Lab Session 10

MA581 : Numerical Computations Lab July-November, 2022 S. Bora

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1. Write a function program $\texttt{G} = \texttt{mycholb(A)}$ that executes the *bordered form* of the Cholesky Decomposition for finding the Cholesky factor of an $n \times n$ positive definite matrix A in $\frac{n^3}{3} + O(n^2)$.

2. Compare the output of $\texttt{G} = \texttt{mycholb(A)}$ with that of the built in Matlab function program `chol` for several different choices of randomly generated positive definite matrices. The following commands may be used to generate them with arbitrary choices of $0 < a < b$, and positive integers n :

   $\gg$ `r = a + (b-a).*rand(n,1); D = diag(r)`
   (Here `r` is a length `n` column vector of values randomly generated from an uniform distribution on the interval `[a,b]` and `D` is an $n \times n$ diagonal matrix with the entries of `r` on the diagonal.)
   $\gg$ `B = randn(n); [Q,R] = qr(B)`
   (Here B is an $n \times n$ matrix containing pseudorandom values drawn from the standard normal distribution and Q is an orthogonal matrix such that $B = QR$ is a QR decomposition of B.)
   $\gg$ `A = Q'*A*Q.`

   (The positive definite matrices are also precisely symmetric matrices with positive diagonal entries and the above commands generate them randomly.)

3. Write a MATLAB function program $[\texttt{Q}, \texttt{R}] = \texttt{cgs(V)}$ to orthonormalize the columns of an $n \times m$ matrix V, $(n \geq m)$ by the Classical Gram Schmidt procedure so that Q is an isometry satisfying

$$\begin{aligned} \text{span}\{Q(:,1)\} &= \text{span}\{V(:,1)\} \\ \text{span}\{Q(:,1), Q(:,2)\} &= \text{span}\{V(:,1), V(:,2)\}, \\ &\vdots \\ \text{span}\{Q(:,1), Q(:,2), \ldots, Q(:,m)\} &= \text{span}\{V(:,1), V(:,2), \ldots, V(:,m)\} \end{aligned}$$

   and R is an upper triangular matrix such that $R(i,j) = \langle V(:,j), Q(:,i) \rangle$.

4. A slight modification of the above program leads to the Modified Gram Schmidt procedure for orthonormalizing the columns of V. Perform this modification to obtain another function program $[\texttt{Q}, \texttt{R}] = \texttt{mgs(V)}$.

5. Write a function program $[\texttt{Q}, \texttt{R}] = \texttt{cgsrep(V)}$ that performs Classified Gram Schmidt with reorthogonalization by making appropriate changes to your function program `cgs`.

   Take care to replace *for loops* by matrix-vector multiplications as far as possible in each of the above programs.