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AP & GP fundamental

* Sequence *

→ collection of elements is ordered in such a way that is has to an identified 1^{st} member, 2^{nd} member

→ if sequence follows a specific pattern then it is known as progression

ex.
1, 2, 3, 4, 5 .... 2-1 = 3-2 = 4-3
2, 4, 6, 8, 10 ... 4-2 = 6-4 = 8-6
3, 6, 9, 12, 15 ... 6-3 = 9-6 = 12-9 } A.P
2, 4, 8, 16, 32 ... 4/2 = 8/4 = 16/8
3, 9, 27, 81, ... 9/3 = 27/9 = 81/27 } G.P

(1) A.P. (Arithmatic Progresion)

~~a+~~

$a, \; a+d, \; a+2d, \; a+3d, \; a+4d, \; \dots \; a+(n-1)d$

↓ 1^{st}, ↓ 2^{nd}, ↓ 3^{rd}, ↓ 4^{th}, ↓ 5^{th}, ↓ n^{th} term

→ n^{th} term of AP $= a+(n-1)d$

d → common difference

* Series *

→ if $a_1, a_2, a_3, a_4 \dots a_n$ be the sequence then the expression $a_1 + a_2 + a_3 + a_4 \dots a_n$ is called series associated with sequence.

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1, 2, 3, 4, 5 → sequence

1 + 2 + 3 + 4 + 5 → series

15 → sum of series

* sum of n terms of A.P. series *

$S_n = a + a+d + a+2d + a+3d \dots + a+(n-1)d$

last term $= a+(n-1)d = L$

2^{nd} last term $= a+(n-2)d = L-d$

3^{rd} last term $= a+(n-3)d = L-2d$

$S_n = a + a+d + a+2d + \dots + L-2d + L-d + L$ — (1)

$S_n = L + L-d + L-2d + \dots + a+2d + a+d + a$ — (2)

→ add both equation

$2S_n = (a+L) + (a+L) + (a+L) + \dots + (a+L)$

$2S_n = n \cdot (a+L)$

$S_n = \frac{n}{2}(a+L)$

$S_n = \frac{n}{2}(a + [a+(n-1)d])$

(2) G.P (geomatric progretion)

$a, ar, ar^2, ar^3, ar^4 \dots ar^{n-1}$

$\downarrow$ 1st, 2nd, 3rd, 4th, 5th ... nth

→ n^{th} term of GP = ar^{n-1}

* sum of n term of G.P. series *

$S_n = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$ — (I)

$r * S_n = ar + ar^2 + ar^3 + ar^4 + ar^5 + \dots + ar^{n-1} + ar^n$ — (2)

→ now subtract both equation

$S_n(1-r) = a - ar^n$

$S_n = \frac{a - ar^n}{1-r}$

$S_n = \frac{a(1-r^n)}{1-r} = -\frac{a(r^n-1)}{-(r-1)} = \frac{a(r^n-1)}{r-1}$

$r < 1$

$S_\infty = \frac{a[1-0]}{1-r} = \frac{a}{1-r}$

Q. $1+2+3+4+\dots+n$ [sum of 1st n natural no]

→ This is A.P. series so.

$$S_n = \frac{n}{2}(2a+(n-1)d)$$
$$= \frac{n}{2}(2+(n-1)1)$$
$$= \frac{n}{2}(n+1)$$
$$= \frac{n(n+1)}{2}$$

Q. $1+3+5+7\dots$ [sum of odd natural number]

→
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{n}{2}[2+(n-1)2]$$
$$= \frac{n}{2}[\cancel{2}+2n-\cancel{2}]$$
$$= \frac{n}{\cancel{2}} \times \cancel{2}n$$
$$= n^2$$

Q. $2+4+6+8+10+\dots$ [sum of even natural no]

→ $$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{n}{2}[2\times 2+(n-1)2]$$
$$= \frac{n}{2}[4+2n-2]$$
$$= \frac{n}{2}[2+2n] \Rightarrow \frac{n}{\cancel{2}} \times \cancel{2}(1+n) \Rightarrow n\cdot(1+n)$$

Q. $1^2+2^2+3^2+4^2+5^2+\dots$ [sum of square of natural number]

→

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2a+(n-1)d]$$

$$(x+1)^3 - x^3 = x^3 + 3x^2 + 3x + 1 - x^3$$

$$= 1 + 3x + 3x^2$$

$x=1$

$$2^3 - 1^3 = 1 + 3\times1 + 3\times1^2 \quad -(1)$$

$x=2$

$$3^3 - 2^3 = 1 + 3\times2 + 3\times2^2 \quad -(2)$$

$x=n$

$$(n+1)^3 - n^3 = 1 + 3n + 3n^2 \quad -(3)$$

→ now sum all these equation

$$(n+1)^3 - 1^3 = 3[1^2+2^2+3^2+\dots+n^2] + 3[1+2+3\dots] + [1+1+1+\dots+n]$$

$$= 3\Sigma n^2 + \frac{3n(n+1)}{2} + n$$

$$3\Sigma n^2 = (n+1)^3 - 1 - \frac{3n(n+1)}{2} - n$$

$= (n+1)\left[(n+1)^2 - \frac{3n}{2}\right] - [n+1]$

$= (n+1)\left[(n+1)^2 - \frac{3n}{2} - 1\right]$

$= (n+1)\left(n^2+2n+\cancel{1} - \frac{3n}{2} \cancel{-1}\right)$

$= (n+1)\left(n^2 + \frac{n}{2}\right)$

$\Rightarrow 3\Sigma n^2 = \frac{(n+1)(n)(2n+1)}{2}$

$= \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

Q. $1^3 + 2^3 + 3^3 + \ldots$ [sum of square of natural number]

$\rightarrow$

$(x+1)^4 - x^4$

$(a+b)^n = {}^nC_0\, a^0 b^n + {}^nC_1\, a^1 b^{n-1} + {}^nC_2\, a^2 b^{n-2}$

$(x+1)^n = {}^nC_0\, x^0 + {}^nC_1\, x^1 + {}^nC_2\, x^2$

$(x+1)^4 - x^4 = {}^4C_0\, x^0 + {}^4C_1\, x^1 + {}^4C_2\, x^2 + {}^4C_3\, x^3$

$+ {}^4C_4\, x^4 - x^4$

$= 1 + 4x + 6x^2 + 4x^3 + \cancel{x^4} - \cancel{x^4}$
$= 4x^3 + 6x^2 + 4x + 1$

$x = 1$

$2^4 - 1^4 = 4 \times 1^3 + 6 \times 1^2 + 4 \times 1 + 1$

$x = 2$

$3^4 - 2^4 = 4 \times 2^3 + 6 \times 2^2 + 4 \times 2 + 1$

$x = n$

$4n^3 + 6n^2 + 4n + 1$

$(n+1)^4 - n^4 = 4[1^3 + 2^3 + 3^3 + \dots n^3] + 6[1^2 + 2^2 + 3^2 \dots n^2] + 4[1 + 2 + 3 + \dots + n] + 1 + 1 + \dots n$

$(n+1)^4 - 1^4 = 4\Sigma n^3 + 6\Sigma n^2 + 4\Sigma n + n$

$4\Sigma n^3 = (n+1)^4 - 1 - 6\Sigma n^2 - 4\Sigma n - n$

$= (n+1)^4 - 1 - \frac{\cancel{6} n(n+1)(2n+1)}{\cancel{6}} - \frac{\cancel{4}^{2} \times n(n+1)}{\cancel{2}} - n$

$= (n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$

$= n+1 \left((n+1)^3 - n(2n+1) - 2n\right) - (n+1)$

$= n+1 \left[(n+1)^3 - n(2n+1) - 2n - 1\right]$

$$= (n+1)\left[n^3 + \cancel{1} + 3n(n+1) - 2n^2 - n - 2n - \cancel{1}\right]$$
$$= (n+1)\left[n^3 + 3n^2 + 3n - 2n^2 - 3n\right]$$
$$= (n+1)\left[n^3 + n^2\right]$$
$$= (n+1)(n^2)(n+1)$$

$$\Sigma n^3 = \frac{(n+1)^2 (n^2)}{4}$$

$$= \left(\frac{(n+1)(n)}{2}\right)^2$$

Q. in an AP 7^{th} term is 40 & 21^{th} term is 124 find 31^{th} term

→

n^{th} term $= a + (n-1)d$

$7^{th} = a + 6d = 40$ — (1)
$21^{th} = a + 20d = 124$ — (2)

$$\cancel{a} + 6d = 40$$
$$\cancel{a} + 20d = 124$$
$$- \quad - \quad -$$
$$-14d = -84$$
$$d = 6$$

→ Put value of d is (1)

$a + 36 = 40$
$a = 4$

for 31^{th} term $a = 4$, $d = 6$, $n = 31$
$= 4 + 30 \times 6$
$= 4 + 180$
$= 184$

Q. sum of series $10 + 84 + 734 + \ldots =$

$\longrightarrow (9^1 + 1) + (9^2 + 3) + (9^3 + 5) + \ldots$

$\underbrace{(9^1 + 9^2 + 9^3 \ldots)}_{G.P} + (1 + 3 + 5 \ldots)$

$= \frac{9(9^n - 1)}{9 - 1} + n^2$

$= \frac{9^{n+1} - 9}{8} + n^2$

$= \frac{9^{n+1} + 8n^2 - 9}{8}$

Q. $4 + 44 + 444 + \ldots + n$

$\longrightarrow$

$4[1 + 11 + 111 + \ldots +]$

$\frac{4}{9}[9 + 99 + 999 + \ldots +]$

$\frac{4}{9}[(10^1 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots +]$

$\frac{4}{9}[(10^1 + 10^2 + 10^3 + \ldots + 10^n) - n]$

$\frac{4}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$

$$= \frac{4}{4}\left[\frac{10^{n+1}-10}{4} - 4n\right]$$

$$= \frac{4}{81}\left[10^{n+1} - 4n - 10\right]$$

Q. A ball is droped from 400m & every time it becomes to 2/3rd of it's height. find total distance coverd by ball before comming to ~~last~~ rest

→

$$= 400 + 2 \times \frac{2}{3} \times 400 + 2 \times \left(\frac{2}{3}\right)^2 \times 400$$

$$= 400\left[1 + 2 \times \frac{2}{3} + 2 \cdot \left(\frac{2}{3}\right)^2 + \ldots\right]$$

$$= 400\left[1 + 2\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 \ldots\right]\right]$$

→ here $r < 1$

$$S_\infty = \frac{a}{1-r} =$$

$$= 400\left[1 + 2\left[\frac{2/3}{1/3}\right]\right]$$

$$= 400[5]$$

$$= 4500 \text{ M}$$