

# GATE 2023 Mechanical Engineering Question Paper Solution

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## General Aptitude (GA)

**Q.1 – Q.5 Carry ONE mark Each**

**Q.1** He did not manage to fix the car himself, so he \_\_\_\_\_ in the garage.

**Answer.**

- (A) got it fixed**
- (B) getting it fixed
- (C) gets fixed
- (D) got fixed

**Solution:-**

The correct option is:

- (A) got it fixed**

**Q.2** Planting : Seed : : Raising : \_\_\_\_\_ (By word meaning)

**Answer.**

- (A) Child**
- (B) Temperature
- (C) Height
- (D) Lift

**Solution:-**

The correct answer is:

(A) Child

**Q.4** The minute-hand and second-hand of a clock cross each other \_\_\_\_\_ times between 09:15:00 AM and 09:45:00 AM on a day.

**Answer.**

(A) 30

(B) 15

(C) 29

(D) 31

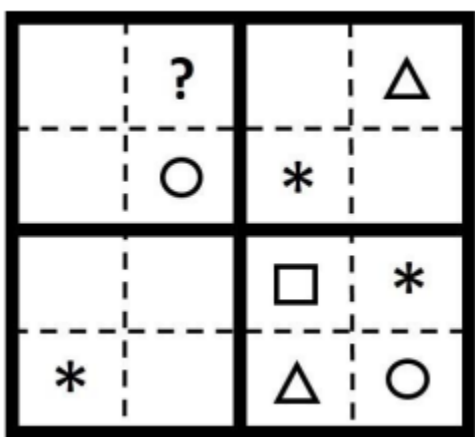
**Q.5**

The symbols  $\bigcirc$ ,  $*$ ,  $\triangle$ , and  $\square$  are to be filled, one in each box, as shown below.

The rules for filling in the four symbols are as follows.

- 1) Every row and every column must contain each of the four symbols.
- 2) Every  $2 \times 2$  square delineated by bold lines must contain each of the four symbols.

Which symbol will occupy the box marked with '?' in the partially filled figure?



**Answer.**

- A) o
- B) \***
- C)  $\Delta$
- D)  $\square$

**Q.6** In a recently held parent-teacher meeting, the teachers had very few complaints about Ravi. After all, Ravi was a hardworking and kind student. Incidentally, almost all of Ravi's friends at school were hardworking and kind too. But the teachers drew attention to Ravi's complete lack of interest in sports. The teachers believed that, along with some of his friends who showed similar disinterest in sports, Ravi needed to engage in some sports for his overall development. Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

**Answer.**

- (A) All of Ravi's friends are hardworking and kind.
- (B) No one who is not a friend of Ravi is hardworking and kind.
- (C) None of Ravi's friends are interested in sports.
- (D) Some of Ravi's friends are hardworking and kind.**

**Solution:-**

The correct answer is:

- (D) Some of Ravi's friends are hardworking and kind.**

**Q.7** Consider the following inequalities

$$p^2 - 4q < 4$$

$$3p + 2q < 6$$

where  $p$  and  $q$  are positive integers.

The value of  $(p + q)$  is \_\_\_\_\_.

**Answer.**

- (A) 2**

- (B) 1
- (C) 3
- (D) 4

**Solution:-**

Let's solve the given inequalities:

1.  $p^2 - 4q < 4$
2.  $3p + 2q < 6$

Let's start with the second inequality, which is simpler:

$$3p + 2q < 6$$

Solve for  $p$ :

$$3p < 6 - 2q$$

$$p < 2 - \frac{2}{3}q$$

Since  $p$  and  $q$  are positive integers, the maximum possible value for  $p$  when  $q = 1$  would be  $p = 1$ .

Now, substitute  $p = 1$  in the first inequality:

$$1^2 - 4q < 4$$

$$1 - 4q < 4$$

Solve for  $q$ :

$$-4q < 3$$

Divide by  $-4$  (reversing the inequality):

$$q > -\frac{3}{4}$$

Since  $q$  is a positive integer, the minimum possible value for  $q$  is 1.

So, we have  $p = 1$  and  $q = 1$ , which gives:

$$p + q = 1 + 1 = 2$$

Therefore, the correct answer is:

(A) 2

**Q.8 Which one of the sentence sequences in the given options creates a coherent Narrative?**

- (i) I could not bring myself to knock.
- (ii) There was a murmur of unfamiliar voices coming from the big drawing room and the door was firmly shut.
- (iii) The passage was dark for a bit, but then it suddenly opened into a bright kitchen.
- (iv) I decided I would rather wander down the passage.

**Answer.**

- (A) (iv), (i), (iii), (ii)
- (B) (iii), (i), (ii), (iv)
- (C) (ii), (i), (iv), (iii)**
- (D) (i), (iii), (ii), (iv)

**Solution:-**

The coherent narrative sequence is:

- (C) (ii), (i), (iv), (iii)

Here's the explanation of the sequence:

- (ii) There was a murmur of unfamiliar voices coming from the big drawing room and the door was firmly shut.
- (i) I could not bring myself to knock.
- (iv) I decided I would rather wander down the passage.
- (iii) The passage was dark for a bit, but then it suddenly opened into a bright kitchen.

**Q.9 How many pairs of sets (S,T) are possible among the subsets of {1, 2, 3, 4, 5, 6} that satisfy the condition that S is a subset of T?**

**Answer.**

**(A) 729**

(B) 728

(C) 665

(D) 664

**Solution:-**

To determine the number of pairs of sets (S, T) where S is a subset of T from the given set {1, 2, 3, 4, 5, 6}, let's consider each element.

For each element in the set, it can either be in the set S, in the set T but not in S, or not in either set. This gives us 3 possibilities for each element.

Since there are 6 elements in the set, the total number of possible pairs (S, T) is  $(3^6 = 729)$ .

Therefore, the correct answer is:

(A) 729

**Q.11 A machine produces a defective component with a probability of 0.015. The number of defective components in a packed box containing 200 components produced by the machine follows a Poisson distribution. The mean and the variance of the distribution are**

**Answer.**

**(A) 3 and 3, respectively**

(B)  $\sqrt{3}$  and  $\sqrt{3}$ , respectively

(C) 0.015 and 0.015, respectively

(D) 3 and 9, respectively

**Solution:-**

The mean ( $\mu$ ) and variance ( $\sigma^2$ ) of a Poisson distribution are both equal to the parameter  $\lambda$ , which represents the average rate of occurrence.

For a Poisson distribution,  $\lambda$  is calculated as the product of the sample size (number of trials) and the probability of success in each trial.

Given the probability of a defective component ( $p$ ) is 0.015, and the number of components ( $n$ ) is 200, we can calculate  $\lambda$  as follows:

$$\lambda = n \times p = 200 \times 0.015 = 3$$

So, the mean ( $\mu$ ) and variance ( $\sigma^2$ ) of the distribution are both 3.

Therefore, the correct answer is:

(A) 3 and 3, respectively

**Q.16** The effective stiffness of a cantilever beam of length  $L$  and flexural rigidity  $EI$  subjected to a transverse tip load  $W$  is\_\_\_\_\_.

**Answer.**

(A)  $3EI/L^3$

(B)  $2EI/L^3$

(C)  $L^3/2EI$

(D)  $L^3/3EI$

**Solution:-**

The effective stiffness of a cantilever beam subjected to a transverse tip load  $W$  can be calculated using the formula:

$$K = \frac{3EI}{L^3}$$

Where:

$K$  = Effective stiffness

$E$  = Young's modulus

$I$  = Moment of inertia of the beam's cross-sectional area

$L$  = Length of the cantilever beam

Comparing the given options with the formula, we can see that the correct answer is:

(A)  $\frac{3EI}{L^3}$

**Q.19** Air (density = 1.2 kg/m<sup>3</sup>, kinematic viscosity = 1.5×10<sup>-5</sup> m<sup>2</sup>/s) flows over a flat plate with a free-stream velocity of 2 m/s. The wall shear stress at a location 15 mm from the leading edge is  $\tau_w$ . What is the wall shear stress at a location 30 mm from the leading edge?

**Answer.**

(A)  $\tau_w / 2$

(B)  $\sqrt{2} \tau_w$

(C)  $2 \tau_w$

(D)  $\tau_w / \sqrt{2}$

**Solution:-**

The relationship between the wall shear stress ( $\tau_w$ ) and the distance from the leading edge ( $x$ ) for laminar flow over a flat plate is given by:

$$\tau_w = \frac{0.664 \cdot \rho \cdot U \cdot \nu}{x}$$

Where:

$\rho$  = Density of air (1.2 kg/m<sup>3</sup>)

$U$  = Free-stream velocity (2 m/s)

$\nu$  = Kinematic viscosity (1.5 × 10<sup>-5</sup> m<sup>2</sup>/s)

$x$  = Distance from the leading edge (in this case, 30 mm = 0.03 m)

Substituting the given values into the formula:

$$\tau_w = \frac{0.664 \cdot 1.2 \cdot 2 \cdot 1.5 \times 10^{-5}}{0.03}$$

$$\tau_w = 0.044 \text{ N/m}^2$$

Now let's consider the options:

(A)  $\frac{\tau_w}{2} = 0.022 \text{ N/m}^2$

(B)  $\sqrt{2} \cdot \tau_w \approx 0.062 \text{ N/m}^2$

(C)  $2 \cdot \tau_w = 0.088 \text{ N/m}^2$

(D)  $\frac{\tau_w}{\sqrt{2}} \approx 0.031 \text{ N/m}^2$

Among the given options, the closest value to the calculated wall shear stress (0.044 N/m<sup>2</sup>) is:

(D)  $\frac{\tau_w}{\sqrt{2}} \approx 0.031 \text{ N/m}^2$

So, the correct answer is option (D).



**Q.21 Consider incompressible laminar flow of a constant property Newtonian fluid in an isothermal circular tube. The flow is steady with fully-developed temperature and velocity profiles. The Nusselt number for this flow depends on**

**Answer.**

- (A) neither the Reynolds number nor the Prandtl number**
- (B) both the Reynolds and Prandtl numbers
- (C) the Reynolds number but not the Prandtl number
- (D) the Prandtl number but not the Reynolds number

**Q.22 A heat engine extracts heat ( $Q_H$ ) from a thermal reservoir at a temperature of 1000 K and rejects heat ( $Q_L$ ) to a thermal reservoir at a temperature of 100 K, while producing work ( $W$ ). Which one of the combinations of [ $Q_H$ ,  $Q_L$  and  $W$ ] given is allowed?**

**Answer.**

- (A)  $Q_H = 2000 \text{ J}$ ,  $Q_L = 500 \text{ J}$ ,  $W = 1000 \text{ J}$
- (B)  $Q_H = 2000 \text{ J}$ ,  $Q_L = 750 \text{ J}$ ,  $W = 1250 \text{ J}$**
- (C)  $Q_H = 6000 \text{ J}$ ,  $Q_L = 500 \text{ J}$ ,  $W = 5500 \text{ J}$
- (D)  $Q_H = 6000 \text{ J}$ ,  $Q_L = 600 \text{ J}$ ,  $W = 5500 \text{ J}$

**Solution:-**

The efficiency ( $\eta$ ) of a heat engine is given by the formula:

$$\eta = \frac{W}{Q_H}$$

Where:

$\eta$  = Efficiency of the heat engine

$W$  = Work output by the heat engine

$Q_H$  = Heat input from the hot reservoir

The efficiency of a heat engine is bounded by the Carnot efficiency, which is the maximum possible efficiency for a heat engine operating between two reservoirs at temperatures  $T_H$  (hot reservoir temperature) and  $T_L$  (cold reservoir temperature):

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H}$$

Given the temperatures  $T_H = 1000 \text{ K}$  and  $T_L = 100 \text{ K}$ , the maximum possible efficiency is:

$$\eta_{\text{Carnot}} = 1 - \frac{100}{1000} = 0.9$$

The efficiency of a heat engine cannot exceed the Carnot efficiency. Therefore, among the given combinations, only the combination in option (B) satisfies this condition:

$$(B) \quad Q_H = 2000 \text{ J}, Q_L = 750 \text{ J}, W = 1250 \text{ J}$$

Let's calculate the efficiency for this option:

$$\eta = \frac{W}{Q_H} = \frac{1250}{2000} = 0.625$$

Since  $\eta < \eta_{\text{Carnot}}$ , option (B) is allowed.

So, the correct answer is:

$$(B) \quad Q_H = 2000 \text{ J}, Q_L = 750 \text{ J}, W = 1250 \text{ J}$$

**Q.23** Two surfaces P and Q are to be joined together. In which of the given joining operation(s), there is no melting of the two surfaces P and Q for creating the joint?

**Answer.**

- (A) Arc welding
- (B) Brazing
- (C) Adhesive bonding**
- (D) Spot welding

**Solution:-**

The process in which there is no melting of the two surfaces being joined is: (C) Adhesive bonding

In adhesive bonding, the surfaces are joined using an adhesive (such as glue) that bonds them together without melting or significant heating of the surfaces. The other options involve heat, melting, or fusing of the surfaces to create the joint.

**Q.25 In a metal casting process to manufacture parts, both patterns and moulds provide shape by dictating where the material should or should not go. Which of the option(s) given correctly describe(s) the mould and the pattern?**

**Answer.**

- (A) Mould walls indicate boundaries within which the molten part material is allowed, while pattern walls indicate boundaries of regions where mould material is not allowed.
- (B) Moulds can be used to make patterns.**
- (C) Pattern walls indicate boundaries within which the molten part material is allowed, while mould walls indicate boundaries of regions where mould material is not allowed.
- (D) Patterns can be used to make moulds.

**Solution:-**

Both the mould and the pattern play important roles in metal casting processes:

A pattern is a replica or model of the final part to be cast. It is used to create the cavity in the mould that will define the shape of the casting. The pattern is typically made from wood, metal, or other suitable materials.

A mould is a cavity or container that is created around the pattern. The molten metal is poured into the mould, and when it cools and solidifies, it takes the shape of the pattern. The mould is usually made from materials like sand, clay, or other refractory materials.

So, the correct option is:

(B) The pattern dictates the shape of the casting, and the mould provides the cavity where the material is poured and solidified.

**Q.26 The principal stresses at a point P in a solid are 70 MPa, -70 MPa and 0. The yield stress of the material is 100 MPa. Which prediction(s) about material failure at P is/are CORRECT?**

**Answer.**

- (A) Maximum normal stress theory predicts that the material fails
- (B) Maximum shear stress theory predicts that the material fails**
- (C) Maximum normal stress theory predicts that the material does not fail**
- (D) Maximum shear stress theory predicts that the material does not fail

**Q.30 The figure shows a block of mass  $m = 20$  kg attached to a pair of identical linear springs, each having a spring constant  $k = 1000$  N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is \_\_\_\_\_ seconds. (Rounded off to two decimal places) Take  $\pi = 3.14$ .**

**Answer. 6.28 sec**

**Solution:-**

The angular frequency ( $\omega$ ) of a mass-spring system is given by:

$$\omega = \sqrt{\frac{k}{m}}$$

The time period ( $T$ ) of the oscillation is the reciprocal of the angular frequency:

$$T = \frac{2\pi}{\omega}$$

Given that  $m = 20 \text{ kg}$  and  $k = 1000 \text{ N/m}$ , we can calculate  $\omega$  and then use it to find  $T$ :

$$\omega = \sqrt{\frac{1000}{20}} = 10 \text{ s}^{-1}$$

Now, calculate the time period:

$$T = \frac{2\pi}{10} = \frac{\pi}{5} \text{ s}$$

To find the time taken by the block to complete ten oscillations, multiply the time period by ten:

$$10 \times \frac{\pi}{5} = \frac{10\pi}{5} = 2\pi = 6.28 \text{ s}$$

Rounded to two decimal places, the time taken is approximately 6.28 seconds.

So, the time taken by the block to complete ten oscillations is approximately 6.28 seconds.

## Q.31

A vector field

$$\mathbf{B}(x, y, z) = x \hat{i} + y \hat{j} - 2z \hat{k}$$

is defined over a conical region having height  $h = 2$ , base radius  $r = 3$  and axis along  $z$ , as shown in the figure. The base of the cone lies in the  $x$ - $y$  plane and is centered at the origin.

If  $\mathbf{n}$  denotes the unit outward normal to the curved surface  $S$  of the cone, the value of the integral

$$\int_S \mathbf{B} \cdot \mathbf{n} \, dS$$

equals \_\_\_\_\_. (Answer in integer)

## Answer.

The surface integral  $\int \mathbf{B} \cdot \mathbf{n} \, dS$  can be calculated using the divergence theorem, which states that the flux of a vector field through a closed surface is equal to the volume integral of the divergence of the field over the region enclosed by the surface.

Mathematically, the divergence theorem can be written as:

$$\int \mathbf{B} \cdot \mathbf{n} \, dS = \iiint (\nabla \cdot \mathbf{B}) \, dV$$

Where  $\nabla \cdot \mathbf{B}$  is the divergence of the vector field  $\mathbf{B}$  and  $dV$  is the differential volume element.

Given the vector field  $\mathbf{B}(x, y, z) = x\mathbf{i} + y\mathbf{j} - 2z\mathbf{k}$ , we need to calculate the divergence  $\nabla \cdot \mathbf{B}$  and then integrate it over the volume of the cone.

The divergence of  $\mathbf{B}$  is given by:

$$\nabla \cdot \mathbf{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

Substituting the components of  $\mathbf{B}$  into the divergence expression:

$$\nabla \cdot \mathbf{B} = 1 + 1 - 2 = 0$$

Since the divergence of  $\mathbf{B}$  is zero, the surface integral becomes:

$$\int \mathbf{B} \cdot \mathbf{n} \, dS = \iiint 0 \, dV = 0$$

Therefore, the value of the integral is 0 (zero).

**Q.32** A linear transformation maps a point  $(x, y)$  in the plane to the point  $(\hat{x}, \hat{y})$  according to the rule  $\hat{x} = 3y$ ,  $\hat{y} = 2x$ . Then, the disc  $x^2 + y^2 \leq 1$  gets transformed to a region with an area equal to \_\_\_\_\_.  
(Rounded off to two decimals) Use  $\pi = 3.14$ .

**Answer. 18.84**

**Solution:-**

Let's analyze the transformation and see how it affects the given disc  $x^2 + y^2 \leq 1$ .

The given transformation equations are:

$$x' = 3y$$

$$y' = 2x$$

For the region  $x^2 + y^2 \leq 1$ , we have a unit disc centered at the origin.

The transformed equations become:

$$x^2 = (3y)^2 = 9y^2$$

$$y^2 = (2x)^2 = 4x^2$$

Substituting the second equation into the first equation:

$$x^2 = 9 \cdot \frac{y^2}{4}$$

$$4x^2 = 9y^2$$

These equations represent an ellipse. The major axis is along the y-axis (vertical) and the minor axis is along the x-axis (horizontal).

The equation of an ellipse with major axis  $a$  and minor axis  $b$  is given by:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Comparing with our equation  $4x^2 = 9y^2$ , we have:

$$\frac{x^2}{(3/2)^2} + \frac{y^2}{2^2} = 1$$

So, the transformed region is an ellipse with a major axis of length  $a = 3$  and a minor axis of length  $b = 2$ .

The area of an ellipse is given by:

$$A = \pi ab$$

Substitute the given values and calculate:

$$A = \pi \cdot 3 \cdot 2 = 6\pi$$

Using  $\pi = 3.14$ , we get:

$$A \approx 6 \cdot 3.14 = 18.84$$

Rounded to two decimal places, the area of the transformed region is approximately 18.84.

Therefore, the area of the transformed region is approximately 18.84 (rounded to two decimal places).

**Q.33** The value of  $k$  that makes the complex-valued function  $f(z) = e^{-kx} (\cos 2y - i \sin 2y)$  analytic, where  $z = x + iy$ , is \_\_\_\_\_.  
(Answer in integer)

**Answer.**  $z = 2$

**Solution:-**

To determine the value of  $k$  that makes the complex-valued function  $f(z) = e^{-kx} (\cos 2y - i \sin 2y)$  analytic, we need to ensure that it satisfies the Cauchy-Riemann equations. The Cauchy-Riemann equations are given by:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Where  $u(x, y)$  represents the real part of  $f(z)$  and  $v(x, y)$  represents the imaginary part of  $f(z)$ .

Given  $f(z) = e^{-kx} (\cos 2y - i \sin 2y)$ , we can separate it into its real and imaginary parts:

$$u(x, y) = e^{-kx} \cos 2y \text{ (real part)}$$

$$v(x, y) = -e^{-kx} \sin 2y \text{ (imaginary part)}$$

Now, let's calculate the partial derivatives:

$$\begin{aligned} \frac{\partial u}{\partial x} &= -ke^{-kx} \cos 2y \\ \frac{\partial v}{\partial y} &= -2e^{-kx} \cos 2y \end{aligned}$$

$$\begin{aligned} \frac{\partial u}{\partial y} &= -2e^{-kx} \sin 2y \\ \frac{\partial v}{\partial x} &= -ke^{-kx} \sin 2y \end{aligned}$$

Now, equate the partial derivatives according to the Cauchy-Riemann equations:

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ -ke^{-kx} \cos 2y &= -2e^{-kx} \cos 2y \end{aligned}$$

From the above equation, we can see that  $k = 2$ .

So, the value of  $k$  that makes the complex-valued function  $f(z)$  analytic is  $k = 2$ .



**Q.35 Consider a counter-flow heat exchanger with the inlet temperatures of two fluids (1 and 2) being  $T_{1, \text{in}} = 300 \text{ K}$  and  $T_{2, \text{in}} = 350 \text{ K}$ . The heat capacity rates of the two fluids are  $C_1 = 1000 \text{ W/K}$  and  $C_2 = 400 \text{ W/K}$ , and the effectiveness of the heat exchanger is 0.5. The actual heat transfer rate is \_\_\_\_\_ kW**

**Answer. 10kW**

**Solution:-**

The actual heat transfer rate ( $Q_{\text{actual}}$ ) in a counter-flow heat exchanger can be calculated using the formula for heat exchanger effectiveness ( $\varepsilon$ ):

$$\varepsilon = \frac{Q_{\text{actual}}}{Q_{\text{max}}}$$

Where:

$Q_{\text{actual}}$  = Actual heat transfer rate

$Q_{\text{max}}$  = Maximum possible heat transfer rate (when one of the fluids gets completely heated or cooled)

The formula for calculating  $Q_{\text{max}}$  is:

$$Q_{\text{max}} = \min(C_1, C_2) \cdot (T_{1, \text{in}} - T_{2, \text{in}})$$

Given the values:

$$T_{1, \text{in}} = 300 \text{ K}$$

$$T_{2, \text{in}} = 350 \text{ K}$$

$$C_1 = 1000 \text{ W/K}$$

$$C_2 = 400 \text{ W/K}$$

$$\varepsilon = 0.5$$

Let's calculate  $Q_{\text{max}}$ :

$$Q_{\text{max}} = \min(1000, 400) \cdot (300 - 350) = 400 \cdot (-50) = -20000 \text{ W}$$

Now, use the effectiveness formula to find  $Q_{\text{actual}}$ :

$$0.5 = \frac{Q_{\text{actual}}}{-20000}$$

$$Q_{\text{actual}} = -20000 \cdot 0.5 = -10000 \text{ W}$$

Since heat transfer rate cannot be negative, we take the absolute value:

$$Q_{\text{actual}} = 10000 \text{ W} = 10 \text{ kW}$$

Therefore, the actual heat transfer rate is 10 kW.

**Q.36**

Which one of the options given is the inverse Laplace transform of  $\frac{1}{s^3 - s}$  ?

$u(t)$  denotes the unit-step function.

**A)**

$$\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t)$$

**B)**

$$\left(\frac{1}{3}e^{-t} - e^t\right)u(t)$$

**C)**

$$\left(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{(t-1)}\right)u(t-1)$$

**D)**

$$\left(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{(t-1)}\right)u(t-1)$$

**Answer. A)****Solution:-**

To find the inverse Laplace transform of a given function, we can use the linearity property and the inverse Laplace transform of elementary functions.

The given function is:

$$F(s) = \frac{1}{s} \cdot \frac{3-s}{s} = \frac{3-s}{s^2}$$

We need to factorize  $F(s)$  into partial fractions to apply the inverse Laplace transform. Let's find the partial fraction decomposition:

$$\frac{3-s}{s^2} = \frac{A}{s} + \frac{B}{s^2}$$

Multiplying both sides by  $s^2$ :

$$3 - s = As + B$$

Comparing coefficients:

$$A = -1, \quad B = 3$$

So, the partial fraction decomposition is:

$$\frac{3-s}{s^2} = -\frac{1}{s} + \frac{3}{s^2}$$

Now, we can use the inverse Laplace transform properties:

1.  $\mathcal{L}^{-1}\{1\} = \delta(t)$ , where  $\delta(t)$  is the Dirac delta function.
2.  $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$
3.  $\mathcal{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{(n-1)!}$  for  $n > 0$

Applying these properties, we get the inverse Laplace transform of  $F(s)$ :

$$\mathcal{L}^{-1}\{F(s)\} = -1 + 3t$$

Now, let's look at the given options:

- (A)  $(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t)u(t)$   
 (B)  $\frac{1}{3}e^{-t} - e^t u(t)$   
 (C)  $(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{t-1})u(t-1)$   
 (D)  $(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{t-1})u(t-1)$

Comparing the options with the calculated inverse Laplace transform  $(-1 + 3t)$ , we see that option (A) matches the result:

$$(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t)u(t)$$

So, the correct answer is option (A).

**Q.41 Which one of the following statements is FALSE?**

- (A) For an ideal gas, the enthalpy is independent of pressure.
- (B) For a real gas going through an adiabatic reversible process, the process equation is given by  $PV^\gamma = \text{constant}$ , where  $P$  is the pressure,  $V$  is the volume and  $\gamma$  is the ratio of the specific heats of the gas at constant pressure and constant volume.**
- (C) For an ideal gas undergoing a reversible polytropic process  $PV^{1.5} = \text{constant}$ , the equation connecting the pressure, volume and temperature of the gas at any point along the process is  $PR = mTV$ , where  $R$  is the gas constant and  $m$  is the mass of the gas.
- (D) Any real gas behaves as an ideal gas at sufficiently low pressure or sufficiently high temperature.

**Q.43 A cylindrical rod of length  $h$  and diameter  $d$  is placed inside a cubic enclosure of side length  $L$ .  $S$  denotes the inner surface of the cube. The view-factor FS-S is**

- (A) 0
- (B) 1
- (C)  $(\pi dh + (\pi d^2)/2)6L^2$
- (D)  $1 - (\pi dh + (\pi d^2)/2)6L^2$**

**Q.45 A CNC machine has one of its linear positioning axes as shown in the figure, consisting of a motor rotating a lead screw, which in turn moves a nut horizontally on which a table is mounted. The motor moves in discrete rotational steps of 50 steps per revolution. The pitch of the screw is 5 mm and the total horizontal traverse length of the table is 100 mm. What is the total number of controllable locations at which the table can be positioned on this axis?**

**Answer.**

(A) 5000 (B) 2 (C) **1000** (D) 200

**Solution:-**

The total number of controllable locations on the axis can be calculated by considering the steps per revolution of the motor, the pitch of the screw, and the total traverse length of the table.

Given:

- Steps per revolution: 50 steps
- Pitch of the screw: 5 mm
- Total traverse length of the table: 100 mm

The total number of controllable locations can be calculated using the formula:

$$\text{Total controllable locations} = \frac{\text{Total traverse length}}{\text{Pitch}} \times \text{Steps per revolution}$$

Substitute the given values:

$$\text{Total controllable locations} = \frac{100 \text{ mm}}{5 \text{ mm/step}} \times 50 \text{ steps/revolution} = 1000 \text{ locations}$$

So, the total number of controllable locations at which the table can be positioned on this axis is: (C) 1000

**Q.46 Cylindrical bars P and Q have identical lengths and radii, but are composed of different linear elastic materials. The Young's modulus and coefficient of thermal expansion of Q are twice the corresponding values of P. Assume the bars to be perfectly bonded at the interface, and their weights to be negligible. The bars are held between rigid supports as shown in the figure and the temperature is raised by  $\Delta T$ . Assume that the stress in each bar is homogeneous and uniaxial.**

Denote the magnitudes of stress in P and Q by  $\sigma_1$  and  $\sigma_2$ ,  
Respectively. Which of the statement(s) given is/are CORRECT?

**Answer.**

- (A) The interface between P and Q moves to the left after heating
- (B) The interface between P and Q moves to the right after heating
- (C)  $\sigma_1 < \sigma_2$
- (D)  $\sigma_1 = \sigma_2$

**Q. 47** A very large metal plate of thickness  $d$  and thermal conductivity  $k$  is cooled by a stream of air at temperature  $T^\infty = 300$  K with a heat transfer coefficient  $h$ , as shown in the figure. The centerline temperature of the plate is  $T_p$ . In which of the following case(s) can the lumped parameter model be used to study the heat transfer in the metal plate?

**Answer.**

- (A)  $h = 10 \text{ Wm}^{-2}\text{K}^{-1}$ ,  $k = 100 \text{ Wm}^{-1}\text{K}^{-1}$ ,  $d = 1 \text{ mm}$ ,  $T_p = 350 \text{ K}$
- (B)  $h = 100 \text{ Wm}^{-2}\text{K}^{-1}$ ,  $k = 100 \text{ Wm}^{-1}\text{K}^{-1}$ ,  $d = 1 \text{ m}$ ,  $T_p = 325 \text{ K}$
- (C)  $h = 100 \text{ Wm}^{-2}\text{K}^{-1}$ ,  $k = 1000 \text{ Wm}^{-1}\text{K}^{-1}$ ,  $d = 1 \text{ mm}$ ,  $T_p = 325 \text{ K}$
- (D)  $h = 1000 \text{ Wm}^{-2}\text{K}^{-1}$ ,  $k = 1 \text{ Wm}^{-1}\text{K}^{-1}$ ,  $d = 1 \text{ m}$ ,  $T_p = 350 \text{ K}$

The lumped parameter model (also known as the lumped capacitance method) is used to analyze transient heat conduction in a solid object when the temperature difference across the object is small and uniform. The condition for using the lumped parameter model is that the Biot number (Bi) should be less than 0.1:

$$Bi = \frac{hd}{k} < 0.1$$

Where:

- $h$  is the convective heat transfer coefficient.
- $d$  is the characteristic length (thickness of the object in this case).
- $k$  is the thermal conductivity of the material.

Let's calculate the Biot number for each given case:

(A)  $Bi = \frac{10 \times 0.001}{100} = 0.0001$  (Lumped parameter model can be used)

(B)  $Bi = \frac{100 \times 1}{100} = 1$  (Lumped parameter model cannot be used)

(C)  $Bi = \frac{100 \times 0.001}{1000} = 0.1$  (Borderline, but typically not used)

(D)  $Bi = \frac{1000 \times 1}{1} = 1000$  (Lumped parameter model cannot be used)

Based on the Biot number criterion, the only case where the lumped parameter model can be used is option (A):

(A)  $h = 10 \text{ W m}^{-2}\text{K}^{-1}, k = 100 \text{ W m}^{-1}\text{K}^{-1}, d = 1 \text{ mm}, TP = 350 \text{ K}$

So, the correct answer is option (A).

**Q.48** The smallest perimeter that a rectangle with area of 4 square units can have is \_\_\_\_\_ units. (Answer in integer)

**Answer.** 8 units

**Solution:-**

Let's denote the length of the rectangle as  $L$  and the width of the rectangle as  $W$ . The area of a rectangle is given by  $A = L \times W$ .

Given that the area is 4 square units ( $A = 4$ ), we have:

$$L \times W = 4$$

To find the perimeter ( $P$ ) of the rectangle, we use the formula for the perimeter of a rectangle:  $P = 2L + 2W$ .

We need to minimize the perimeter while keeping the area constant. This can be achieved when the rectangle is a square (i.e.,  $L = W$ ) because a square has the smallest perimeter for a given area among all rectangles.

Substitute  $L = W$  into the area equation:

$$L \times L = 4$$

$$L^2 = 4$$

Take the square root of both sides:

$$L = 2$$

Now, we can calculate the perimeter:

$$P = 2L + 2W = 2(2) + 2(2) = 4 + 4 = 8$$

So, the smallest perimeter that a rectangle with an area of 4 square units can have is 8 units.

**Q.49 Consider the second-order linear ordinary differential equation  $x^2 d^2y/dx^2 + x dy/dx - y = 0$ ,  $x \geq 1$  with the initial conditions  $y(x = 1) = 6, dy/dx|_{x=1} = 2$ . The value of  $y$  at  $x = 2$  equals \_\_\_\_\_. (Answer in integer)**



**Answer. 9**

**Solution:-**

Therefore, the general solution of the differential equation is:

$$y(x) = c_1 x^1 + c_2 x^{-1} = c_1 x + \frac{c_2}{x}$$

Using the initial condition  $y(x = 1) = 6$ :

$$6 = c_1 + c_2$$

Now, differentiate  $y$  with respect to  $x$  and use the initial condition  $\left. \frac{dy}{dx} \right|_{x=1} = 2$ :

$$\begin{aligned} \frac{dy}{dx} &= c_1 - \frac{c_2}{x^2} \\ \left. \frac{dy}{dx} \right|_{x=1} &= c_1 - c_2 = 2 \end{aligned}$$

We have a system of equations:

$$c_1 + c_2 = 6$$

$$c_1 - c_2 = 2$$

Adding the two equations gives:

$$2c_1 = 8$$

$$c_1 = 4$$

Substitute  $c_1 = 4$  into the first equation:

$$4 + c_2 = 6$$

$$c_2 = 2$$

So, the particular solution for the given initial conditions is:

$$y(x) = 4x + \frac{2}{x}$$

Now, we can find the value of  $y$  at  $x = 2$ :

$$y(2) = 4(2) + \frac{2}{2} = 8 + 1 = 9$$

**Therefore the value of  $y$  at  $x=2$  is 9.**

**Q.50** The initial value problem  $dy/dt + 2y = 0$ ,  $y(0) = 1$  is solved numerically using the forward Euler's method with a constant and positive time step of  $\Delta t$ . Let  $y_n$  represent the numerical solution obtained after  $n$  steps. The condition  $|y_{n+1}| \leq |y_n|$  is satisfied if and only if  $\Delta t$  does not exceed \_\_\_\_\_. (Answer in integer)

**Answer.** 0.999 to 1.001

**Q.51** The atomic radius of a hypothetical face-centered cubic (FCC) metal is  $(\sqrt{2}/10)$  nm. The atomic weight of the metal is 24.092 g/mol. Taking Avogadro's number to be  $6.023 \times 10^{23}$  atoms/mol, the density of the metal is \_\_\_\_\_ kg/m<sup>3</sup>. (Answer in integer)

**Answer.** 2490 to 2510

**Q.52** A steel sample with 1.5 wt.% carbon (no other alloying elements present) is slowly cooled from 1100 °C to just below the eutectoid temperature (723 °C). A part of the iron-cementite phase diagram is shown in the figure. The ratio of the pro-eutectoid cementite content to the total cementite content in the microstructure that develops just below the eutectoid temperature is \_\_\_\_\_. (Rounded off to two decimal places)

**Answer.** 0.53 to 0.55

**Q.53** A part, produced in high volumes, is dimensioned as shown. The machining process making this part is known to be statistically in control based on sampling data. The sampling data shows that D1 follows a normal distribution with a mean of 20 mm and a standard deviation of 0.3 mm, while D2 follows a normal distribution with a

mean of 35 mm and a standard deviation of 0.4 mm. An inspection of dimension C is carried out in a sufficiently large number of parts. To be considered under six-sigma process control, the upper limit of dimension C should be \_\_\_\_\_ mm. (Rounded off to one decimal place)

**Answer. 16.4 to 16.6**

**Q.56** An optical flat is used to measure the height difference between a reference slip gauge A and a slip gauge B. Upon viewing via the optical flat using a monochromatic light of wavelength  $0.5 \mu\text{m}$ , 12 fringes were observed over a length of 15 mm of gauge B. If the gauges are placed 45 mm apart, the height difference of the gauges is \_\_\_\_\_ m.

**Answer. 8.999 to 9.001**

**Q.57** Ignoring the small elastic region, the true stress ( $\sigma$ ) – true strain ( $\epsilon$ ) variation of a material beyond yielding follows the equation  $\sigma = 400\epsilon^{0.3}$  MPa. The engineering ultimate tensile strength value of this material is \_\_\_\_\_ MPa.

**Answer. 206 to 207**

**Q.59** A cylindrical bar has a length  $L = 5 \text{ m}$  and cross section area  $S = 10 \text{ m}^2$ . The bar is made of a linear elastic material with a density  $\rho = 2700 \text{ kg/m}^3$  and Young's modulus  $E = 70 \text{ GPa}$ . The bar is suspended as shown in the figure and is in a state of uniaxial tension due to its self-weight. The elastic strain energy stored in the bar equals \_\_\_\_\_ J. (Rounded off to two decimal places) Take the acceleration due to gravity as  $g = 9.8 \text{ m/s}^2$

**Answer. 2.00 to 2.16**

**Q.60** A cylindrical transmission shaft of length 1.5 m and diameter 100 mm is made of a linear elastic material with a shear modulus of 80 GPa. While operating at 500 rpm, the angle of twist across its length is found to be 0.5 degrees. The power transmitted by the shaft at this speed is \_\_\_\_\_ kW. (Rounded off to two decimal places)  
Take  $\pi = 3.14$ .

**Answer. 237 to 240**

**Q.61** Consider a mixture of two ideal gases, X and Y, with molar masses  $\bar{M}_X = 10$  kg/kmol and  $\bar{M}_Y = 20$  kg/kmol, respectively, in a container. The total pressure in the container is 100 kPa, the total volume of the container is 10 m<sup>3</sup> and the temperature of the contents of the container is 300 K. If the mass of gas-X in the container is 2 kg, then the mass of gas-Y in the container is \_\_\_\_ kg. (Rounded off to one decimal place) Assume that the universal gas constant is 8314 J kmol<sup>-1</sup>K<sup>-1</sup>

**Answer. 3.9 to 4.1**

**Q.62** The velocity field of a certain two-dimensional flow is given by  $\mathbf{V}(x, y) = k(x\hat{i} - y\hat{j})$  where  $k = 2 \text{ s}^{-1}$ . The coordinates  $x$  and  $y$  are in meters. Assume gravitational effects to be negligible. If the density of the fluid is 1000 kg/m<sup>3</sup> and the pressure at the origin is 100 kPa, the pressure at the location (2 m, 2 m) is \_\_\_\_\_ kPa. (Answer in integer)

**Answer. 83.999 to 84.001**

