CS & IT ENGINEERING

Discrete Mathematics

Graph Theory

Lecture_ 05

By- Satish Yadav Sir

Topics to be Covered

Topic Bipartite Graph

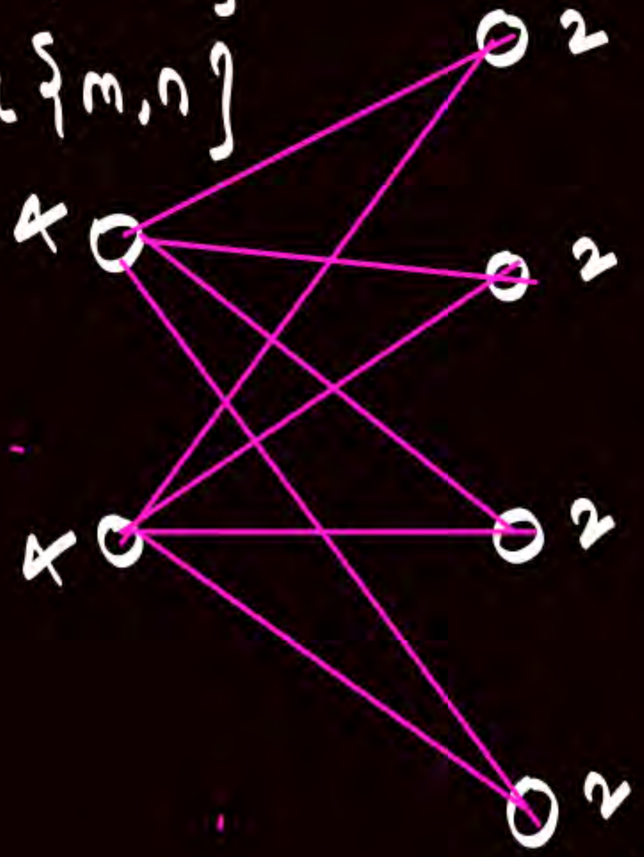
Bipartite Graph

Complete bipartite Graph ($K_{m,n}$)

$|V_1| = m \quad |V_2| = n.$

$\delta(K_{m,n}) = \min\{m,n\}$

$\Delta(K_{m,n}) = \max\{m,n\}$



$K_{2,4}.$

$\delta(K_{2,4}) = 2.$

$\Delta(K_{2,4}) = 4.$

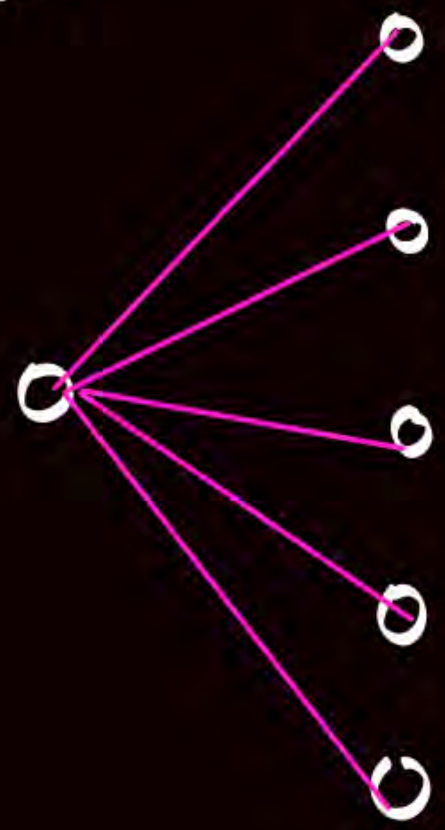
Total vertices $= m + n.$

Total edges $= m \times n.$

Star Graph ($K_{1,n-1}$)

6 vertices → Star Graph.

$K_{1,5}$



$\delta(K_{1,n-1}) = 1.$

$\Delta(K_{1,n-1}) = n-1$

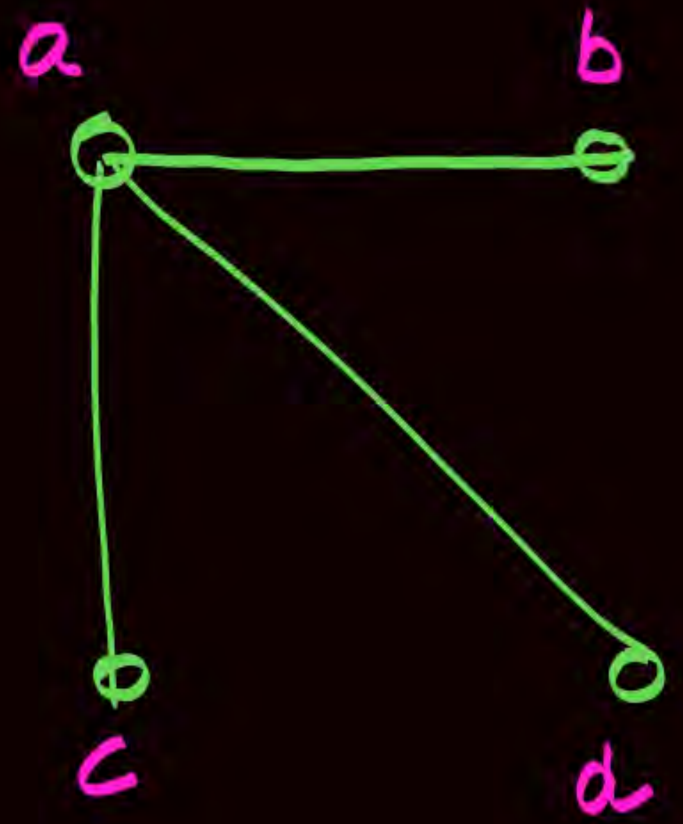
Complement Graph ($\bar{G}$)

$V(G) = V(\bar{G})$

$G + \bar{G} = K_n$

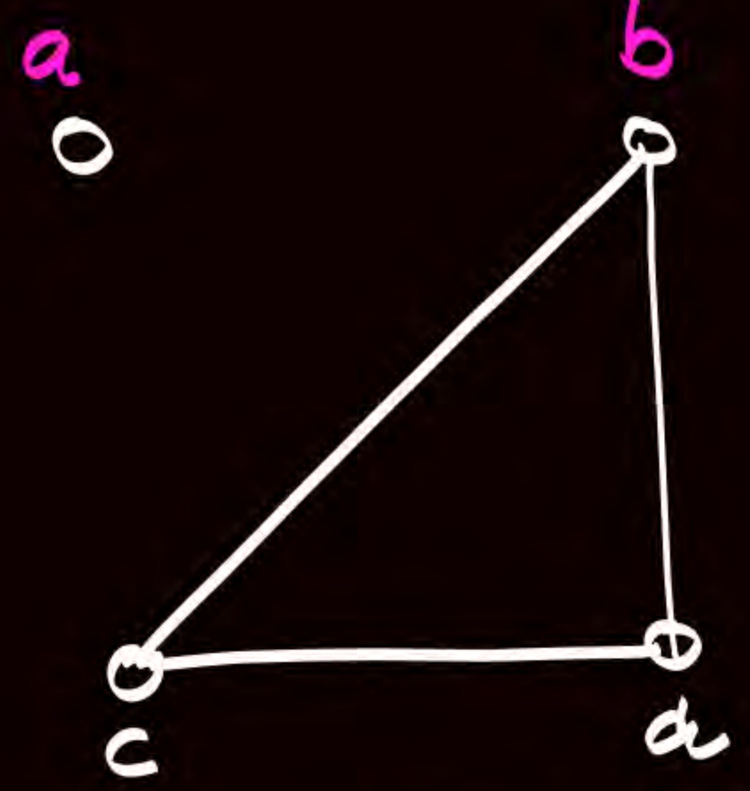
$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

$$\boxed{e(\bar{G}) = \frac{n(n-1)}{2} - e(G)}$$



G

edges → present
absent

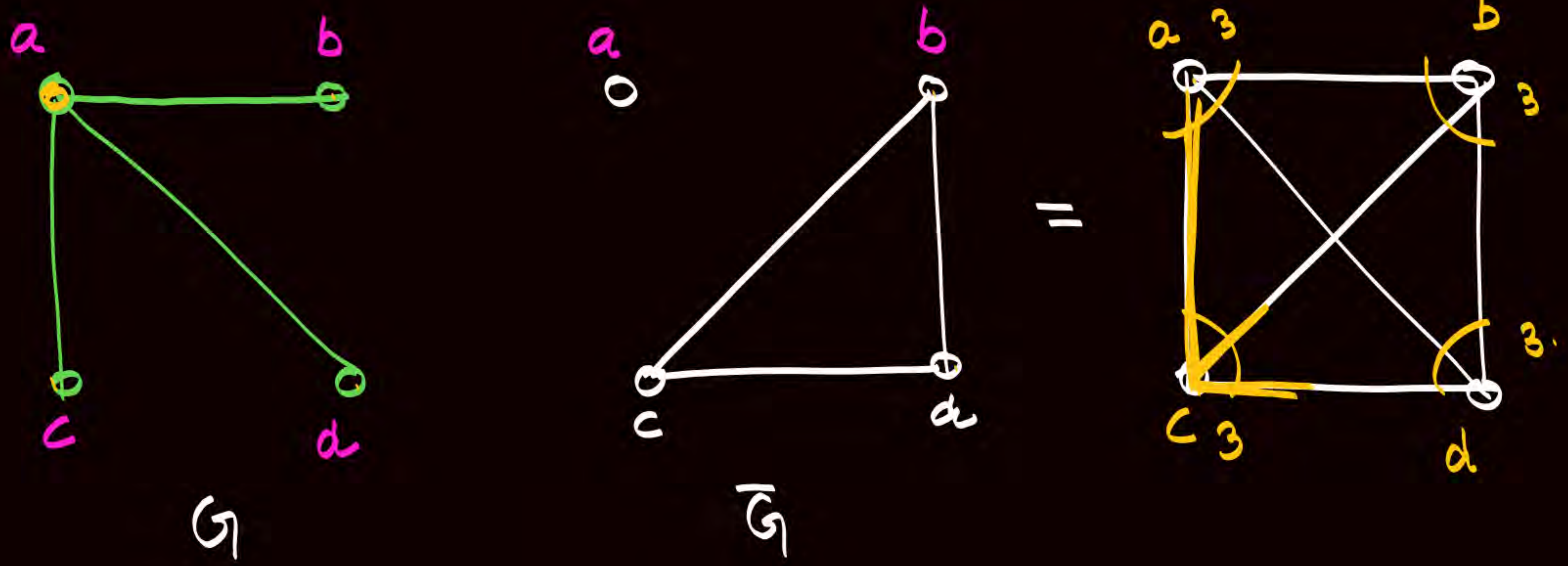


$\bar{G}$

edges → absent
present.

Complement Graph ($\bar{G}$)

$G + \bar{G} = K_n$.



Total vertices $= 6$

$K_n \quad n-1, n-1, n-1 \ldots\ldots n-1.$

$G \to d_1, d_2, d_3 \ldots\ldots d_n.$

$\overline{G} \quad n-1-d_1, n-1-d_2, n-1-d_3 \ldots n-1-d_n.$

$K_6 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5$

$G \to 5, 2, 2, 2, 2, 1.$

$\overline{G} \quad 0, 3, 3, 3, 3, 4.$

$G \rightarrow 3, \quad 3, \quad 3, \quad 1.$

no simple Graph.

$n-1, n-1 \cdots 1$

$\bar{G} \rightarrow$ N.A.

#Q. The compliment of a graph, G, of order n, denoted $\overline{G}$, has the same vertex set as G with $E(\overline{G}) = E(Kn) - E(G)$. If every vertex of G has an odd degree, except for one, how many vertices have odd degree in $\overline{G}$?

Thm 2: no. of ODV in graph will always be even

n ↓ odd.

K_n (e) (e) (e) (e) . . . (e)
n-1, n-1, n-1, n-1 . . . n-1

G e. $\underbrace{o \; o \; o \; o \; o \; o}_{\text{even}}$

Total vertices = odd.

$\overline{G}$ e $\boxed{o, o, o \; . \; . \; . \; . \; o.}$

n-1

#Q. The compliment of a graph, G, of order n, denoted G, has the same vertex set as G with $E(\overline{G}) = E(K_n) - E(G)$. If every vertex of G has an odd degree, except for one, how many vertices have odd degree in $\overline{G}$?

Ans: n-1.

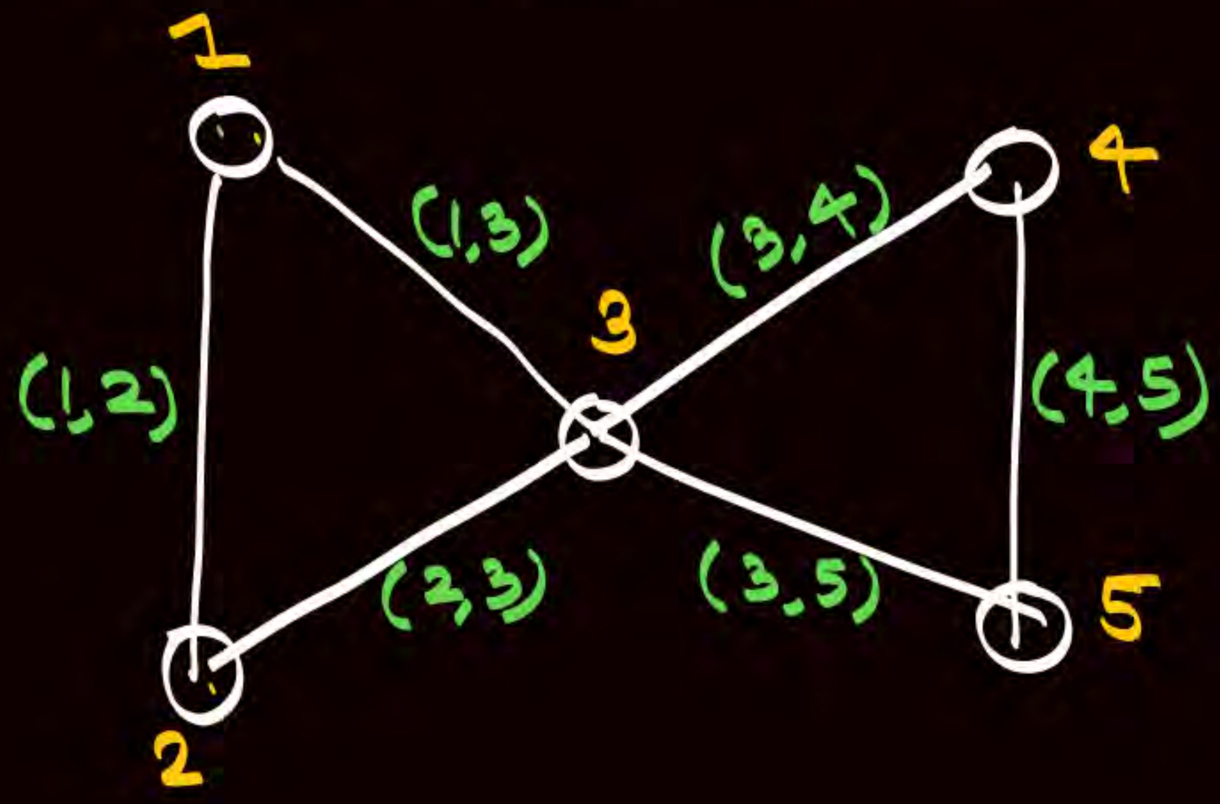
	e	e	e	e	e
K_5	4	4	4	4	4
G	4,	1,	1,	1,	1.
$\overline{G}$	0,	3,	3,	3,	3.

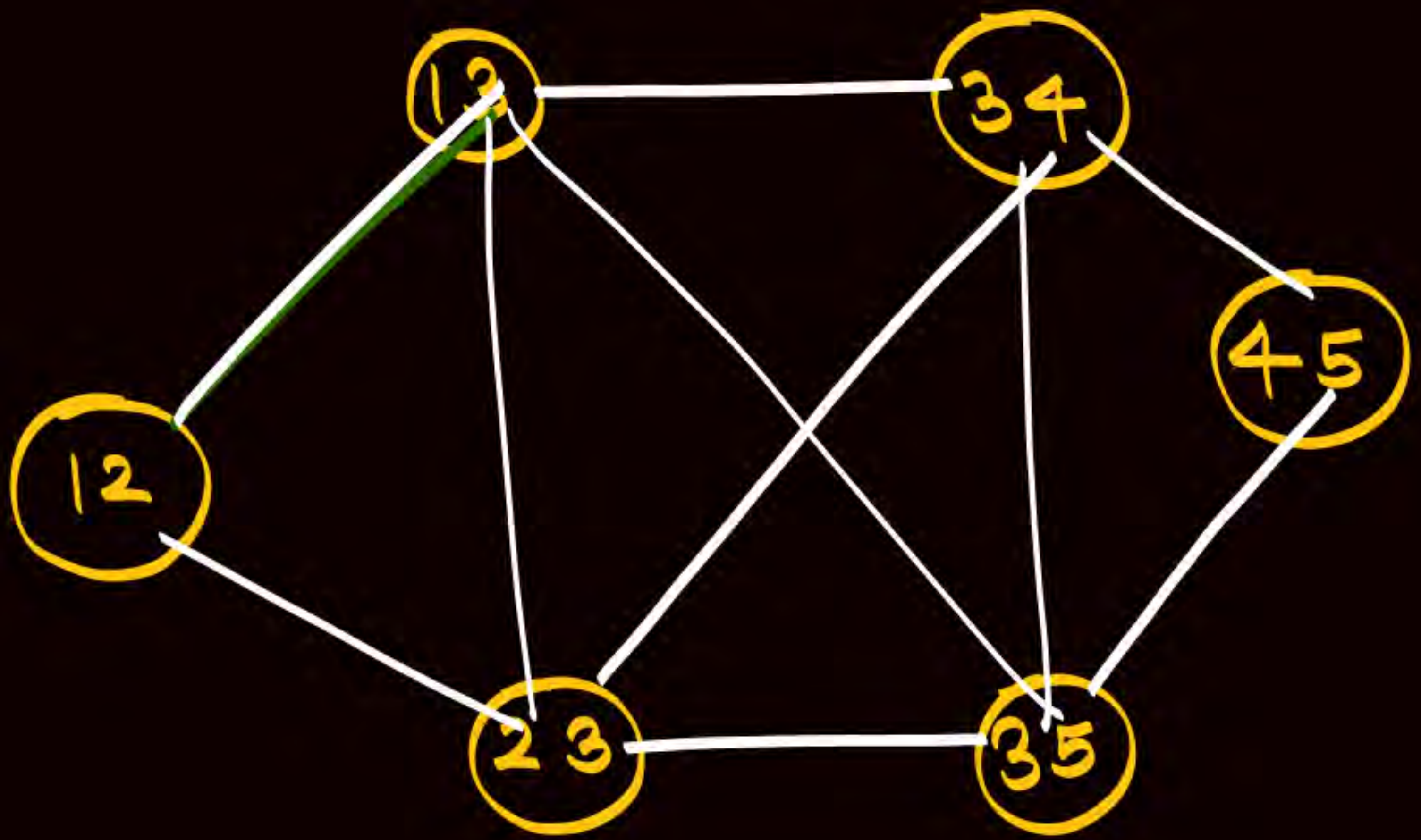
G: 4, $\boxed{1, 1, 1, 1}$ even

odd

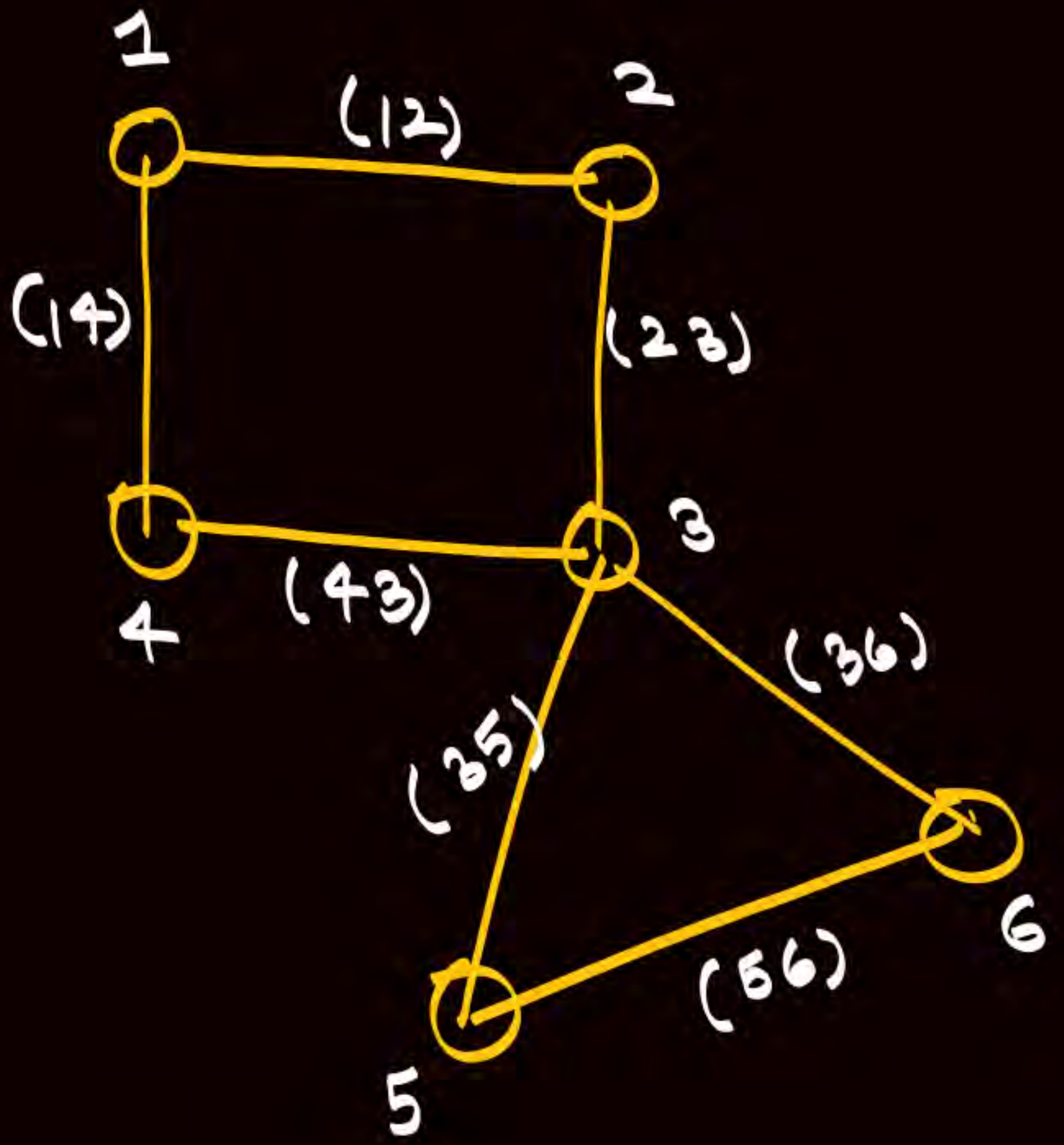
Total vertices = 5

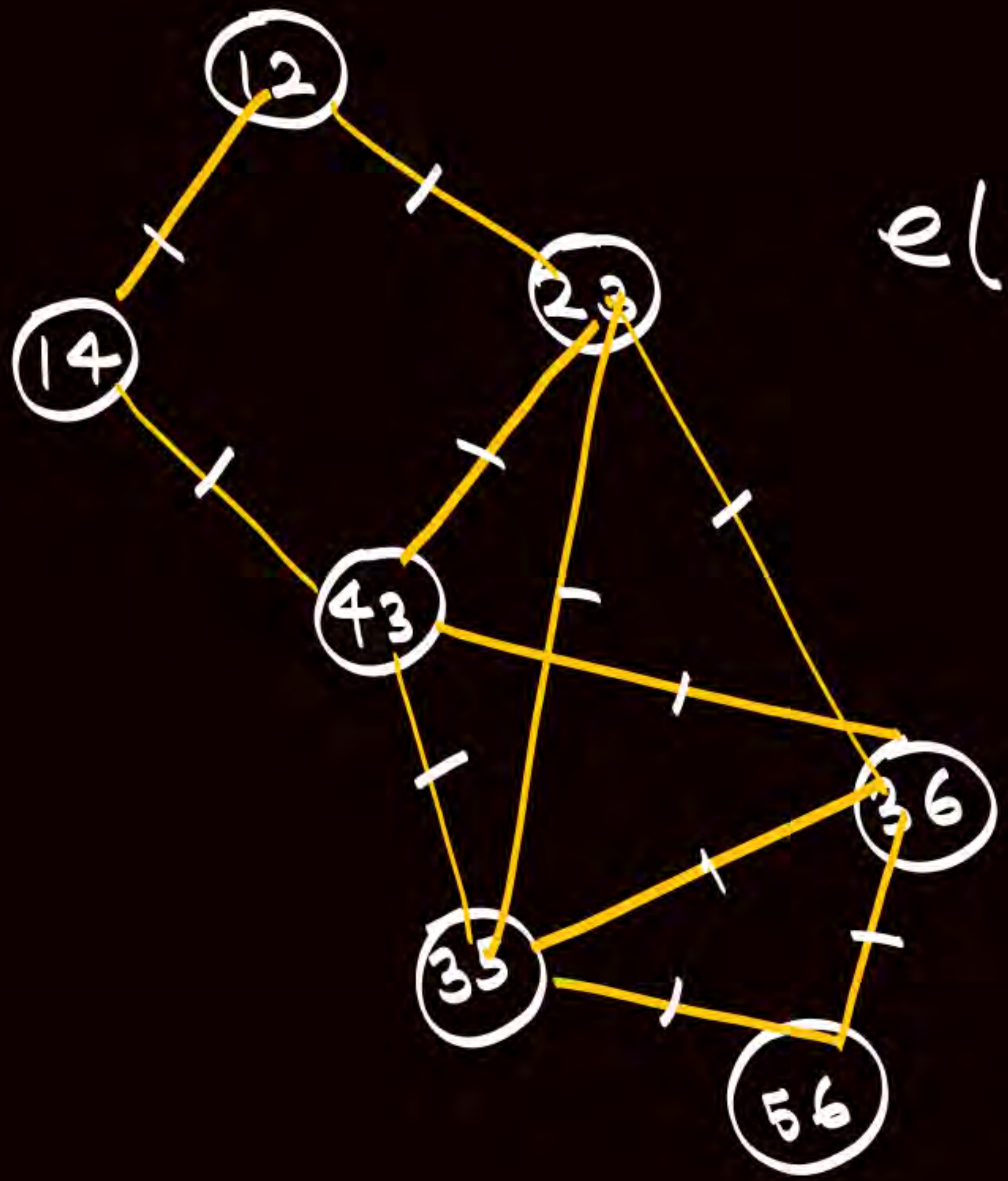
Line Graph (L(G))



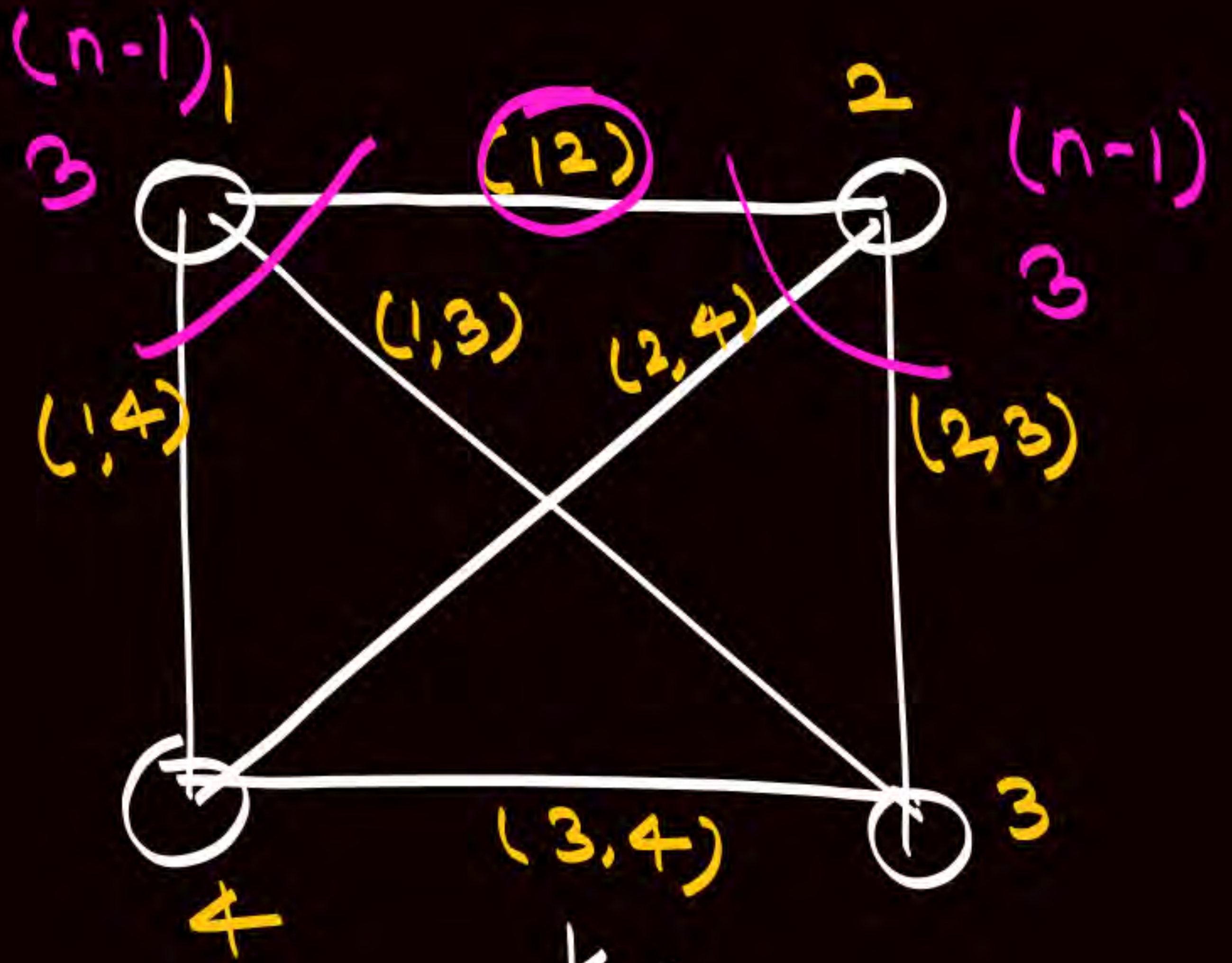


edge ⟶ vertex.

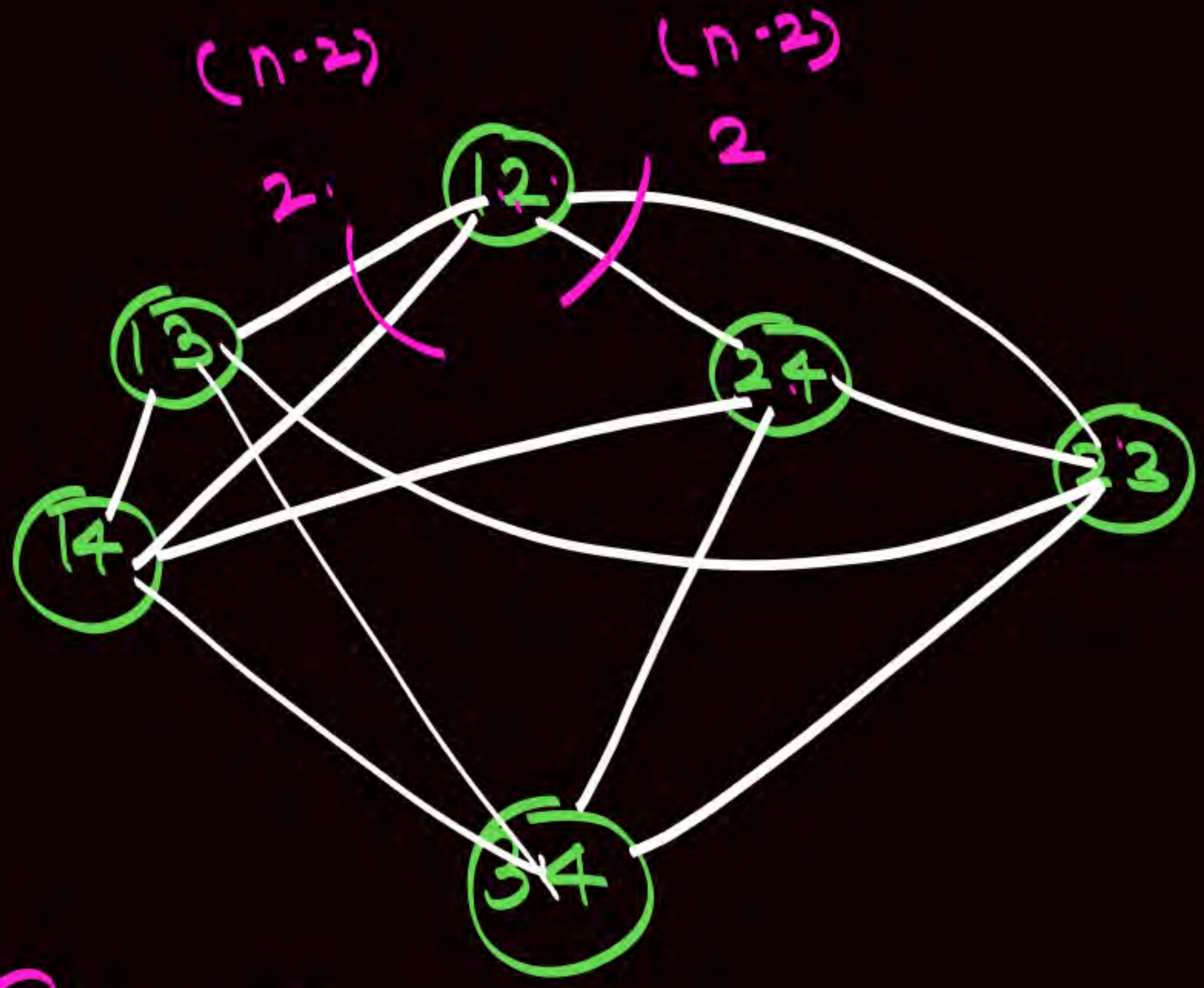




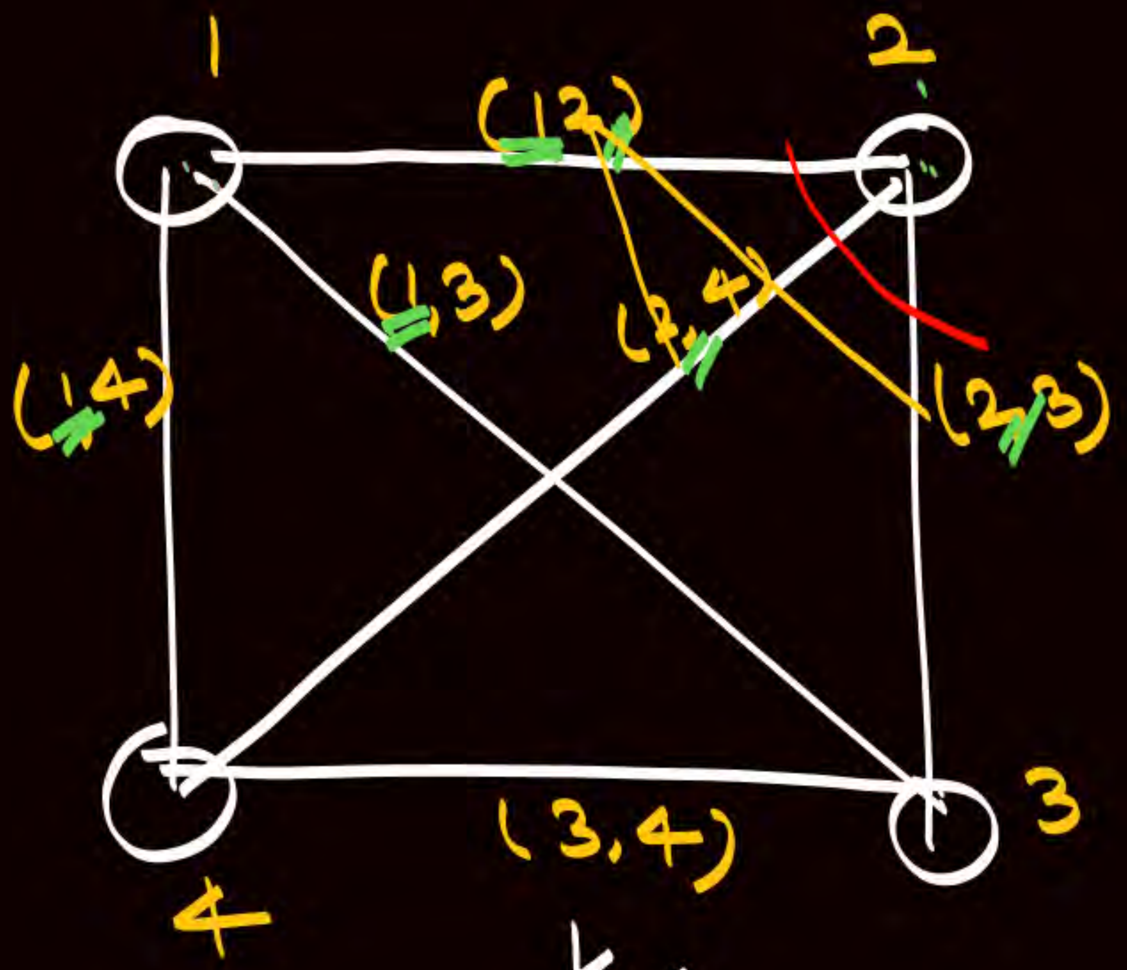
$e(L(G)) = 11.$



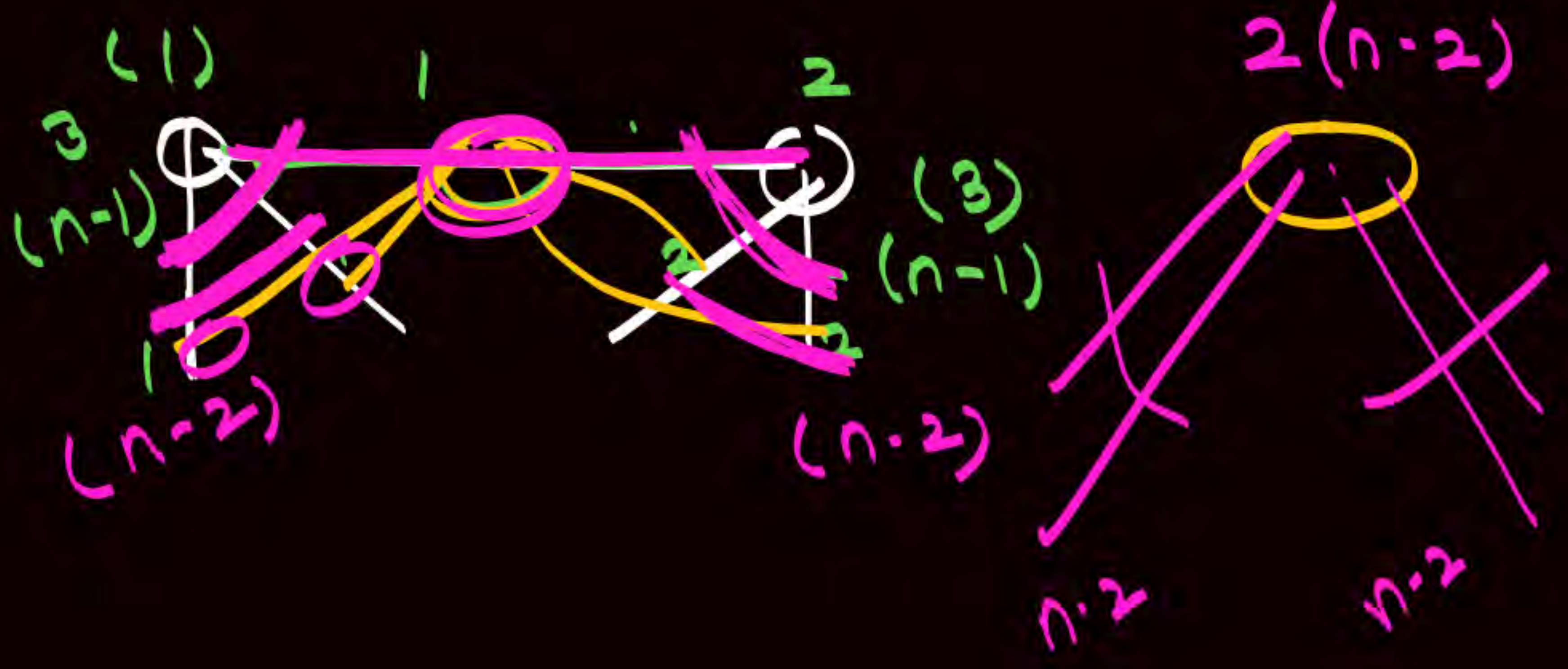
K4.



n



K4.



K4

$n = 4$.

what will be degree of each vertex of $L(K_n) = ?$

A) $n-1$

b) $2(n-1)$

c) $n-2$

d) $2(n-2)$

Q:

$$V(G) = 10$$

$$e(G) = 5$$

$$e(\bar{G}) = ?$$

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

$$5 + x = \frac{10 \cdot 9}{2}$$

$$x = 45 - 5 = 40$$

$$\boxed{e(\bar{G}) = 40}$$

THANK - YOU

Slide