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Modifications of the optimal velocity traffic model to include delay due to driver reaction time

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Abstract

Straightforward inclusion of a delay time due to driver reaction time in the optimal velocity (OV) model reveals an unphysical sensitivity to driver reaction times. For delay times of nearly 1 s, which are typical for most drivers, oscillations in vehicle velocity induced by encountering a slower vehicle grow until limited by non-linear effects. Simulations demonstrate that unrealistically small delay times are needed for lengthy platoons of vehicles to avoid collisions. This is a serious limitation of the OV model. Other models, such as the inertial car-following model, allow somewhat larger delay times, but also show unphysical effects. Modifications of the OV model to overcome this deficiency are demonstrated. In addition, unphysical short-period oscillations of vehicle velocity are eliminated by introducing partial car-following into the model. Traffic jams are caused primarily by the delay due to driver reaction time in the modified OV model.

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Keywords: Traffic model; Traffic flow; Traffic dynamics; Congested flow

1. Introduction

One of the fundamental problems in traffic flow is that above a critical density of vehicles (typically 20–30 vehicles/km/lane) flow is unstable [1,2]. A small perturbation can induce large oscillations in vehicle speed and headway with the concomitant reduction in average speed and flow [3]. Highway capacity is generally in excess of

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2000 vehicles/h/lane, but in the congested state, flow can drop to one-half (or less) of the maximum value (see Fig. 1d of Ref. [2] for empirical data).

Several microscopic models have been proposed to reproduce observed macroscopic features of traffic flow. The optimal velocity (OV) model, introduced by Bando et al. [4], has received considerable attention. In the OV model, the acceleration of the nth vehicle is determined by the difference between the actual velocity, v_n, and an optimal velocity $V(\Delta x_n)$, which depends on the headway Δx_n to the car in the front:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \frac{1}{\tau}(V(\Delta x_n) - v_n)\,, \tag{1}$$

where

$$V(\Delta x_n) = V_0\left[\tanh\left(\frac{\Delta x_n - D}{b} - C_1\right) + C_2\right], \qquad \Delta x_n = x_{n-1} - x_n \tag{2}$$

and x_n is the position of the nth car; x_{n-1} is the position of the preceding car. The length of the vehicles is $D = 5$ m and τ is a time constant representative of the vehicle dynamics. The length scale is b while V_0, C_1 and C_2 are constants.

Another rather simple model, called the inertial car-following model, describes the acceleration of a vehicle in terms of its position and velocity relative to the vehicle it follows [5]. Both models reproduce some salient qualitative features of traffic flow and congestion. A review of traffic models can be found in Refs. [6,7].

As originally formulated neither of these models, OV or inertial car-following model, account for driver response time, which has been found to be about a second [8–10]. [1, 2] The purpose of this paper is to determine the effect of realistic time delays on the behavior of traffic models. The maximum size of a platoon of vehicles that avoids collisions (the "safe platoon") is calculated as a measure of stability. A previous study of the effects of explicit delay on the OV model [11] concluded that delay times are not large enough to be significant and can be taken into account by simply redefining the sensitivity parameter. The present work reaches different conclusions.

In the OV model, when we include time delay t_d (representing driver reaction time), the equation for the velocity $v_n(t)$ of the nth vehicle is given by

$$\tau\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} + v_n(t) = V(\Delta x_n(t - t_d))\,, \tag{3}$$

where $\Delta x_n(t - t_d)$ is the headway to the preceding vehicle, evaluated at an earlier time. A specific example of the right-hand side of Eq. (1) has been given by Sugiyama [12] (without the time delay):

$$V(\Delta x) = 16.8[\tanh(0.086(\Delta x - 25)) + 0.913] \tag{4}$$

with all quantities expressed in metric units. The parameters were chosen to reproduce the average velocity vs. headway characteristics of a Japanese freeway. We take

[1] The delay time reported in Refs. [8,9] includes mechanical delay as well as driver reaction time, which can vary with traffic conditions and previous driver actions. According to the National Safety Council, the average reaction time of drivers is $\frac{3}{4}$ second. See http://www.carcarecouncil.org/sre0-sec.htm.

[2] In Ref. [10], 0.7 s is considered the minimum driver reaction time for braking.

$\tau = 0.5$ s, which is a realistic time constant for vehicle response in a first-order dynamical description. This is the same parameterization investigated by Bando et al. [11]. The plan of the paper is to describe simulations with delay included straightforwardly (Eq. (3)) in Section 2 and then to modify the OV model to accommodate delay significantly better in Section 3. Conclusions are drawn in Section 4.

2. Simulations

In this section, we investigate a platoon of vehicles all spaced with the same headway and traveling at the same speed initially, and examine the effect of delay on the response of cars following a slower lead vehicle. This procedure has been shown by Nagatani to induce traffic jams [13]. In contrast to analyses of Bando et al. [11], we consider an open system rather than a closed system on a ring. In both instances, single-lane unidirectional flow with no passing and no exit or entry of other vehicles (such as at an on-ramp) is assumed.

First consider a small delay, 0.1 s. Calculations show that a platoon as large as 100 vehicles (the most we consider) is stable, albeit with some oscillations in velocity. The initial headway is 25 m and the speed is 15.34 m/s while the lead vehicle travels at a steady 14 m/s. If the delay time is increased to 0.3 s, which is still well below typical driver reactions times, only the first 14 cars avoid a collision. The 15th vehicle experiences a headway less than the length of the vehicles, 5 m. Similar behavior holds for 0.5 s delay. In this instance, however, only the first six cars do not collide (Fig. 1).

A plot of the number of vehicles in a platoon before a collision, i.e., the safe platoon, as a function of delay time reveals an abrupt change between $t_d = 0.2$ and 0.25 s (Fig. 2, for the purpose of illustration the results for smaller τ are also shown). For delay less than about 0.2 s, platoons of 100 vehicles involve no collisions, but for larger delays only much smaller platoons are safe. Bando et al. [11] noted a change in stability at $t_d = 0.22$ s and Nagatani and Nakanishi [14], who included delay in

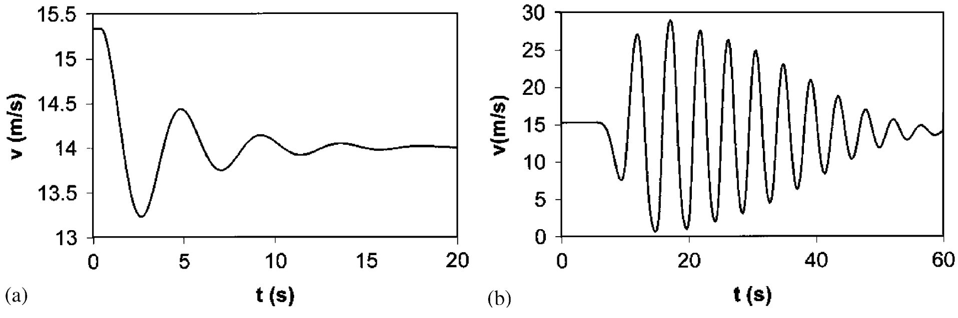

Fig. 1. Response of the first car in the platoon (a) and the last car (6th) before collision (b). Initial headway = 25 m and speed = 15.34 m/s as the platoon approaches the lead vehicle traveling at 14 m/s. Delay time $t_d = 0.5$ s and vehicle time constant (reciprocal of sensitivity) $\tau = 0.5$ s.

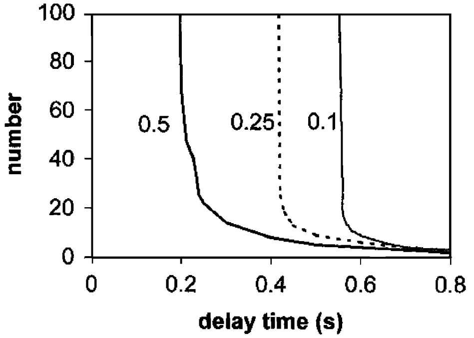

Fig. 2. The maximum number of vehicles in a platoon before a collision occurs as a function of delay time. Only 100 vehicles are considered. Initial headway = 25 m and speed = 15.34 m/s as the platoon approaches the lead vehicle traveling at 14 m/s. Results for three different values of the time constant representing vehicle response are shown (0.1, 0.25 and 0.5 s), although only $\tau = 0.5$ s is realistic.

a different fashion, demonstrated similar effects. Thus, simulations show that the OV model is unrealistically sensitive to delay time. Decreasing τ shifts the safe platoon size curves as shown in Fig. 2. However, such small values of the time constant are not considered realistic.

For comparison purposes, let us briefly consider the inertial car-following model [5,15]. It can be summarized as follows:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = A\left(1 - \frac{\Delta x_n^0}{\Delta x_n}\right) - \frac{Z^2(-\Delta v_n)}{2(\Delta x_n - D)} - kZ(v_n - v_{per}) , \tag{5}$$

where $\Delta v_n = v_{n-1} - v_n$. The desired headway is $\Delta x_n^0 = v_n T + D$, where T is 1–2 s and the permitted speed is v_{per} (taken to be 25 m/s). Parameters $A = 3$ m/s^2 and $k = 2$ s^{-1}. Also,

$$Z(v) = \begin{cases} v, & v > 0 , \\ 0, & v < 0 . \end{cases} \tag{6}$$

In Fig. 3, the maximum number of vehicles avoiding a collision under the same conditions as in Fig. 2 is shown for two values of T, the first is determined by setting T such that $\Delta x_n^0 = v_n T + D$ for $\Delta x_n^0 = 25$ m and $v_n = 15.34$ m/s, making all cars at equilibrium initially. The safe platoon size for this model shows the same type of curve as found for the OV model, but shifted by about 0.1 s to higher values of delay time. If T is taken to be 2 s (as suggested by Tomer et al.) and the initial vehicle headway set at the equilibrium value 35.68 m, the curve is shifted even higher. Thus, the sensitivity to delay time is less in the inertial car-following model than in the OV model.

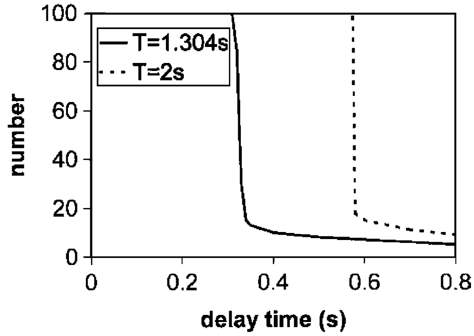

Fig. 3. Safe platoon size vs. delay time t_d for two different headway times for the interial car-following model of Ref. [5]. Same initial velocities as in Fig. 2. For $T = 1.304$ s, the headway is the same also. For $T = 2$ s, the initial headway is longer (38.64 m).

3. Modifications of the OV model

Bando et al. [11] suggested that one replace τ by $\tau + t_d$ in the original OV model to account for the effects of driver reaction time, implying that a plot of safe platoon size vs. $\tau + t_d$ should not depend strongly on the underlying values of τ. Such a plot is shown in Fig. 4 for three values of τ (0.1, 0.25 and 0.5 s). It can be seen that this approximation is not exceedingly accurate.

In place of redefining τ, a simple modification can be made that preserves the equilibrium properties of the OV model (i.e., velocity vs. headway) and permits reasonable values of t_d. The optimal velocity function for time-varying situations is modified by replacing $V(\Delta x_n(t-t_d))$ with $V(\Delta x_n(t-t_d)+t_d\Delta v_n(t-t_d))$. This assumes that drivers can sense rate of change of headway in addition to headway, an assumption inherent in the inertial car-following model and other models. Eq. (3) now reads

$$\tau \frac{dv_n(t)}{dt} + v_n(t) = V(\Delta x_n(t - t_d) + t_d \Delta v_n(t - t_d)). \tag{7}$$

For small t_d the argument of the right-hand side closely approximates $\Delta x_n(t)$ as in the original OV model. For larger delays, simulations show that for the conditions of Fig. 2, platoons of 100 vehicles do not experience collisions for $t_d \leqslant 1.0$ s, which is a substantial improvement. However, unrealistic high-frequency oscillations (period of only a few seconds) occur in OV model simulations for small (Fig. 5) or vanishing t_d (see, for example, Fig. 3 of Ref. [3]). Similar oscillations can be found in simulations using the inertial car-following model, but not in ordinary car-following models [9,16] where acceleration is proportional to $\Delta v_n(t-t_d)$. Attempting to maintain a Δx_n vs. Δv_n relation apparently causes rapid accelerations, which produce velocities that overshoot the desired velocity, giving roughly sinusoidal oscillations. Simulations also show the development of non-linear oscillations in which vehicle velocities fluctuate wildly from near the maximum velocity (32.1 m/s) to almost zero (interpreted by Mitarai and Nakansihi [3] as alternating sequences of jams and free flow).

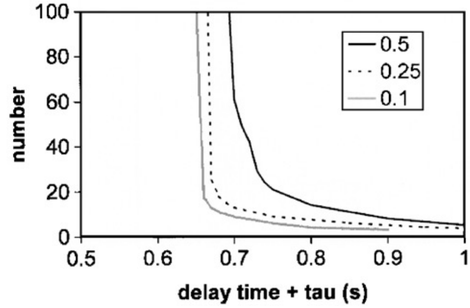

Fig. 4. Safe platoon size of Fig. 2 plotted against $\tau + t_d$ for various values of τ (0.1, 0.25 and 0.5 s). This figure pertains to the OV model.

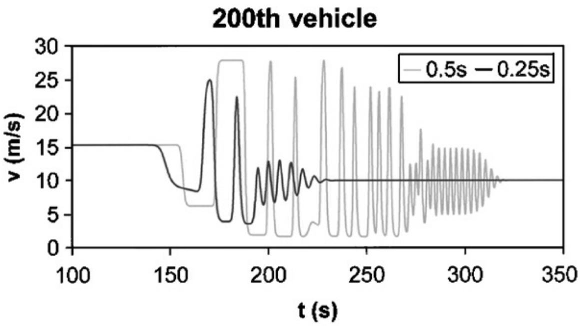

Fig. 5. Velocity of the 200th vehicle of a platoon with initial headway 25 m and velocity 15.34 m/s as a function of time for two values of delay $t_d = 0.25$ and 0.5 s with $\tau = 0.5$ s. The lead vehicle travels at 10 m/s. The argument of the optimal velocity function is $\Delta x_n(t - t_d) + t_d \Delta v_n(t - t_d)$ in these calculations. See Eq. (7).

In the OV model, if we modify the desired velocity [right-hand side of Eq. (7)] to be $v_{n-1}(t-t_d)$ when $V(\Delta x_n(t-t_d)+t_d\Delta v_n(t-t_d))$ exceeds both $v_n(t)$ and $v_{n-1}(t-t_d)$, the high-frequency oscillations can be eliminated. Now let

$$V_{OV} = V(\Delta x_n(t - t_d) + t_d \Delta v_n(t - t_d)). \tag{8}$$

Then Eq. (7) is replaced by

$$\tau \frac{dv_n(t)}{dt} + v_n(t) = V_{desired} \,, \tag{9a}$$

where

$$V_{desired} = \begin{cases} V_{OV}, & V_{OV} < v_n(t)\,, \\ \min\{V_{OV}, v_{n-1}(t - t_d)\}, & V_{OV} > v_n(t)\,. \end{cases} \tag{9b}$$

This modification turns the OV model into a car-following-like model under certain conditions when a vehicle is accelerating to catch the preceding vehicle. [Note that instead of acceleration being proportional to $v_{n-1}(t-t_d) - v_n(t-t_d)$ as in conventional

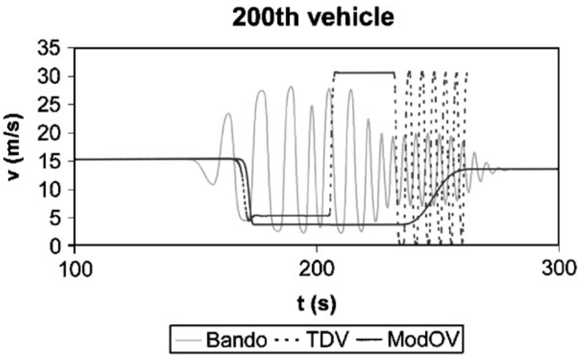

Fig. 6. Velocity of the 200th vehicle of a platoon with initial headway 25 m and velocity 15.34 m/s as a function of time. The lead vehicle decelerates from 15.34 to 13.34 m/s in the first 20 s and then remains at a constant speed. The response for the original OV model [Eq. (4), denoted "Bando"] the time-delayed optimal velocity model [Eq. (7), denoted "TDV"] of Fig. 5, and the modified optimal velocity model [Eq. (9), denoted "ModOV"] which combines the TDV with partial car following. $\tau = 0.5$ s.

car-following models, in the modified OV model it is proportional to $v_{n-1}(t-t_d)-v_n(t)$.] It eliminates the serious overshoot that occurs in the original OV model. A comparison of the OV model with no delay, the delay model given by Eq. (7), and the modified delay model of Eq. (9a) is shown in Fig. 6. The velocity of the 200th vehicle is calculated for the various models. To simulate a typical traffic situation, we take the lead vehicle to decelerate from the common initial speed of the platoon before traveling at a constant speed. Initially, the vehicles are at an equilibrium headway (25 m) and velocity (15.34 m/s). The 20-s deceleration of the lead vehicle at 0.1 m/s^2 is enough to induce congested flow in all three models. The OV model with no delay shows a sequence of jams and free flow followed by nearly sinusoidal oscillations, which eventually die out. The delay OV model [Eq. (7)] with $t_d = 0.75$ s gives a jam, then free flow at about 30 m/s, and subsequently wild oscillations. If the car-following modification is made [Eq. (9b)], a jam taking about a minute to traverse occurs before the vehicle smoothly establishes a constant speed (the lead vehicle's velocity of 13.54 m/s) with no wild oscillations. The final headway is somewhat greater than the equilibrium value for the OV model (27.0 rather than 23.6 m) due to the partial car-following properties of the modified model (Fig. 7a). In Fig. 7b the velocity–headway trajectories for the 100th and the 200th vehicles are compared to the optimal velocity function (OFV) given by Eq. (4). When vehicles decelerate, their velocities are higher than that determined by the OFV for the same headways, an effect we attribute to delay. Accelerating vehicles have larger headway at a given velocity due to both delay and the partial car-following property of the model.

To show that the results obtained so far are not peculiar to velocities near the inflection point in the OVF, we show in Fig. 8a a jam formed at higher initial velocity. In this case, the lead vehicle decelerates from 22.15 m/s (30 m headway) to 20.15 m/s in 20 s and then maintains a constant velocity. The first 10 cars decelerate slowly with only mild fluctuations of velocity, but a jam forms by the 25th vehicle. The jam

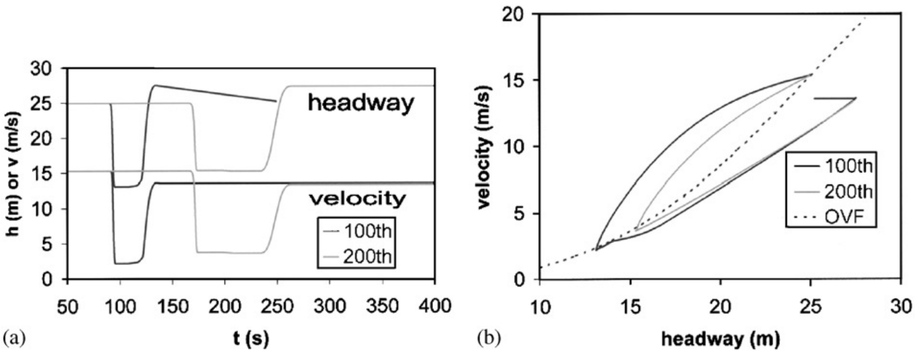

Fig. 7. (a) The headway and velocity of the 100th and 200th vehicles calculated with the ModOV model for a delay of 0.75 s and $\tau = 0.5$ s. (b) Plots of velocity vs. headway for 100th and 200th vehicles. The optimal velocity function (OFV) of Ref. [12] is shown for comparison.

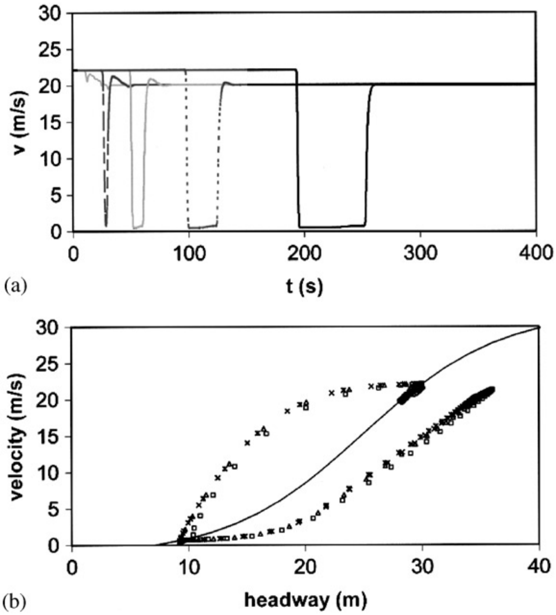

Fig. 8. (a) The velocity of various vehicles ($n = 10, 25, 50, 100, 200$) in a platoon with the initial headway 30 m and velocity 22.15 m/s. The lead vehicle decelerates from 22.15 to 20.15 m/s in 20 s and then remains at a steady velocity. The results calculated with the ModOV model for a delay of 0.75 s and $\tau = 0.5$ s. (b) Plots of velocity vs. headway for the vehicles in (a). The optimal velocity function (OFV) of Ref. [12] is shown for comparison.

continues to grow with the number of vehicles. The time spent in the jam is also about a minute by the 200th car. Trajectories in velocity–headway space are displayed in Fig. 8b. The spread of headways at a given velocity are not unlike those observed empirically (see Ref. [12]).

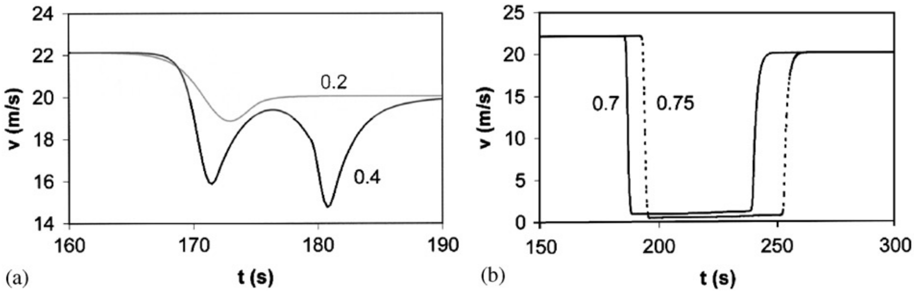

Fig. 9. The response of the 200th vehicle for the conditions in Fig. 8 for different values of delay t_d: (a) small values (0.2 and 0.4 s) and (b) values typical of driver reaction times (0.7 and 0.75 s).

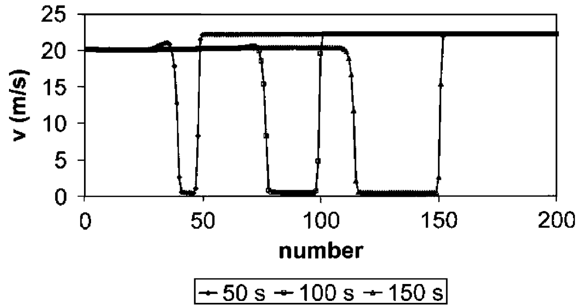

Fig. 10. Same conditions as in Fig. 8 with $t_d = 0.75$ s and $\tau = 0.5$ s. Velocity vs. car number at $t = 50, 100$ and 150 s. Note that the first car ($n = 1$) is at the left and the last at the right ($n = 200$).

Formation of jams in the modified OV model depends strongly on the value of t_d. For $t_d = 0.2$ s, the 200th vehicle (under the conditions of Fig. 8) shows only a small dip in velocity; no real jam forms (see Fig. 9a). However, for more realistic delays around $\frac{3}{4}$ s, full-fledged jams develop (Fig. 9b). *Thus, traffic jams develop because of delay due to driver reaction time in the modified OV model.*

In no case is there significant overshoot or the appearance of subsequent jams in Fig. 9 because of the partial car-following property. Like any car-following model, however, the model breaks down if the headway to the preceding vehicle becomes too large. Clearly, if a vehicle is 1 km back of another, the driver will not adjust his/her speed according to it.

In Fig. 10 snapshots of v_n vs. n at various times show the growing width of the jam. Prior to the jam, the flux of vehicles is 22.15/30/s but only about 20.15/35/s downstream, so it is easy to see why the jam grows. From a plot of vehicle velocity vs. position (Fig. 11a) it can be seen that the front of the jam propagates upstream at about 9 m/s. The back also propagates upstream, but at a lower speed (6 m/s). In Fig. 11b, headway vs. car number is shown. The headway actually increases somewhat downstream of the jam relative to upstream, although the vehicles are traveling at a

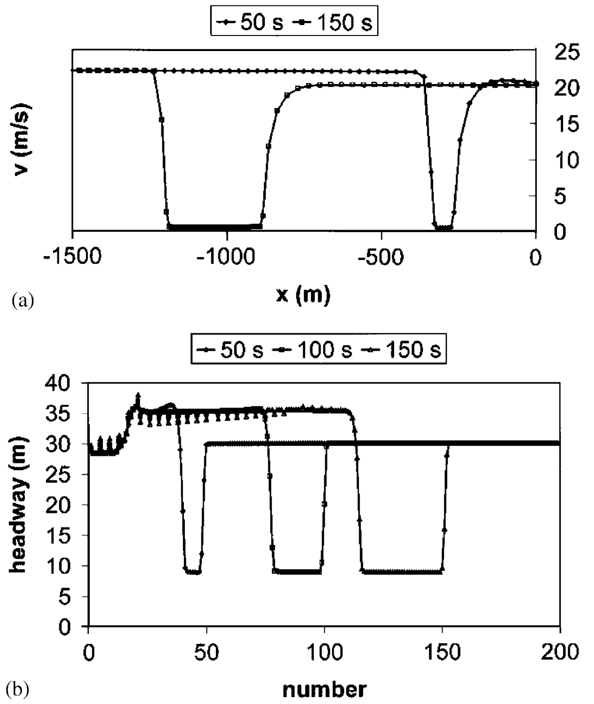

Fig. 11. Formation of a jam in real space showing (a) vehicle velocity vs. position for $t = 50$ and 150 s. In this panel, the first car is at the right and the 200th at the left; (b) headway vs. car number at different times (50, 100 and 150 s). Same conditions as in Fig. 10.

slower speed. Thus, the modified OV model does not, strictly speaking, give a jamming transition [13] because the headway is higher than that given by the optimal velocity function [Eq. (4)] and is not unique. Nor does the model appear to give a sequence of jams and free flow due to a single perturbation. Of course, multiple perturbations could lead to multiple jams.

4. Conclusions

Simulations and analysis of the optimal velocity model of traffic dynamics have been performed to study the effects of delay due to driver reaction times. Simulations show that the stability of a platoon of vehicles is dependent on the platoon size as well as delay time. In general, the OV model displays an unrealistically strong sensitivity to delay. For a delay time of 0.3 s, only platoons of 14 or fewer vehicles avoid collisions. It appears that the parameters given by Sugiyama [12] are inconsistent with the straightforward introduction of reasonable delay times into the OV model. In contrast to the conclusions reached by Bando et al. [11], the present work concludes that a simple redefinition of the sensitivity parameter $(1/\tau)$ is not adequate. It is demonstrated that realistic delay times t_d can be safely introduced if modifications of the OV model are made. The unphysical high-frequency oscillations in vehicle velocity predicted by models that try to maintain a strict headway–velocity relation are eliminated in the modified model. The modified model displays formation of a jam, but not the sequence of jammed and free flow described by Mitarai and Nakansihi [3].

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