EECS 442 Computer Vision, Fall 2012

Homework 1 Solution

Problem 1

(a) To construc the rotation matrix R, we write the cooridantes of the basis vectors of W in terms of those of C so that any point in W coordinates can be transformed into C coordinates.

$$\begin{aligned} W_x &= \frac{-\sqrt{2}}{2}\,C_x + 0\,C_y + \frac{-\sqrt{2}}{2}\,C_z \\ W_y &= 0\,C_x + 1\,C_y + 0\,C_z \\ W_z &= \frac{\sqrt{2}}{2}\,C_x + 0\,C_y + \frac{-\sqrt{2}}{2}\,C_z \end{aligned}$$

These three vectors will be the three columns of the rotation matrix encoding the orientation of camera frame C relative to the world frame W. Once the camera is rotated, world points must be translated by d along the z-axis so that the world origin will be located at coordinate $[0,0,d]^T$ relative to C. The final change of coordinate transformation is

$${}^C_W TR = \begin{bmatrix} \frac{-\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Check that ${}^C_W TR\, O^W_W = {}^C_W TR[0,0,0,1]^T = [0,0,d,1]^T = O^C_W$, and ${}^C_W TR\, O^W_C = {}^C_W TR[d\frac{\sqrt{2}}{2}, 0, d\frac{\sqrt{2}}{2}, 1]^T = [0,0,0,1]^T = O^C_C$.

(b) Assume in the world coordinates W, $a^W = [0,p,q]^T, b^W = [0,p+1,q]^T, c^W = [0,p+1,q+1]^T, d^W = [0,p,q+1]^T$. Transforming a^W, b^W, c^W, d^W into camera coordinates C by ${}^C_W TR$,

$$\begin{bmatrix} \frac{-\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ p & p+1 & p+1 & p \\ q & q & q+1 & q+1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{q}{\sqrt{2}} & \frac{q}{\sqrt{2}} & \frac{q+1}{\sqrt{2}} & \frac{q+1}{\sqrt{2}} \\ p & p+1 & p+1 & p \\ -\frac{q}{\sqrt{2}}+d & -\frac{q}{\sqrt{2}}+d & -\frac{q+1}{\sqrt{2}}+d & -\frac{q+1}{\sqrt{2}}+d \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

we get $a^C = [\frac{q}{\sqrt{2}}, p, -\frac{q}{\sqrt{2}}+d]^T, b^C = [\frac{q}{\sqrt{2}}, p+1, -\frac{q}{\sqrt{2}}+d]^T, c^C = [\frac{q+1}{\sqrt{2}}, p+1, -\frac{q+1}{\sqrt{2}}+d]^T$, and $d^C = [\frac{q+1}{\sqrt{2}}, p, -\frac{q+1}{\sqrt{2}}+d]^T$. Observe that we have unit length for all four edges, and $\frac{\pi}{2}$ for all four angles in camera coordinates. This is still a square in the camera coordinates and has unit area.

(c) Assume two parallel line segments $\overrightarrow{a^W b^W}, \overrightarrow{c^W d^W}$ in the world coordinates W, where $a^W = [x_1, y_1, z_1]^T, b^W = [x_1+p, y_1+q, z_1+r]^T, c^W = [x_2, y_2, z_2]^T$, and $d^W = [x_2+pt, y_2+qt, z_1+rt]^T$. Similar to (b), by transforming a^W, b^W, c^W, d^W into camera coordinates C, we get

$$
\begin{aligned}
a^C &= [-\frac{1}{\sqrt{2}}x_1 + \frac{1}{\sqrt{2}}z_1,\ y_1,\ \ -\frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{2}}z_1 + d]^T \\
b^C &= [-\frac{1}{\sqrt{2}}(x_1+p) + \frac{1}{\sqrt{2}}(z_1+r),\ y_1+q,\ \frac{1}{\sqrt{2}}(x_1+p) - \frac{1}{\sqrt{2}}(z_1+r) + d]^T \\
c^C &= [-\frac{1}{\sqrt{2}}x_2 + \frac{1}{\sqrt{2}}z_2,\ y_2,\ \ -\frac{1}{\sqrt{2}}x_2 - \frac{1}{\sqrt{2}}z_2 + d]^T \\
d^C &= [-\frac{1}{\sqrt{2}}(x_2+pt) + \frac{1}{\sqrt{2}}(z_2+rt),\ y_2+qt,\ \frac{1}{\sqrt{2}}(x_2+pt) - \frac{1}{\sqrt{2}}(z_2+rt) + d]^T.
\end{aligned}
$$

We get

$$
\begin{aligned}
\overrightarrow{a^C b^C} &= [-\frac{1}{\sqrt{2}}p + \frac{1}{\sqrt{2}}r,\ q\ ,-\frac{1}{\sqrt{2}}p - \frac{1}{\sqrt{2}}r] \\
\overrightarrow{c^C d^C} &= [-\frac{1}{\sqrt{2}}tp + \frac{1}{\sqrt{2}}tr,\ tq\ ,-\frac{1}{\sqrt{2}}tp - \frac{1}{\sqrt{2}}tr] = t\overrightarrow{a^C b^C}.
\end{aligned}
$$

Thus $\overrightarrow{a^C b^C}$ and $\overrightarrow{c^C d^C}$ are parallel in the camera coordinates.

(d) Assume a^W, b^W have the same coordinates as in (b). We have $\overrightarrow{a^W b^W} = [0,1,0]^T = \overrightarrow{a^C b^C}$. Thus $\overrightarrow{ab}$ has the same orientation in both reference systems.

Problem 2

The projections $p' = [x', y']^T$ can be calculated by

$$x' = f'\frac{x}{z},\ y' = f'\frac{y}{z}.$$

Therefore, the projections of points on line Q are given by

$$x' = f'\frac{1}{t},\ y' = f'\frac{1}{t}.$$

Substituting $t = -1$ and ∞, the two endpoints are $[f'\frac{1}{-1}, f'\frac{1}{-1}]^T = [-f, -f]^T$ and $[f'\frac{1}{-\infty}, f'\frac{1}{-\infty}]^T = [0,0]^T$.

Problem 3

(a) $l = x_1 \times x_2 = [1, 1, -4]^T$.

(b) $l' = x_1' \times x_2' = Hx_1 \times Hx_2' = [137.10, 62.54, -396.76]^T$.

(c) Given x_1, and x_2 lie on l, we have $x_1^T l = x_2^T l = 0$ and hence $(x_1^T - x_2^T)l = 0$. Similarly, we have $(x_1'^T - x_2'^T)l' = 0$. Combining the two equations we obtain

$$(x_1'^T - x_2'^T)l' = (x_1^T - x_2^T)l.$$

Since $x_1' = Hx_1$ and $x_2' = Hx_2$, we have

$$
\begin{aligned}
(Hx_1{}^T - Hx_2{}^T)l' &= (x_1^T - x_2^T)l \\
(x_1{}^T - x_2{}^T)H^T l' &= (x_1^T - x_2^T)l.
\end{aligned}
$$

Assuming $x_1 \neq x_2$, we get $l' = H^{-T}l$.