MCAC 301: Design and Analysis of Algorithms

Neelima Gupta

ngupta@cs.du.ac.in

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Linear Search : Correctness

Let us rewrite the linear search algorithm of chapter 2 as follows:

input : Array: $A[1 \dots n], Key$
output: index of first occurrence of key if it is found, 0 otherwise

```
i = 1
while i ≤ n do
    if A[i] = key then
        return i
    end
    else i++
end
return 0
```

Steps to Prove

- Loop Invariance
- Prove the Loop Invariance using Mathematical Induction
- Use the Loop Invariance to prove the correctness

Loop Invarinace

```
i = 1
while i ≤ n do
    if A[i] = key then
        return i
    end
    else i++
end
return 0
```

Hypothesis H(r) : When the control reaches the "While" statement for the r^{th} time,

1. $i = r$
2. key $\notin A[1 \ldots r-1]$

$\forall \; 1 \leq r \leq n+1$

Proof Of Loop Invariance

Hypothesis $H(r)$: When the control reaches the "While" statement for the r^{th} time,

1. $i = r$
2. key $\notin A[1 \ldots r-1]$

Proof by induction on r.

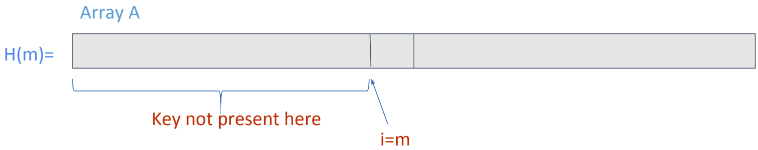
1. Base Case: when $r = 1$, the claim holds vacuously.
2. Induction Hypothesis: $H(m) \Rightarrow H(m+1)$.

Induction Hypothesis

$i = 1$
while $i \leq n$ **do**
  if $A[i] = key$ **then**
    return i
  end
  else $i{+}{+}$
end
return 0

Hypothesis $H(r)$: When the control reaches the "While" statement for the r^{th} time,

1. $i = r$
2. key $\notin A[1 \ldots r-1]$



Induction Hypothesis

```
i = 1
while i ≤ n do
    if A[i] = key then
        return i
    end
    else i++
end
return 0
```

Hypothesis H(r) : When the control reaches the "While" statement for the r^{th} time,

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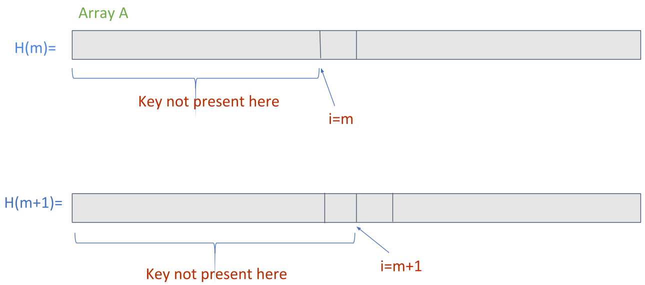
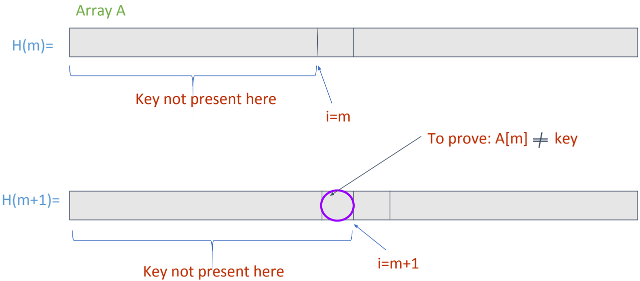


Induction Hypothesis

$i = 1$
while $i \leq n$ **do**
  if *A[i]* $=$ *key* **then**
    return i
  end
  else $i{+}{+}$
end
return 0

Hypothesis $H(r)$: When the control reaches the "While" statement for the r^{th} time,

1. $i = r$
2. key $\notin A[1 \ldots r-1]$



Proof of Correctness

```
i = 1
while i ≤ n do
    if A[i] = key then
        return i
    end
    else i++
end
return 0
```

Hypothesis H(r) : When the control reaches the "While" statement for the r^{th} time,

1. $i = r$
2. key $\notin A[1 \ldots r-1]$

Suppose that the test condition in "While" statement is executed exactly k times. i.e. body of the loop is executed $k-1$ times.

1. Case 1: $k \leq n$. In this case, $i = k$ and $A[1 \ldots k-1]$ does not contain the key. Also, the while loop terminated because $A[i](i.eA[k]) = key$. Thus the value returned in statement 4 is the position of the first occurrence of the key in the array.
2. Case 2: $k = n+1$. By loop invariant hypothesis, $i = n+1$ and $A[1 \ldots n]$ does not contain the key. Since $i = n+1$, control goes to statement 8 and the algorithm returns 0.