CS & IT ENGINEERING

Discrete Mathematics

GRAPH THEORY

Lecture-3

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Topics to be Covered

- **Topic** Degree Sequemce ✓
- **Topic** Havell Hakimi thm ✓
- **Topic** Types of Graphs ✓

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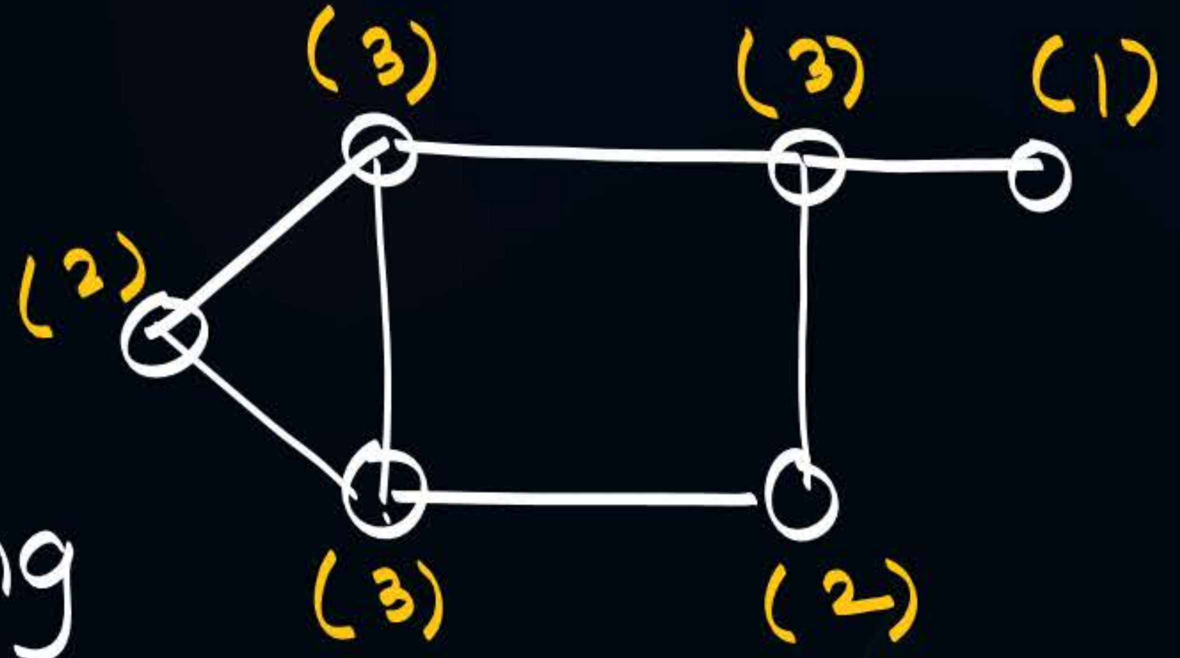
Graph Theory

Degree sequence

writing degrees of all vertices either in increasing or decreasing order.



→ 3, 3, 3, 2, 2, 1

OR

→ 1, 2, 2, 3, 3, 3.

Graph Theory

What will be total no. of edges 5, 2, 2, 2, 2, 1 ?

$$\sum d(v_i) = 2e$$

$$5+2+2+2+2+1 = 2e$$

$$14 = 2e$$

$$e = 7$$

5, 2, 2, 2, 2, 1.

Total vertices = 6.

$e = 7$.

Graph Theory

what will be total no. of edges $3, 3, 3, 1$?

$$\boxed{\sum d(v_i) = 2e}$$

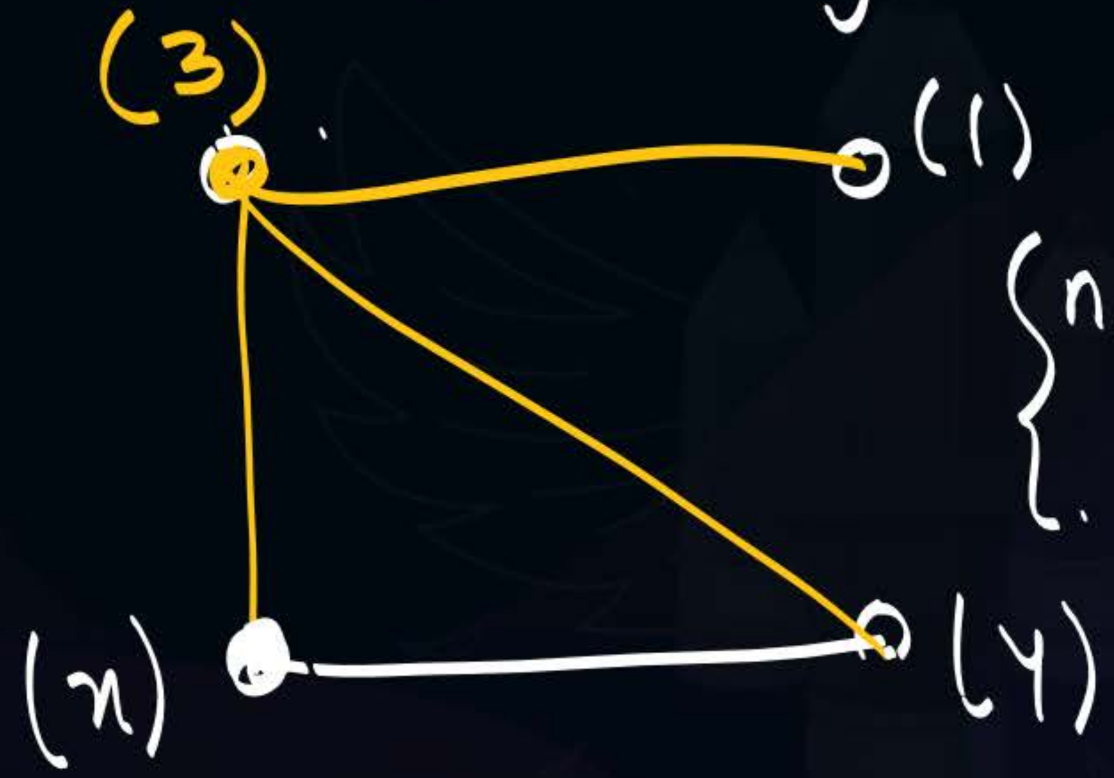
$$3+3+3+1 = 2e$$

$$10 = 2e$$

$$e = 5$$

$\Rightarrow 3, 3, 3, 1$

Total no. of vertices = 4.



{no simple Graph.

$5, 2, 2, 2, 2, 1 \longrightarrow$ simple Graph.

$3, 3, 3, 1 \longrightarrow$ no simple Graph.

$\boxed{\text{Degree sequence} \longrightarrow \text{Simple Graph.}}$

$\hookrightarrow$ Graphical sequence.

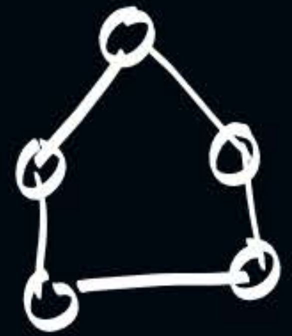
Graph Theory

Graphical sequence? (MSQ)

A) 5, 4, 3, 2, 1.

B) 4, 4, 3, 2, 1.

C) 2, 2, 2, 2, 2. ✓



D) 1 1 1 1 1 1. ✓

A) 5, 4, 3, 2, 1

Thm: no. of odd degree vertices must be even.

no simple Graph.

reason 2: $n = 5$

$\Delta(G) \le n-1 \le 4$. ✗

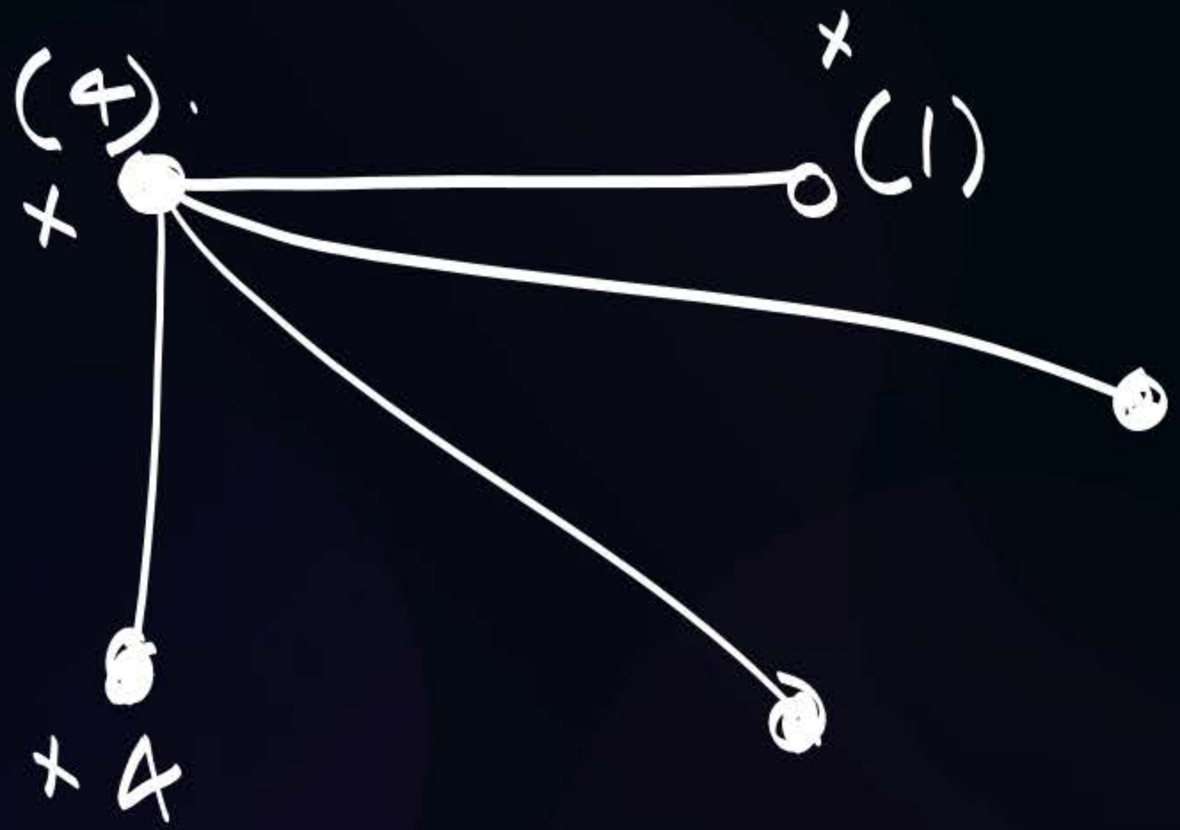
Graph Theory

4, 4, 3, 2, 1

Thm 2 ✓

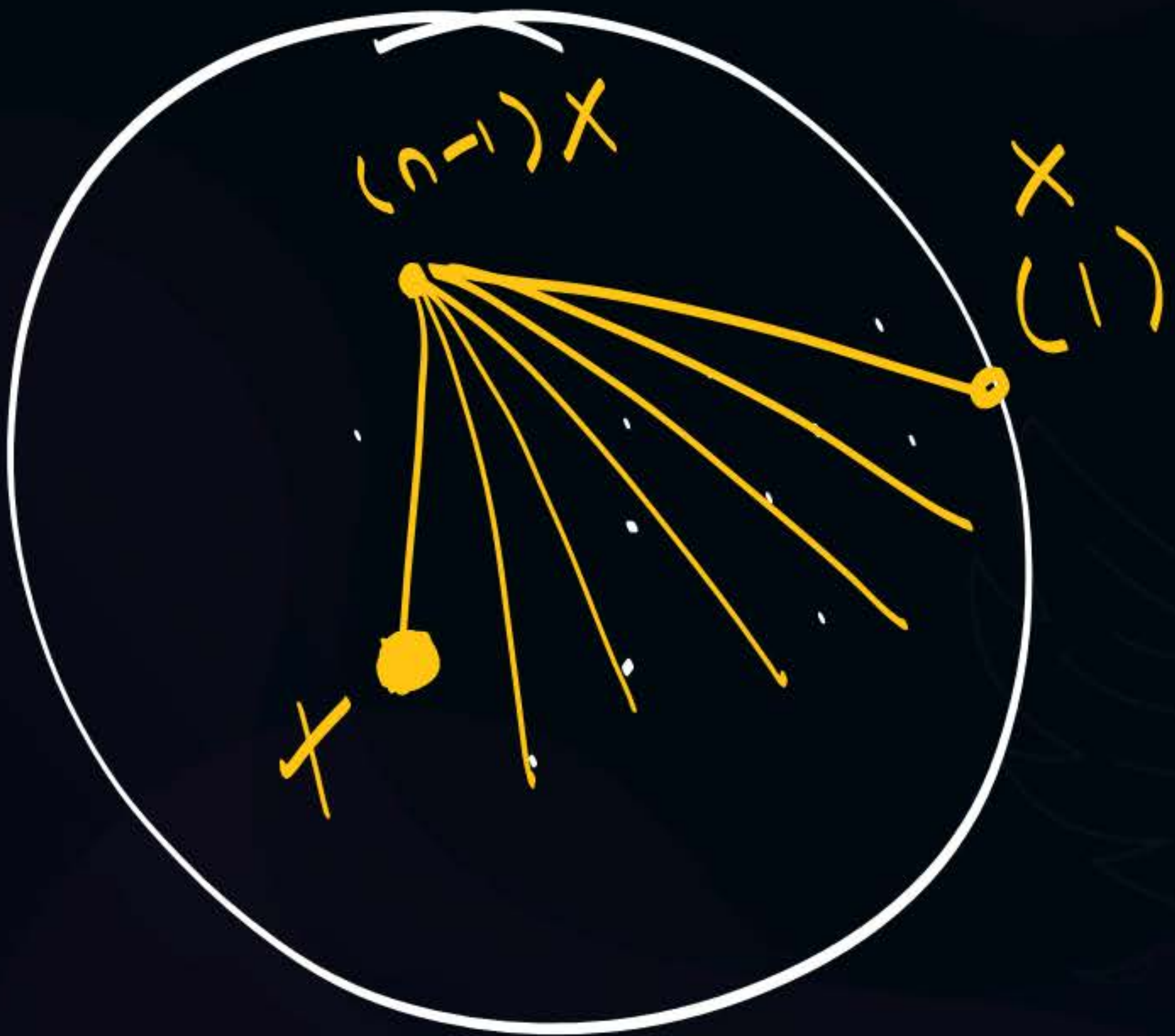
Thm 3 ✓

Total vertices = 5



Graph Theory

$\Rightarrow \boxed{n-1,\ n-1\ \dots\dots\ 1.}$ → not be Graphical.



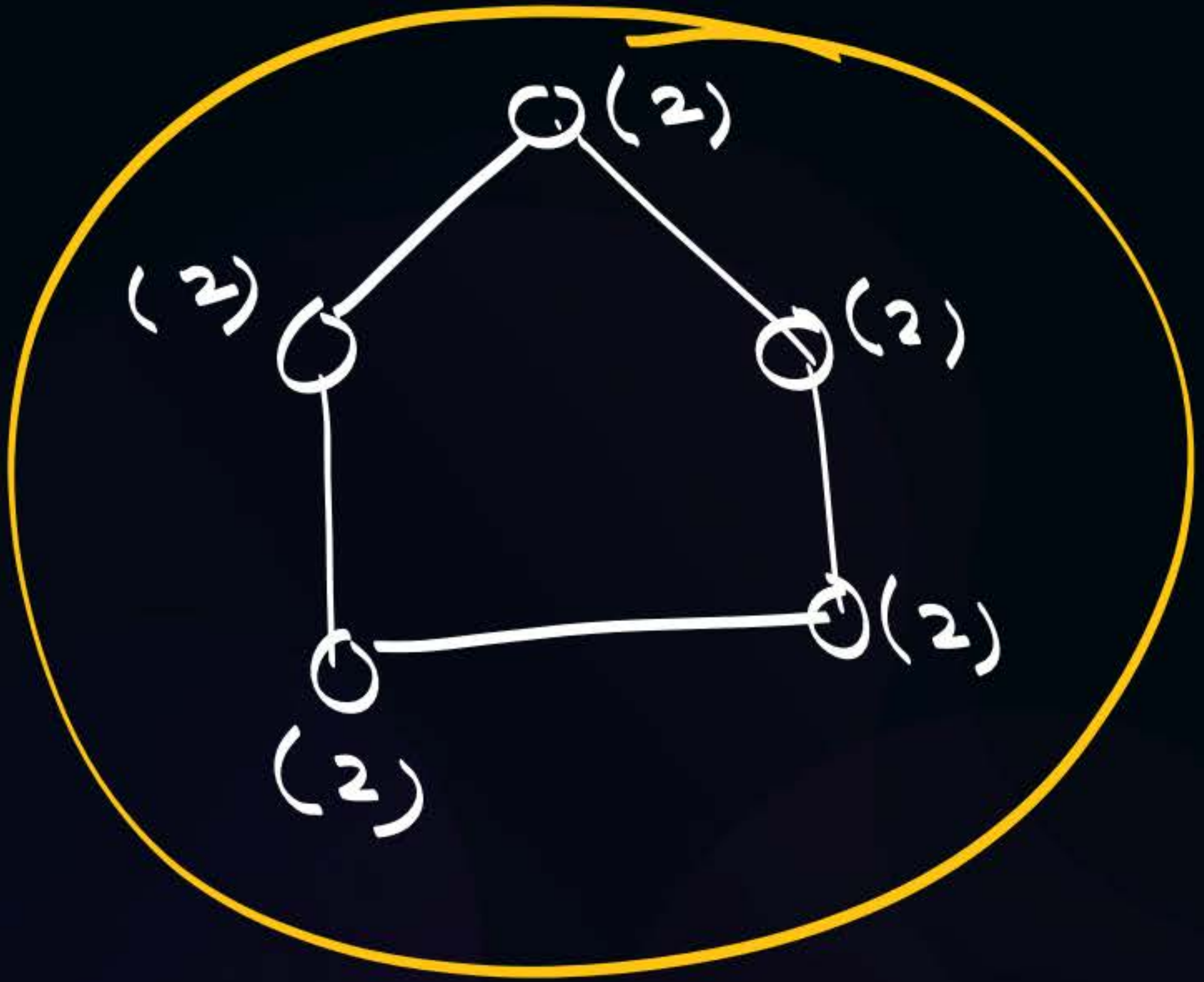
may ← → maynot

$\Rightarrow n-1,\ n-1,\ \dots\dots$

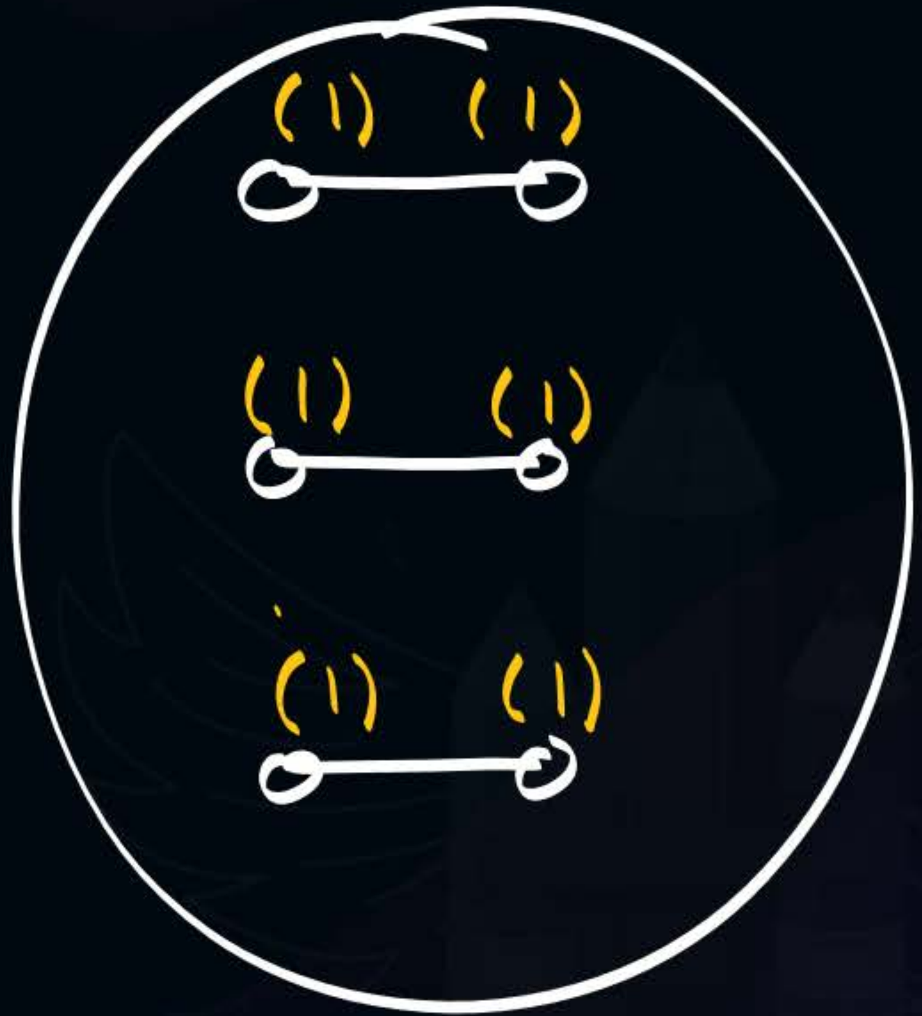
3, 3, 3, 3

Graph Theory

2, 2, 2, 2, 2.

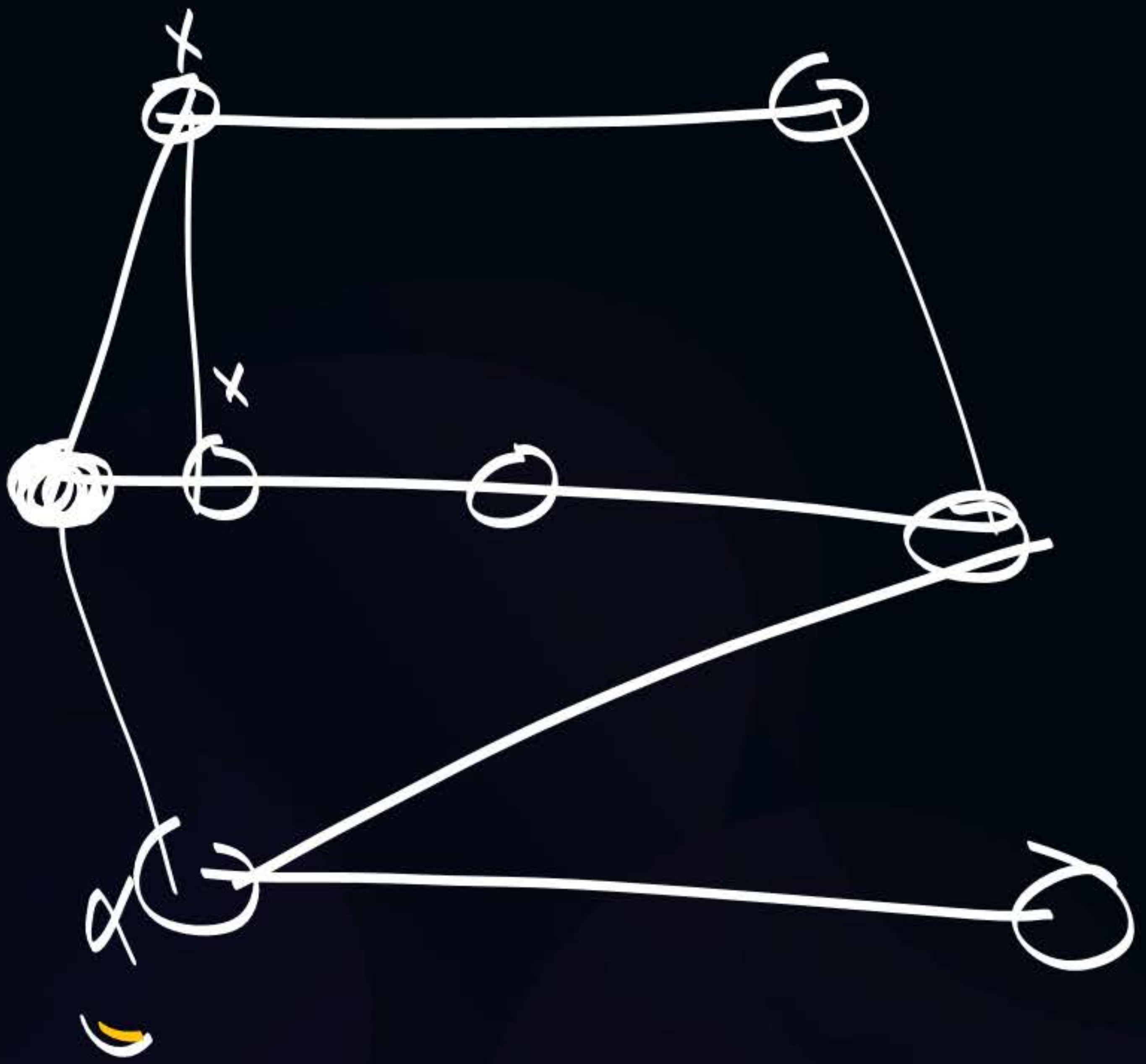


1 1 1 1 1 1.



Graph Theory

3 3 3 3 3 2 2 1.



Graph Theory

$\cancel{3}$ 3 3 3 3 2 2 1.

2, 2, 2, 3 2 2 1.

$\cancel{3}$, 2 2 2 2 2 1.

1 1 1 2 2 1.

$\cancel{2}$ 2 1 1 1 1.

1 0 1 1 1

count

cut

dlt by 1.

ordering

10111

→ 11110

#Q. Apply the Havel-Hakimi result to determineif the following sequences are graphic:

(a) (1,1,1,1,2,2,2,2,2,2)

(b) (1,1,1,2,2,2,3,3,4,7)

(c) (0,1,2,3,4,4)

(d) (1,1,2,2,2,2,3,3)

(e) (1,3,3,4,5,5,5,5,5)

(f) (1,2,3,4,4,5,6,7)

~~7~~, 4, 3, 3, 2, 2, 2, 1, 1, 1

3 2 2 1 1 1 0 1 1

~~3~~ 2 2 1 1 1 1 1 0

1 1 0 1 1 1 1 0

1 1 1 1 1 1 0 0

~~3~~ 3 2 2 2 2 1 1

2 1 1 2 2 1 1

2 2 2 1 1 1 1

#Q. Apply the Havel-Hakimi result to determineif the following sequences are graphic:

(a) (1,1,1,1,2,2,2,2,2,2)

(b) (1,1,1,2,2,2,3,3,4,7)

(c) (0,1,2,3,4,4)

(d) (1,1,2,2,2,2,3,3)

(e) (1,3,3,4,5,5,5,5,5)

(f) (1,2,3,4,4,5,6,7)

$\cancel{7}, 6, 5, 4, 4, 3, 2, 1$

$\cancel{5}\ 4\ 3\ 3\ 2\ 1\ 0$

$\cancel{3}\ 2\ 2\ 1\ 0\ 0$

$1\ 1\ 0\ 0\ 0$

#Q. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order.

Which of the following sequences $\boxed{\text{can not be}}$ the degree sequence of any graph?

I. 7,6,5,4,4,3,2,1 ✓

II. 6,6,6,6,3,3,2,2 ✗

III. 7,6,6,4,4,3,2,2 ✓

IV. 8,7,7,6,4,2,1,1 ✗

MCQ [2010]

A. I and II

B. III and IV

C. IV only

D. II and IV

$\cancel{6}$ 6 6 6 3 3 2 2

5 5 5 2 2 1 2

$\cancel{5}$ 5 5 2 2 2 1

$\cancel{4}$ 4 1 1 1 1

3 0 0 0 1

3 1 0 0 0 ✗

[NAT]

#Q. For which integers x $(0 \le x \le 7)$, if any, is the sequence 7, 6, 5, 4, 3, 2, 1, x graphical?

Ans: 4

$\rightarrow$ 7, 6, 5, 4, 3, 2, 1, x.

$0 \le x \le 7$

x can not be odd.

$x = 2$ $\qquad$ $x = 4$ $\qquad$ $x = 6$.

x has to be even.

7, 6, 5, 4, 3, 2, 2, 1

$x = 0, 2, 4, 6$.

x can not be 0.

Graph Theory

$x = 0$

$$7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ \overset{x}{\downarrow}\ 0$$

$\Delta(G) \le 6,\ n = 7$

0

Graph Theory

Degree sequence :

1) Thm 2.

2) Thm 3

3) $n-1, n-1 \cdots 1$.

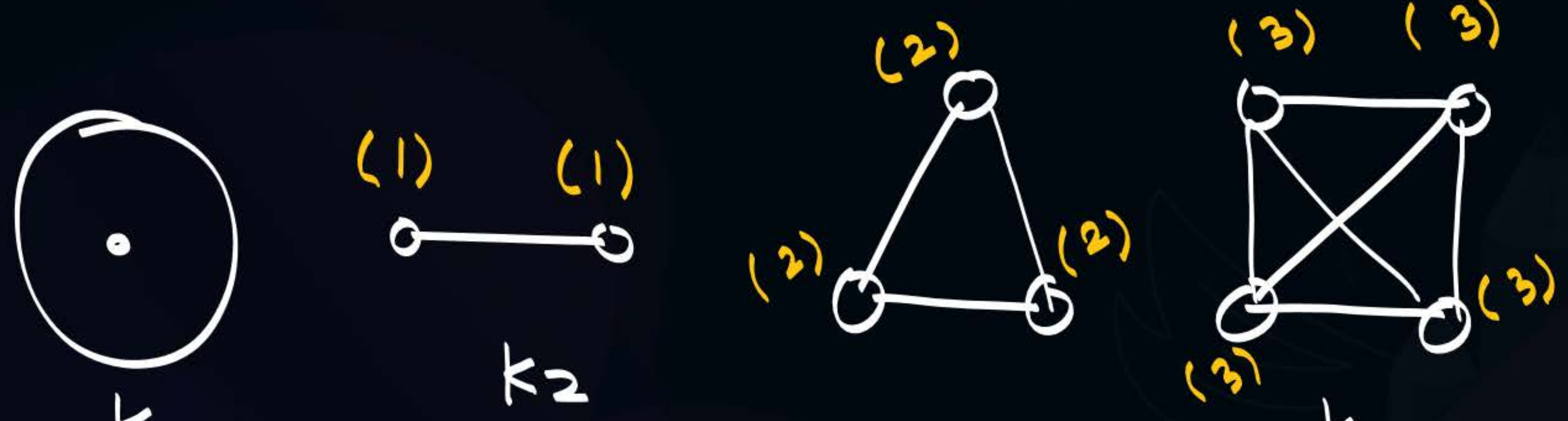
4) Havell- Hakimi Thm.

Graph Theory

* Degrees of all vertices must be n-1.

Types of Graphs:

Complete Graph (K_n) $(n \geq 1)$



K_1 K_2 K_3 K_4

Graph Theory

$$\sum d(v_i) = 2e$$

$$n \times (n-1) = 2e$$

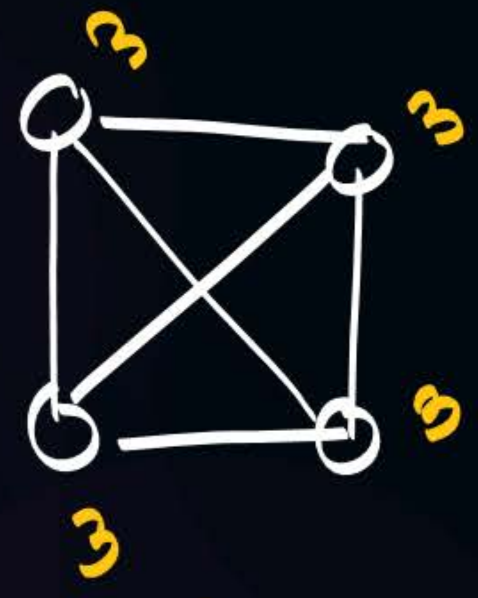
$$\boxed{e = \frac{n(n-1)}{2}}$$

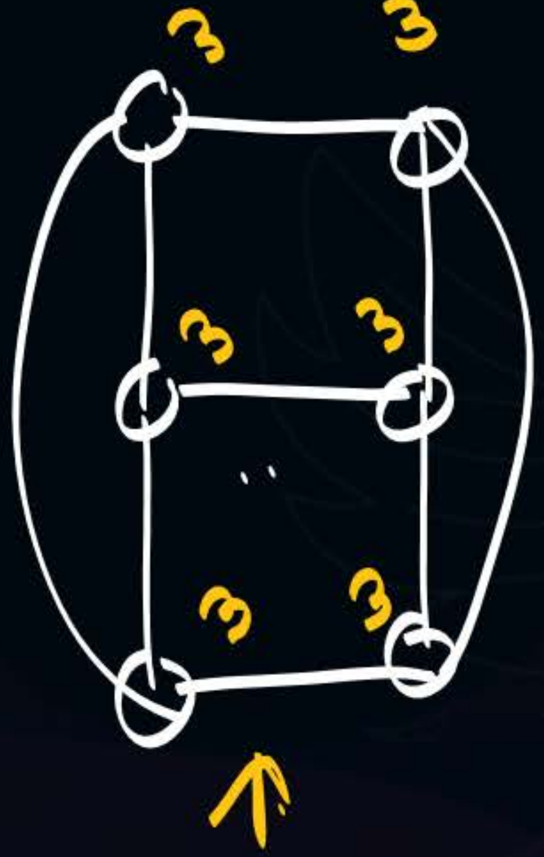
$$* \quad \delta(G) = \frac{2e}{n} = \Delta(G) = n-1$$

Graph Theory

Regular Graph ($\delta(G) = \frac{2e}{n} = \Delta(G)$)

Degrees of all vertices are same.





(T)

all $K_n \rightarrow$ Regular Graphs.

all Regular Graph $\rightarrow K_n$

(False)

eg:

THANK - YOU