

Team notebook

CatsOnTrees

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```
const int maxn = 2e5 + 5;
vecl a(maxn);
vecl t[4*maxn];
void merge(vecl& temp1, vecl& temp2,
           vecl& final){
    int i = 0, j = 0;
    while(i < sz(temp1) && j <
          sz(temp2)){
```

```

    if(temp1[i] <= temp2[j]) {
        final.pb(temp1[i]);
        i++;
    } else {
        final.pb(temp2[j]);
        j++;
    }
}
while(i < sz(temp1)){
    final.pb(temp1[i]);
    i++;
}
while(j < sz(temp2)){
    final.pb(temp2[j]);
    j++;
}
}

void build(int ind, int tl, int tr){
    if(tl == tr){
        t[ind].pb(a[tl]);
        return;
    }
    int tm = (tl + tr) / 2;
    build(2 * ind, tl, tm);
    build(2 * ind + 1, tm + 1, tr);
    merge(t[2 * ind], t[2 * ind + 1],
        t[ind]);
}

int query(int ind, int tl, int tr, int
l, int r, int valuetoCompare){ //
    query for elements strictly greater
    than k
    if(l > r){
        return 0;
    }
}

```

```

    if(l == tl && r == tr){
        return t[ind].end() -
            upper_bound(all(t[ind]),
                valuetoCompare);
    }
    int tm = (tl + tr) / 2;
    return (query(2 * ind, tl, tm, l,
        min(r, tm), valuetoCompare) +
        query(2 * ind + 1, tm + 1,
            tr, max(l, tm + 1), r,
                valuetoCompare));
}

```

1.2 policybased

```

/*
find_by_order(k): return iterator to
    k'th element(counting from zero)
order_of_key(k) : number of items < k in
    O(logn) time.
*/
include <ext/pb_ds/assoc_container.hpp>
include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update>
    ordered_set;

```

1.3 segmentTree

```

/*

```

```

Usage: Node* tr = new Node(v, 0, sz(v));
finds max in a range, [L, R)
*/
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf;
    assert(s < i);
    return (void*)&buf[i -= s];
}
void operator delete(void*) {}

const int inf = 1e9;
struct Node {
    Node *l = 0, *r = 0;
    ll lo, hi, mset = inf, madd = 0,
        val = -inf;
    Node(int lo, int
        hi):lo(lo),hi(hi){} // Large
        interval of -inf
    Node(vll& v, int lo, int hi) :
        lo(lo), hi(hi) {
        if (lo + 1 < hi) {
            int mid = lo + (hi
                - lo)/2;
            l = new Node(v,
                lo, mid); r =
                new Node(v,
                    mid, hi);
            val = max(l->val,
                r->val);
        }
        else val = v[lo];
    }
    ll query(int L, int R) {

```

```

    if (R <= lo || hi <= L)
        return -inf;
    if (L <= lo && hi <= R)
        return val;
    push();
    return max(l->query(L, R),
        r->query(L, R));
}
void set(int L, int R, ll x) {
    if (R <= lo || hi <= L)
        return;
    if (L <= lo && hi <= R)
        mset = val = x, madd = 0;
    else {
        push(), l->set(L,
            R, x),
            r->set(L, R, x);
        val = max(l->val,
            r->val);
    }
}
void add(int L, int R, ll x) {
    if (R <= lo || hi <= L)
        return;
    if (L <= lo && hi <= R) {
        if (mset != inf)
            mset += x;
        else madd += x;
        val += x;
    }
    else {
        push(), l->add(L,
            R, x),
            r->add(L, R, x);

```

```

        val = max(l->val,
            r->val);
    }
}
void push() {
    if (!l) {
        int mid = lo + (hi
            - lo)/2;
        l = new Node(lo,
            mid); r = new
            Node(mid, hi);
    }
    if (mset != inf)
        l->set(lo,hi,mset),
        r->set(lo,hi,mset),
        mset = inf;
    else if (madd)
        l->add(lo,hi,madd),
        r->add(lo,hi,madd),
        madd = 0;
}
};

```

1.4 simple_segtree

```

const ll ninf = -1e17; const ll inf =
    1e17;
struct Segtree{ int left(int v){ return
    v+1;}
    int right(int v, int tl, int tr){
        int mid = (tl + tr)/2; return v +
        2*(mid - tl + 1);}
    vll node; int sz;

```

```

Segtree(int n) { node.assign(2*n,
    inf); sz = n; }
// return min value possible
ll query(int l, int r, int v = 0,
    int tl = 0, int tr = -1){
    if(tr == -1) tr = sz-1;
    if(r < tl || l > tr) return inf;
    if(l <= tl && tr <= r) return
        node[v];
    int mid = (tl + tr)/2;
    return min(query(l,r,left(v), tl,
        mid),
        query(l,r,right(v,tl,tr),
            mid+1, tr)); }
ll update(int x, ll val, int v = 0,
    int tl = 0, int tr = -1){
    if(tr == -1) tr = sz-1;
    if(x < tl || x > tr) return
        node[v];
    if(x == tl && x == tr) return
        node[v] = val;
    int mid = (tl + tr)/2;
    return node[v] =
        min(update(x,val,left(v), tl,
            mid),
            update(x,val,right(v,tl,tr),
                mid+1, tr)); } };

```

1.5 treap

```

struct Node {
    char val;
    int weight, size;
    Node *left, *right;

```

```

Node(char c) : val(c),
              weight(rand()), size(1),
              left(NULL), right(NULL) {}
} *root;

inline int size(Node *treap) {
    return treap ? treap->size : 0;
}

void split(Node *treap, Node *&left,
           Node *&right, int val) {
    if (!treap) {
        left = right = NULL;
        return;
    }
    if (size(treap->left) < val) {
        split(treap->right, treap->right,
              right, val -
                size(treap->left) - 1);
        left = treap;
    } else {
        split(treap->left, left,
              treap->left, val);
        right = treap;
    }
    treap->size = 1 + size(treap->left)
                + size(treap->right);
}

void merge(Node *&treap, Node *left,
           Node *right) {
    if (left == NULL) {
        treap = right;
        return;
    }
    if (right == NULL) {
        treap = left;

```

```

        return;
    }

    if (left->weight < right->weight) {
        merge(left->right, left->right,
              right);
        treap = left;
    } else {
        merge(right->left, left,
              right->left);
        treap = right;
    }
    treap->size = 1 + size(treap->left)
                + size(treap->right);
}

ostream& operator<<(ostream &os, Node
*n) {
    if (!n) return os;
    os << n->left; os << n->val; os <<
        n->right;
    return os;
}

void solve() { // USAGE:
    // get integer n, q, and s
    for(auto c: s) merge(root, root, new
        Node(c));
    while(q--) {
        int l, r; cin >> l >> r;
        Node *a, *b;
        split(root, a, b, l - 1);
        Node *c, *d;
        split(b, c, d, r - l + 1);
        merge(root, a, d);
        merge(root, root, c);
    }
}

```

```

        cout << root << nl;
    }

```

1.6 treapLazy

```

struct Node {
    char val;
    int weight, size;
    Node *left, *right;
    int toinvert;
    Node(char c) : val(c),
                  weight(rand()), size(1),
                  left(NULL), right(NULL),
                  toinvert(0) {}
} *root;

inline int size(Node *treap) {
    return treap ? treap->size : 0;
}

void push(Node *treap) {
    if(treap == NULL) return;
    if(treap->toinvert == 0) return;
    Node *temp = treap->left;
    treap->left = treap->right;
    treap->right = temp;
    treap->toinvert = 0;
    if(treap->left != NULL)
        treap->left->toinvert ^= 1;
    if(treap->right != NULL)
        treap->right->toinvert ^= 1;
}

void split(Node *treap, Node *&left,
           Node *&right, int val) {
    if (!treap) {
        left = right = NULL; return;
    }

```

```

    }
    push(treap);
    if (size(treap->left) < val) {
        split(treap->right, treap->right,
              right, val -
              size(treap->left) - 1);
        left = treap;
    } else {
        split(treap->left, left,
              treap->left, val);
        right = treap;
    }
    treap->size = 1 + size(treap->left)
        + size(treap->right);
}

void merge(Node *&treap, Node *left,
Node *right) {
    if (left == NULL)
        treap = right; return;
    if (right == NULL)
        treap = left; return;
    push(left); push(right);
    if (left->weight < right->weight) {
        merge(left->right, left->right,
              right); treap = left;
    } else {
        merge(right->left, left,
              right->left); treap = right;
    }
    treap->size = 1 + size(treap->left)
        + size(treap->right);
}

void solve() { //USAGE:
    //get integers n,q and string s

```

```

    for(auto c: s) merge(root, root, new
        Node(c));
    while(q--) {
        int l, r; cin >> l >> r;
        Node *a, *b;
        split(root, a, b, l - 1);
        Node *c, *d;
        split(b, c, d, r - l + 1);
        c->toinvert ^= 1;
        merge(root, a, c);
        merge(root, root, d);
    }
    cout << root << nl;
}

```

2 DP-Optimizations

2.1 convexhull

```

const ll is_query = -(1LL<<62);
struct line {
    ll m, b;
    mutable function<const line*>
        succ;
    bool operator<(const line& rhs)
        const {
        if (rhs.b != is_query)
            return m < rhs.m;
        const line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m -
            m) * x;
    }
}

```

```

    }
};

struct dynamic_hull : public
    multiset<line> { // will maintain
    upper hull for maximum
        const ll inf = LLONG_MAX;
        bool bad(iterator y) {
            auto z = next(y);
            if (y == begin()) {
                if (z == end())
                    return 0;
                return y->m ==
                    z->m && y->b <=
                    z->b;
            }
            auto x = prev(y);
            if (z == end()) return
                y->m == x->m && y->b
                <= x->b;
            /* compare two lines by
            slope, make sure
            denominator is not 0 */
            ll v1 = (x->b - y->b);
            if (y->m == x->m) v1 =
                x->b > y->b ? inf :
                -inf;
            else v1 /= (y->m - x->m);
            ll v2 = (y->b - z->b);
            if (z->m == y->m) v2 =
                y->b > z->b ? inf :
                -inf;
            else v2 /= (z->m - y->m);
            return v1 >= v2;
        }
        void insert_line(ll m, ll b) {

```

```

    auto y = insert({ m, b });
    y->succ = [=] { return
        next(y) == end() ? 0 :
        &*next(y); };
    if (bad(y)) { erase(y);
        return; }
    while (next(y) != end() &&
        bad(next(y)))
        erase(next(y));
    while (y != begin() &&
        bad(prev(y)))
        erase(prev(y));
}
ll eval(ll x) { //maximum at
    point x
    auto l =
        *lower_bound((line) {
            x, is_query });
    return l.m * x + l.b;
}
};

```

2.2 divideNconquer

```

//dp[i][j] = min of
    dp[ i - 1 ][k] + C [k][j] forall k
    < j,
//and optk[i][j] <= optk[i][j+1]
//optk is optimial k that gives you
    answer.
// compute dp_cur[l], ... dp_cur[r]
    (inclusive)
// C(a,c) + C(b,d) <= C(a,d) + C(b,c)
    for all a<=b<=c<=d

```

```

void compute(int l, int r, int optl, int
    optr) {
    if (l > r)
        return;
    int mid = (l + r) >> 1;
    pair<long long, int> best =
        {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid,
        optr); k++) {
        best = min(best, {(k ?
            dp_before[k - 1] : 0) + C(k,
            mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}
// notebook-generator ./ --author
    "CatsOnTrees" --initials UTP --size
    12 --columns 3 --paper a4paper

```

2.3 knuth

```

//dp[i][j] = min { dp[i][k] + dp[k][j] +
    C(i, j) } : for i <= k < j
// O(n^3) -> O(n^2) if optk(i,j-1) <=
    opt(i,j) <= opt(i+1,j)
// criteria to see:
/* for a<=b<=c<=d
1) C(b,c) <= C(a,d)
2) C(a,c) + C(b,d) <= C(a,d) + C(b,c)
eg: C(i,j) = cost of arr[i..j] if all
    elements +ve */

```

```

int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];
    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };
    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i]
            according to the problem
    }
    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k
                <= min(j-1, opt[i+1][j]);
                k++) {
                if (mn >= dp[i][k] +
                    dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] +
                        dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }
    cout << dp[0][N-1] << endl;
}

```

2.4 matrixExponentiation

```

vector<vector<int>> indentify(int n) {
    vector<vector<int>> i(n,
        vector<int>(n, 0));
    for(int j = 0; j < n; j++) {
        i[j][j] = 1;
    }
    return i;
}

ll mod_mul(ll a, ll b){
    a = a%M; b = b%M;
    return ((a*b)%M + M)%M;
}

vector<vector<int>>
mul(vector<vector<int>>& a,
    vector<vector<int>>& b, int mod) {
    int n = sz(a);
    vector<vector<int>> toreturn(n,
        vector<int>(n, 0));
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++) {
            int ans = 0;
            for(int k = 0; k < n; k++) {
                ans += mod_mul(a[i][k] ,
                    b[k][j]);
                ans = ans%mod;
            }
            toreturn[i][j] = ans;
        }
    }
    return toreturn;
}

vector<vector<int>>
expo(vector<vector<int>> &mat, int
    pow, int mod) {

```

```

    int n = mat.size();
    if(pow == 0) return indentify(n);
    vector<vector<int>> temp =
        indentify(n);
    auto Exp = expo(mat, pow/2, mod);
    if(pow % 2) {
        temp = mul(temp, mat, mod);
        vector<vector<int>> result = Exp;
        result = mul(result, result, mod);
        temp = mul(temp, result, mod);
    } else {
        vector<vector<int>> result = Exp;
        result = mul(result, result, mod);
        temp = mul(temp, result, mod);
    }
    return temp;
}

```

2.5 sosDP

```

for(int i = 0; i < (1<<N); ++i) F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask =
    0; mask < (1<<N); ++mask){
    if(mask & (1<<i)) F[mask] +=
        F[mask^(1<<i)];
}

```

3 Geometry

3.1 convexhull

```

vector<pair<int,int>> vec;
vector<pair<int,int>> hull;
typedef pair<int,int> pi;
pi operator+(pi a,pi b) {
    return {a.first+b.first,
        a.second+b.second};
}
pi operator-(pi a,pi b) {
    return {a.first-b.first,
        a.second-b.second};
}
int operator&(pi a,pi b) {
    return a.first*b.first +
        a.second*b.second;
}
int operator*(pi a,pi b) {
    return a.first*b.second -
        a.second*b.first;
}

void solve()
{
    int n;
    cin>>n;
    vec.clear();
    hull.clear();
    for(int i=0;i<n;i++)
    {
        int u,v;
        cin>>u>>v;
        vec.push_back({u,v});
    }
    sort(all(vec));
    for(int rep=0;rep<2;rep++)
    {
        int lim=sz(hull);

```

```

for(auto &p:vec)
{
    while(sz(hull)>=lim+2)
    {
        auto a=hull.end()[-2];
        auto b=hull.end()[-1];
        if(((b-a)*(p-a)) <= 0)
            break;
        hull.pop_back();
    }
    hull.push_back(p);
}
hull.pop_back();
reverse(all(vec));
}

```

3.2 point2d

```

struct point2d {
    ll x, y;
    point2d() {}
    point2d(ll x, ll y): x(x), y(y) {}
    point2d& operator+=(const point2d
        &t) {
        x += t.x; y += t.y;
        return *this;
    }
    point2d& operator-=(const point2d
        &t) {
        x -= t.x; y -= t.y;
        return *this;
    }
    point2d& operator*=(ll t) {

```

```

        x *= t; y *= t;
        return *this;
    }
    point2d& operator/=(ll t) {
        x /= t; y /= t;
        return *this;
    }
    point2d operator+(const point2d &t)
        const {
        return point2d(*this) += t;
    }
    point2d operator-(const point2d &t)
        const {
        return point2d(*this) -= t;
    }
    point2d operator*(ll t) const {
        return point2d(*this) *= t;
    }
    point2d operator/(ll t) const {
        return point2d(*this) /= t;
    }
    int operator^(const point2d &t)
        const {
        return x*t.y - y*t.x;
    }
    int operator|(const point2d &t)
        const {
        return x*t.x + y*t.y;
    }
    bool operator<(const point2d&u) const
        {
        return (x<u.x) || (x==u.x &&
            y<u.y);
    }

```

```

    int cross(const point2d &t1,const
        point2d &t2,const point2d &t3)
        const {
        return (t2-t1)^(t3-t1);
    }
    int dot(const point2d &t1,const
        point2d &t2,const point2d &t3)
        const {
        return (t2-t1)|(t3-t1);
    }
};

```

3.3 Useful fnx

```

// line intersection:
point2d intersect(point2d a1, point2d
    d1, point2d a2, point2d d2) {
    return a1 + cross(a2 - a1, d2) /
        cross(d1, d2) * d1;
}
//line segment intersection (a,b), (c,d):
int sgn(const long long& x)
{ return x >= 0 ? x ? 1 : 0 : -1; }
bool inter1(ll a, ll b, ll c, ll d) {
    if (a > b) swap(a, b);
    if (c > d) swap(c, d);
    return max(a, c) <= min(b, d);
}
bool check_inter(const pt& a, const pt&
    b, const pt& c, const pt& d) {
    if (cross(a-c, d-c) == 0 &&
        cross(b-c, d-c) == 0)
        return inter1(a.x, b.x, c.x, d.x) &&
            inter1(a.y, b.y, c.y, d.y);
}

```



```

return sgn(cross(b-a, c-a)) !=
    sgn(cross(b-a, d-a)) &&
    sgn(cross(d-c, a-c)) !=
    sgn(cross(d-c, b-c));
}

```

4 Graphs

4.1 2sat

```

/*
(x    y)    (x    y) so add edge
    between ~x ---> y and ~y ---> x
id[x] < id[y] => x = false
Both variables must have the same value
is equivalent to:
(x    y) (x    y).
*/
struct two_sat { int n;
    vector<vector<int>> g, gr; // gr is
        the reversed graph
    vector<int> comp, topological_order,
        answer; // comp[v]: ID of the SCC
        containing node v
    vector<bool> vis;
    two_sat() {}
    two_sat(int _n) { init(_n); }
    void init(int _n) {
        n = _n;
        g.assign(2 * n, vector<int>());
        gr.assign(2 * n, vector<int>());
        comp.resize(2 * n);
        vis.resize(2 * n);

```

```

        answer.resize(2 * n);
    }
    void add_edge(int u, int v) {
        g[u].push_back(v);
        gr[v].push_back(u);
    }
    // For the following three functions
    // int x, bool val: if 'val' is
    // true, we take the variable to be
    // x. Otherwise we take it to be x's
    // complement.
    // At least one of them is true
    void add_clause_or(int i, bool f,
        int j, bool g) {
        add_edge(i + (f ? n : 0), j + (g
            ? 0 : n));
        add_edge(j + (g ? n : 0), i + (f
            ? 0 : n));
    }
    // Only one of them is true
    void add_clause_xor(int i, bool f,
        int j, bool g) {
        add_clause_or(i, f, j, g);
        add_clause_or(i, !f, j, !g);
    }
    // Both of them have the same value
    void add_clause_and(int i, bool f,
        int j, bool g) {
        add_clause_xor(i, !f, j, g);
    }
    // Topological sort
    void dfs(int u) {
        vis[u] = true;
        for (const auto &v : g[u])
            if (!vis[v]) dfs(v);

```

```

        topological_order.push_back(u);
    }
    // Extracting strongly connected
    // components
    void scc(int u, int id) {
        vis[u] = true;
        comp[u] = id;
        for (const auto &v : gr[u])
            if (!vis[v]) scc(v, id);
    }
    // Returns true if the given
    // proposition is satisfiable and
    // constructs a valid assignment
    bool satisfiable() {
        fill(vis.begin(), vis.end(),
            false);
        for (int i = 0; i < 2 * n; i++)
            if (!vis[i]) dfs(i);
        fill(vis.begin(), vis.end(),
            false);
        reverse(topological_order.begin(),
            topological_order.end());
        int id = 0;
        for (const auto &v :
            topological_order)
            if (!vis[v]) scc(v, id++);
        // Constructing the answer
        for (int i = 0; i < n; i++) {
            if (comp[i] == comp[i + n])
                return false;
            answer[i] = (comp[i] > comp[i
                + n] ? 1 : 0);
        }
        return true;
    }
}

```

```
};
```

4.2 bellmanford

```
struct edge
{ int a, b, cost;};
int n, m, v;
vector<edge> e;
const int INF = 1000000000;
void solve(){
    vector<int> d (n, INF);
    d[v] = 0;
    vector<int> p (n, - 1);
    int x;
    for (int i=0; i<n; ++i){
        x = -1;
        for (int j=0; j<m; ++j)
            if (d[e[j].a] < INF)
                if (d[e[j].b] > d[e[j].a] +
                    e[j].cost)
                {
                    d[e[j].b] = max (-INF, d[e[j].a] +
                        e[j].cost);
                    p[e[j].b] = e[j].a;
                    x = e[j].b;
                }
    }
    if (x == -1)
        cout << "No negative cycle from "
            << v;
    else
    {
        int y = x;
        for (int i=0; i<n; ++i)
```

```
        y = p[y];
        vector<int> path;
        for (int cur=y; ; cur=p[cur])
        {
            path.push_back (cur);
            if (cur == y && path.size() > 1)
                break;
        }
        reverse (path.begin(), path.end());
        cout << "Negative cycle: ";
        for (size_t i=0; i<path.size(); ++i)
            cout << path[i] << ' ';
    }
}
```

4.3 dijkstra

```
vector<ll> dist;
vector<bool> vis;
//Single source shortest path algorithm
void dijkstra(vvpll& graph, ll start){
    int n = graph.size();
    vis.assign(n,false);
    dist.assign(n,1e18);
    //priority queue stores distance
    , current
    priority_queue<pair<ll,ll>, vvpll,
        greater<pair<ll,ll>>> peq;
    dist[start] = 0;
    vis[start] = 0;
    peq.push(MP(0,start));
    while(!peq.empty()){
        ll curr = peq.top().S;
        ll currdist = peq.top().F;
```

```
        peq.pop();
        if(vis[curr]) continue;
        vis[curr] = true;
        //update all the children
        for(auto cpx: graph[curr]){
            if(vis[cpx.F])
                continue;
            ll newDist =
                currdist+cpx.S;
            //relaxation
            if(dist[cpx.F] >
                newDist){
                dist[cpx.F] =
                    newDist;
                peq.push(MP(newDist,
                    cpx.F));
            }
        }
    }
}
```

4.4 dinics

```
struct Dinics{
    struct Edge{
        int to, revidx;
        ll cap, ocap;
        Edge(int to, int revidx,
            ll cap, ll
            ocap):to(to),
            revidx(revidx),
            cap(cap), ocap(ocap){}
        Edge(){}
        ll flow(){
            return max(ocap - cap,
                0ll);
```

```

    }
};
vector<vector<Edge>> adj;
vector<int> level, next;
int n;
Dinics(int n):n(n){
    level.assign(n, 0),
    next.assign(n,0);
    adj.assign(n,
        vector<Edge>(0));
}
void addEdge(int u, int v, ll
    cap, ll rev = 0){
    adj[u].push_back(Edge(v,
        adj[v].size(), cap,
        cap));
    adj[v].push_back(Edge(u,
        adj[u].size() - 1,
        rev, rev));
}
ll dfs(int curr, int t, ll flow){
    // cout<<curr<<" "<<t<<"
    // "<<flow<<"\n";
    if(curr == t || !flow)
        return flow;
    for(int& i = next[curr]; i
        < adj[curr].size();
        i++){
        Edge &edge = adj[curr][i];
        if(level[edge.to] !=
            level[curr]+1)
            continue;
        ll actualflow;
        actualflow = dfs(edge.to,
            t, min(flow,

```

```

            edge.cap));
        if(actualflow){
            edge.cap -= actualflow;
            adj[edge.to][edge.revidx].cap
                += actualflow;
            return actualflow;
        }
    }
    return 0;
}
ll calc(int src, int t){
    ll flow = 0;
    const ll inf = 1e16;
    //capacity scaling
    for(int L = 30; L >= 0;
        L--){ do{
        level.assign(n, 0);
        next.assign(n, 0);
        //level assignment
        queue<int> q;
        level[src] = 1;
        q.push(src);
        while(!q.empty() &&
            !level[t]){
            int curr = q.front();
            q.pop();
            for(int i = 0; i <
                adj[curr].size(); i++){
                Edge &e = adj[curr][i];
                if(!level[e.to] && (e.cap
                    >> L)){
                    level[e.to] = level[curr]
                        + 1;
                    q.push(e.to);
                }
            }
        }
    }
}

```

```

        //flows
        ll curflow;
        while(curflow = dfs(src,
            t, inf)) flow +=
            curflow;
    } while(level[t] != 0);
}
return flow;
}
};

```

4.5 DSU

```

class DSU{ // 1 based
public:
    vector<ll> par, sz;
    ll maxSize; //size of the max
        componenet
    ll numc; //number of connected
        components
    DSU(ll n){
        par.assign(n+1,-1);
        sz.assign(n+1,1);
        for(int i = 1; i <= n;
            i++) par[i] = i;
        maxSize = 1; numc = n;
    }
    ll findRoot(ll node){
        if(par[node] == node)
            return node;
        return par[node] =
            findRoot(par[node]);
    }
    void merge(ll a, ll b){

```

```

    a = findRoot(a); b =
        findRoot(b);
    if(a != b){
        if(sz[a] < sz[b])
            swap(a,b);
        par[b] = a; sz[a]
            += sz[b];
        maxSize =
            max(maxSize,sz[a]);
        numc--;
    }
};

```

5 Maths

5.1 BetterCRT

```

int normalize(int x,int m) {
    x %= m; if(x<0) x+= m; return x;
}
// add gcd(a,b,x,y) extended
// x = a[0] mod n[0], a[1] mod n[1] ...
// works for non coprime n.
pair<int, int> CRT(vector<int>& a,
    vector<int>& n)
{
    int len = a.size();
    for(int i = 0; i < len; i++)
    {
        a[i] = normalize(a[i],n[i]);
    }
    int ans = a[0];

```

```

int lcm = n[0];
for(int i = 1; i < len; i++){
    int x1,y1;
    int d = gcd(lcm, n[i], x1, y1);
    if((a[i]-ans)%d != 0)
    {
        return {-1,-1};
    }
    ans = normalize(ans + x1 * (a[i]
        - ans) / d % (n[i] / d) *
        lcm, lcm * n[i]/d);
    lcm = (lcm*n[i])/__gcd(lcm, n[i]);
}
return {ans,lcm};
}

```

5.2 eulerToitientRootn

```

// counts the number of integers between
// 1 and n inclusive, which are coprime
// to n
int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

```

5.3 extendedgcd

```

int gcd(int a, int b, int& x, int& y) {
    x = 1, y = 0;
    int x1 = 0, y1 = 1, a1 = a, b1 = b;
    while (b1) {
        int q = a1 / b1;
        tie(x, x1) = make_tuple(x1, x - q
            * x1);
        tie(y, y1) = make_tuple(y1, y - q
            * y1);
        tie(a1, b1) = make_tuple(b1, a1 -
            q * b1);
    } return a1;
}

int modinv(int a, int m){
    int x, y;
    int g = gcd(a, m, x, y);
    if(g != 1) return -100;
    else return (x%m + m)%m;
}

```

5.4 linearSieve

```

const int maxn = 100000;
int lp[maxn+1], mobius[maxn+1];
int twopow[maxn+1];
vector<int> primes(0);
void lsieve(){
    lp[1] = 1;
    mobius[1] = 1;
    for(int i = 2; i <= maxn; i++){
        if(lp[i] == 0){

```

```

    lp[i] = i;
    primes.push_back(i);
}
for(int j = 0;
    (j<primes.size()) &&
    primes[j]<=lp[i] &&
    i*primes[j] <= maxn;
    j++){
    lp[i*primes[j]] =
        primes[j];
}
if(lp[i] == i) mobius[i] =
    -1;
else {
    int x = i/lp[i];
    if(x%lp[i] == 0)
        mobius[i] = 0;
    else mobius[i] =
        mobius[x]*mobius[lp[i]];
}
}
}

```

5.5 mobiphi

```

void mobiphin(int n) {
    vector<int> phi(n + 1);
    vector<int> mu(n + 1);
    for (int i = 0; i <= n; i++){
        phi[i] = i; mu[i] = 1;
    }
    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;

```

```

        if((j/i)%i == 0) mu[j] = 0;
        mu[j] = -mu[j];
    }
}
}

```

6 strings

6.1 manacher

```

/*
 * For each position in a string,
 * computes p[0][i] = half length of
 * longest even palindrome around pos i,
 * p[1][i] = longest odd (half rounded
 * down). where i is right center
 * (verify)
 * O(n)
 */
vi<vi> manacher(const string& s) {
    int n = sz(s);
    vi<vi> p = {vi(n+1), vi(n)};
    rep(z,0,2) for (int i=0,l=0,r=0;
        i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t,
            p[z][l+t]);
        int L = i-p[z][i], R =
            i+p[z][i]-!z;
        while (L>=1 && R+1<n &&
            s[L-1] == s[R+1])
            p[z][i]++, L--,
                R++;
        if (R>r) l=L, r=R;
    }
}

```

```

    }
    return p;
}

```

6.2 trie

```

typedef struct trie{
    typedef struct node{
        node* nxt[2]; int cnt = 0;
        node(){
            nxt[0] = nxt[1] = NULL;
            cnt = 0;
        }
    }Node;
    Node* head;
    trie() { head = new Node(); }
    void insert(int x){
        Node* cur = head;
        for(int i = 30; i >= 0;
            i--){
            int b = (x >> i) & 1;
            if(!cur -> nxt[b])
                cur -> nxt[b] =
                    new Node();
            cur = cur -> nxt[b];
            cur -> cnt++;
        }
    }
    void remove(int x){
        Node* cur = head;
        for(int i = 30; i >= 0;
            i--){
            int b = (x >> i) & 1;
            cur = cur -> nxt[b];

```

```

        cur -> cnt--;
    }
}
int maxxor(int x){
    Node* cur = head;
    int ans = 0;
    for(int i = 30; i >= 0; i--){
        int b = (x >> i) & 1;
        if(cur -> nxt[!b] && cur
            -> nxt[!b] -> cnt > 0)
        {
            ans += (1LL << i);
            cur = cur -> nxt[!b];
        }
        else
            cur = cur -> nxt[b];
    }
    return ans;
}
}Trie;

```

6.3 Z algo

// z[i] is the length of the longest string that is,
 // at the same time, a prefix of s and
 // a prefix of the suffix of s starting at i

```

vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {

```

```

        if (i <= r)
            z[i] = min (r - i + 1, z[i - 1]);
        while (i + z[i] < n && s[z[i]] ==
            s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    } // for number of occurrences of t
    in s
    return z; //str = s+$+t
}

```

7 Trees

7.1 centroid decomposition

```

vll tree;
//return distance between a and b
int dist(int a, int b){
    // return depth[a] + depth[b] -
    2*depth[lca];
}
//find depth and parent
void dfs1(int v, int p, int d = 0){
    depth[v] = d;
    for(ll &nbr: tree[v])
        {if(nbr == p) continue; dfs1(nbr,
            v, d+1);}
}
vll centroidTree; //actual tree
int croot; //centroid root
vector<bool> vis; //helper
vll cpar; vll sz;

```

```

//construct centroid tree
int markSizeOfUnvisited(int v, int p =
    -1){
    sz[v] = 1;
    for(int nbr: tree[v]){
        if(!vis[nbr] && nbr != p){
            sz[v] +=
                markSizeOfUnvisited(nbr,
                    v);
        }
    }
    return sz[v];
}

int findCentroid(int v, int p, int num){
    for(int nbr: tree[v]){
        if(!vis[nbr] && nbr != p
            && sz[nbr] > num/2){
            return findCentroid(nbr,
                v, num);
        }
    }
    return v;
}

void constructCentroidTree(int v, int p){
    markSizeOfUnvisited(v, p);
    int cvex = findCentroid(v, p,
        sz[v]);
    if(p == -1) croot = cvex;
    else{
        cpar[cvex] = p;
        centroidTree[cvex].push_back(p);
        centroidTree[p].push_back(cvex);
    }
    vis[cvex] = true;
    for(int nbr: tree[cvex]){

```

```

        if(vis[nbr]) continue;
        constructCentroidTree(nbr,
                               cvex);
    }
}
vll opt;
int query(int vx){ int res = 1e8;
    for(int px = vx; px != -1; px =
        cpar[px]){
        res = min(res,
            (int)opt[px] +
            dist(px, vx));
    } return res;
}
void update(int vx){
    for(int px = vx; px != -1; px =
        cpar[px]){
        opt[px] =
            min((int)opt[px],
                dist(vx, px));
    }
}
void init(int n){
    tree.assign(n, vll(0));
    centroidTree.assign(n, vll(0));
    cpar.assign(n, -1); sz.assign(n,
        0);
    vis.assign(n, false);
}
void solve(){
    int n, m; cin>>n>>m; init(n);
    for(int i = 0; i < n-1; i++){
        int u, v; cin>>u>>v; u--;
        v--;
        tree[u].push_back(v);

```

```

        tree[v].push_back(u);
    }
    constructCentroidTree(0,-1);
    //constructs tree
    // other dfs stuff
    const int inf = 1e8;
    opt.clear(); opt.resize(n, inf);
    update(0); //color node 0
    for(int i = 0; i < m; i++){
        int type; cin>>type; int
            vx; cin>>vx; vx--;
        if(type == 1) update(vx);
        else cout<<query(vx)<<"\n";
    }
}

```

7.2 dsu_on_trees

```

vector<int> ansval(maxn, 0);
map<int, int> nmaps[maxn];

void dfs2(int curr, int par){
    map<int, int> &currmap =
        nmaps[curr];
    int bestans = 0;
    for(int child: tree[curr])
        if(child != par){
            dfs2(child, curr);
            auto &cmap = nmaps[child];
            bestans = max(bestans,
                ansval[child]);
        }
    if(cmap.size() >
        currmap.size()){

```

```

        swap(cmap,
            currmap);
    }
    // merge cmap to currmap
    for(auto temp: cmap){
        currmap[temp.first]
            += temp.second;
        if(currmap[temp.first]
            > 1){
            bestans =
                max(bestans,
                    temp.first);
        }
    }
    currmap[colors[curr]]++;
    if(currmap[colors[curr]] > 1)
        bestans = max(bestans,
            colors[curr]);
    ansval[curr] = bestans;

    if(par != -1 && bestans != 0){
        ans[minmax(curr, par)] =
            max(ans[minmax(curr,
                par)], bestans);
    }
}

```

7.3 hld

```

//import lca
//import segment tree
const int maxn = 200005;
vll tree;

```

```

vll par;
ll sz[maxn], pos[maxn], moola[maxn],
depth[maxn];
ll heavy[maxn], chain[maxn];
int num = 0;
//assigns first parent, subtree size,
heavy child and depth
int dfs(int curr, int p, int d = 0){
    par[curr][0] = p; sz[curr] = 1;
    depth[curr] = d;
    int maxchild = -1, msize = 0;
    for(auto &child: tree[curr]){
        if(child == p) continue;
        sz[curr] += dfs(child,
            curr, d+1);
        if(sz[child] > msize){
            maxchild = child;
            msize =
                sz[child];
        }
    }
    heavy[curr] = maxchild;
    return sz[curr];
}
//assign pos (in segtree), and chaintop
void decompose(int curr, int p, bool
    isheavy = false){
    pos[curr] = num++;
    if(isheavy == true) chain[curr] =
        chain[p];
    else chain[curr] = curr;

    if(heavy[curr] != -1){
        decompose(heavy[curr],
            curr, true);
    }
}

```

```

    }
    for(auto &child: tree[curr]){
        if(child == p || child ==
            heavy[curr]) continue;
        decompose(child, curr,
            false);
    }
}

int query(Node* stree, int from, int p){
    int res = 0;
    while(chain[from] != chain[p]){
        int top = pos[chain[from]];
        int till = pos[from];
        res =
            max(stree->query(top,
                till+1), res);
        //jump to the above one
        from = chain[from];
        from = par[from][0];
    }
    int top = pos[p];
    int till = pos[from];
    res = max(res, stree->query(top,
        till+1));
    return res;
}

//call dfs(0,-1) then
decompose(0,-1,false)
//then query(node, child, lca), segtree
is of size n (vertices)

```

7.4 lca

```

#define sz(x) (int)(x).size()
template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1,
        V) {
        for (int pw = 1, k = 1; pw
            * 2 <= sz(V); pw *= 2,
            ++k) {
            jmp.emplace_back(sz(V) -
                pw * 2 + 1);
            rep(j,0,sz(jmp[k]))
                jmp[k][j] = min(jmp[k -
                    1][j], jmp[k - 1][j
                    + pw]);
        }
    }
    T query(int a, int b) {
        assert(a < b); // or
        return inf if a == b
        int dep = 31 -
            __builtin_clz(b - a);
        return min(jmp[dep][a],
            jmp[dep][b - (1 <<
                dep)]);
    }
};

struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;
    //pass in adjacency list, 0-based
    tree, root at 0

```



```

LCA(vector<vi>& C) : time(sz(C)),
    rmq((dfs(C,0,-1), ret)) {}
void dfs(vector<vi>& C, int v,
    int par) {
    time[v] = T++;
    for (int y : C[v]) if (y
        != par) {
        path.push_back(v),
        ret.push_back(time[v]);
        dfs(C, y, v);
    }
}
int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) =
        minmax(time[a],
            time[b]);
    return path[rmq.query(a,
        b)];
}
//dist(a,b){return depth[a] +
    depth[b] - 2*depth[lca(a,b)];}
};

```

8 X Extras

8.1 customHash

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t
        x) {
        //
        http://xorshift.di.unimi.it/splitmix64.c

```

```

        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) *
            0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) *
            0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM
        = chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x +
            FIXED_RANDOM);
    }
};

```

8.2 dynamicConnectivity

```

// DSU with Rollback
vll par; vll sz;
vector<pll> hist; int cc = 0;
int findRoot(int v){
    while(par[v] != v) v = par[v];
    return v;
}
void unite(int u, int v){
    u = findRoot(u); v = findRoot(v);
    if(u != v){
        cc--;
        if(sz[u] > sz[v]) swap(u,
            v);
        hist.push_back({~v,
            sz[v]}); sz[v] +=

```

```

        sz[u];
        hist.push_back({u,
            par[u]}); par[u] = v;
    }
}
void rollback(int save){
    while((int)hist.size() > save){
        int v = hist.back().first;
        int val =
            hist.back().second;
        hist.pop_back();
        if(v < 0){
            cc++; sz[~v] = val;
        } else {
            par[v] = val;
        }
    }
}

int n, m, q;
void init(){
    par.assign(n, 0);
    iota(par.begin(), par.end(), 0);
    sz.assign(n, 1); cc = n; }

vll answer;
void dnc(int l, int r, vector<ppll>&
    redge){
    vector<ppll> partialoverlap;
    int save = hist.size();
    for(auto &val: redge){
        int cl = val.first.first;
        int cr = val.first.second;
        int u = val.second.first;
        int v = val.second.second;
        if(cr <= 1 || cl >
            r){/*nothing*/}
        else if(cl <= 1 && r < cr){
            unite(u, v);
        } else {

```

```

        partialoverlap.push_back(val);
    }
}
if(l == r){
    //answer query:
    answer[l] = cc;
} else {
    int mid = (l+r)/2;
    dnc(l, mid,
        partialoverlap);
    dnc(mid+1, r,
        partialoverlap);
}
rollback(save);
}
int main(){
    cin>>n>>m>>q; init();//initialize
        dsu
    vector<pair<pll, ll>> edgelist;
    //already existing edges
    for(int i = 0; i < m; i++){
        int u,v;cin>>u>>v;u--;v--;
        if(u > v) swap(u, v);
        edgelist.push_back({{u,v},
            0});
    }
    answer.assign(q+1, -1);
    // if u,v then remove else add
    edge
    for(int i = 1; i <= q; i++){
        int u,v; cin>>u>>v; u--;
        v--;
        if(u > v) swap(u, v);
        edgelist.push_back({{u,v},
            i});
    }

```

```

    }
    sort(edgelist.begin(),
        edgelist.end());
    int esize = edgelist.size();
    vector<ppll> rangeedge;
    for(int i = 0; i < esize; i++){
        auto &val = edgelist[i];
        int u = val.first.first, v
            = val.first.second;
        int aq = val.second; int
            en;
        if(i+1>=esize || val.first
            !=
            edgelist[i+1].first){
            en = q+1;
        } else {
            en =
                edgelist[i+1].second;
            i++;
        }
        rangeedge.push_back({{aq,en},
            {u,v}});
    }
    dnc(0,q, rangeedge);
    for(int i = 0; i <= q; i++){
        cout<<answer[i]<<" ";
        //number of cc
    }
}

```

8.3 mathstuff

```

/*
Burnside lemma:
-----

```

$(1/n) * \text{sum}(c[0] \dots c[n-1])$
 where n ways to change positions and
 c[k] is
 combinations that remain unchanged when
 k'th way applied
 c[0] = total combinations usually

Derangement rec:

$D[n] = (n-1)(D[n-1] + D[n-2])$

$D[1] = 0, D[2] = 1$

Catalan Number

$2n_C_n - 2n_C_(n+1)$

$C[n]$ = no. of binary trees of n nodes (l
and r diff)

$C[n]$ = no of trees of n+1 nodes (l and r
same)

Fermats theorem

$a^p = a \pmod p$

(when a,p coprime)

so $a^{(p-1)} \pmod p = 1$

Wilson's Theorem

if p is prime then

$(p-1)! = -1 \text{ or } p-1 \pmod p$

Game Theory

Parital Games - Eg: Chess, TicTacToe

Impartial - Moves only depend on state of the game.

Types of Impartial Games -

- a) Normal Game: Player who plays the last move wins
- b) Misere Game: Player who plays the last move loses

NIM GAME- Impartial Normal Game;

rules-

Stone piles: [a₁, a₂, a₃..... a_n]

In each turn, a player : choose one pile

-> remove atleast one or more stones

The player who takes the last stone wins.

Alice moves first.

If $XOR(a_1, a_2, \dots, a_n) == 0$

Bob wins

else

Alice wins

What is Sprague-Grundy Theorem?

Suppose there is a composite game (more than one sub-game) made up of N

sub-games and two players, A and B.

Then Sprague-Grundy Theorem says that if both A and B play optimally (i.e., they don't make any mistakes), then

the player starting first is guaranteed to win if the XOR of the Grundy numbers of position in each sub-games at the beginning of the game is non-zero. Otherwise, if the XOR evaluates to zero, then player A will lose definitely, no matter what.

We can apply Sprague-Grundy Theorem in any impartial game and solve it. The basic steps are listed as follows:

- Break the composite game into sub-games.
- Then for each sub-game, calculate the Grundy Number at that position.
- Then calculate the XOR of all the calculated Grundy Numbers.
- If the XOR value is non-zero, then the player who is going to make the turn (First Player) will win else he is destined to lose, no matter what.

A Dynamic Programming

(Memoization-based) approach to

calculate Grundy Number of a Game

Game Description-

Just like a one-pile version of Nim, the game starts with a pile of n stones, and the player to move may take any positive number of stones.

The last player to move wins. Which player wins the game?

A Function to calculate Mex of all the values in that set

This function remains same

*/

```
int calculateMex(unordered_set<int> Set)
{
    int Mex = 0;
    while (Set.find (Mex) !=
           Set.end())
        Mex++;
}
```

```
    return (Mex);
}

// A function to Compute Grundy Number
// of 'n'
// Only this function varies according
// to the game
int calculateGrundy(int n, int Grundy[])
{
    if (n == 0)
        return (0);
    if (Grundy[n] != -1)
        return (Grundy[n]);
    unordered_set<int> Set; // A Hash
    Table
    for (int i=0; i<=n-1; i++)
        Set.insert(calculateGrundy(
            Grundy[i]));

    // Store the result
    Grundy[n] = calculateMex (Set);
    return(Grundy[n]);
}

//Driver program to test above functions
int main() {
    int n = 10;
    // An array to cache the
    // sub-problems so that
    // re-computation of same
    // sub-problems is avoided
    int Grundy[n+1];
    memset (Grundy, -1,
            sizeof(Grundy));
    printf ("%d", calculateGrundy(n,
        Grundy));
    return (0);
}
```

}

8.4 random rng

```
//include <random> and <chrono>
mt19937
rng(chrono::steady_clock::now().time_since_epoch().count());
// mt19937 rng((uint64_t) new char);
```

8.5 submaskOfBitmask

```
for (int m=0; m<(1<<n); ++m)
    for (int s=m; s; s=(s-1)&m)
        ... s and m ...
```

8.6 template

```
#define rep(1, a, b) for(int i = a; i < (b); ++i)
#define MOD2 1000000009 #define MOD3 10000000021
#define MOD4 10000000033 #define MOD5 10000000087
```

```
#define MOD6 10000000093 #define MOD7 10000000097
const ll RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct chash { ll operator()(ll x) const { return x ^ RANDOM; } };
gp_hash_table<ll, ll, chash> dp;
int main(){
    ios::sync_with_stdio(false);
    cin.tie(0);
    return 0;
}
```
