CS & IT ENGINEERING

Discrete Mathematics

Graph Theory

Lecture_ 06

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Topics to be Covered

Topic Types Of Graphs – II.

Topic Hypercube.

Isomorphic Graph: (same graph but diff representation)

G_1, G_2 are isomorphic to each other, if G_1, G_2 will have $\boxed{\text{1:1 correspondance}}$

Same no. of vertices.
same no. of edges.
same degree sequence.

Topic: Graph Theory

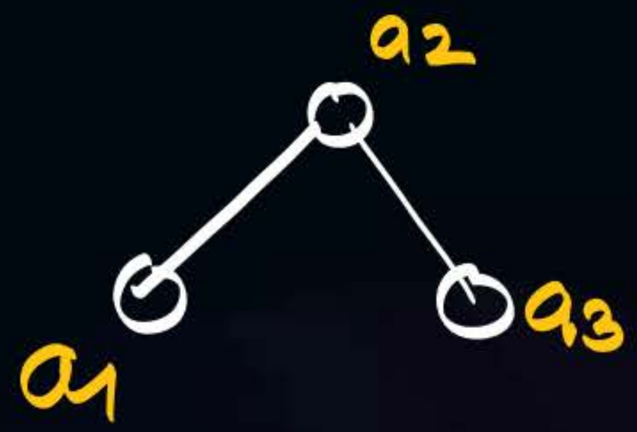
Isomorphic Graph: (same graph but diff representation)

$G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

$f : G_1 \to G_2$ $f : V_1 \to V_2$ $f : E_1 \to E_2$

(1:1 correspondance)

Isomorphic Graph: (same graph but diff representation)



$f: G1 \to G2$

$f: V1 \to V2$

$f: E1 \to E2$

$f: V1 \to V2$

$a1 \to b1$

$a2 \to b2$

$a3 \to b3$

$f: E1 \to E2$

$(a1, a2) \to (b1, b2)$

$(a2, a3) \to (b2, b3)$

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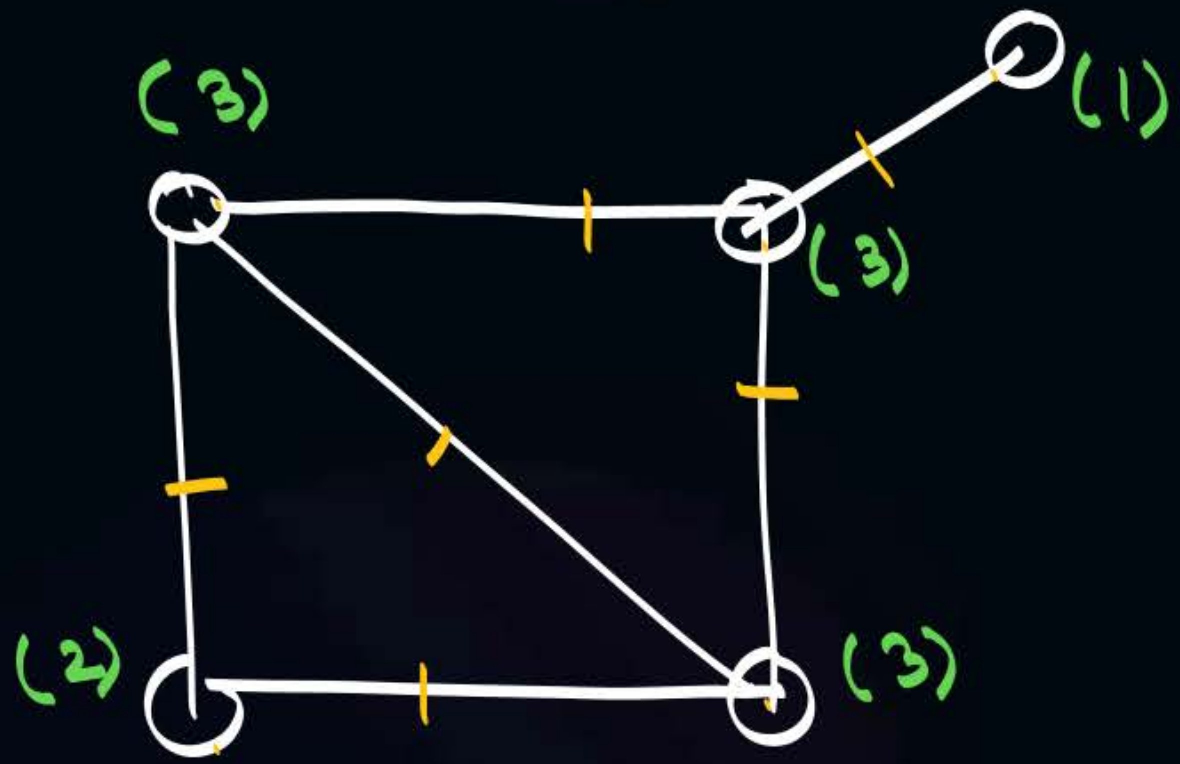
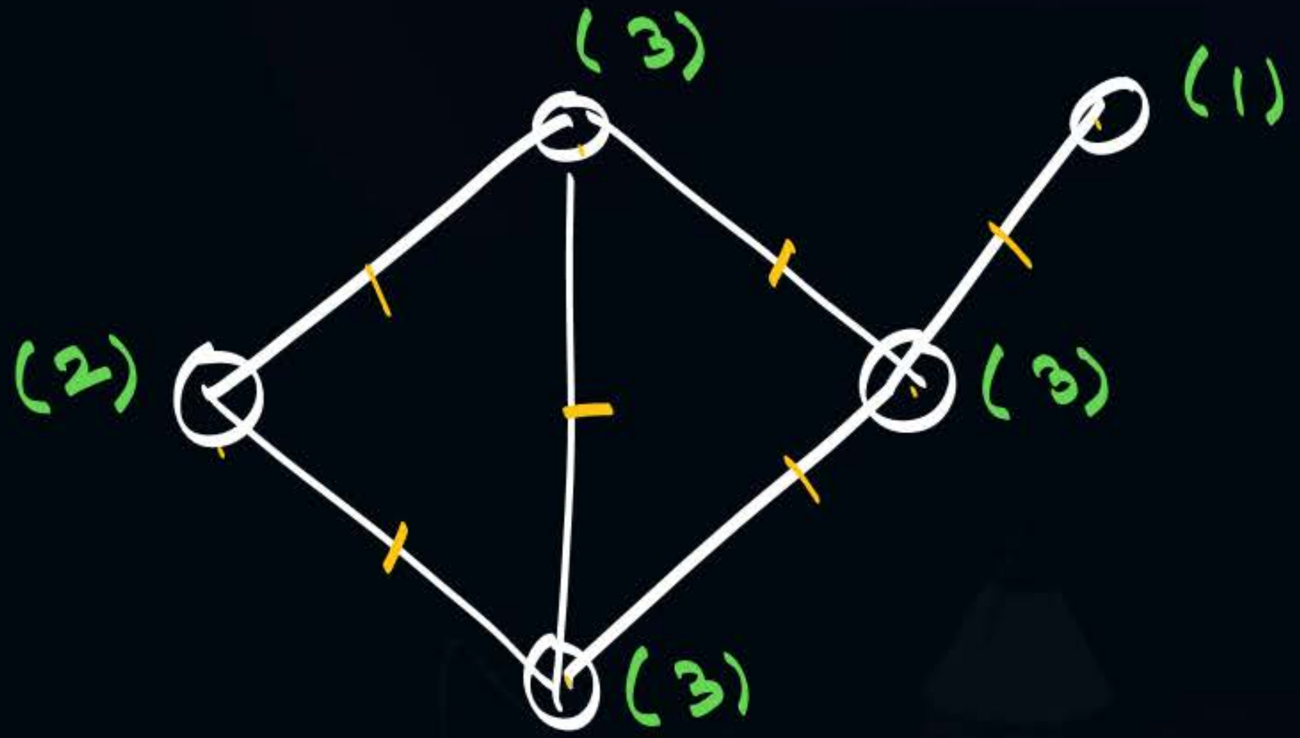
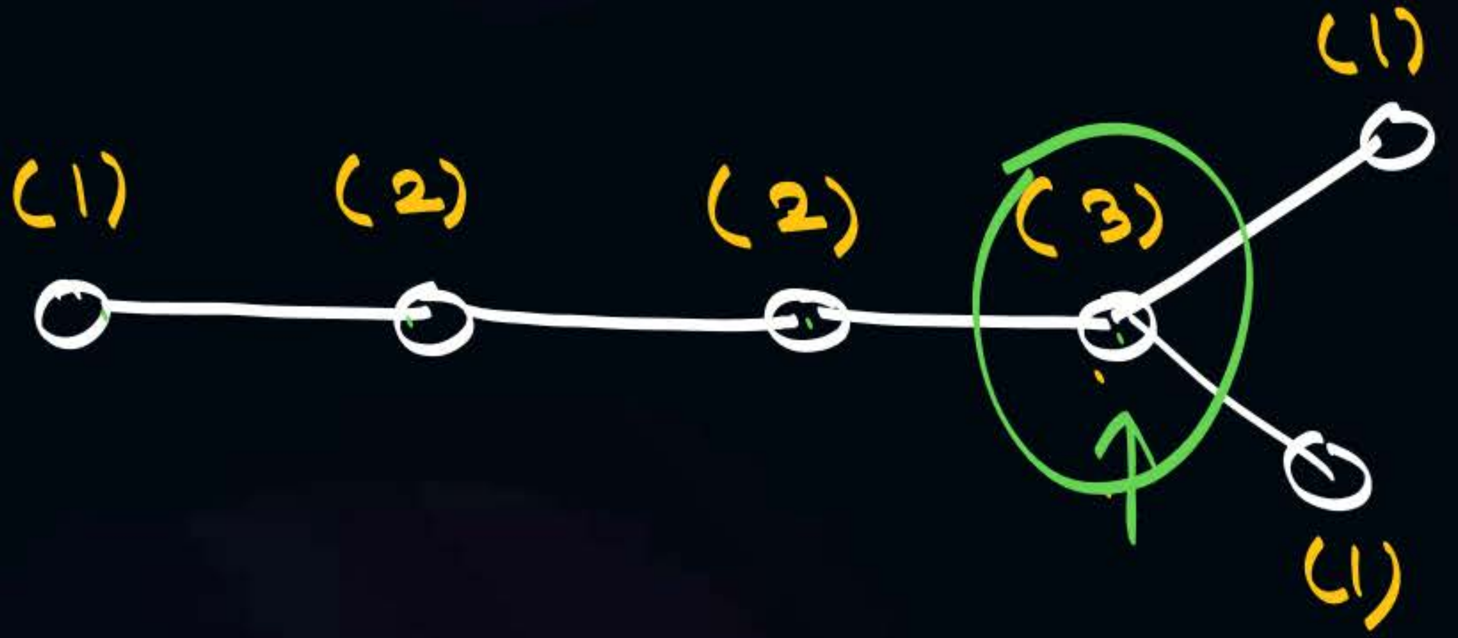


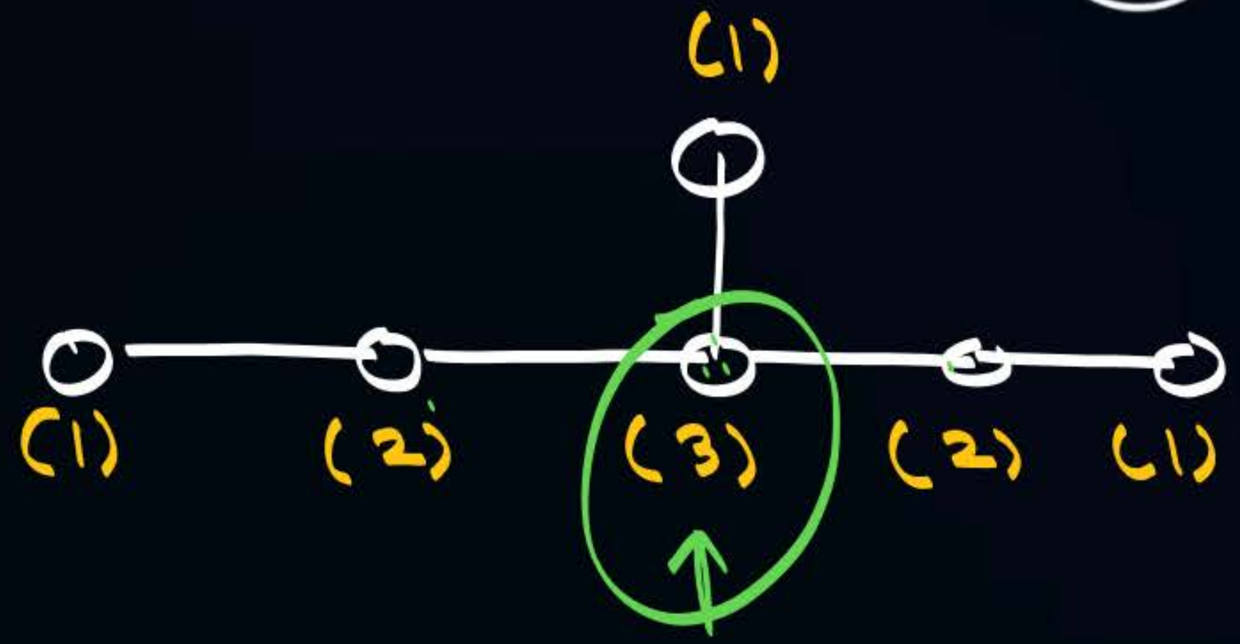
$n = 5$
$e = 6$
3 3 3 2 1



$n = 5$
$e = 6$
3 3 3 2 1

Topic: Graph Theory





3 2 2 1 1 1

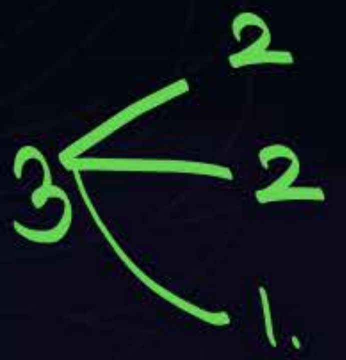
$n = 6$

$e = 5$

3 2 2 1 1 1

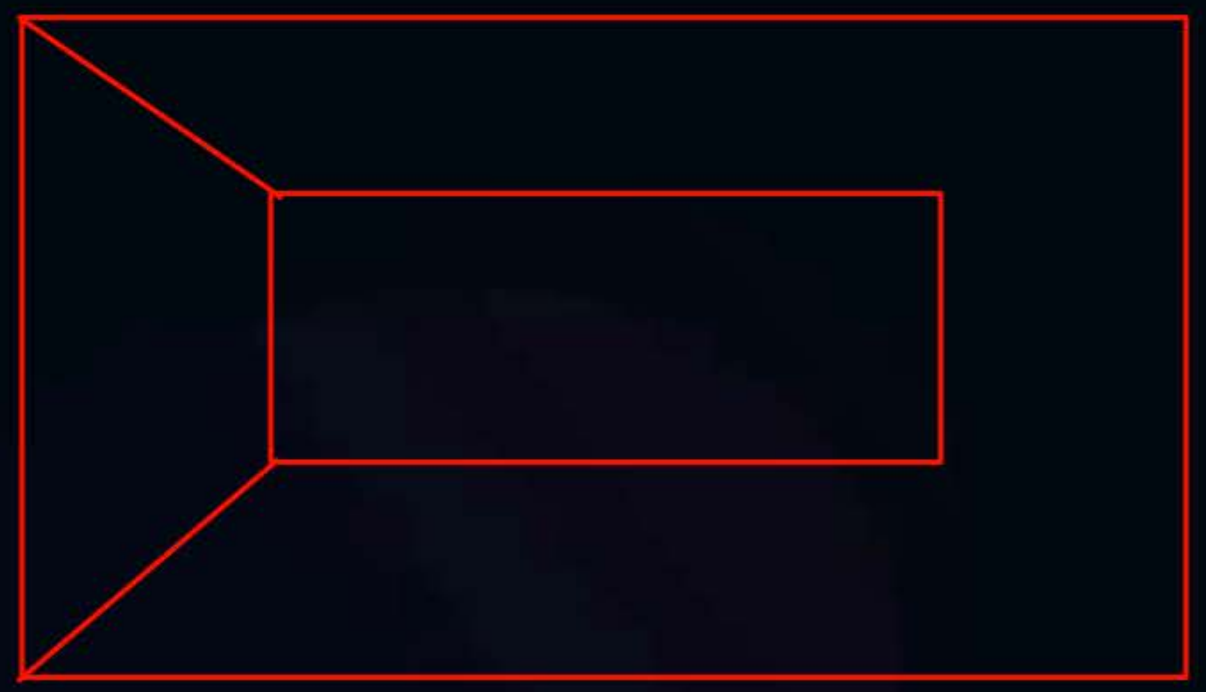
$n = 6$

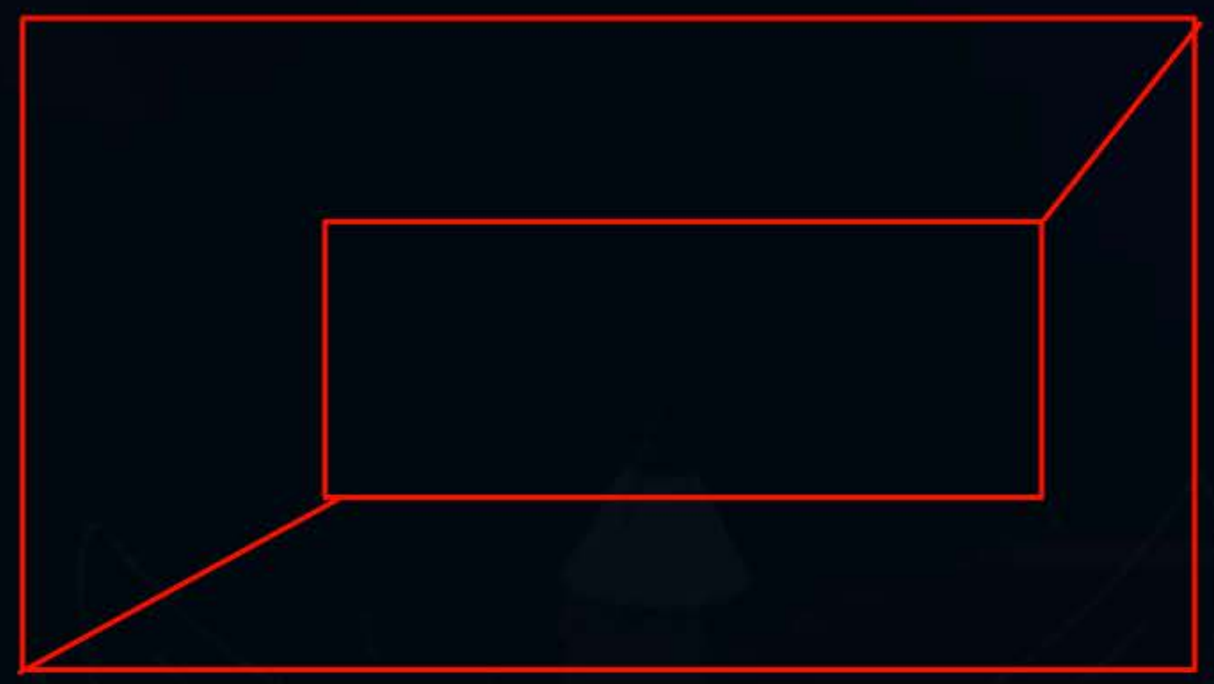
$e = 5$



G_1, G_2 are isomorphic then they will have same degree sequence, same no. of vertices & edges

but viceversa is not true.





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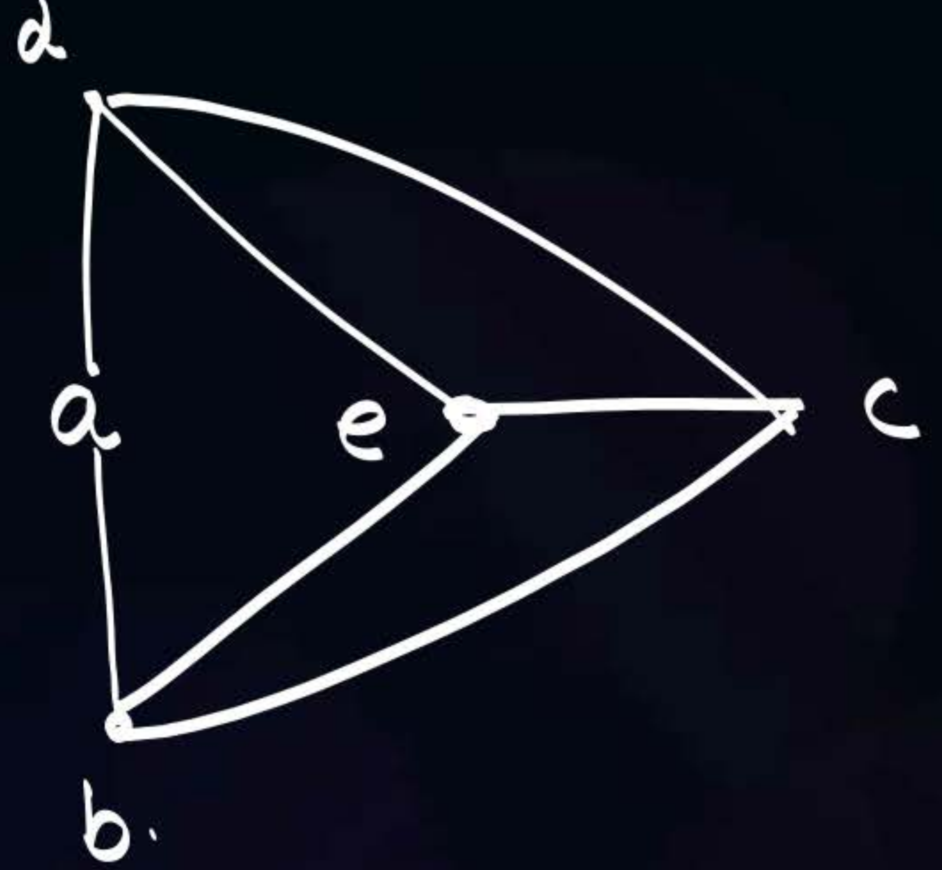
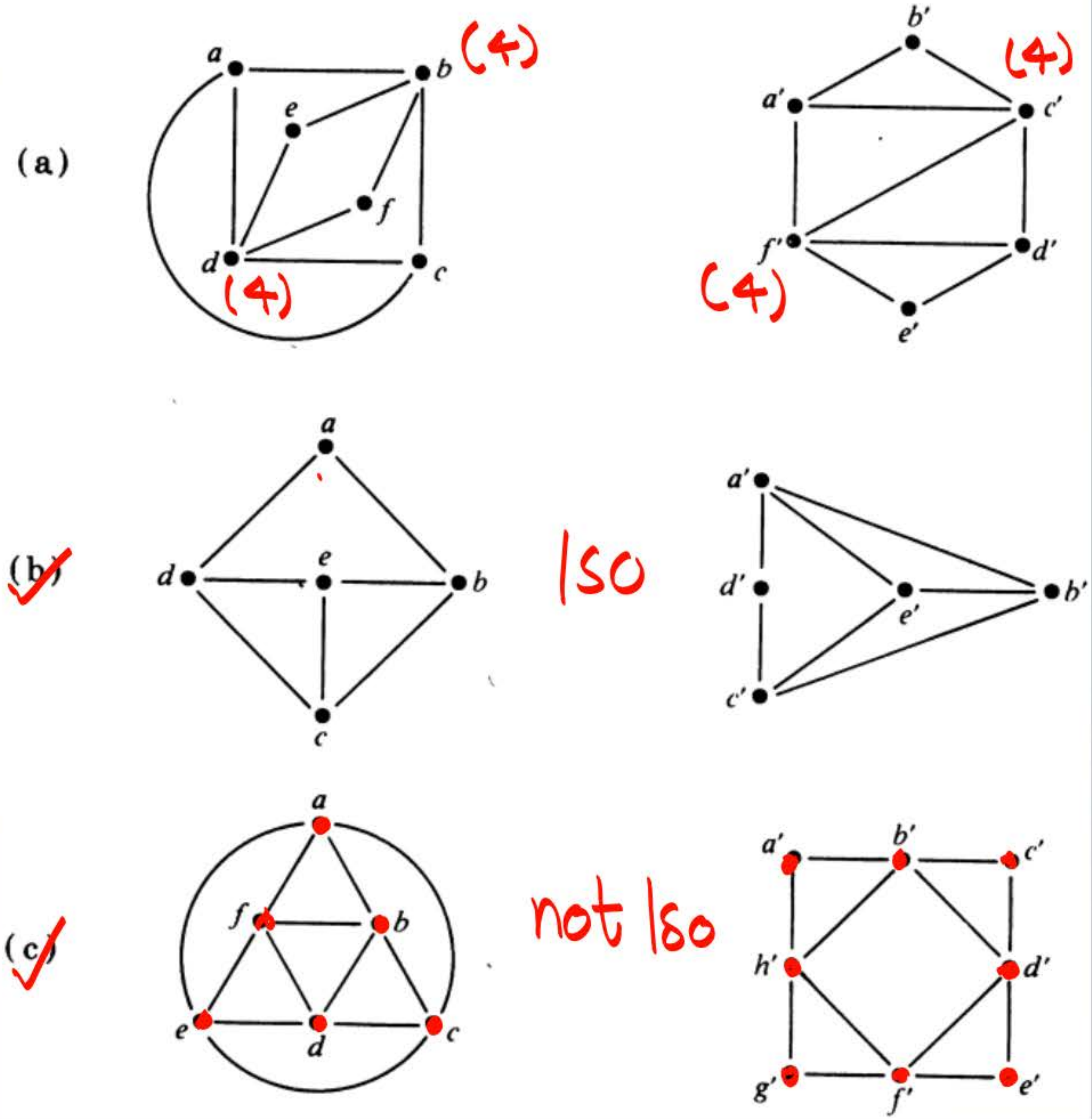
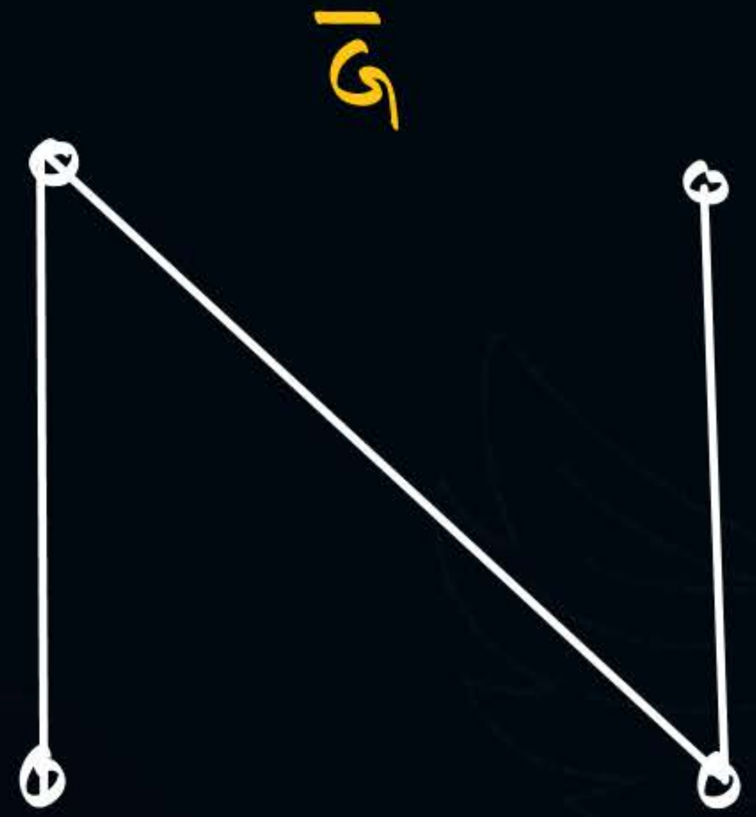




Topic: Graph Theory

Self-complement ($G \equiv \bar{G}$)

when a graph is $\boxed{\text{isomorphic}}$ to its own complement



Topic: Graph Theory

$$V(G) = V(\overline{G})$$

$$E(G) = E(\overline{G})$$

$$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$$

$$\downarrow \qquad \downarrow$$

$$e + e = \frac{n(n-1)}{2}$$

$$2e = \frac{n(n-1)}{2}$$

$$\boxed{e = \frac{n(n-1)}{4}}$$

$n = 3 \quad e = \frac{n(n-1)}{4} = \frac{3 \cdot 2}{4}$ ×

$n = 4 \quad e = \frac{n(n-1)}{4} = \frac{\cancel{4} \cdot 3}{\cancel{4}} = 3$

$n = 5 \quad e = \frac{n(n-1)}{4} = \frac{5 \cdot 4}{4} = 5$

$n = 6 \quad e = \frac{n(n-1)}{4} = \frac{6 \cdot 5}{4}$ ×

$(n = e)$

$C_n \quad (n \geq 3)$

$\downarrow$

C_5

$n = 5$

$\downarrow$

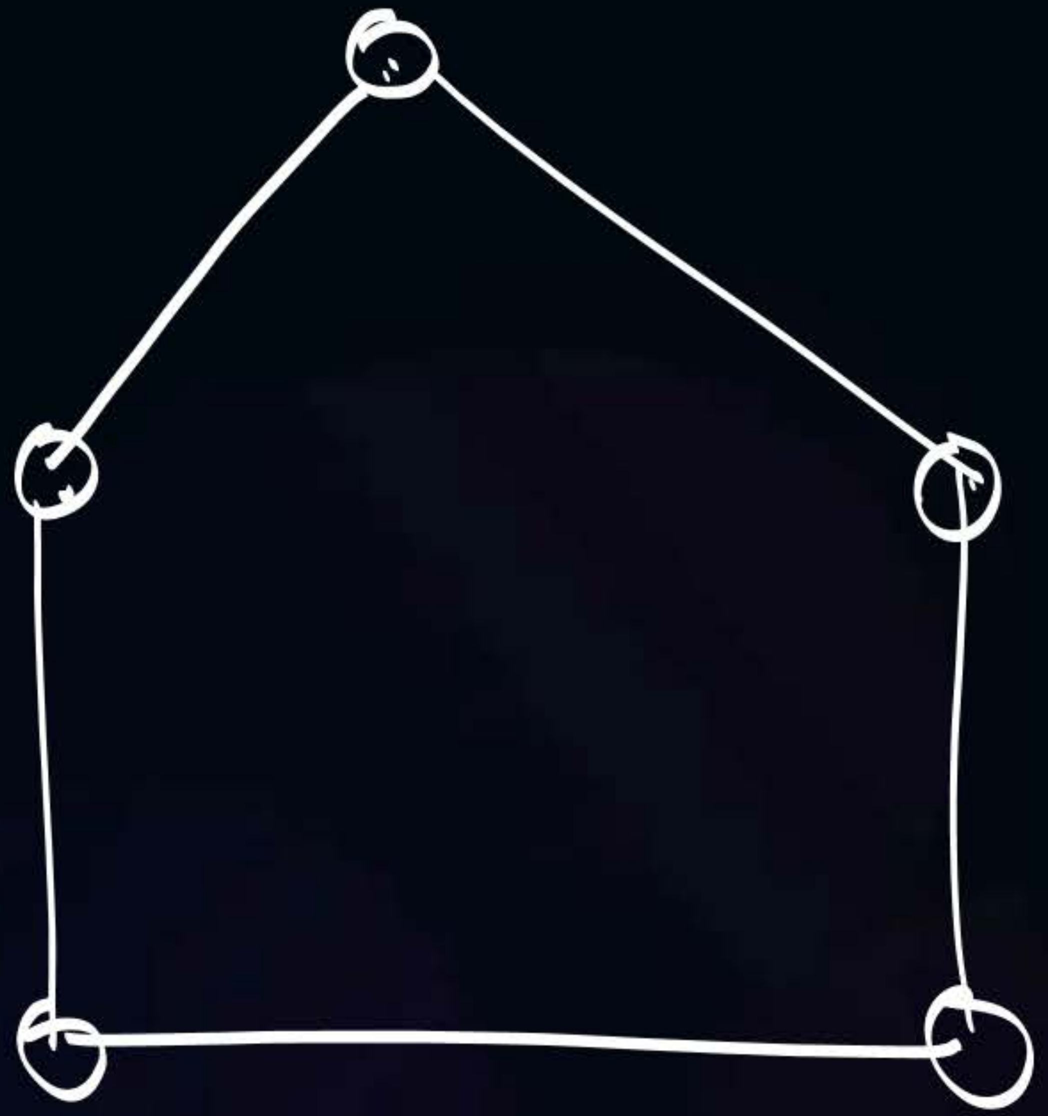
[Self-complement]

$n = 4 \quad e = \frac{n(n-1)}{4} = \frac{\cancel{4} \cdot 3}{\cancel{4}} = 3$

$n = 5 \quad e = \frac{5 \cdot 4}{4} = 5 \longrightarrow (C_5)$

Topic: Graph Theory

C_5



$\overline{C_5} = C_5$

C_5 is only cycle graph which is self complement

Topic: Graph Theory

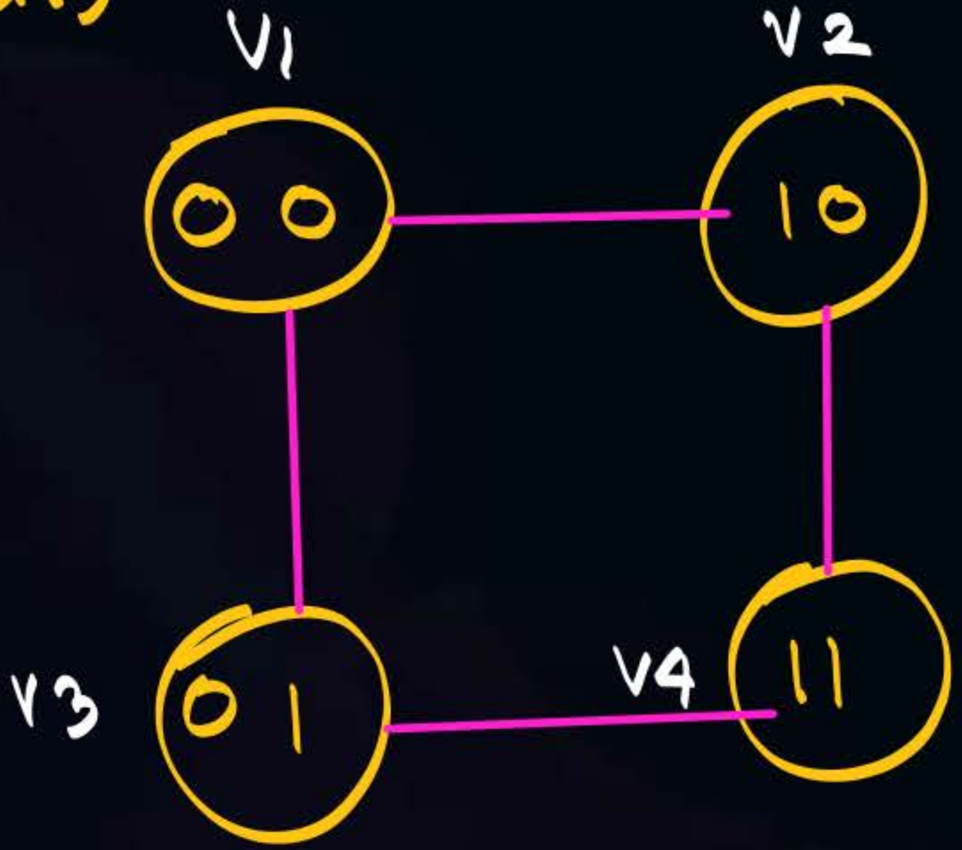
Consider a Graph vertices are represented as n-bit signal. two vertices are adjacent, if there bit position changes by 1-bit.

what will be total no. of edges in G? $(n \geq 1)$

Graph vertices are represented as n-bit signal.

$n = 2$ (2-bit) Total vertices $= 2^2 = 4$.

Every vertex of graph is represented as 2-bit signal.



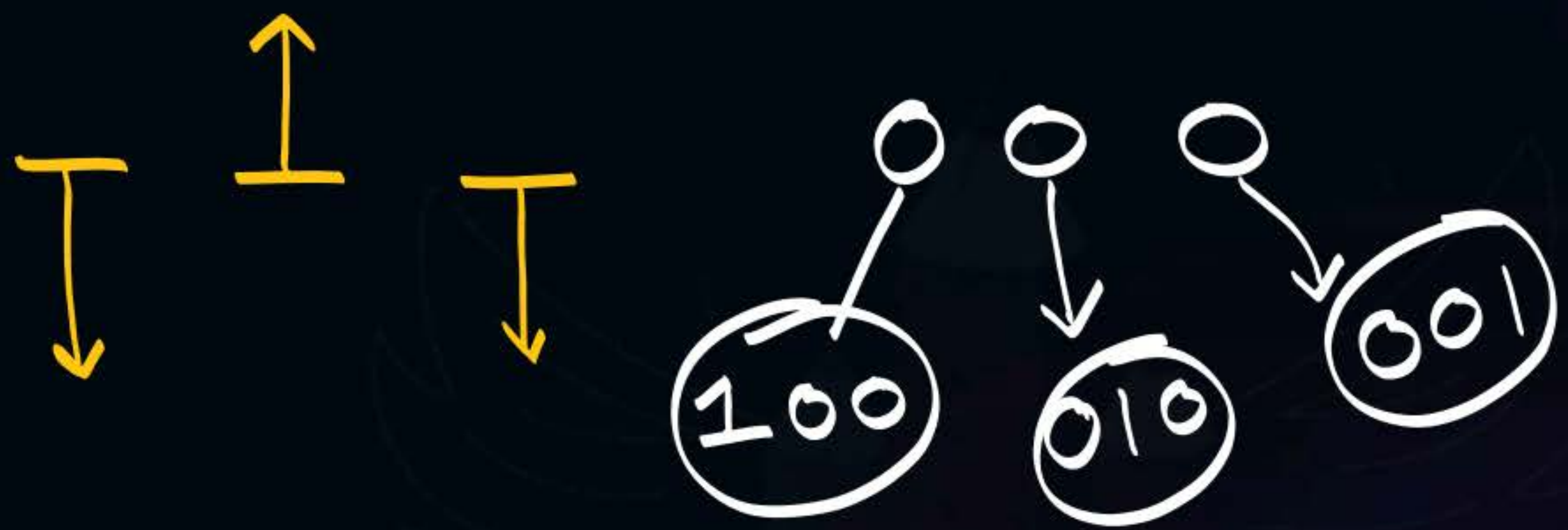
* Degree of each vertex will be 2.

Topic: Graph Theory

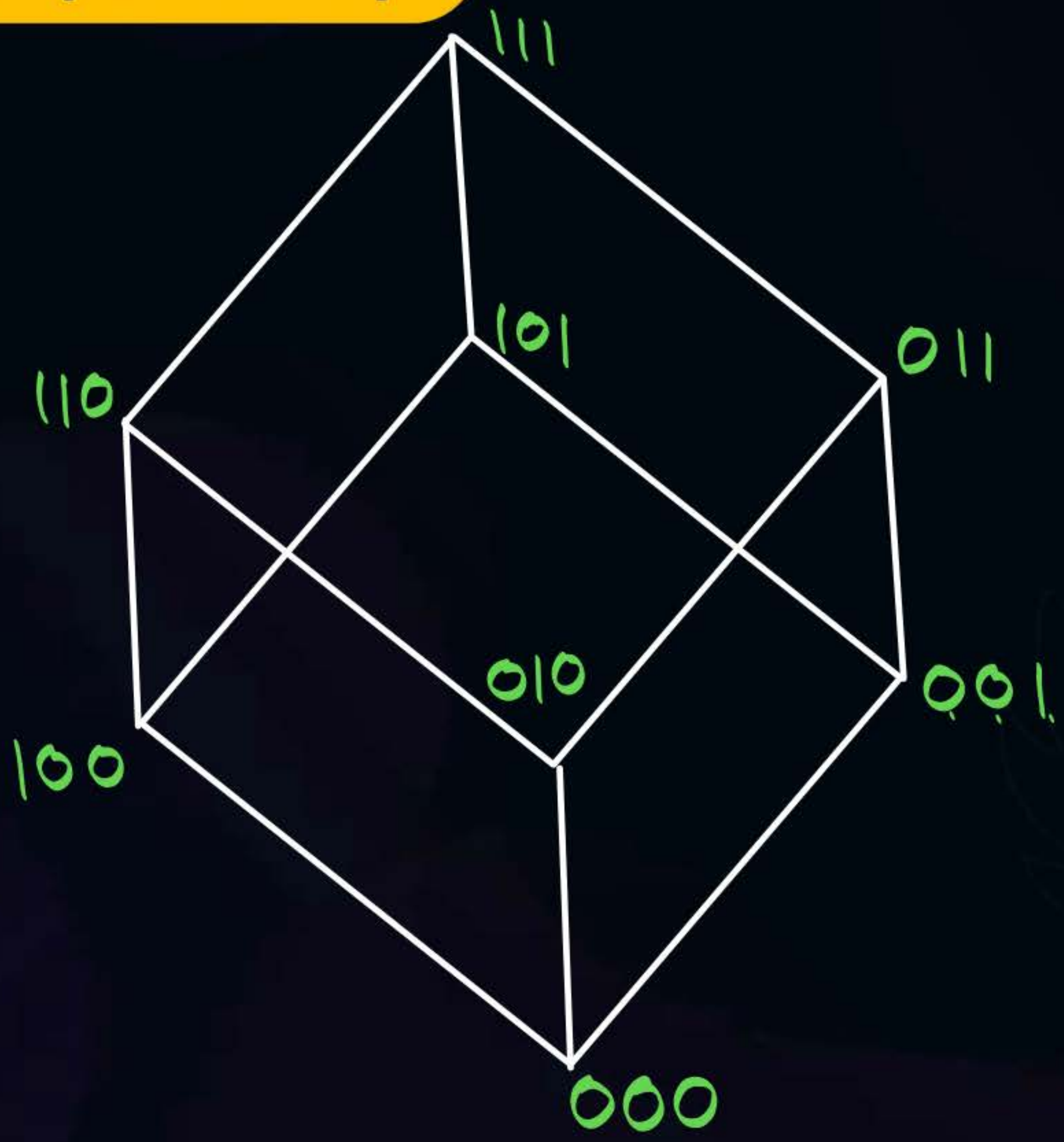
$n = 3$ (3-bit)

Total vertices $= 2^3 = 8$

Degree of each vertex will be 3.



Topic: Graph Theory



$n \rightarrow$ bit signal.

Total vertices $= 2^n$

Degree of each vertex will be n.

$$\sum d(v_i) = 2e$$

$$2^n \times n = 2e$$

$$\boxed{e = n \cdot 2^{n-1}}$$

Topic: Graph Theory

Hypercube (Q_n)

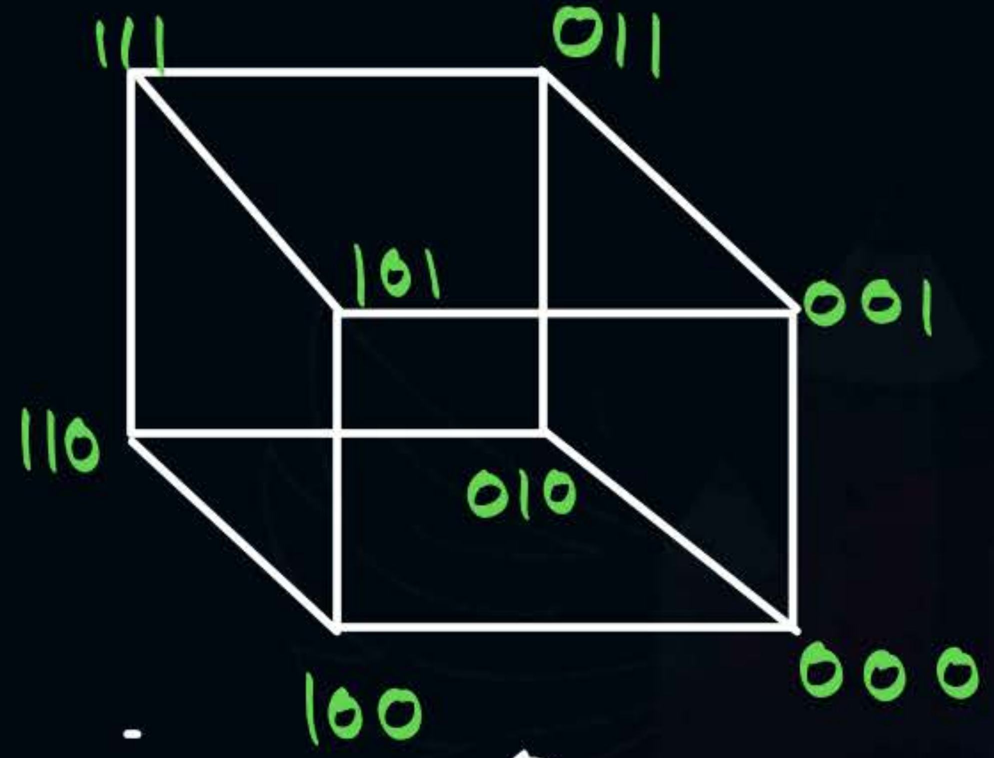
↳ n-bit signal

$V(Q_n) = 2^n$

$E(Q_n) = n \cdot 2^{n-1}$

Q_1

Q_2



Q_3

THANK - YOU