CS & IT ENGINEERING

Discrete Mathematics

Graph Theory

Lecture_08

By- Satish Yadav Sir

Recap of Previous Lecture

Topic Connected / Disconnected

Topics to be Covered

Topic Tree

Topic Properties Of Disconnected

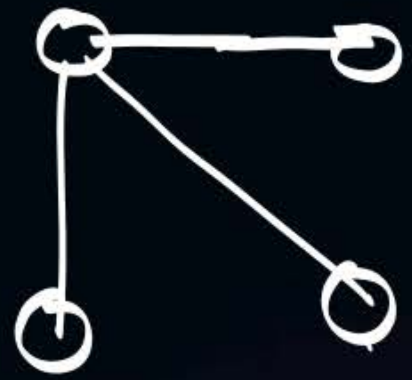
Topic: Graph Theory

(False)

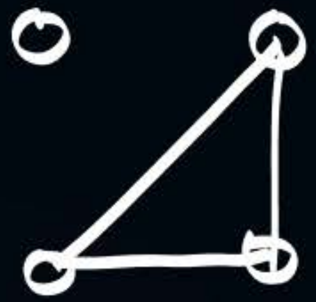
S1: if G is connected then $\bar{G}$ will also be connected.

S2: if G is disconnected then $\bar{G}$ will be connected (True)

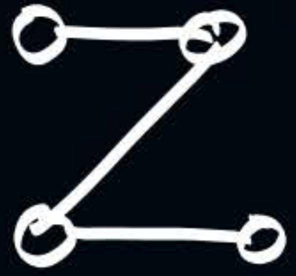
Topic: Graph Theory



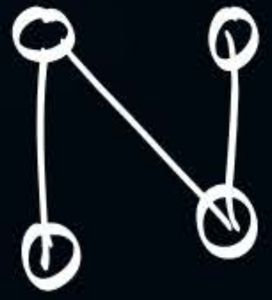
$G \downarrow C$



$\bar{G} \downarrow D.C.$



$G \downarrow C$



$\bar{G} \downarrow C.$

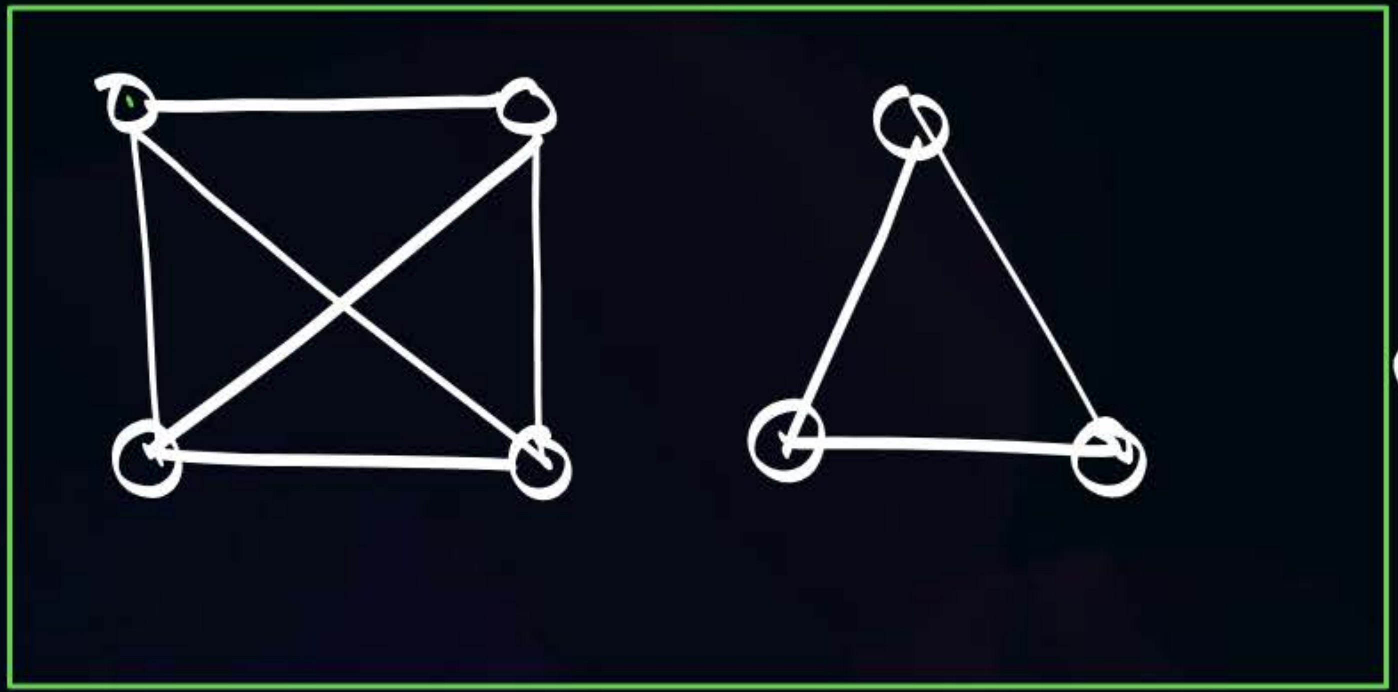
$G \longrightarrow \bar{G}$

$G \downarrow C \qquad \bar{G}: C/D.C.$

Topic: Graph Theory

$G = (V, E)$

$|V| = 7$



$\overline{G}$

Path is available betn all pair of vertices.

Topic: Graph Theory

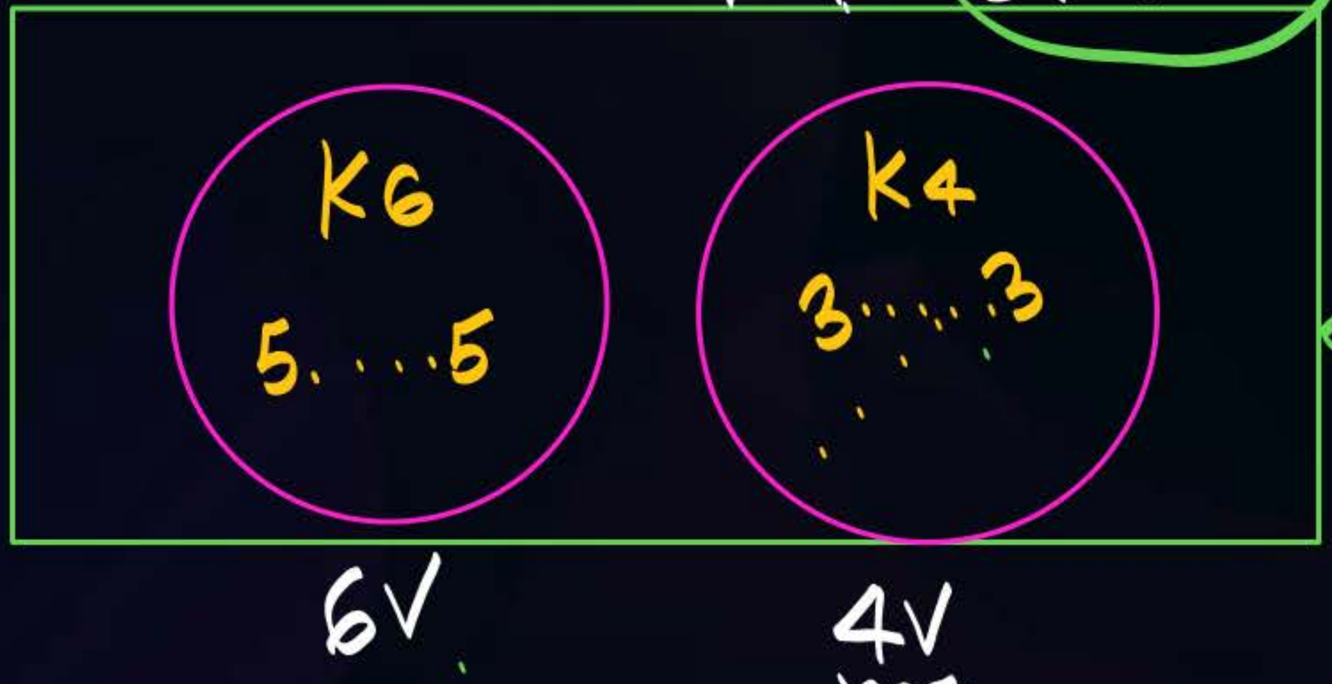
K_{10}

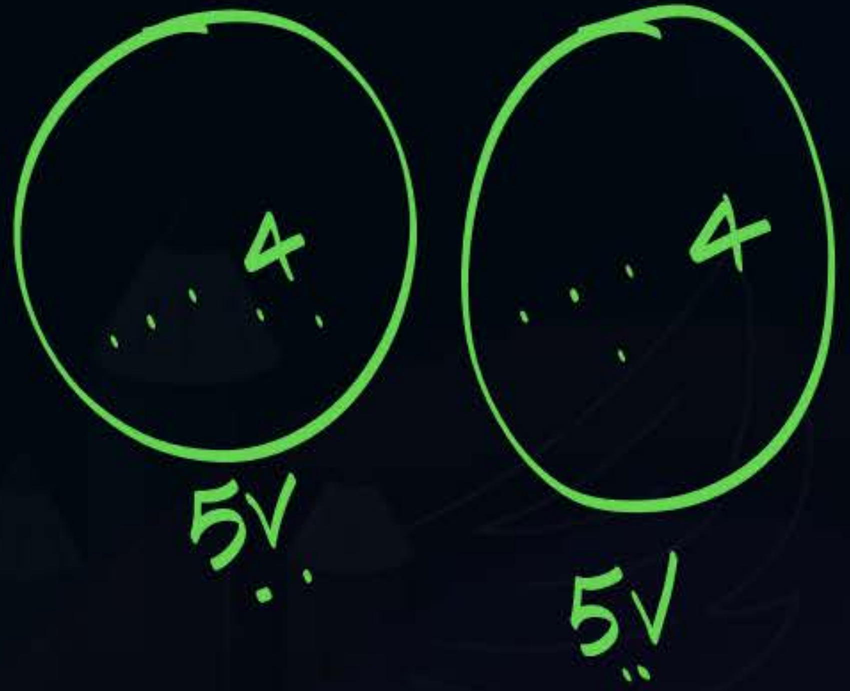
Consider a Graph is having 10 vertices, $\delta(G) \geq 5$ then G is

let's assume disconnected Graph ($k \geq 2$)

$k = 2.$

$n = 10 \quad \delta(G) \geq 5$





min. degree: 3

Assumption is wrong.

Hence connected Graph.

Thm: if $\delta(G) \geq \frac{n-1}{2}$ then G is connected (True)

S_2: if G is connected then $\delta(G) \geq \frac{n-1}{2}$ (False)

$\downarrow$

C_{11} then $\delta(G) \geq 5$

$\downarrow$

2.

Topic: Graph Theory

C / D.C.

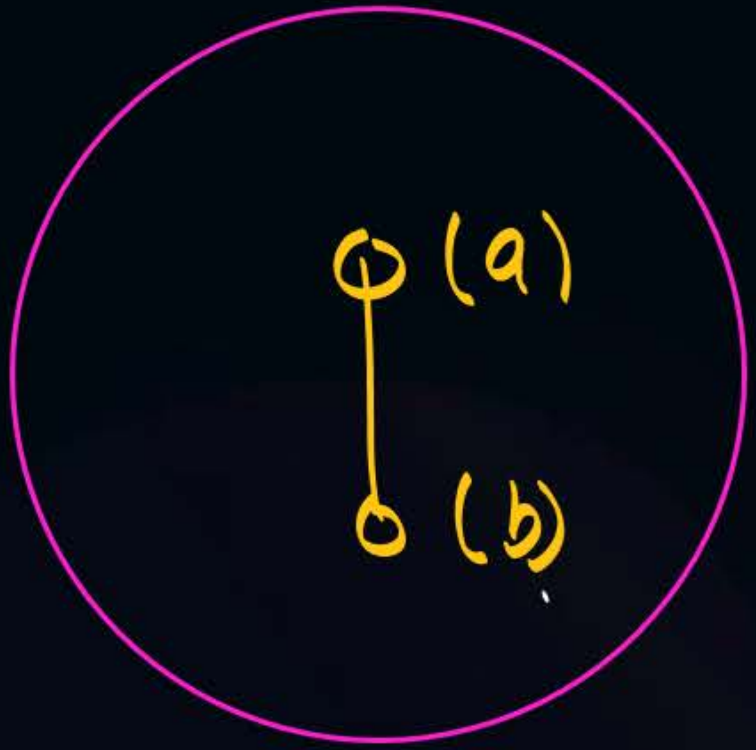
Consider a Graph, contains $\boxed{\text{exactly 2 odd degree vertices}}$ {a} {b}
then there is a path between a and b (True)

G → connected → Stmt → True.

G → disconnected
↓
a, b will be in same component
↓
True.

Exactly 2 odd degree vertices.

Topic: Graph Theory



Exactly 2 odd degree vertices.
a, b.

Disconnected
↓
a, b will lie in same Component

Path is available. betⁿ a & b.

$G = (V, E)$

Range of edges: $(k=1)$

$1 \equiv 2 \equiv 3$

Tree.

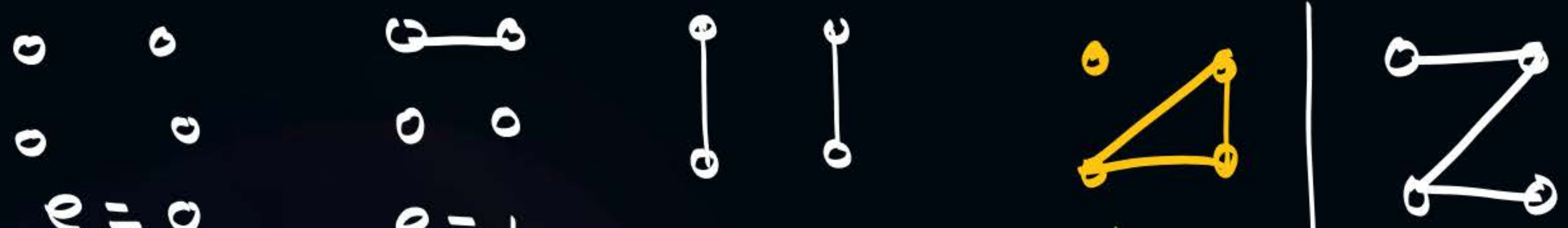
1. min → $\left\{ \begin{array}{l} n-1 \\ + \\ \text{connected.} \end{array} \right. \leq \boxed{e \leq \frac{n(n-1)}{2} \ \{\max\}}$

2. G is connected & does not contains cycle. (Acyclic Graph)

3. unique Path betⁿ all pair of vertices.

Topic: Graph Theory

$n = 4$ $\quad$ $n = 4$ $\quad$ $n = 4$ $\quad$ $n = 4$

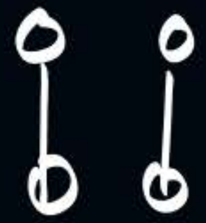


$e = 0$ × $\quad$ $e = 1$ × $\quad$ $e = 2$ $\quad$ $e = 3$ | $e = 3$

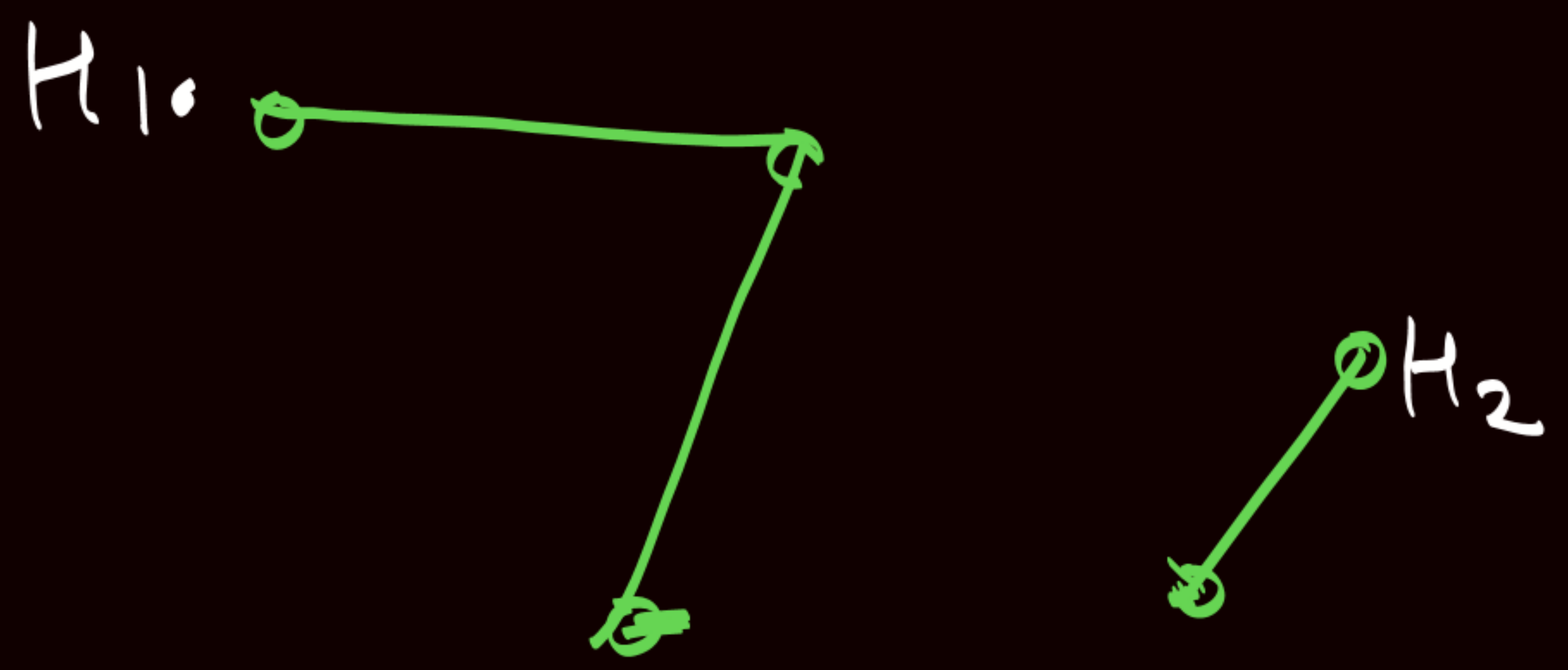
G is having n-1 edges then G is connected (False)

G is connected & n-1 edges then it is min.

G is acyclic Graph then it will be n-1 edges (F)

$n = 4$  → Acyclic Graph.

G is acyclic connected then it will have n-1 edges (T)


H1
H2

Topic: Graph Theory

Total vertices $= n$.

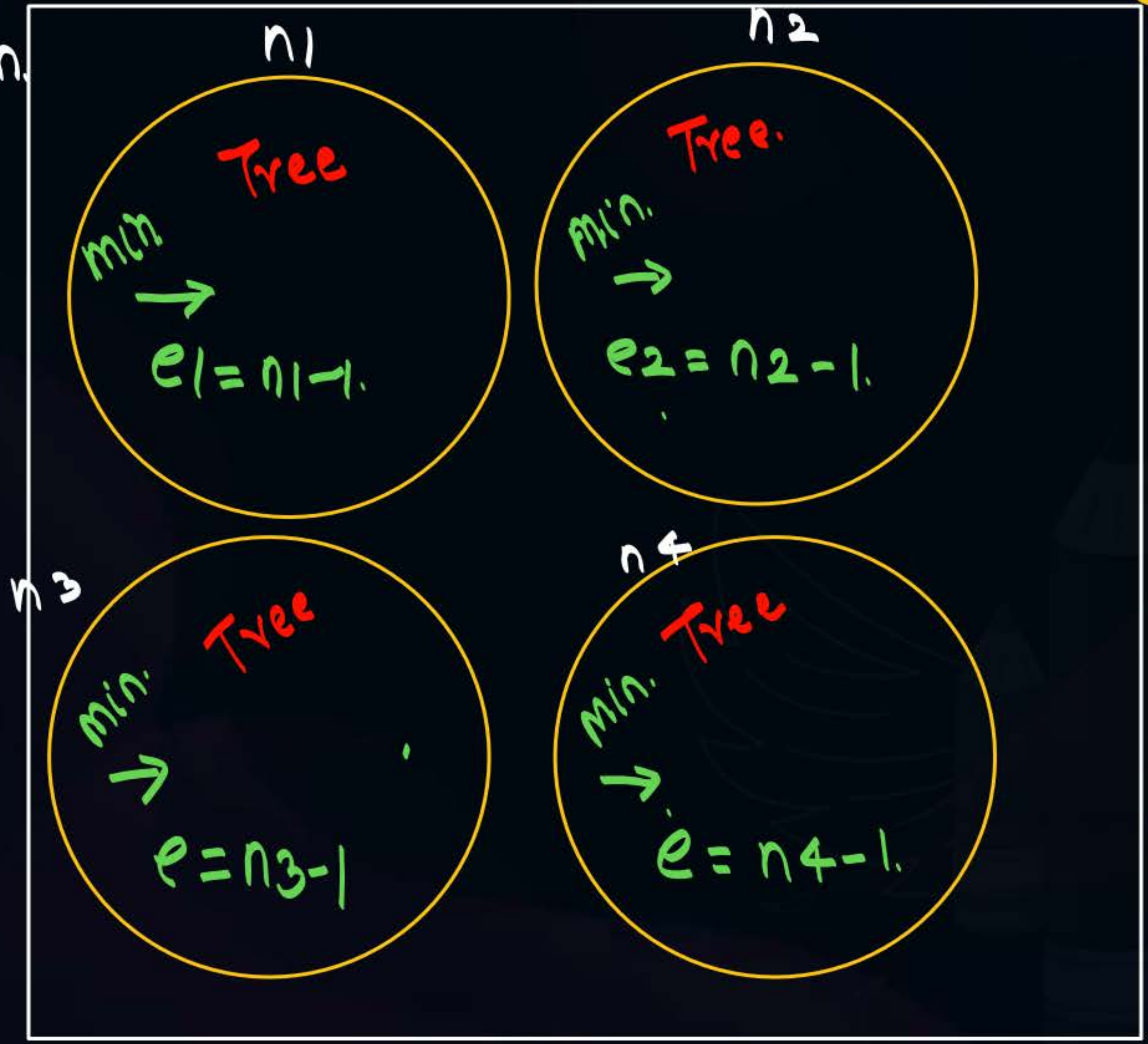
$K = 4$.

min. no. of edges.

$e = n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1$

$= \underline{n_1 + n_2 + n_3 + n_4} - 4$.

$= n - 4 = n - K$.



min. no. of edges.

Forest

$e = n - K$.

Topic: Graph Theory

$n=10$

Consider a disconnected graph, then what will be min. no. of edges.

min

$n=10 \quad k=2 \quad e=8$

$n=10 \quad k=3 \quad e=7$

⋮

$n=10 \quad k=10 \quad e=0$.

Range of edges ($k \geq 2$)

$$n-k \leq e \leq \frac{(n-k)(n-k+1)}{2}$$

$$\boxed{\sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}}$$

Topic: Graph Theory

Consider a Graph of 10 vertices & $\boxed{\text{4 components}}$ what will be max. no. of edges?

$n=10 \quad k=4$

max. $e = \dfrac{(n-k)(n-k+1)}{2}$

$= \dfrac{(10-4)(10-4+1)}{2}$

$= \dfrac{6 \cdot 7}{2} = 21.$

min $e = n-k.$

$e = 10-4 = 6.$

Topic: Graph Theory

Consider a $\boxed{\text{Disconnected}}$ Graph of 10 vertices, what will be max. no. of edges?

&

min.

max — max.

$n = 10 \quad K = 2$

$n = 10 \quad K = 3$

$n = 10 \quad K = 10$

min

$n = 10 \quad K = 2$

$n = 10 \quad K = 3$

$\vdots$

$n = 10 \quad K = 10$

Topic: Graph Theory

Consider a disconnected Graph having 4 components & 20 vertices

What will be max. no. of edges?

In 1st component → 5V

2nd —— " ——→ 6V

3rd —— " ——→ 4V

4th —— " ——→ 5V

5V	6V	4V	5V
$\frac{5\cdot\cancel{4}^{2}}{\cancel{2}}$	$\frac{\cancel{6}^{3}\cdot 5}{\cancel{2}}$	$\frac{\cancel{4}^{2}\cdot 3}{\cancel{2}}$	$\frac{5\cdot\cancel{4}^{2}}{\cancel{2}}$
$e_1=10$	$e_2=15$	$e_3=6$	$e_4=10$

$$e = 10+15+6+10 = 41.$$

THANK - YOU