Monte Carlo Valuation of European Call Options

Technical Documentation Mathematical Model

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1 Introduction

1.1 Project Objective

The primary objective of this project is to estimate the fair value of a **European Call Option** on Apple Inc. (AAPL) utilizing Monte Carlo simulation methods.

1.2 Option Definitions

- **European Call Option:** Grants the holder the right, without obligation, to purchase the underlying asset at a fixed Strike Price (K) only on the expiration date (T).
- **American Call Option:** Differing from the European style, this option allows for exercise at any point up to maturity, offering distinct advantages for dividend capture strategies.

2 Methodology: Monte Carlo Simulation

The Monte Carlo simulation is a computational algorithm that utilizes random sampling to model stochastic processes. It quantifies uncertainty by generating a distribution of possible outcomes to derive a probabilistic fair value.

2.1 Key Advantages

Monte Carlo methods are preferred over analytical models (such as Black-Scholes) in three specific scenarios:

1. **Path Dependency**
   Analytical models often look only at the starting and end points. Monte Carlo simulates the entire price trajectory, making it essential for instruments like *Asian Options*, where the payoff depends on the average price over time.
2. **Complex Payoff Rules**
   It adeptly handles exotic options with conditional execution, such as *Barrier Options* (active only if price hits X or stays below Y). Deriving closed-form mathematical formulas for these conditional constraints is often impossible.
3. **High Dimensionality**
   When valuing *Basket Options* (a derivative based on a portfolio of assets like AAPL, GOOGL, and MSFT), the mathematics involves solving differential equations for multiple variables. Monte Carlo simulations efficiently manage these multi-variable environments without exponential complexity.

3 The Payoff Function

The value of a call option at maturity (T) depends on where the stock price (S_T) finishes relative to the strike price (K). The payoff formula is:

$$C_T = \max(S_T - K, 0) \quad (1)$$

- If $S_T > K$: The option is *In-The-Money* (Profit $= S_T - K$).
- If $S_T \leq K$: The option is *Out-Of-The-Money* (Worthless, Payoff $= 0$).

4 Stochastic Process: Geometric Brownian Motion

We assume the stock price follows a **Geometric Brownian Motion (GBM)**. This is the standard model used in the Black-Scholes framework. The continuous-time Stochastic Differential Equation (SDE) is:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{2}$$

Where:

- μ is the drift (expected return, replaced by risk-free rate r for valuation).
- σ is the volatility of the stock.
- W_t is a Wiener process (Brownian motion).

5 Monte Carlo Discretization

To simulate this on a computer, we apply **Itô's Lemma** to solve the SDE and discretize it over small time steps (Δt). The formula used in the Python script is:

$$S_{t+\Delta t} = S_t \cdot \exp\left((r - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}Z\right) \tag{3}$$

Where Z is a random draw from the standard normal distribution: $Z \sim \mathcal{N}(0, 1)$.

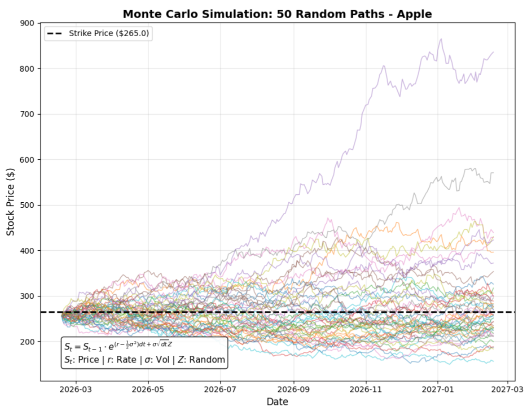
6 Valuation Methodology

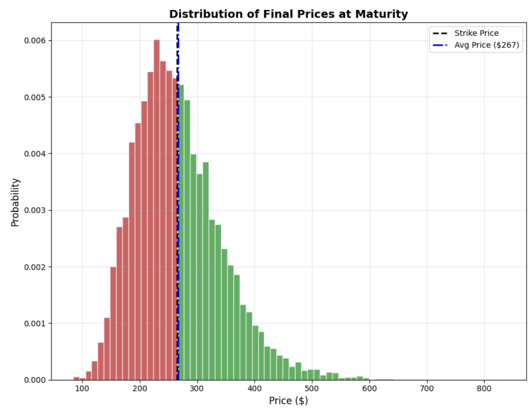
The simulation performs the following steps 10,000 times:

1. Generate a random price path from $t = 0$ to $t = T$.
2. Calculate the price S_T at maturity.
3. Calculate the payoff $\max(S_T - K, 0)$.

The fair option price is the average of these payoffs, discounted back to the present value:

$$V_0 = e^{-rT} \cdot \frac{1}{N} \sum_{i=1}^{N} \text{Payoff}_i \tag{4}$$



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SIMULATION RESULTS REPORT
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Current Stock Price:   $255.78
Target Strike Price:   $265.0
Fair Option Price:     $29.43
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Probability of Profit (Green Area): 45.4%
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