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# Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB

Project Report

Motorway Traffic Simulation Using Agent-based Modeling and Cellular Automata

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## ABSTRACT

In this report, we used high-volume of GPS mobile sensor traffic data to estimate the traffic density. One model is constructed based on principles of agent-based modelling techniques, and one model, Phase Transition Model (PTM), is used as comparison. The ground-true density calculated from the data is set as the benchmark for the test. All the estimation results from these two models are compared with ground-true density profile. The results show that both of the models can successfully capture the main characteristics of the density pattern. However, the PTM has more accurate simulation outcome with around 0.18 of relative error. In contrast, the agent-based model has a slightly higher relative error but considering the small amount of input data requirement, we tend to conclude that the model still qualifies the simulation outcome and serve the estimation purposes well. We think the agent-based model has certain merits for future works and further development. Finally, a table of pros and cons of both models is generated as to show the direct contrast between these two models.

## INDIVIDUAL CONTRIBUTION

**Junqing Tang** has been the major author and conductor of the project, main duties includes the coding process, literature review, communication, final report preparation, etc.

**Jingya Yan** has provided useful ideas and discussion at the early-stage of the project.

## 1. INTRODUCTION AND MOTIVATIONS

Traffic simulation and prediction are playing indispensable roles in transport science. Particularly, the way we predict and estimate traffic flows falls into the purpose that how to efficiently forecast or analyse the traffic jams in either one single truck way or our entire transport system. Traffic jam has always been a “headache” for most of international mega cities, even though national governments keep investing significant grants to upgrade and reinforce their traffic infrastructures (Figure.1). In order to provide better and more comprehensive explanation of real-time traffic performance, scientists and engineers have been continuously developing traffic models and methods throughout centuries.



Figure.1. Massive traffic jam in Bangkok city. (Source: <http://gpssystems.net/5-worlds-worst-traffic-jams-avoid/>)

This dissertation delivered results of highway traffic simulations using Agent-based modelling techniques, particularly, the cellular automata has been applied. The authors applied one distinctive traffic models and constructed one simulation model to compare the estimation results. The model we applied is the Phase Transition Model, known as

PTM. PTM is a set of well-defined differentiation equations, which accredited as having satisfactory effects on traffic density estimation. Due to the nature of the PTM, we tend to describe such method of modelling as “top-to-bottom” simulation method, since the differentiation equations in PTM are formulated to cope with the ground-true situation from a holistic mathematical perspective.

The constructed model is based on agents. In contrast, agent-based modelling techniques are well studied and we tend to describe such method as “bottom-to-top” simulation method (as we only set rules for each individual agents and let them behave, and we study the overall performance), therefore, it is worthy to know the difference between these two types of distinctive methods

## 2. DESCRIPTION OF THE DATA

The dataset involved and analysed in our work contain high time-resolution vehicle trajectories collected by the Next Generation SIMulation (NGSIM) for Federal Highway Administration (NGSIM, 2006). Data reflects the travel on northbound direction of interstate 80 in Emeryville, California on April 13, 2005. The collecting equipment was video cameras mounted on a 30-story building. The total of 45 minutes of data is available and subdivided into three 15-minutes datasets with high recoding precision of every 0.1s. We took the first 15-minutes data ([I 80 4:00-4:15 dataset](#)) as our study subject for our model implementation and calibration, and the second 15-mins data for model validation purpose. The site was approximately 1650 feet in length (about 500m). Within each dataset, vehicles are selected. Vehicles on HOV lane #1 and merging lane #6 are eliminated, as they are not representative.

### 2.1 Preliminary analysis of datasets

In preliminary data analysis, direct and basic information of vehicle can be revealed. Taking one particular vehicle as an example, the following Figure.2 shows vehicle speed variation and acceleration against sampling time period. It can be seen from the graph that most high speed occur after around 40 seconds, and each significant variation in

speed corresponds to a significant change in acceleration as shown. These two quantities have great impacts on driving behaviours. Therefore, the main characteristics of these two profiles need to be well understood. Also, Figure.3 demonstrates the trajectory data plot for 80 vehicles to outline vehicle-moving pattern in temporal-spatial domain. It is clear to observe wave propagation in vehicle trajectories plot.

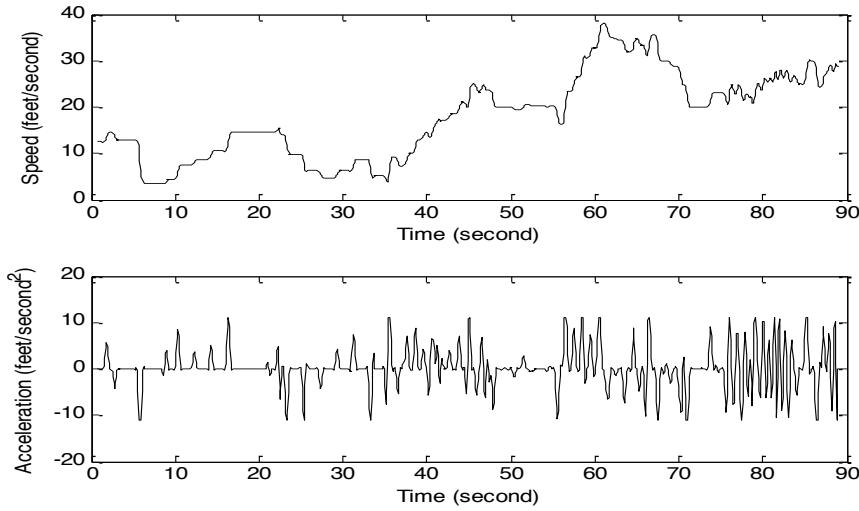


Figure.2 Speed and acceleration profile of one single vehicle from I80 4:00-4:15 dataset

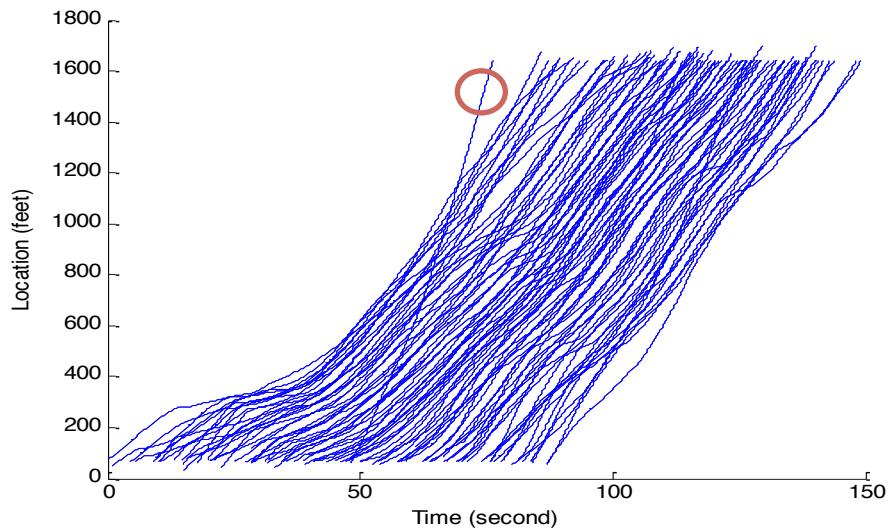


Figure.3 Corresponding vehicle trajectory profile for 80 vehicles from I80 4:00-4:15 dataset

Note that the particular vehicle trajectory highlighted in the red circle is one vehicle from the HGV lane, which has much higher speed than other congested lanes (as mentioned above, we eliminated the data from HGV lane). The general information about speed and acceleration in each lane can also be analysed. The outcomes have been illustrated in Table.1 and plotted in Figure.4.

Table.1 Mean speed and acceleration for each lane in 15-minutes data

Traffic quantities	Mean Speed (Feet/second)	Mean Acceleration (Feet/second square)
Lane #1	51.7	0.172
Lane #2	23.9	0.026
Lane #3	23.7	0.043
Lane #4	20.3	-0.035
Lane #5	22.3	-0.004
Lane #6	21.5	0.106

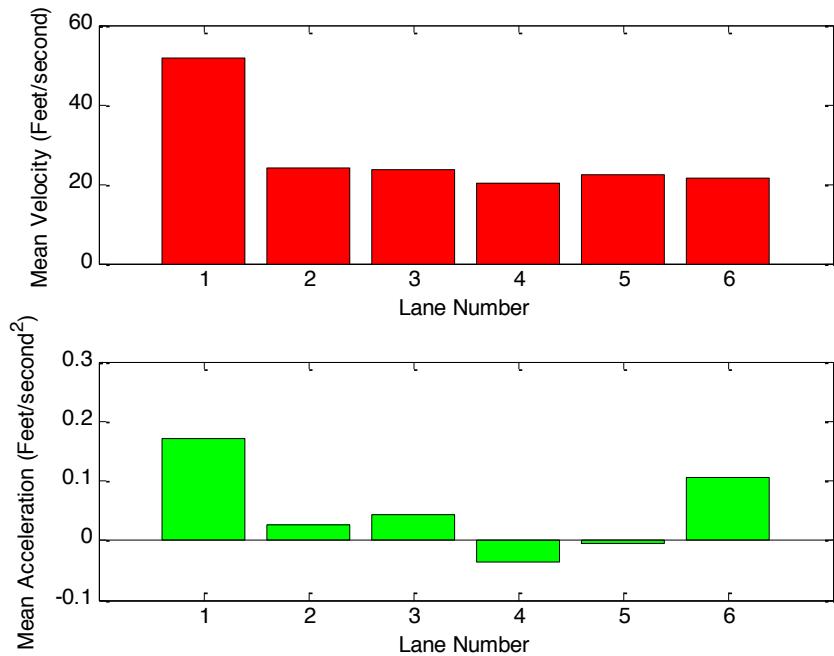


Figure.4 Mean speed and acceleration plots for each lane in I 80 4:00-4:15 dataset

### 3. DESCRIPTION OF THE MODEL

#### 3.1 The Phase Transition Model (PTM)

This model is known as the higher-order traffic hydrodynamic model as it captures second-order variations of traffic in addition to the average speed and density, and motivated by the experimental observation that “when the vehicle density exceeds certain critical value, the density-flow pairs are scattered in a two-dimensional region, instead of forming a one-to-one relationship”. (Piccoli, Han and et al, 2011)

Transition means the model consists two distinctive phase in traffic behaviours: the congested phase and the uncongested phase. According to outline by Piccoli, Han and et al (2011), in the uncongested phase, the dynamic is governed by the first-order model, LWR model.

$$\begin{cases} \frac{\partial k(t, x)}{\partial t} + \frac{\partial Q(k(t, x))}{\partial x} = 0 \\ v = v(k) \end{cases} \quad (1)$$

Where  $k$  is the density,  $v$  is the speed,  $t$  is the time and  $x$  is the location of the vehicle. One may note that these conservation laws are same as the formulae shown in LWR model section, which indicates that the PTM is same as LWR model when traffic is uncongested. In contrast, the congested phase is governed and characterised by the conservation laws shown below

$$\begin{cases} \frac{\partial k(t, x)}{\partial t} + \frac{\partial Q(k(t, x))}{\partial x} = 0 \\ \frac{\partial q(t, x)}{\partial t} + \frac{\partial [(q - q^*) \cdot v]}{\partial x} = 0 \\ v = v(k, q) \end{cases} \quad (2)$$

Where, the speed  $v(k, q)$  depends both on the local density and  $q$  which describes the deviation or perturbation from the equilibrium state of the traffic.  $q^*$  is a given constant value.

The formation of component  $v(k, q)$  can be various. Two forms of this term can be expressed as shown below.

$$v(k, q) = \left(1 - \frac{k}{k_{jam}}\right) \cdot \frac{q}{k} \quad (3)$$

Where, the term  $k_{jam}$  denotes the jam density. Another can be expressed as:

$$v(k, q) = A(k_{jam} - k) + B(q - q^*)(k_{jam} - k) \quad (4)$$

Where A and B are positive parameters. The conservation laws for congested phase can be flexible and can be chosen from different second-order model equations such as Aw-Rascle-Zhang equations (Aw and Rascle, 2000; Zhang, 2002. Cited by Piccoli, Han and et al, 2011). Moreover, if taking the reaction time of drivers into account, the Siebel and Mauser (2006) source term can be introduced (Piccoli, Han and et al, 2011). Then the PDE for congested phase can be upgraded as

$$\frac{\partial q(t, x)}{\partial t} + \frac{\partial[(q - q^*) \cdot v]}{\partial x} = \frac{q^* - q}{T - \tau} \quad (5)$$

Piccoli, Han and et al (2011) have presented well-developed descriptions and explanations about estimating traffic quantities using PTM in their paper named “*Second-order Models and Traffic Data from Mobile Sensors*”. In this paper, the quantities  $k$  and  $q$  are well-estimated mathematically according to three different situations: PTM with source term and strongly stable traffic, PTM with source and less stable traffic and PTM without source term. By introducing two additional variables  $\hat{k} = k_{jam} - k$  and  $\hat{q} = q - q^*$ , Piccoli, Han and et al proposed the equations and descriptions for traffic quantities density and perturbation in these three situations. In this dissertation, it is assumed that the traffic is strongly stable and traffic quantities were estimated by PTM with Siebel and Mauser source term, which gives the equation (7). And then density and perturbation can be deduced and expressed as

$$\hat{k} = k_{jam} - k = \frac{1}{A} (v + (T - \tau) \frac{Dv}{Dt}) \quad (6)$$

$$\hat{q} = q - q^* = -\frac{A(T - \tau)}{B} \frac{\frac{Dv}{Dt}}{v + (T - \tau) \frac{Dv}{Dt}}$$
(7)

In this report, the value of  $\tau$  is set to be 1 second, and  $T = 2/3$  seconds, thus term  $T - \tau$  is equal to  $-1/3$  seconds.

### 3.2 The Agent-based Model

The model we constructed was based on cellular automata, which in the case, each cell is acting as an individual agent. Agents can self-update along the time scale. In order to simplify the process, the driving behaviour of each vehicle in each agent was set according to the classic car-following behaviour, which is typically what the vehicles would react in the reality.

The principle of our model is that:

- Each vehicles will react according to the former vehicles in front of it
- They keep a safety distance between each two vehicles
- If there would be sufficient space in front of one vehicle, this vehicle will automatically fill in that space as following the former car. No space would be left blank intentionally between vehicles.

According to the principles we set for the agent, the agents have two dimensions, namely spatial dimension and temporal dimension. In terms of traffic density, we looked into the number of vehicles in each agent. The equation (8) provides the method for calculating the density based on the number of vehicles in each agent.

$$\rho = \frac{N}{L}$$
(8)

Where the  $N$  represents the number of vehicles in one agent, and the  $L$  is the distance/length that agent covers. For each agent, there would be 3 quantities to govern the number of vehicles in the agent at the time  $t_i$ : (a) the original number of vehicles at agent  $C_{i-1,j}$  at time  $t_{i-1}$ , (b) inflow from former agent  $C_{i-1,j-1}$  from time  $t_{i-1}$  to  $t_i$  and (c) outflow to  $C_{i-1,j+1}$  at time from  $t_{i-1}$  to  $t_i$ . The reasoning mechanism can be illustrate as following Figure.5.

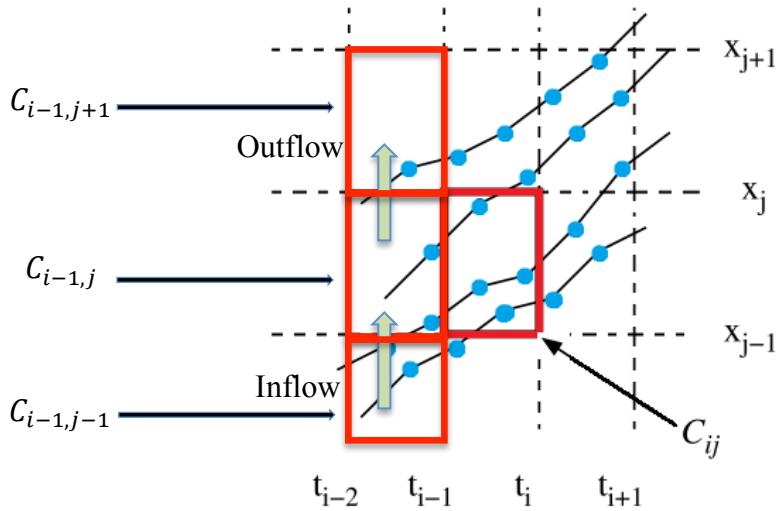


Figure.5 Subdivided bins/cells of study area (Piccoli, Han and et al, 2011).

Therefore, the number of vehicles in agent  $C_{i,j}$  can be expressed as following equation:

$$C_{i,j} = C_{i-1,j} + \text{Inflow} - \text{Outflow} \quad (9)$$

## 4. IMPLEMENTATION

For model implementation, we divided the entire study area into cells (agents) with spatial length of 70feet (around 21m) and temporal length of 4 seconds. For PTM, the parameter A and B of PTM, in this project, were taken as 200 and 500 respectively for model calibration. The first and last 100 feet (around 30 meters) of the study area were cut to avoid unrepresentative data.

In Agent-based model, the boundary conditions of the data are provided. The boundary condition sets at the initial temporal dimension (data array in  $t=0-4s$  cells) and the initial spatial dimension (data array in  $x=0-70\text{feet}$  cells). Also, the maximum vehicles number in one agent can be calculated as we assuming the average vehicle length. The average vehicle length and the safety space needed ahead has been assumed as around 15 feet, multiplying the number of lanes (4 study lanes) and divided by the spatial dimension of the cell, which finally gives the maximum number in each agent is around 18 vehicles.

We firstly have done the model calibration using first 15mins data, and comparing the performance of two models, and we used second 15mins data for model validation to accredit our agent-based model.

## 5. SIMULATION, RESULTS AND DISCUSSION

### 5.1 Model calibration on traffic density estimation

By counting the number of vehicles falls into each cell, we can apply equation (8) to calculate the traffic density of each cell.

However, before this, one of the most important traffic quantities must be obtained for further investigation, namely, the jam density. Jam density plays a critical role in traffic simulation. It determines the critical level of density at which the overall traffic starts to show the sign of congestion. By plotting the Flow-Density fundamental diagram of the study data, one can easily estimate the jam density of the ground-true data. As shown in following Figure.6, our study traffic data has a jam density of 0.26 veh/foot. Please note

that for Agent-based model, if we use jam density 0.26 veh/foot and reversely deduce the maximum number of vehicle in one cell, it also gives around 18 vehicles, which sufficiently accredit our presumption in agent-based model.

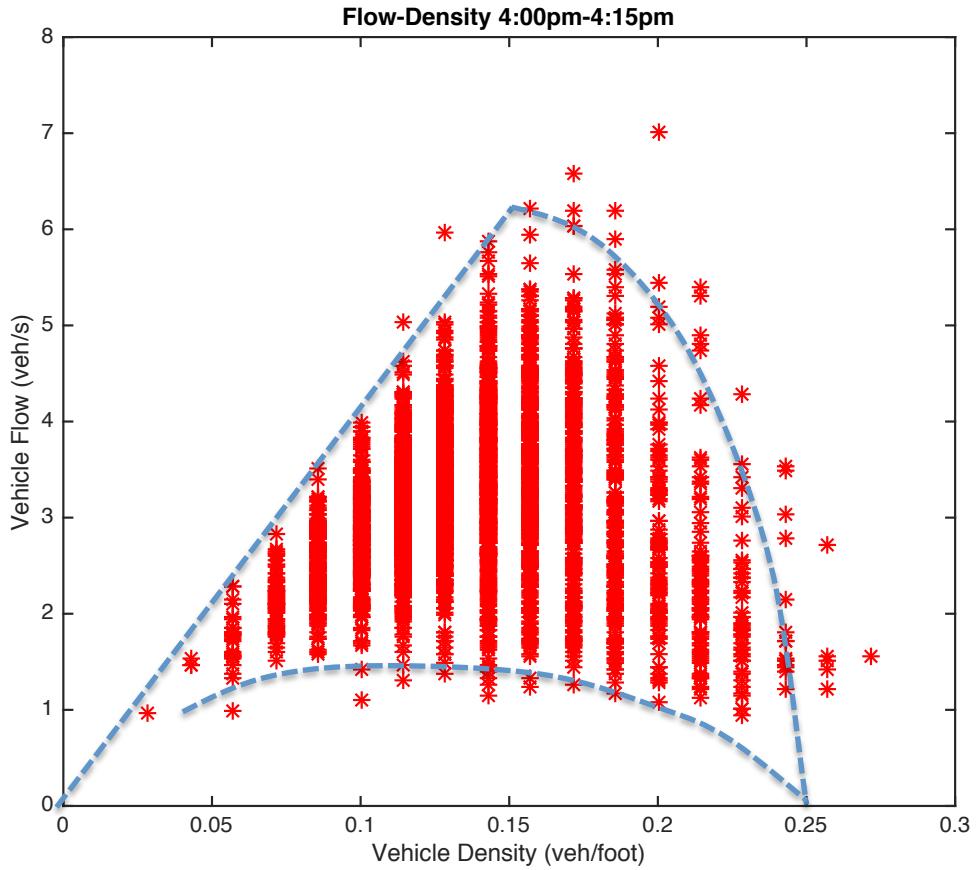


Figure.6 the Flow-Density plot of the first 15mins (4: 00pm - 4: 15pm) traffic data

By taking jam density as 0.26 veh/foot, the estimated traffic density of the data (PTM) can be calculated and plotted as shown in following. Figure. 7 shows the ground-truth density, PTM estimated density and agent-based model estimated density. It can be seen that the both PTM and our agent-based model can capture the main characteristics of the ground-truth traffic in a certain level of satisfaction. There are three main congestion waves during first 15mins of recordings: the first wave starts from around 100 seconds, and the second wave and third wave start at around 140s and 170s respectively.

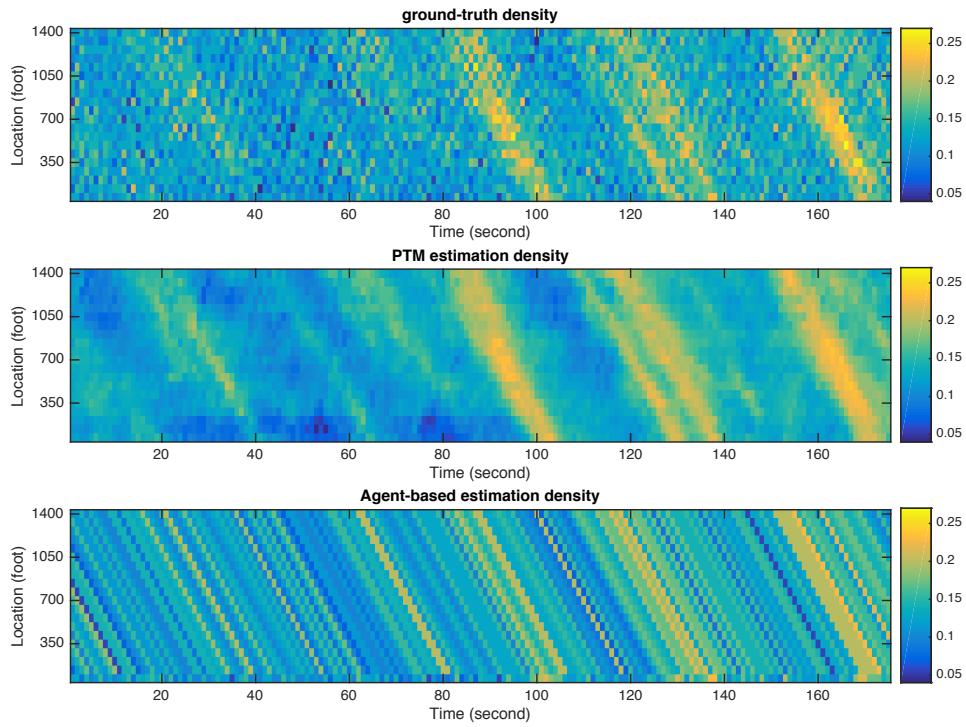


Figure. 7 ground-true, PTM estimation and Agent-based estimation on density profiles

In terms of estimation accuracy, we calculated the mean relative error for each cell for two estimation cases. Table.2 illustrates the calculated mean relative error for PTM and Agent-based model. Apparently, the PTM has better estimation result than Agent-based model does (around 24%), as it has smaller relative error of around 19%

Table.2 the mean relative errors for each model estimation result

Mean relative error/Models	PTM	Agent-based Model
Mean relative error in cells	0.188	0.239

To show it in another perspective, we plot the mean density on entire study area at each time interval, with estimated results from two models as well. The Figure. 8 outlines the basic profile of density variation on entire study motorway along the time.

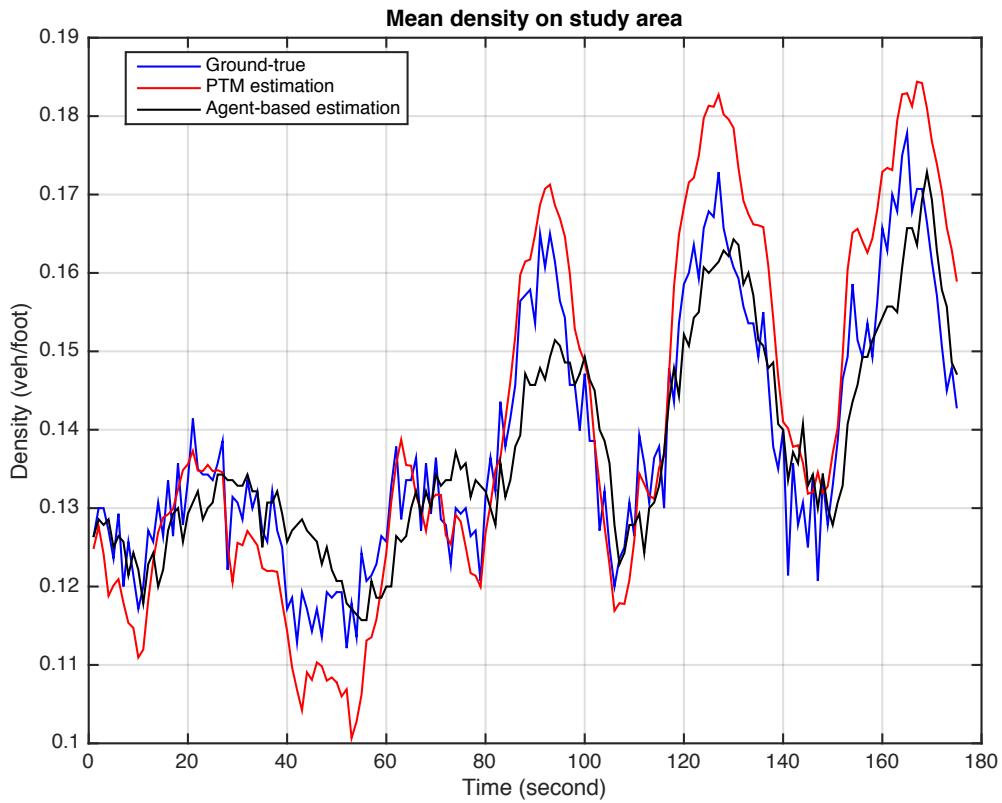


Figure. 8 Mean density on study area along time and estimation results

## 5.2 Model validation

In this section, we used second 15mins data to perform the model validation. Model validation merely taken place in Agent-based model with nothing changed. The following figures show the results of the simulation. Again, the simulation results tend to be at a satisfactory level. The relative error has been computed as 0.258. The results can validate the feasibility of proposed agent-based model.

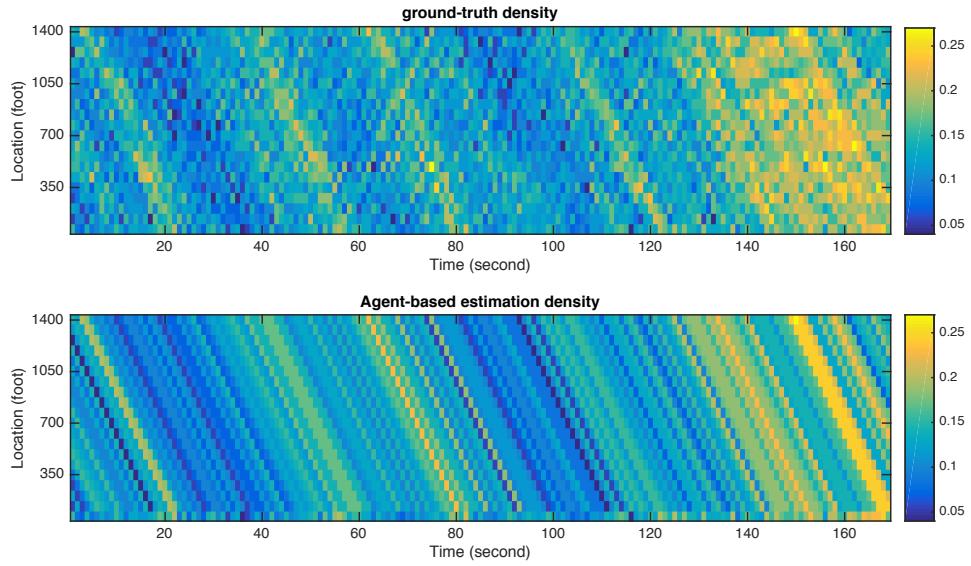


Figure. 9 The simulation results of agent-based model in 4:15pm-4:30pm dataset

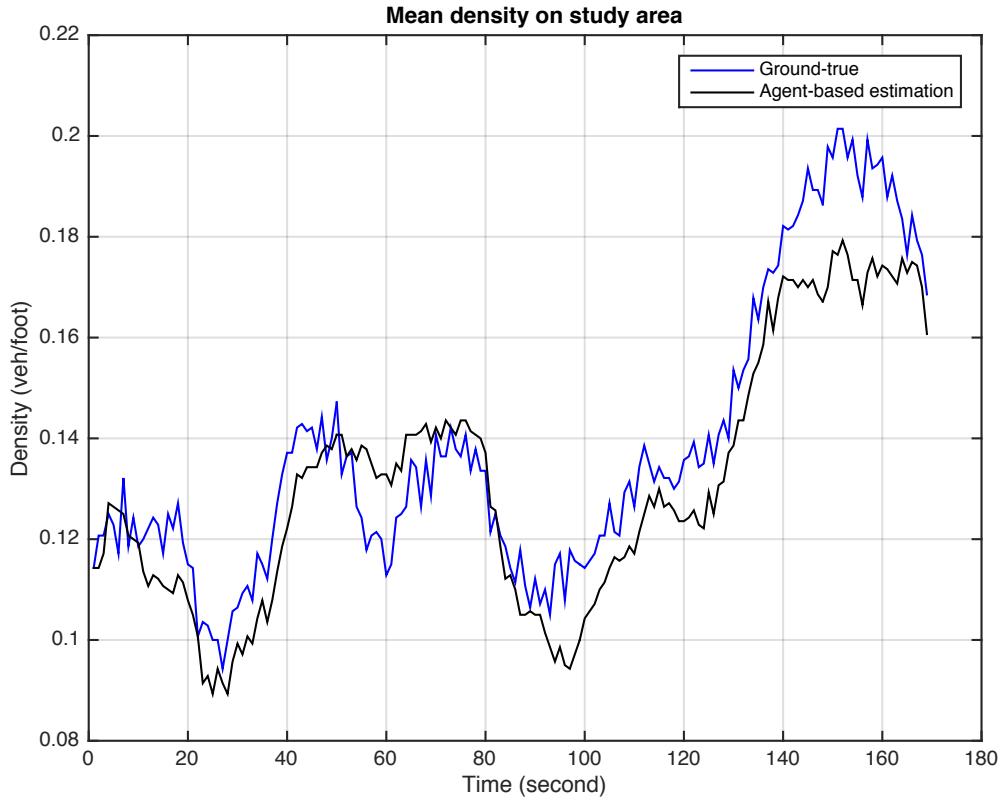


Figure. 10 Mean density of ground-true values and estimated values by agent-based model

## 6. SUMMARY

In this report, we have examined two models for traffic simulation purpose. One is the well-documented Phase Transition Model; another is constructed and proposed by authors based on Agent-based Modelling principles. One traffic quantity, traffic density, has been used as study object to perform the model comparison.

In terms of the relative error of simulation results, the PTM has obvious advantages in simulation (relatively small error obtained). However, as for capturing the overall pattern of the density profile, both model presented estimations at satisfactory level. Following Table. 3 shows the summarised pros and cons of both models.

Table.3 Comparison between PTM and Agent-based Model

Models/Remarks	Advantages	Disadvantages
PTM	<ul style="list-style-type: none"><li>More accurate estimation of traffic quantities</li><li>Mathematically well-defined and well-documented</li></ul>	<ul style="list-style-type: none"><li>Rely on large amount of data</li><li>Need to fully understand the mathematical mechanism of the model</li><li>Not flexible to be modified according to other parameters such as driving behaviours</li></ul>
Agent-based model	<ul style="list-style-type: none"><li>Do not need large amount of data, only need to provide boundary conditions for simulation</li><li>The principle of model is simply and easily understandable</li><li>Flexible, can be modified according to different purposes and driving behaviours</li></ul>	<ul style="list-style-type: none"><li>The accuracy is relatively low as having relatively high simulation errors</li><li>Rely on several pre-assumptions and simplifications</li><li>Could be case-sensitive</li></ul>

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