Memorization in Neural Networks: Does the Loss Function Matter?

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- Zhang et al. [11] showed that SGD-based training of neural networks drives the training accuracy to 100% even in case of randomly labelled data.
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- None of the standard regularization methods such as weight decay, drop-out, etc. seem effective in resisting such overfitting.
- Many other studies (e.g., [1, 5, 3, 4]) throw interesting light on the dynamics of this memorization process and what it means for generalization in deep networks

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- The choice of loss function can be critical in determining this.
- None of the studies on memorization investigate this.

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- We formally define what 'resisting memorization' means and provide some theoretical justification for the empirical results

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- Design of algorithms for robust learning when training data has randomly corrupted labels, is also a much studied problem
- Here our interest is in the inherent ability of a loss function to resist overfitting of training data when labels are randomly altered.

Notation

- $\mathcal{X} \subseteq \mathbb{R}^n$: feature space;
- $\mathcal{Y} = \{1, \ldots, K\}$ where K: number of classes
- $S = \{\boldsymbol{x}_i, y_i^{cl}\}_{i=1}^{\ell}$: Original training set
- $S_\eta = \{\boldsymbol{x}_i, y_i\}_{i=1}^{\ell}$: Training set with randomly altered labels:

$$y_i = \begin{cases} y_i^{cl} & \text{with probability } 1 - \eta \\ j \in \mathcal{Y} - \{y_i^{cl}\} & \text{with probability } \frac{\eta}{K-1} \end{cases} \quad (1)$$

  where η is referred to as the noise rate.
- h_η: classifier function (with softmax layer as output layer) learnt by an algorithm with S_η as training data
- To define loss functions we take y_i to be one-hot vector. ($\boldsymbol{e}^k$ represents class-k).

Notation (contd.)

- Let $J_1 = \frac{1}{\ell} \sum_{i=1}^{\ell} \mathbb{I}_{[h_\eta(\boldsymbol{x}_i)=y_i]}$
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- This is the accuracy of h_η on the same training set but computed with respect to the original labels.
- Relative values of J_1 and J_2 can give interesting insights into how different loss functions behave.

Loss Functions We Compare

- We consider 3 loss functions in this paper: CCE, MSE, and Robust Log Loss (RLL).

$$\begin{aligned}
\mathcal{L}_{CCE}(\mathbf{h}(\mathbf{x}), \mathbf{e}^k) &= -\sum_i e_i^k \, \log\left(h_i(\mathbf{x})\right) = -\log(h_k(\mathbf{x})) \\
\mathcal{L}_{MSE}(\mathbf{h}(\mathbf{x}), \mathbf{e}^k) &= \sum_i \left(h_i(\mathbf{x}) - e_i^k\right)^2 \\
\mathcal{L}_{RLL}(\mathbf{h}(\mathbf{x}), \mathbf{e}^k) &= \log\left(\frac{\alpha+1}{\alpha}\right) - \log(\alpha + h_k(\mathbf{x})) \\
&\quad + \sum_{j \neq k} \frac{1}{K-1} \log(\alpha + h_j(\mathbf{x}))
\end{aligned}$$

where $\alpha > 0$ is a parameter of the RLL.

Loss Functions (contd.)

- CCE and MSE are fairly commonly used loss functions for classification and regression tasks.
- RLL is a **symmetric** loss.

Symmetric Loss

A loss function, L, is **symmetric** if $\exists\ C \in \mathbb{R}_{++}$ such that:

$$\sum_{j=1}^{K} L(h(\boldsymbol{x}), j) = C,\ \forall h, \boldsymbol{x} \tag{2}$$

We next present some empirical results comparing these three losses in their ability to resist memorization

Experimental Setup

Datasets:

- MNIST [8]
- CIFAR-10 [7]

Networks:

- Inception-Lite (same as that used in [11]) for CIFAR-10
- ResNet-32 for CIFAR-10
- ResNet-18 for MNIST

Experimental Setup (contd.)

Details about network training:

Inception-Lite

- SGD(lr $= 10^{-2}$, momentum=0.9)
- number of epochs = 100
- learning rate reduced by 0.95 factor every epoch

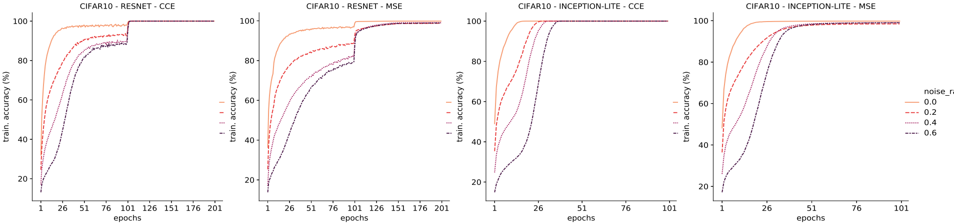
ResNet-32

- SGD(lr $= 10^{-1}$, momentum=0.9, weight_decay=10^{-4})
- number of epochs = 200
- learning rate reduced by 0.1 factor at epochs 100 and 150

ResNet-18

- Adam(lr $= 10^{-3}$)
- number of epochs = 200

Results on CIFAR-10

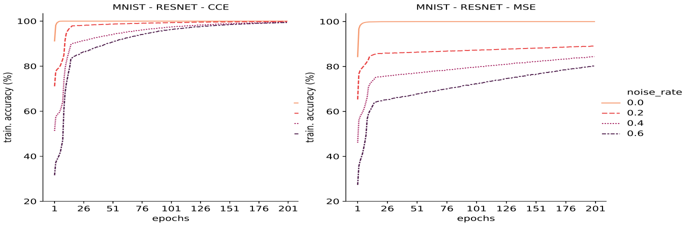


(a) *CCE* **(b)** *MSE* **(c)** *CCE* **(d)** *MSE*

Figure 1: Training set accuracies for ResNet-32 ((a) & (b)) & Inception-Lite ((c) & (d)) trained on CIFAR-10 with CCE and MSE losses for for $\eta \in \{0., 0.2, 0.4, 0.6\}$

- CCE and MSE achieve 100% training accuracy (irrespective of noise rate) thus showing they memorize random labels.

Results on MNIST

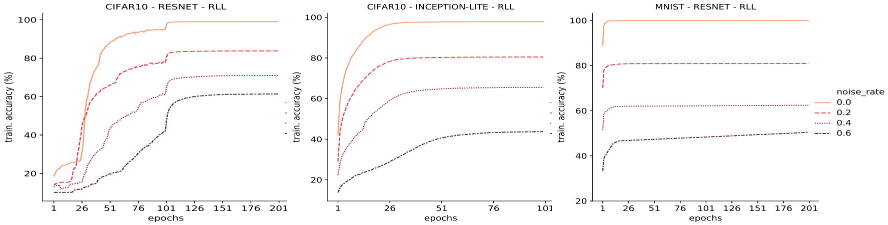


(a) *CCE* **(b)** *MSE*

Figure 2: Training set accuracies for ResNet-18 trained on MNIST with CCE and MSE losses for $\eta \in \{0., 0.2, 0.4, 0.6\}$

- CCE overfits even with this smaller network. MSE also achieves high trainig accuracy irrespective of amount of noise.

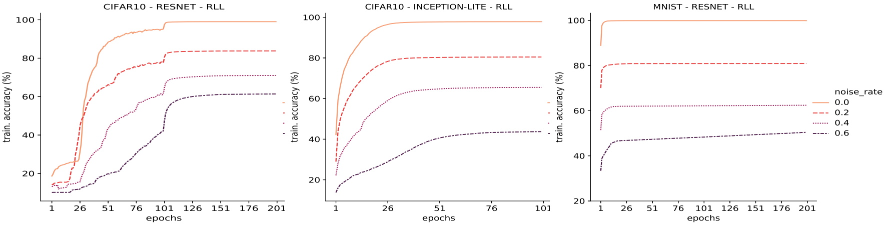
Results on CIFAR-10 & MNIST - with RLL



(a) $ResNet - 32$ **(b)** $Inception - Lite$ **(c)** $ResNet - 18$

Figure 3: Training set accuracies for networks trained on CIFAR-10 ((a) & (b)) and MNIST ((a)) with RLL for $\eta \in \{0., 0.2, 0.4, 0.6\}$

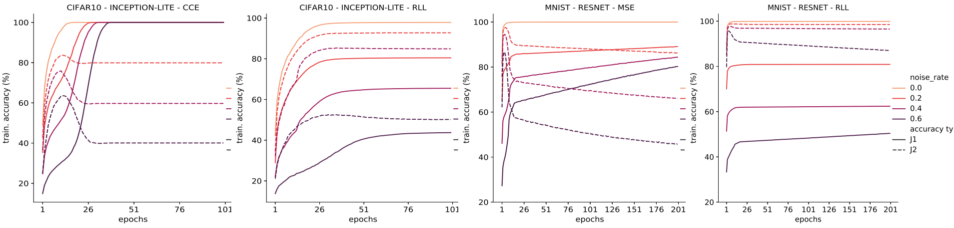
Results on CIFAR-10 & MNIST - with RLL



(a) $ResNet-32$ **(b)** $Inception-Lite$ **(c)** $ResNet-18$

- Training accuracies of RLL saturate much below 100% – the higher the noise rate the lower the training accuracy
- We can see from Figure 4 that the training accuracy of RLL saturates to almost $(1-\eta) \times 100\%$ (η :– noise rate). Now the network does not overfit. In addition, it is as if we have almost inferred the noise rate!

Results for J_1 and J_2 accuracy - with RLL



(a) *CCE* **(b)** *RLL* **(c)** *MSE* **(d)** *RLL*

Figure 5: J_1 and J_2 accuracies for $\eta \in \{0., 0.2, 0.4, 0.6\}$ (Solid lines show J_1 accuracy; dashed lines show J_2 accuracy)

- For RLL, the J_2 is always above J_1 curve – showing RLL resists overfitting to noisy labels
- For CCE and MSE, the J_1 curve eventually goes above J_2 – showing the network overfits the noisy labels as epochs progress

Resisting Memorization

- Recall: S is the original training data and S_η is the data with randomly altered labels; $\mathcal{D}$ and $\mathcal{D}_\eta$ are the corresponding distributions.
- Let h and h_η denote the classifier learned by an algorithm when given S and S_η as training data, respectively.

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- Let h and h_η denote the classifier learned by an algorithm when given S and S_η as training data, respectively.
- We say that an algorithm **resists memorization** if

$$\frac{1}{m}\sum_{i=1}^{m}\mathbb{I}_{\{h(\boldsymbol{x}_i)=y_i^{cl}\}} = \frac{1}{m}\sum_{i=1}^{m}\mathbb{I}_{\{h_\eta(\boldsymbol{x}_i)=y_i^{cl}\}}$$

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- This is a reasonable formalization for 'resisting memorization'
- Note that the J_2 accuracy defined earlier is the quantity on RHS above. The accuracy for $\eta = 0$ would be the quantity on LHS above.
- For RLL, as we saw, the J_2 accuracy is mostly close to the accuracy achieved when $\eta = 0$

Resisting Memorization (contd.)

Theorem

Let $\mathcal{L}$ be a symmetric loss, $\mathcal{D}$ and $\mathcal{D}_\eta$ as defined above. Assume $\eta < \frac{K-1}{K}$. Let $y_{\boldsymbol{x}}^{cl}$ and $y_{\boldsymbol{x}}$ denote the original and noisy labels corresponding to the pattern $\boldsymbol{x}$. The risk of h over $\mathcal{D}$ and $\mathcal{D}_\eta$ is $R_{\mathcal{L}}(h) = \mathbb{E}_{\mathcal{D}}[\mathcal{L}(h(\boldsymbol{x}), y_{\boldsymbol{x}}^{cl})]$ and $R_{\mathcal{L}}^{n}(h) = \mathbb{E}_{\mathcal{D}_\eta}[\mathcal{L}(h(\boldsymbol{x}), y_{\boldsymbol{x}})]$ resp. Then, given any two classifiers h_1 and h_2, if $R_{\mathcal{L}}(h_1) < R_{\mathcal{L}}(h_2)$, then $R_{\mathcal{L}}^{\eta}(h_1) < R_{\mathcal{L}}^{\eta}(h_2)$ and vice versa.

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- For symmetric losses relative risks of two classifiers are same both with and without noise.
- So, symmetric losses can resist memorization (if we can get minimum of risk)

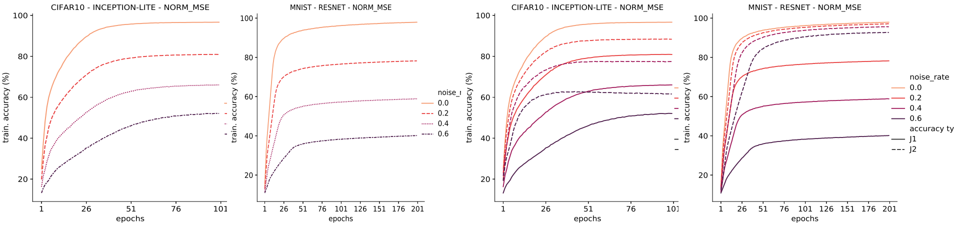
Resisting Memorization: Symmetric Losses

- As is easy to see, the symmetry condition implies that the loss function is bounded.
- Given a bounded loss function we can satisfy the symmetry condition by 'normalizing' it. Given a bounded loss, L, define $\bar{L}$, by

$$\bar{L}(h(X), j) = \frac{L(h(X), j)}{\sum_s L(h(X), s)} \tag{3}$$

- $\bar{L}$ satisfies the symmetry condition (Equation 2). As mentioned earlier, CCE loss is unbounded and hence normalization would not turn it into a symmetric loss. However, we can normalize MSE loss.

Results for J_1 and J_2 accuracy - with Norm. MSE



(a) *Norm.MSE* **(b)** *Norm.MSE* **(c)** *Norm.MSE* **(d)** *Norm.MSE*

Figure 6: Train. accuracy and J_1 & J_2 accuracies for Inception-Lite ((a) & (c)) & ResNet-18 ((b) & (d)) trained on CIFAR-10 and MNIST resp. for $\eta \in \{0., 0.2, 0.4, 0.6\}$ (Solid lines show J_1 accuracy; dashed lines show J_2 accuracy)

- Once we normalize MSE, it no longer overfits the data with random labels; it behaves more like RLL now.

Conclusions

- The phenomenon of memorization in deep networks has received a lot of attention because it raises important questions on how to understand generalization abilities of these networks.
- In this work we have shown through empirical studies that changing the loss function alone can significantly change this memorization.
- We showed this with the symmetric loss function, RLL, and we have provided some theoretical analysis to explain the empirical results.
- The results presented here suggest that choice of loss function can play a critical role in overfitting by deep networks.
- We feel it is important to further investigate the nature of different (symmetric) loss functions for a better understanding of robust learning.

Thank You
Any Questions?

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