CS & IT ENGINEERING

Discrete Mathematics

GRAPH THEORY

Lecture-2

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Topics to be Covered

- Topic: Thm 5
- Topic: Number Of Graphs
- Topic: Degree Sequence


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Topic : Basics Of Graphs

Thm 1: $\sum d(v_i) = 2e$ $\qquad$ $\sum d(v_i) = \text{even}$

Thm 2: no. of odd degree vertices must be even.

Thm 3: max. degree $\leq n-1$.

Thm 4: max no. of edges $\leq \frac{n(n-1)}{2}$.

Thm 5: $2^{n(n-1)/2}$.

n v with e edges. $\frac{n(n-1)}{2} C_e$.

Topic : Basics Of Graphs

Total vertices = n.

n = 4

max. degree $\leq n-1$.

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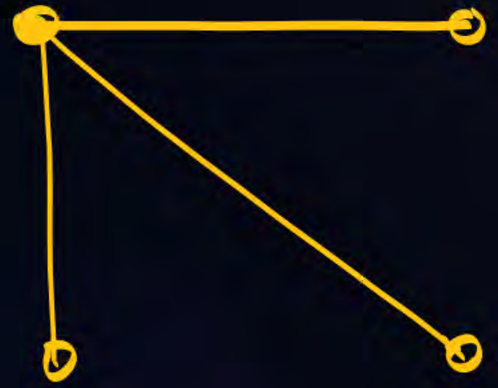

Topic : Basics Of Graphs

$n = 4$

$e = 3$

$n = 4$

$e = 3$

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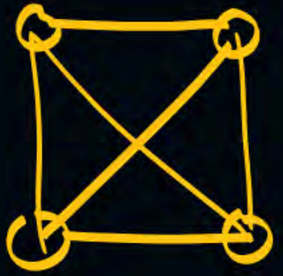


$e = 6$

$n = 4$

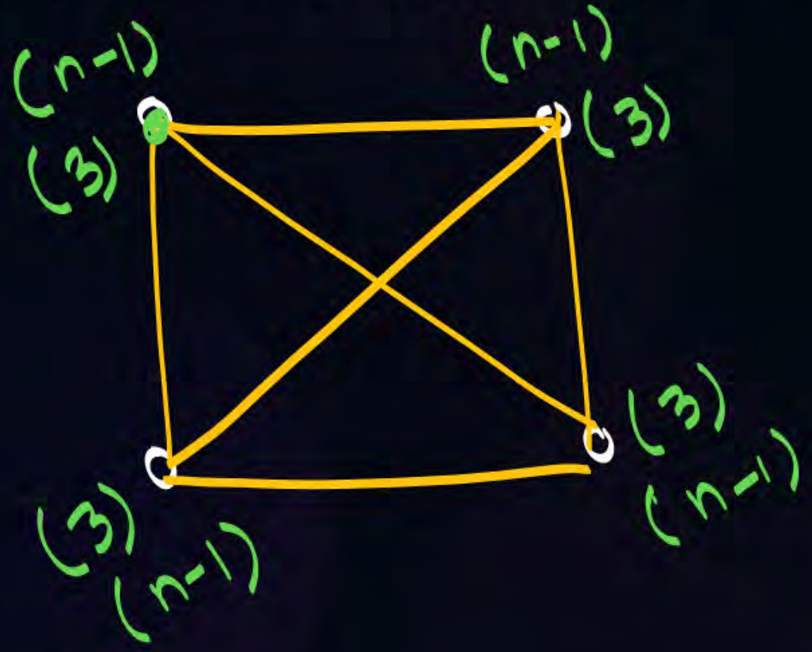
$$e \leq \frac{n(n-1)}{2}$$

$$e \leq \frac{4 \cdot 3}{2} = 6$$

Topic : Basics Of Graphs

Thm 4: maximum no. of edges $\leq \frac{n(n-1)}{2}$.

Total vertices $= n = 4$.

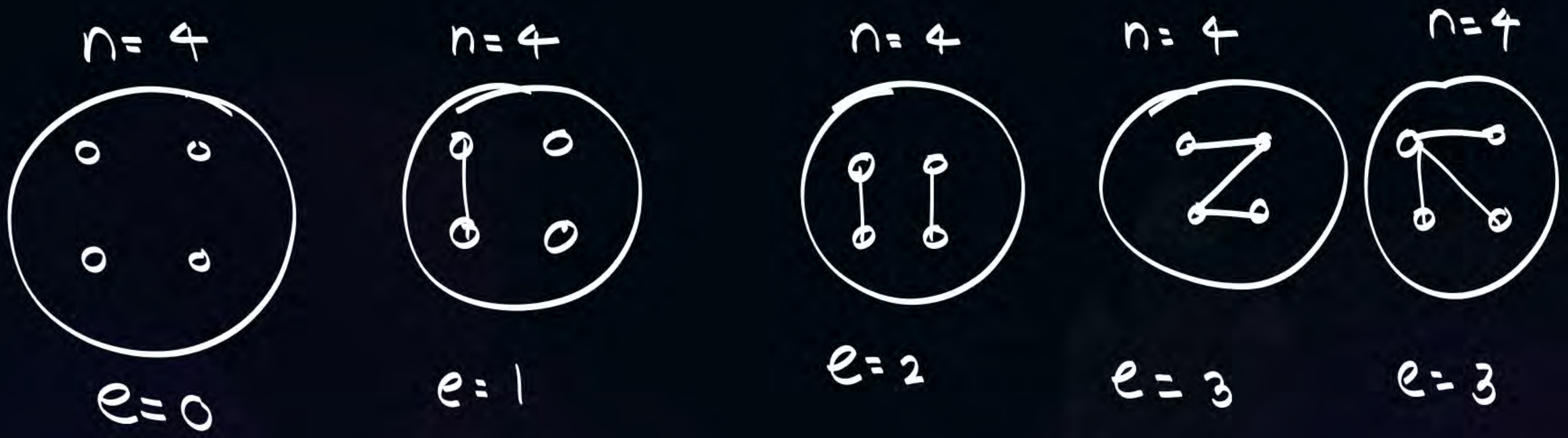


$n \times (n-1) = 2e$

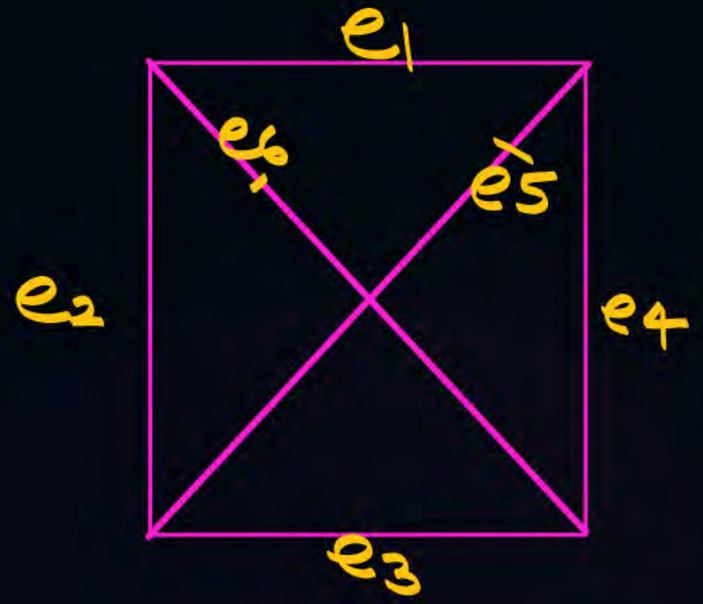
$e = \frac{n(n-1)}{2}$.

Topic : Basics Of Graphs

Total no. of Graphs are possible with 4 vertices.



Topic : Basics Of Graphs



Topic : Basics Of Graphs

Total vertices = n.

Total no. of graphs are possible $2^{\frac{n(n-1)}{2}}$

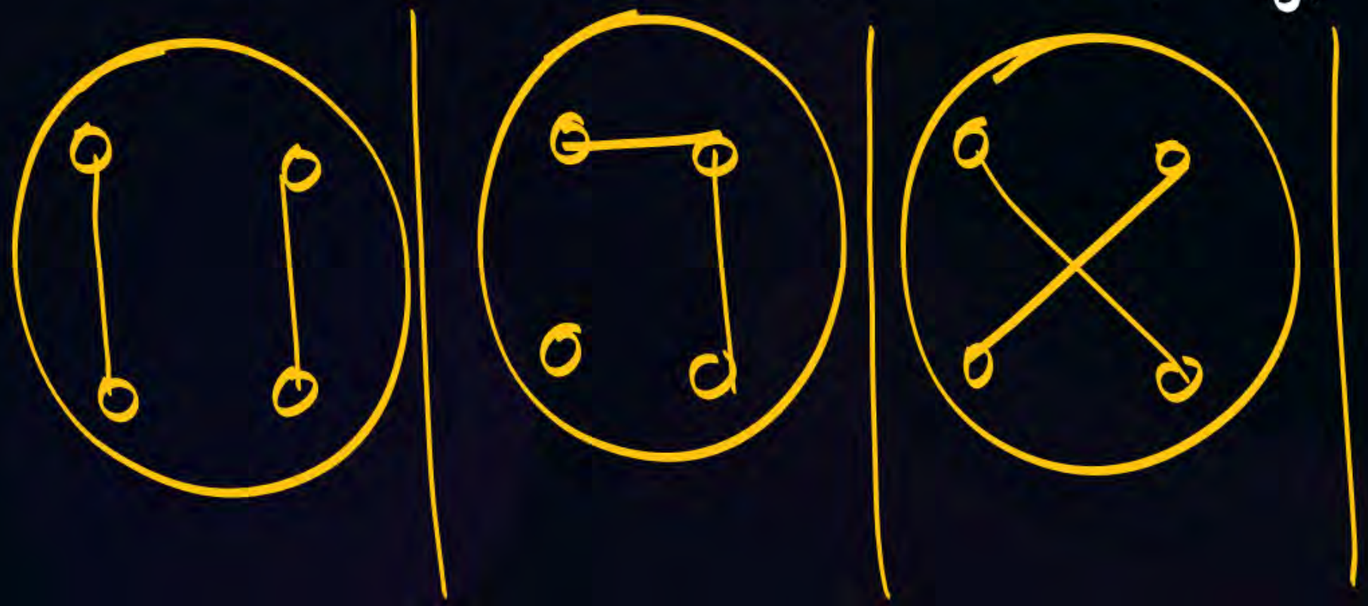
* How many graphs are possible with 6 vertices.

$$2^{\frac{6(6-1)}{2}} = 2^{15}$$

Topic : Basics Of Graphs

{e1, e2, e3, e4, e5, e6}. Ans: $6C_2$

* How many graphs are possible with 4 vertices & 2 edges ?



{S1, S2, S3 . . . . S6}.

In a class of 6 students how many ways we can select 2 students ?

Ans: $6C_2$.

Topic : Basics Of Graphs

How many graphs are possible with n vertices. & e edges.

$$^{\frac{n(n-1)}{2}}C_{e}$$

Topic : Basics Of Graphs

How many graphs are possible with 4 vertices & atleast 2 edges.

$$^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6.$$

Topic : Basics Of Graphs

$$^6C_0 + {}^6C_1 + \underbrace{{}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6}_{x} = 2^6$$

4 v with atleast 2 edges $\Rightarrow 2^6 - {}^6C_0 - {}^6C_1$

4v with almost 2 edges $\Rightarrow {}^6C_0 + {}^6C_1 + {}^6C_2$

Topic : Basics Of Graphs

Total graphs 4v $2^{\frac{4(3)}{2}}$ 2^6

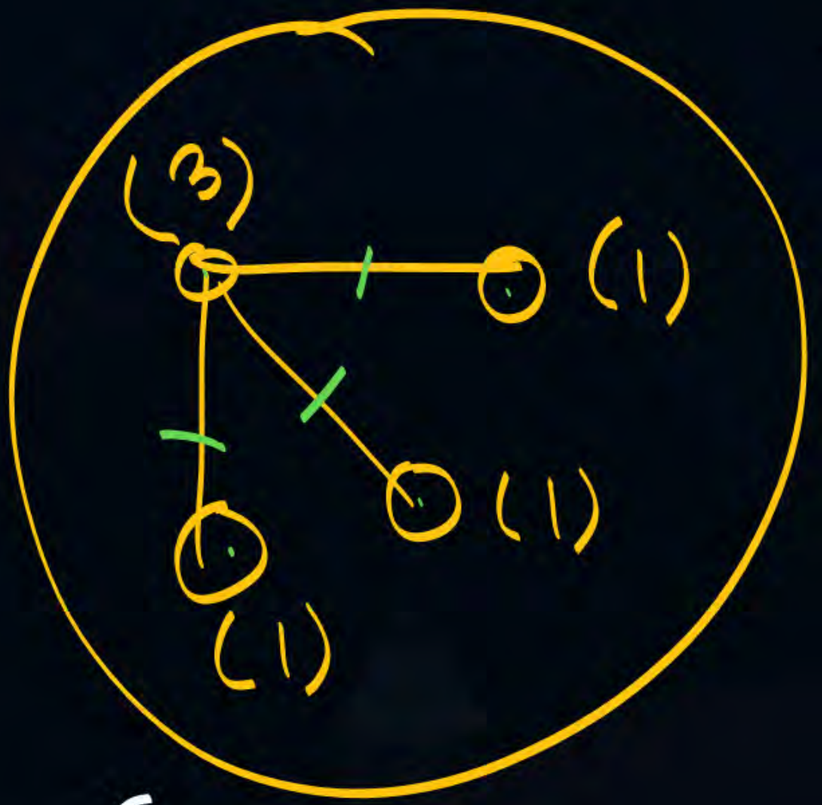
Total graphs 4v with exactly 2 edges. ${}^6C_2$

Total no. of graphs 4v with atleast 2 edges

${}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

Topic : Basics Of Graphs

Thm 1 : $\sum d(v_i) = 2e$.



$n = 4$ $e = 3$

possible degrees 1, 3.

3 vertices of degree 1.

$n = 14 \quad e = 30$

possible degrees 4 & 5

how many vertices of degree 5 ?

Solt^n: Assume x no. of vertices of degree 5.

$$\Sigma d(v_i) = 2e$$

$$(14-x)\times 4 + 5x = 2e = 2\times 30$$

$$56 - 4x + 5x = 60$$

$$x = 4$$

[NAT]

#Q. A certain graph G has order 14 and size 27. The degree of each vertex of G is 3, 4 or 5. There are six vertices of degree 4. How many vertices of G have degree 3 and how many have degree 5?

$5v \longrightarrow$ Degree 3

Assume :

x vertices of degree 3

$\Sigma d(v_i) = 2e$

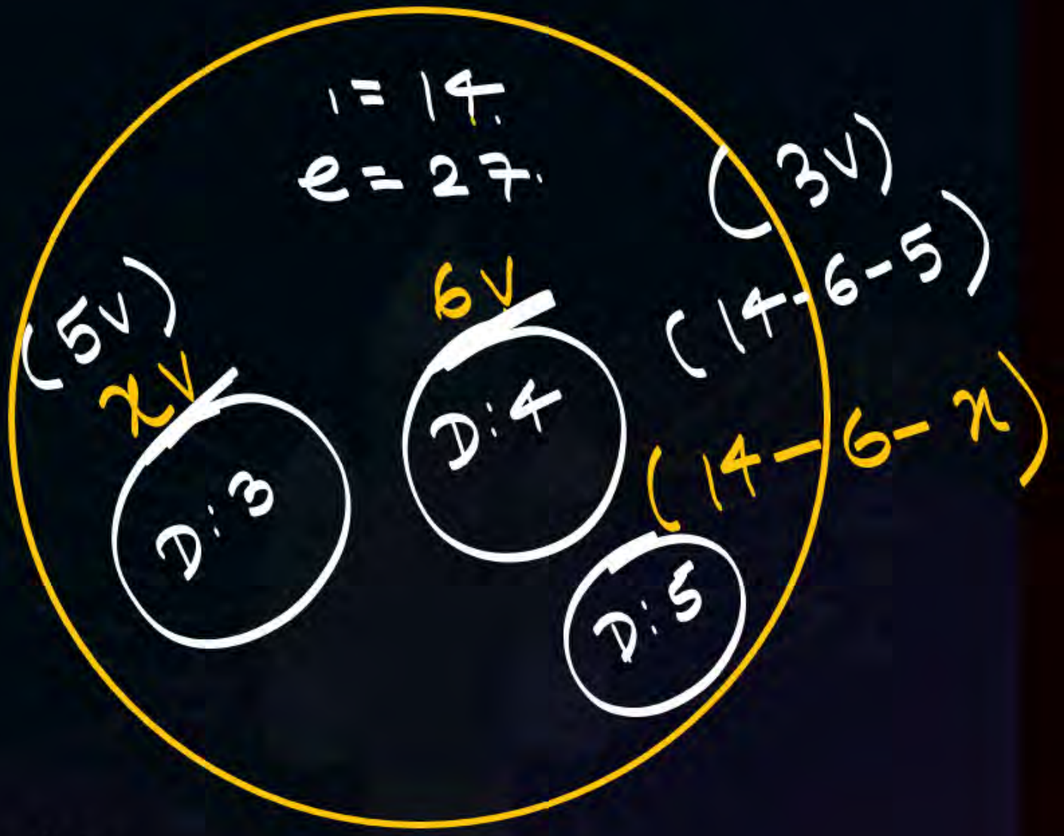
$3 \times x + 6 \times 4 + 5(14 - 6 - x) = 2 \cdot 27$

$3x + 24 + 5(8 - x) = 54$

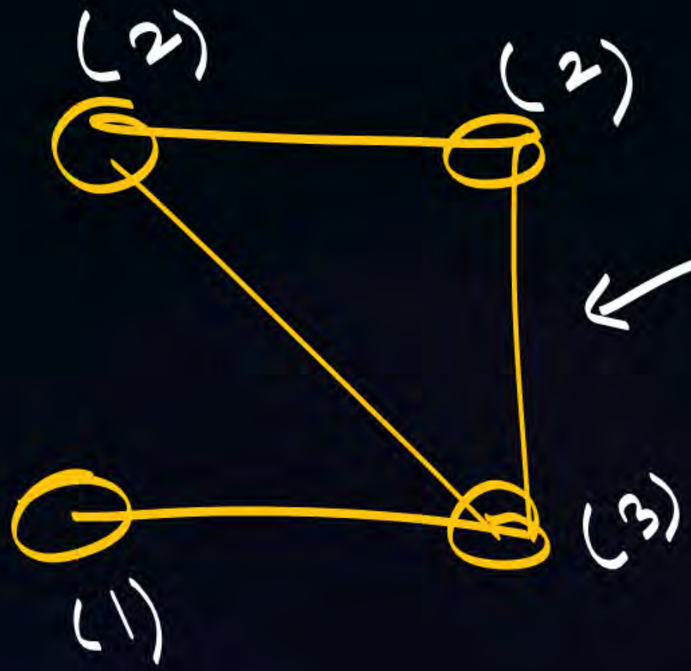
$3x + 24 + 40 - 5x = 54$

$-2x = -64 + 54 = -10$

$x = 5$



Topic : Basics Of Graphs



n = 4 e = 4.

1 vertex = Degree : 1

2 vertices => Degree 2

1 vertex => Degree 3.

1, 2, 3.

Topic : Basics Of Graphs

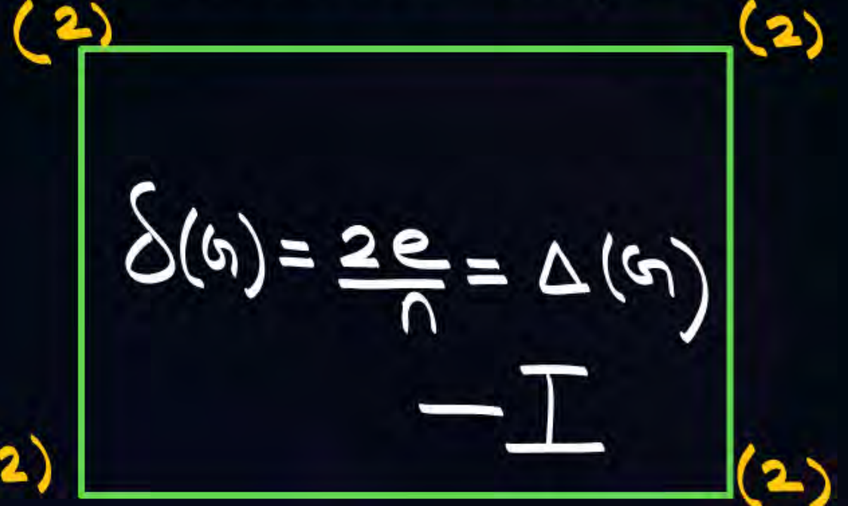
Consider a graph of 100 vertices and degree of each vertex will be atleast 3

what will be no. of edges?

Topic : Basics Of Graphs

minimum degree ($\delta(G)$)

maximum degree ($\Delta(G)$)



$$\delta(G) = \frac{2e}{n} = \Delta(G)$$

— I

$\delta(G) = 2 \quad \Delta(G) = 3$

$$\delta(G) < 2e/n < \Delta(G)$$

— II

$$\frac{\sum d(v_i)}{n} = \frac{2e}{n} = 2.$$

$\delta(G) = 2 \quad \Delta(G) = 2$ avg. degree $= \frac{\text{Total degrees}}{\text{Total vertices}} = \frac{2+2+2+2}{4} = \frac{\sum d(v)}{n}$

Topic : Basics Of Graphs

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$

↑ atleast

Topic : Basics Of Graphs

[GATE]

Consider a Graph with 25 edges & degree of each vertex will be atleast 3

then what will be max. no. vertices.

$\delta(G) = 3 \quad n = 25$

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$

$$\delta(G) \leq \frac{2e}{n} \qquad 3 \leq \frac{2 \times 25}{n} \qquad n \leq \frac{50}{3}$$

$n = 16$

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

$2^{n(n-1)/2}$

$\frac{n(n-1)}{2}$ c e.

$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$

THANK - YOU