

add (val)
remove ()] $\Rightarrow \boxed{O(\log n)}$, $n = \text{no. of elements in PQ.}$

peek ()
size ()] $\Rightarrow \textcircled{O(1)}$ Constant time *

arr:

✓ 0	✓ 1	✓ 2	✓ 3	• 4	• 5	✗ 6	• 7	• 8	• 9	✗ 10	• 11	• 12
<u>12</u>	<u>62</u>	<u>22</u>	<u>15</u>	<u>37</u>	<u>99</u>	<u>11</u>	<u>37</u>	<u>98</u>	<u>67</u>	<u>31</u>	<u>84</u>	<u>99</u>

K=4

✗
Implement ⇒

Given \rightarrow ① K sorted array
 ② $k=3$

arr =

✓ 0	✓ 1	✓ 2	✓ 3	✓ 4	✓ 5	✓ 6	✓ 7	✓ 8
3	2	4	1	6	5	7	9	8

			3	5	4	6		
←	→							
0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9

res =

Queue size

Queue ops

tmp

n \rightarrow add $\rightarrow \log n \rightarrow n \log n$

kn \rightarrow add $\rightarrow \log kn$
 $\rightarrow n \log(kn)$

Output
 Sorted
 $k=3$

$n \log(k+1)$ $<$ $n \log n$

```
public static int[] sort(int arr[], int k){
    PriorityQueue<Integer> pq = new PriorityQueue<>();

    int tmp = 0;
    int res[] = new int[arr.length];
    for(int idx = 0 ; idx < arr.length ; idx++){
        if(pq.size() < k+1){
            pq.add(arr[idx]);
        }else{
            res[tmp] = pq.remove();
            pq.add(arr[idx]);
            tmp++;
        }
    }

    while(pq.size() > 0){
        res[tmp] = pq.remove();
        tmp++;
    }

    return res;
}
```

Lists

0 1 2 3 4
 10 20 30 40 50
 0 1 2 3 4 5 6
 5 7 9 11 13 15 17
 0 1 2
 1 2 3
 0 1
 32 39

Output
 6 C
 =

$l_i - d_i - val$
 0 - 0 - 10
 1 - 1 - 7

Output
↳ Complete sorted List

Pq → val min

$n \log k$

1-1-7

li-di-val

0-0-10
1-1-7
~~2-2-3~~
3-0-32

1 2 3 5

```
ArrayList<Integer> rv = new ArrayList<>();
```

```
PriorityQueue<Pair> pq = new PriorityQueue<>();
```

```
for(int idx = 0 ; idx < lists.size() ; idx++){
    pq.add(new Pair(idx,0,lists.get(idx).get(0)));
}
```

```
while(pq.size() > 0){
    Pair minPair = pq.remove();
```

```
    rv.add(minPair.val);
```

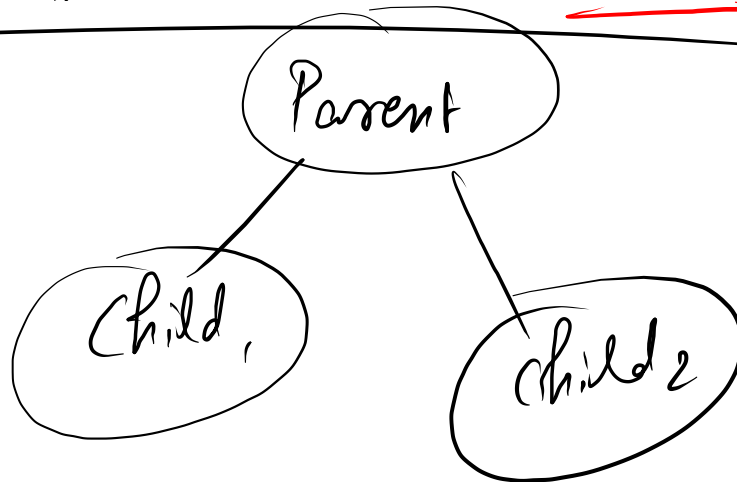
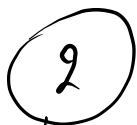
min?

```
    if(minPair.di+1 < lists.get(minPair.li).size()){
        minPair.di = minPair.di+1;
        minPair.val = lists.get(minPair.li).get(minPair.di);
        pq.add(minPair);
    }
}
```

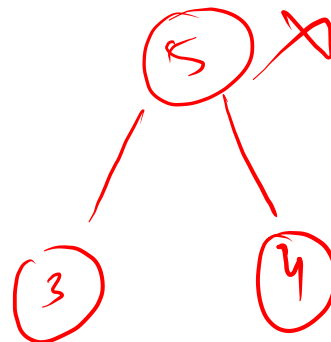
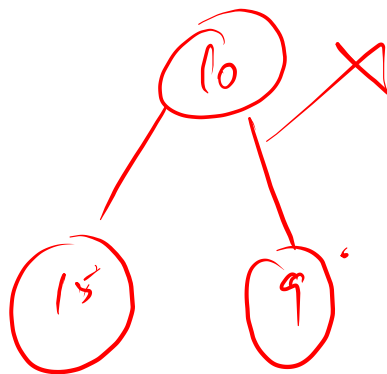
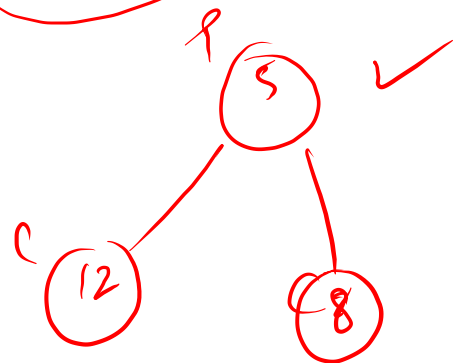
```
return rv;
```

Priority Queue

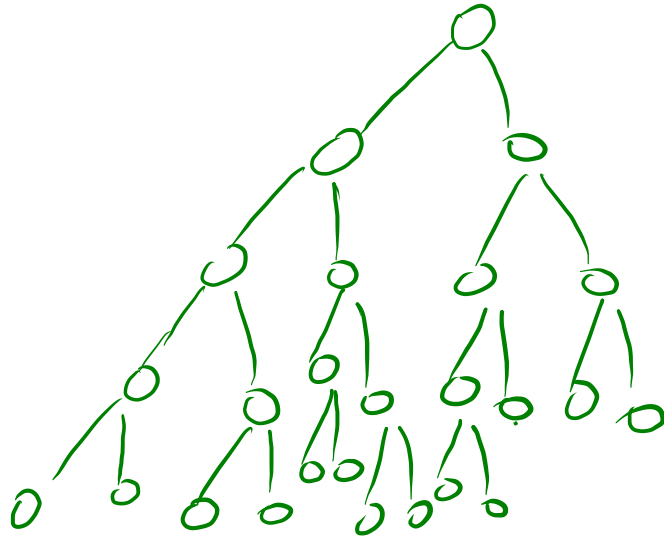
parent must always contain highest priority element



min - Priority



CBT



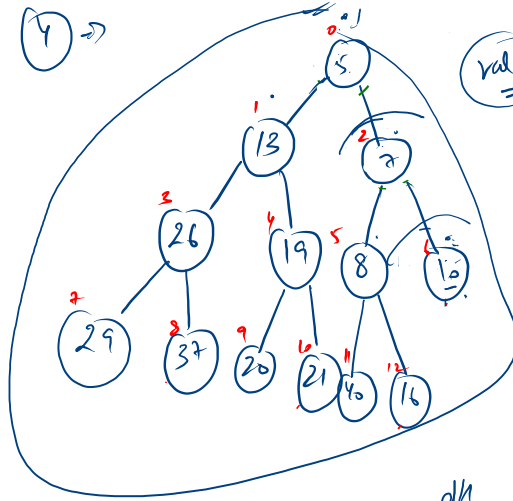
Complex Binary Tree

↳ only last level can be incomplete

↳ addition, L to R

0	1	2	3	4	5	6	7	8	9	10	11	12	13
4	13	7	29	19	8	5	29	37	20	21	40	16	10

$\underline{cidx} \rightarrow pidx$
 $2 \rightarrow 0$
 $9 \rightarrow 4$
 $10 \rightarrow 4$
 \underline{idx}
 $\hookrightarrow lidx = (2 \cdot idx) + 1$
 $\hookrightarrow ridx = (2 \cdot idx) + 2$
 \underline{cidx}
 $\hookrightarrow pidx = \frac{(cidx - 1)}{2}$



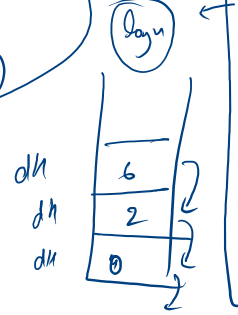
$val = 4$

```

public int remove() {
    if(data.size() == 0){
        System.out.println("Underflow");
        return -1;
    }
    int val = data.get(0);
    data.set(0, data.get(data.size()-1));
    data.remove(data.size()-1);
    downHeapify(0);
    return val;
}

```

$\Rightarrow \underline{minIdx = 6}$



```

public void downHeapify(int idx){
    int lidx = (2*idx)+1;
    int ridx = (2*idx)+2;
    int minIdx = idx;
    if(lidx < data.size() && data.get(lidx) < data.get(minIdx)){
        minIdx = lidx;
    }
    if(riidx < data.size() && data.get(riidx) < data.get(minIdx)){
        minIdx = ridx;
    }
    if(minIdx != idx){
        int val = data.get(idx);
        int minVal = data.get(minIdx);
        data.set(idx, minVal);
        data.set(minIdx, val);
        downHeapify(minIdx);
    }
}

```

$\underline{\text{H.O.P.}}$ \checkmark \checkmark
 $\Rightarrow \underline{\text{B.T.}}$ \checkmark \checkmark
 $(?)$

$(data[cidx] < data[pidx])$
 $\checkmark 0 < 28$
 $\times 2 > 27$