Lab Session 3

MA581 : Numerical Computations Lab S. Bora August 17, 2021

Switch to `format long e` for all experiments

1. For a given function f write a function program `y = newton(f,g,x,N,tol)` that performs Newton iterations to find a column vector `y` of length `N` such that $|$`f(y(N))`$| <$ `tol` with initial guess `x`. The `f` and `g` should be the function handles of f and its first derivative.

   Organize your program in such a way that it terminates if either $|$`f(y)`$| <$ `tol` or the iterations exceed N. If $|$`f(y(N))`$| \geq$ `tol`, and the iterations exceed N, then the program should produce an error message "zero could not be found for the given tolerance within the given iterations".

2. Find $2^{\frac{1}{4}}$ correct up to 7 decimal places by using the function $f(x) = x^4 - 2$ and initial estimate $x = 1$ by

   [a] Bisection [b] Regula-Falsi [c] Secant [d] Newton

   methods. Also perform fixed point iterations with

   [e] $g(x) = \frac{x}{3} + \frac{4}{3x^3}$ [f] $g(x) = \frac{3x}{4} + \frac{1}{2x^3}$

   Record the number of iterations in each case and answer the following questions.

   (i) Does the number of iterations necessary to produce the desired accuracy in theory match with your output for Bisection method?

   (ii) Does any method fail to converge? If so, provide justification from theory for the failure.

   (iii) Which method requires the least number of iterations?

   (iv) Does Newton's Method exhibit quadratic convergence? Justify your answer with data.

3. Let $a > 0$. Then the square root $\alpha = \sqrt{a}$ is the zero of $f(x) = x^2 - a$. The Newton's method yields

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right),\ n = 0, 1, 2, \ldots.$$

   This scheme converges globally, that is, $x_n \longrightarrow \sqrt{a}$ for any $x_0 > 0$. To verify global convergence, compute $\sqrt{173373}$ for various values of x_0. Compare your computed result with the result obtained by using MATLAB command `sqrt(173373)`. Determine the number of iterations required to achieve $|x_n - \sqrt{173373}| \leq$ `tol`, for `tol` $= 10^{-8}, 10^{-12}$. Use the same starting guess in each case. Do the results show quadratic order of convergence?

4. The natural frequencies of vibration of a uniform beam of unit length, clamped on one end and free on the other, satisfy the equation $\tan(x)\tanh(x) + 1 = 0$. Use a zero finder (your own program) to determine the smallest positive solution of this equation. Compare your result with that obtained by using MATLAB function `fzero`.

5. Write a function program `[y,T] = NewtonS(f,g,x,iter,tol)` to find an approximate solution `y` $=$ (`y(1)`, `y(2)`) of a nonlinear system of equations $f(u,v) = 0$ via Newton's method. The inputs should be the function handles of `f` and its Jacobian matrix `g`, initial guess `x`, maximum number of iterations `N` and tolerance `tol` such that `norm(f(y(1), y(2)))` $<$ `tol`.

   Your program should terminate if either the iterations exceed `N` or `norm(f(y(1), y(2)))` $<$ `tol`. If termination happens because the norm of `f(y(1), y(2))` is not less than `tol` but the iterations exceed N, then the program should provide an error message "zero could not be found for the given tolerance within the given iterations".

6. Sketch the two curves on the u-v plane and find all solutions exactly via simple algebra.

   (a)

$$\begin{aligned} u^2 + v^2 &= 1 \\ (1-u)^2 + v^2 &= 1 \end{aligned}$$

   (b)

$$\begin{aligned} u^2 + 4v^2 &= 4 \\ v^2 + 4u^2 &= 4 \end{aligned}$$

   Denoting each system by $F(u,v) = 0$, perform iterations of Newton's Method until `norm`$(F(u,v)) < 10^{-7}$ with starting guess $x = (1/2, 1/2)$. Report the number of iterations required and the computed value of (`u`, `v`) in each case.

Submit a livescript program that contains all comments, answers and codes necessary to produce the required output in it. Also the answers should be correctly numbered. The filename of the program should be rollnumber-MA581-Lab3.mlx.

*** End ***