12.6.5.12

EE24BTECH11041 - Mohit

1) Find the maximum and minimum values of $x+\sin 2x$ on $(0, 2\pi)$
   Solution:-
   a) Theoritical Solution:-

$$y = x + \sin 2x \tag{1.1}$$
$$\tag{1.2}$$

Differentiating on both side,

$$\frac{dy}{dx} = 1 + 2\cos 2x \tag{1.3}$$

Putting $\frac{dy}{dx} = 0$

$$1 + 2\cos 2x = 0 \tag{1.4}$$
$$\cos 2x = -\frac{1}{2} \tag{1.5}$$
$$x = \frac{1}{2}\cos\left(-\frac{1}{2}\right) \tag{1.6}$$

The value of x we get in interval in $(0, 2\pi)$,

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \tag{1.7}$$

Again differentaiting , Equation 1.6 ,

$$\frac{d^2y}{dx^2} = -4\sin 2x \tag{1.8}$$

Putting the value of x in Equation 1.8 ,We will get $-4\sin 2x$ negative for $x = \frac{\pi}{3}, \frac{4\pi}{3}$ and positive for $x = \frac{2\pi}{3}, \frac{5\pi}{3}$
Hence, we will get maxima for $x = \frac{\pi}{3}, \frac{4\pi}{3}$ and minima for $x = \frac{2\pi}{3}, \frac{5\pi}{3}$
Value of Function when maxima occur is is 1.91 and 5.05 and when maxima occur is 1.23 and 4.36 .
But we have to consider initial and final values of Function in interval $(0, 2\pi)$.
Value of function at $x = 0$ is 0 and at $x = 2\pi$ is 6.29 Therefore, minimum value of function is 0 and maximum value 2π.

CODING LOGIC:-
Define the function:

$$f(x) = x + \sin(2x) \tag{1.9}$$

Set parameters for gradient descent and ascent:

a) Learning rate (η) = 0.01
b) Maximum iterations (max_iter) = 10,000
c) Tolerance = 1×10^{-6}

Initialize starting points

$$\text{start\_points} = \text{linspace}(0, 2\pi, 50) \tag{1.10}$$

A range of 50 equally spaced starting points between 0 and 2π is chosen to search for critical points.

Use C functions for gradient descent and ascent
And Value of x changes as:-

$$\eta = 0.01 \tag{1.11}$$

$$x_{n+1} = x_n + \eta \frac{dy}{dx_n} \text{ ,when gradient is positive} \tag{1.12}$$

$$x_{n+1} = x_n - \eta \frac{dy}{dx_n} \text{ ,when gradient is negative} \tag{1.13}$$

Change in value of x depends on gradient ,
Find critical points - For each starting point, we use both gradient descent and ascent to find the corresponding critical points:
Classify critical points:

a) Minima are identified by checking the slope:

$$f(x + 1 \times 10^{-3}) - f(x - 1 \times 10^{-3}) > 0 \tag{1.14}$$

b) Maxima are identified by :-

$$f(x + 1 \times 10^{-3}) - f(x - 1 \times 10^{-3}) < 0 \tag{1.15}$$