

<del>5</del>	<del>0</del>	<del>1</del>	<del>2</del>	4	5	<del>6</del>	<del>7</del>	8	9
0	1	2	3	4	5	6	7	8	9

✓ (0,1)

✓ (1,2)

✓ (4,8)

✓ (5,6)

✓ (2,3)

✓ (6,8)

[visualize] (6,1)

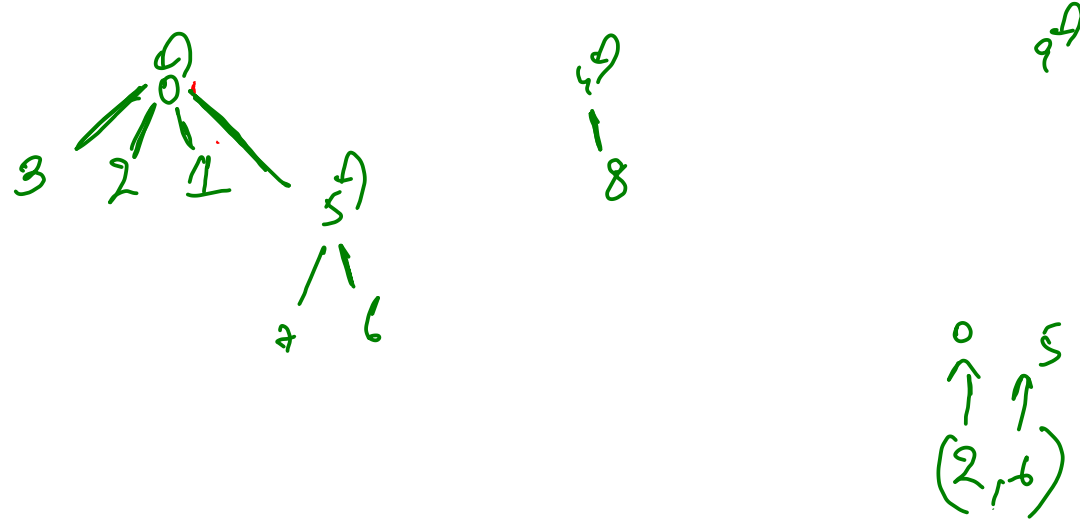
↓ 6  
5 0

[Parent]

[vertex]

data =

0	1	2	3	4	5	6	7	8	9
0	0	0	0	9	0	5	5	4	9



```
public static void main(String[] args) {
    UnionFind obj = new UnionFind(10);

    obj.union(0, 1);
    obj.union(1, 2);
    obj.union(4, 8);
    obj.union(5, 6);
    obj.union(2, 3);
    obj.union(6, 7);

    System.out.println(obj.isConnected(2, 1));
    System.out.println(obj.isConnected(2, 6));
    System.out.println(obj.isConnected(8, 3));
}
```

Quick Union

```
public static class UnionFind{
    int []data;

    public UnionFind(int vtces){
        data = new int[vtces];

        for(int i = 0 ; i < vtces ; i++){
            data[i] = i;
        }

        public void union(int vtx1,int vtx2){
            int rootv1 = find(vtx1);
            int rootv2 = find(vtx2);

            if(rootv1 == rootv2){
                // do nothing
            }else{
                data[rootv2] = rootv1;
            }
        }

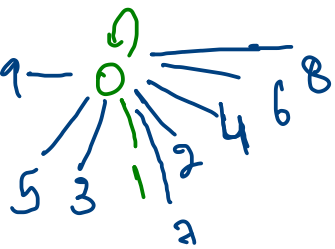
        public int find(int vtx){ // returns root
            if(data[vtx] == vtx){
                return vtx;
            }

            return find(data[vtx]);
        }

        public boolean isConnected(int v1,int v2){ // part of same comp. or not
            return find(v1) == find(v2);
        }
    }
}
```

0 1 2 3 4 5 6 7 8 9

0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---



find(0)	0	✓ (8, 9)
find(1)	0	✓ (7, 9)
find(2)	0	✓ (6, 9)
find(3)	0	✓ (5, 9)
find(4)	0	✓ (4, 9)
find(5)	0	✓ (3, 9)
find(6)	0	✓ (2, 9)
find(7)	0	✓ (1, 9)
find(8)	0	✓ (0, 9)
find(9)	0	

$O(\log n)$  =  $\Rightarrow$  path compression

$\Downarrow$

$O(\log n)$

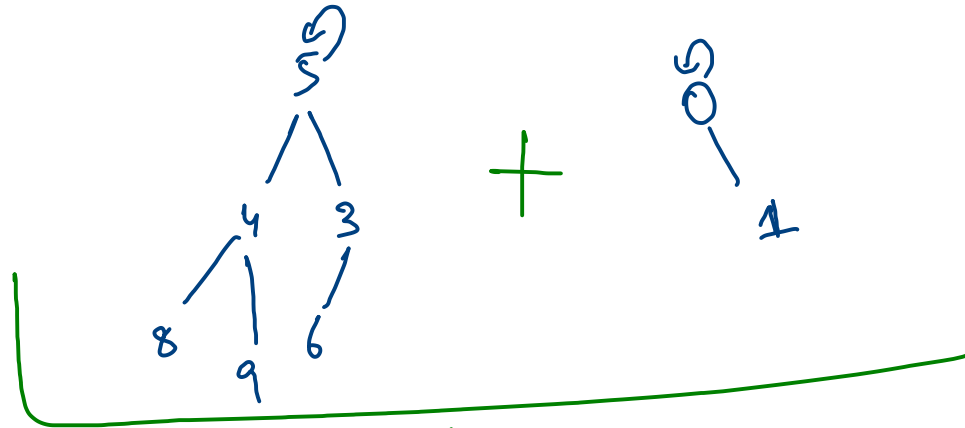
```

public int find(int vtx){ // returns root
    if(data[vtx] == vtx){
        return vtx;
    }
    return data[vtx] = find(data[vtx]);
}

```

Union by Rank

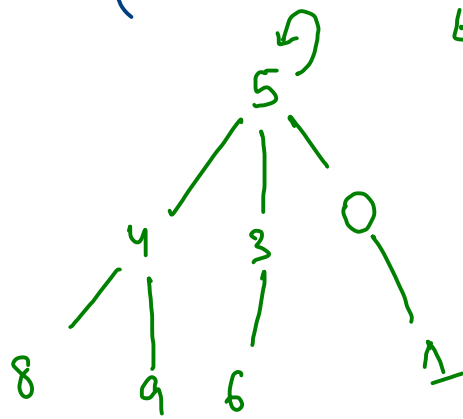
Edge (3, 1)  
↓ ↓  
5 0



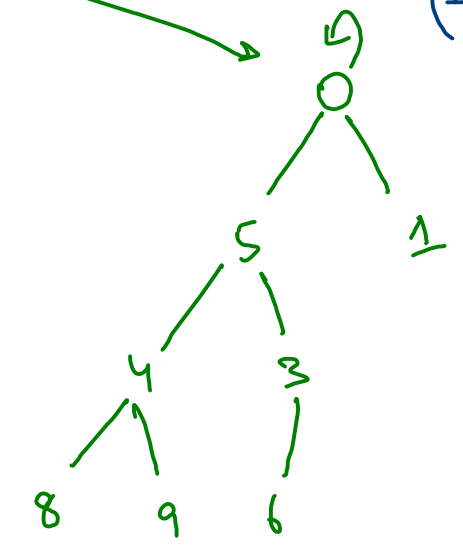
~~find~~  
↑  
union

✓✓

(A)



(B)



Pos:

0	1	2	3	4	5	6	7	8	9
0	1	3	3	3	4	7	3	8	9

Rank:

0	1	2	3	4	5	6	7	8	9
1	1	1	3	2	1	1	2	1	1

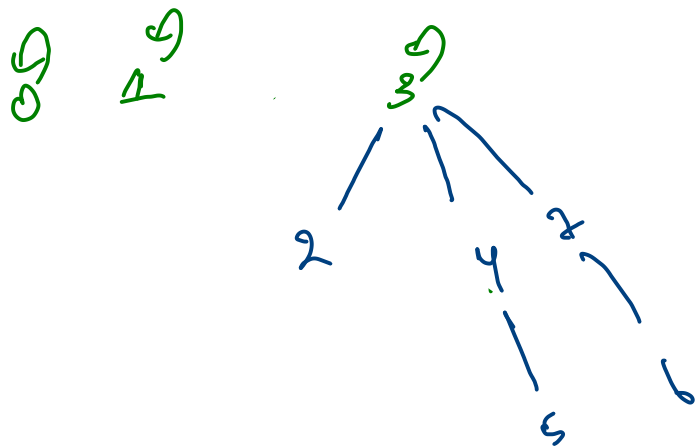
(3, 2) ✓

(4, 5) ✓

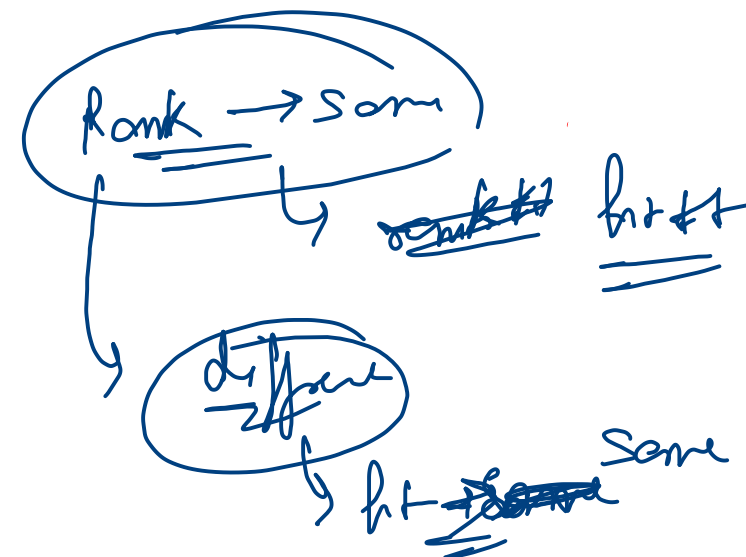
(2, 5) ✓

(7, 6) ✓

(5, 4) ✓



8 9



Union find  $\Rightarrow$  Brute Force

Time Complexity  $\rightarrow O(N)$

Only Path Compression  $\rightarrow$  Time Complexity  $\rightarrow O(\log n)$

only Union by Rank  $\rightarrow$  Time Complexity  $\rightarrow O(\log n)$

Path Compression + Union by Rank  $\rightarrow$  Time Complexity  $\Rightarrow O(\alpha(N))$

$O(1)$

Inverse Ackermann  
function

Average  $\rightarrow$  Constant

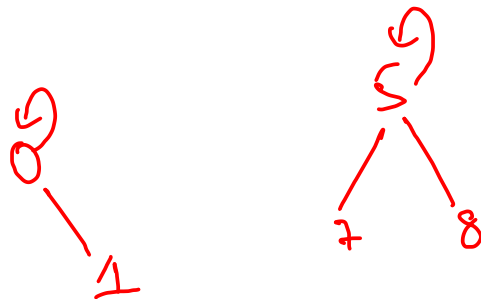
$n$

	0	1	2	$m$
0	<del>0</del>	<del>1</del>	0	
1	0	0	<del>0</del>	
2	0	<del>0</del>	<del>0</del>	

$r, c$   
 $\hookrightarrow \text{idx} \Rightarrow r \cdot m + c$

positions =  $[[0,0],[0,1],[1,2],[2,1]]$   $[2,2]$

$\downarrow$  0  
 $\downarrow$  1  
 $\downarrow$  5  
 $\downarrow$  7  
 $\downarrow$  8



Count = ~~0~~ ~~1~~ ~~2~~ ~~1~~ ~~2~~ ~~7~~ ~~4~~ ~~8~~ <sup>2</sup>

$\downarrow$

4

✓  
par =

0	1	2	3	4	5	6	7	8
0	0	-1	-1	-1	5	-1	5	5

✓  
rank =

0	1	2	3	4	5	6	7	8
1	0	0	0	0	<del>1</del>	0	0	0

$n = 3$   
 $m = 3$

7 9 3 9  
 5 2 1 1

0	0	1	1
3	1	1	1
6	1	1	1

-1	2	2	3	-1	2	-1	2	-1
0	1	2	3	4	5	6	7	8

0	0	1	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8

Count = 0  
 1  
 2  
 3  
 4  
 5  
 6  
 7  
 8  
 9  
 10

Input

3  
 3  
 [[0,1],[1,2],[2,1],[1,0],[0,2],[0,0],[1,1]]

Output

[1,2,3,4,3,2,-1]

Expected

[1,2,3,4,3,2,1]

```

ArrayList<Integer> res = new ArrayList<>();
for(int pos[] : positions){
    int r = pos[0] , c = pos[1];
    int idx = r*m + c;
    if(mat[r][c] != 1){
        mat[r][c] = 1;
        obj.init(idx);

        int ni = (r-1)*m + c , ei = (r)*m + (c+1) , si = (r+1)*m + (c) , wi = (r)*m + (c-1);
        if(r-1 >= 0 && mat[r-1][c] == 1){
            obj.union(idx,ni);
        }
        if(c+1 < m && mat[r][c+1] == 1){
            obj.union(idx,ei);
        }
        if(r+1 < n && mat[r+1][c] == 1){
            obj.union(idx,si);
        }
        if(c-1 >= 0 && mat[r][c-1] == 1){
            obj.union(idx,wi);
        }
    }
    res.add(obj.count);
}

```





$n=3$   
 $m=3$

	0	1	2
0	0	1	1
1	3	0	5
2	6	0	0

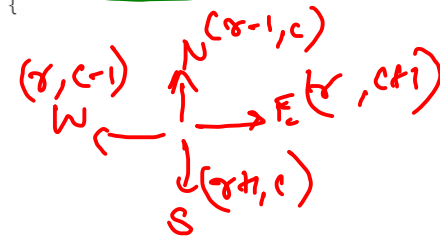
for

0	1	2	3	4	5	6	7	8
-1	2	2	-1	2	2	-1	2	-1

rank

0	1	2	3	4	5	6	7	8
0	0	1	0	0	0	0	0	0

Count = 2 3 4 3 2  
idx = 4  
r: 1, c: 1



```
class UnionFind{
    int par[];
    int rank[];
    int count;
```

```
UnionFind(int vtces){
    par = new int[vtces];
    rank = new int[vtces];
    count = 0;
    Arrays.fill(par, -1);
}

void init(int v){
    count++;
    par[v] = v;
}

void union(int v1, int v2){
    int rt1 = find(v1);
    int rt2 = find(v2);
    if (rt1 != rt2){
        count--;
        int rank1 = rank[v1];
        int rank2 = rank[v2];

        if (rank1 < rank2){
            par[rt1] = rt2;
        } else if (rank1 > rank2){
            par[rt2] = rt1;
        } else{
            par[rt2] = rt1;
            rank[rt1]++;
        }
    }
}

int find(int v){
    if (par[v] == v){
        return v;
    }
    return par[v] = find(par[v]);
}
```

```
public List<Integer> numIslands2(int n, int m, int[][] positions) {
    UnionFind obj = new UnionFind(m*n);
    int mat[][] = new int[n][m];

    ArrayList<Integer> res = new ArrayList<>();
    for (int pos[] : positions){
        int r = pos[0], c = pos[1];
        int idx = r*m + c;
        if (mat[r][c] != 1){
            mat[r][c] = 1;
            obj.init(idx);

            int ni = (r-1)*m + c, ei = (r)*m + (c+1), si = (r+1)*m + (c), wi = (r)*m + (c-1);
            if (r-1 >= 0 && mat[r-1][c] == 1){
                obj.union(idx, ni);
            }
            if (c+1 < m && mat[r][c+1] == 1){
                obj.union(idx, ei);
            }
            if (r+1 < n && mat[r+1][c] == 1){
                obj.union(idx, si);
            }
            if (c-1 >= 0 && mat[r][c-1] == 1){
                obj.union(idx, wi);
            }
        }
        res.add(obj.count);
    }
    return res;
}
```

