Lab Session 7

MA581: Numerical Computations Lab S. Bora September 14, 2022

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1. Let $y(x)$ be a solution curve of the IVP $\frac{dy}{dx} = f(x, y)$, $y(u) = v$, where $f : [a, b] \mapsto \mathbb{R}$ is continuous and $u \in [a, b]$.

   (a) Write a MATLAB function program `[x,y] = Eulerforward(f,int,ics,m)` to solve the IVP via the forward-Euler method. The outputs should be a column $x = [x_1 \ \cdots \ x_m]^T$ of m equidistant points $x_i \in [a, b]$ such that $x_i = u$ and $x_m = b$ and a column vector $y = [y_1 \ \cdots \ y_m]^T$ such that $y_i \approx y(x_i)$. The input `f` should be the function handle, `int` = [`a b`] and `ics` = [`u`, `v`].

   (b) Write a MATLAB function program `[x,y] = RungeKuttasecond(f,int,ics,m)` to generate the same output as in part (a) from the same inputs via the second order Runge-Kutta method with trapezoid steps.

   (c) Write a MATLAB function program `[x,y] = RungeKuttafourth(f,int,ics,m)` to generate the same output as in part (a) from the same inputs via the fourth order Runge-Kutta method.

2. The problems $y' = \cos x$, $y(0) = 0$ and $y' = \sqrt{1 - y^2}$, $y(0) = 0$ have the same solution on the interval $[0, \pi/2]$.

   (a) Use second and fourth order Runge-Kutta codes written above to compute the solutions to both the problems and plot them graphically. Make one plot of solutions for both problems via second order Runge-Kutta method and another plot for the solutions via fourth order Runge-Kutta method.

   (b) What happens to the computed solutions if the interval is changed to $[0, \pi]$?

   (c) What happens on the interval $[0, \pi]$ if the second problem is changed to $y' = \sqrt{|1 - y^2|}$, $y(0) = 0$?

   Write a report of your experiments, that answers all queries and mentions the number of points `m` used in each case.

3. Consider the n-by-n system of first order ODEs $\frac{d\mathbf{y}}{dx} = \mathbf{f}(x, \mathbf{y})$, where $\mathbf{f} : [a, b] \times \mathbb{R}^n \mapsto \mathbb{R}^n$ is continuous and $u \in [a, b]$. Let $\mathbf{y}(x) := \{y_1(x), \ldots, y_n(x)\}$ where $y_i : [a, b] \mapsto \mathbb{R}$, $i = 1, \ldots, n$, be a unique solution of the associated IVP satisfying the initial condition $\mathbf{y}(u) = \mathbf{v}$, for $u \in [a \ b]$.

   Write a MATLAB function program `[x,Y] = Eulerforwardvec(f,int,ics,m)` to solve the above IVP for $n = 2$ via the vector version of forward-Euler method. It should generate a column vector $x = [x_1 \ \cdots \ x_m]^T$ of m equidistant points in $[a, b]$ such that $x_1 = u$ and $x_m = b$ and an $2 \times m$ matrix Y such that $Y(:, i) \approx \mathbf{y}(x_i)$. The input `f` should be the function handle, `int` = [`a b`] and `ics` = [`u`, `v`].

4. Solve the IVP $y'' + \sin xy' + x^2 y = x^2 \sin x$, $y(0) = 0, y'(0) = 1$, numerically by converting it into an IVP associated with a 2-by-2 first order ODE and using `[x,Y] = Eulerforwardvec(f,int,ics,m)` with `int` = [`0, pi/2`] and $m = 1000$. Plot the solution curve.

Submit a livescript program that contains all comments, answers and codes necessary to produce the required output. Ensure that the answers are correctly numbered. The filename of the livescript program should be Your-rollnumber-MA581-Lab6.mlx

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