM(n-1,mem); a
    }
    if(n-2 >= 0){
        tways += climbStairsM(n-2,mem); b
    }
    if(n-3 >= 0){
        tways += climbStairsM(n-3,mem); c
    }
    return mem[n] = tways;
}

```



```

public static int climbStairsT(int n){
    int mem[] = new int[n+1];

    for(int i = 0 ; i <= n ; i++){
        if(i == 0){
            mem[i] = 1;
            continue;
        }

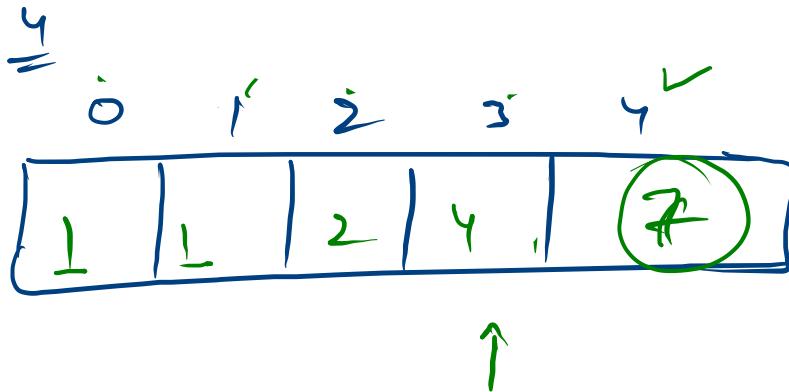
        int tways = 0;

        if(i-1 >= 0){
            tways += mem[i-1];
        }
        if(i-2 >= 0){
            tways += mem[i-2];
        }
        if(i-3 >= 0){
            tways += mem[i-3];
        }

        mem[i] = tways;
    }

    return mem[n];
}

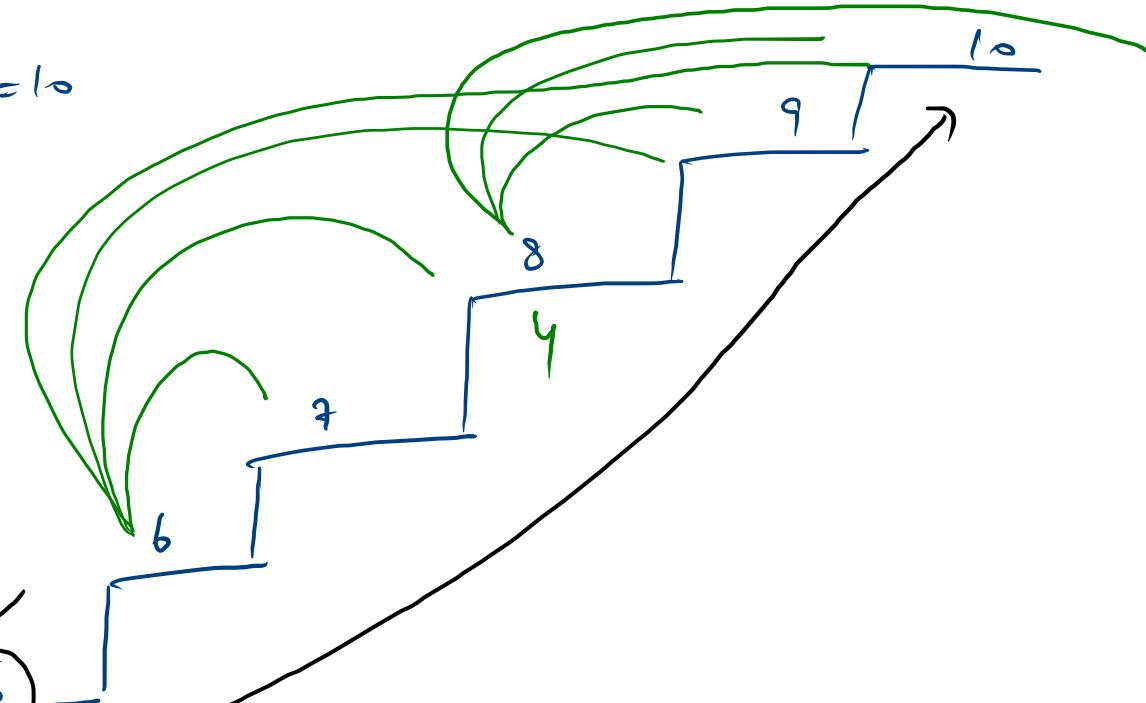
```



## TMoreS - VARIABLE JUMP

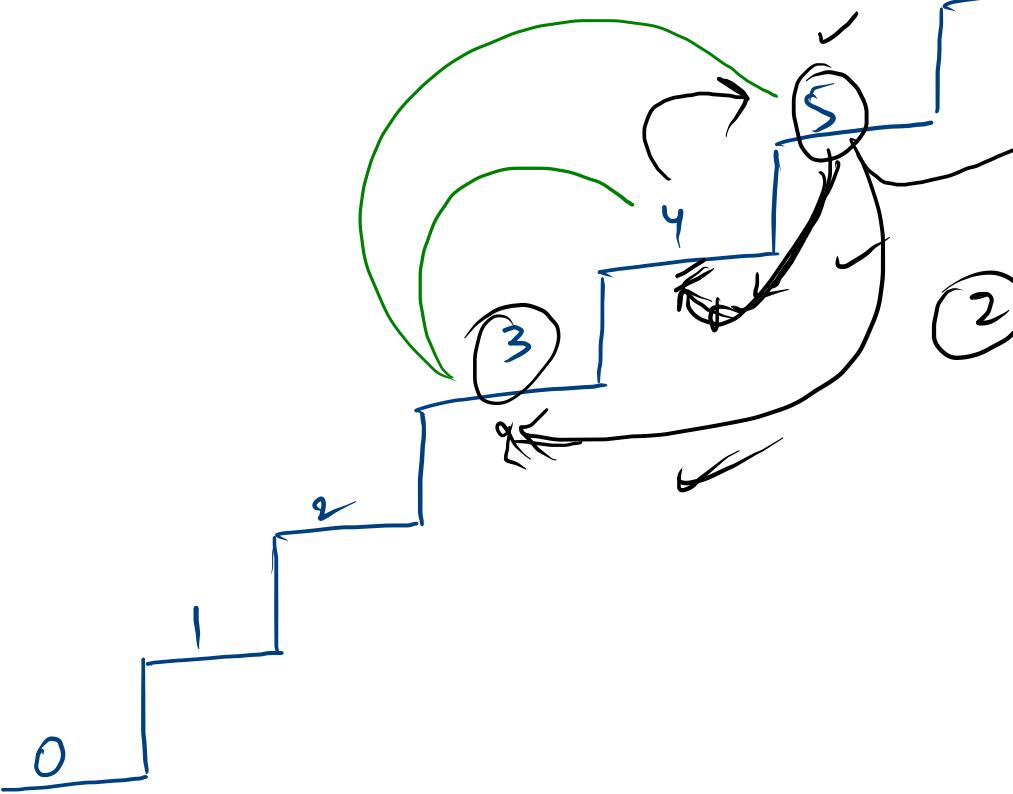
0	1	2	3	4	5	6	7	8	9
3	3	0	2	1	2	4	2	0	0

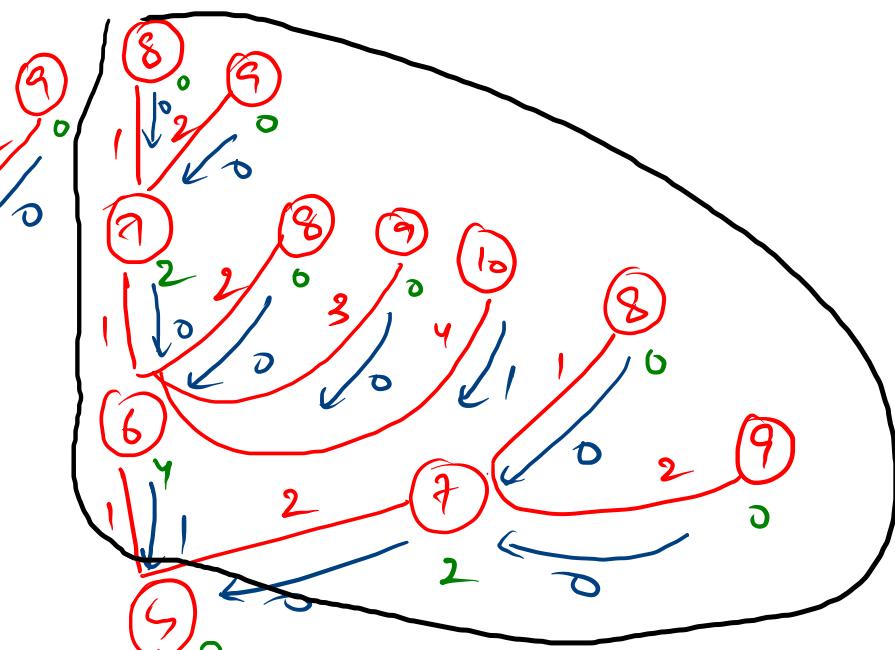
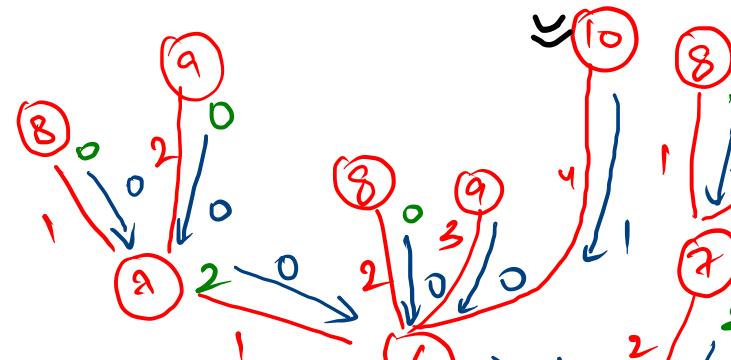
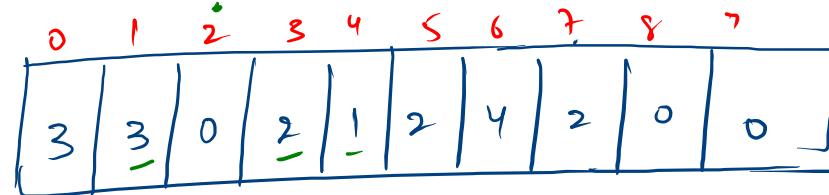
$m = 10$



$0^{th} \rightarrow 10^{th}$

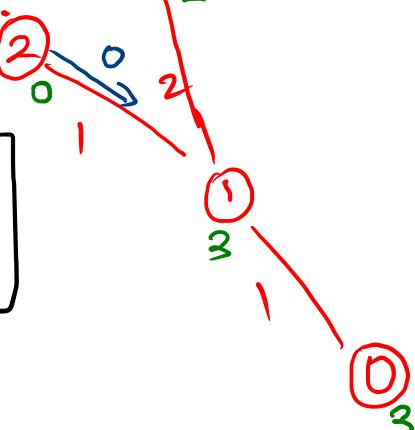
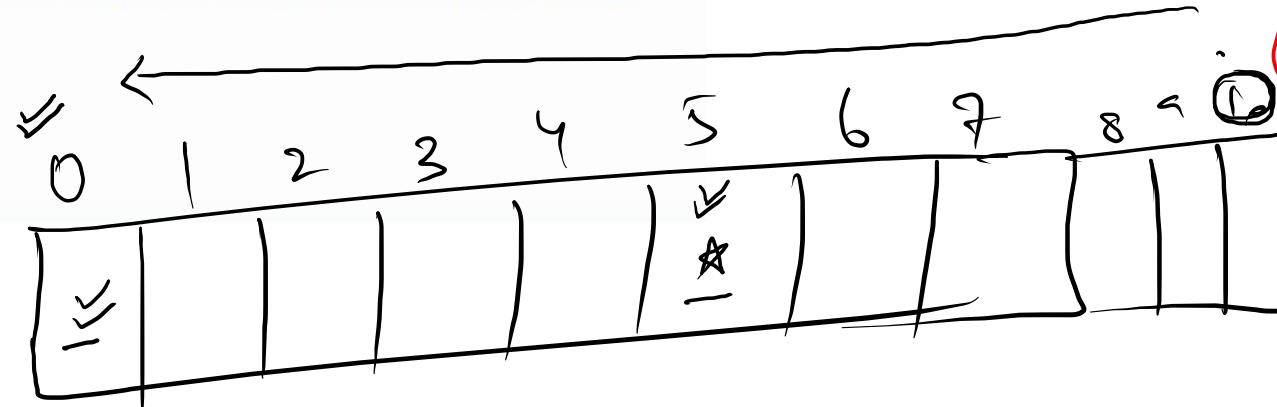
Ways





```
public static int ClimbStairsVarJumpRec(int i,int moves[]){
    if(i == moves.length){
        return 1;
    }
    int tways = 0;
    int maxJump = moves[i];
    for(int len = 1 ; len <= maxJump ; len++){
        if(i+len <= moves.length){
            tways += ClimbStairsVarJumpRec(i+len,moves);
        }
    }
    return tways;
}
```





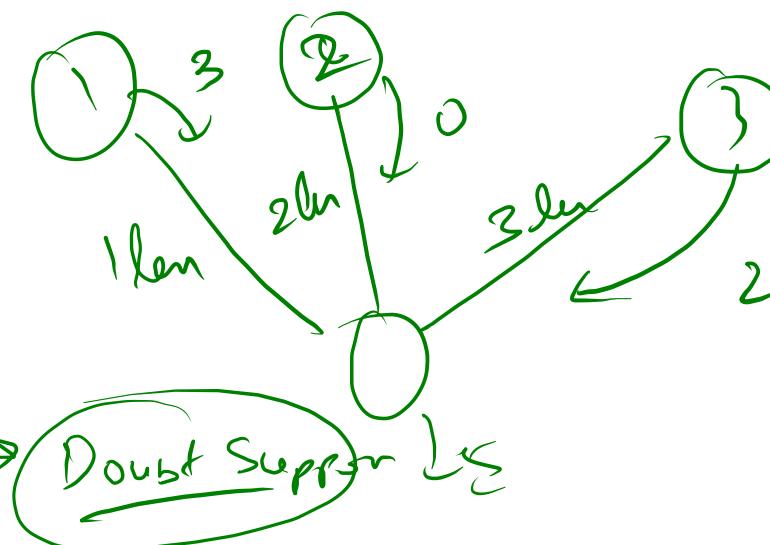
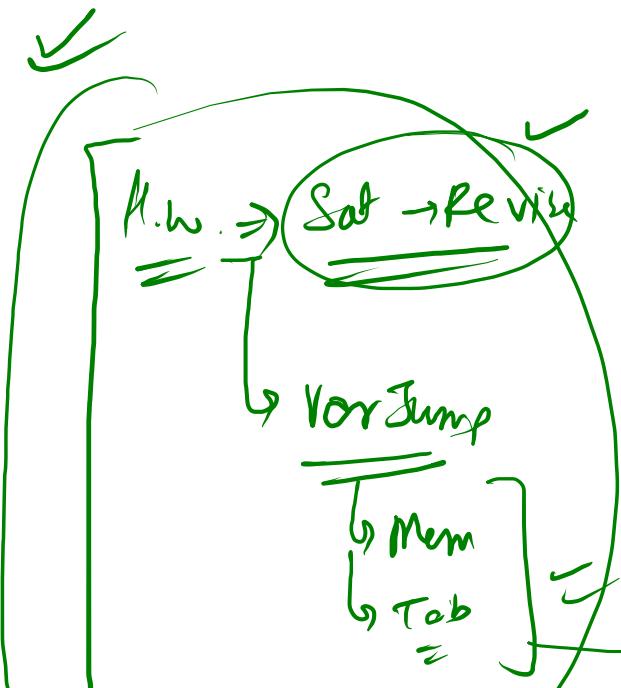
memo =

0	1	2	3	4	5	6	7	8	9
3	3	0	2	1	2	0	2	0	0

memo =

0	1	2	3	4	5	6	7	8	9	10
5	3	0	2	1	1	1	0	0	0	1

- ①  $\text{mem} \rightarrow [n+1]$
- ② order of solution  
right to left
- ③ find ans  $\rightarrow \text{mem}[0]$



~~May~~ May 12 — May

1/2

June 1

July - 1

Aug 1

3.5

4

5