12.9.4.15

EE24BTECH11041 - Mohit

1) Find the equation of a curve passing through the point $(0,0)$ and whose differential equation is $y^{'} = e^x \sin x$
Solution:-

The given differential equation is:

$$y^{'} = e^x \sin x \tag{1.1}$$

Integrating both sides:

$$y = \int e^x \sin x dx \tag{1.2}$$

Using integration by parts:

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx \tag{1.3}$$

Let I = $\int e^x \sin x dx$. Then:

$$I = e^x \sin x - \int e^x \cos x dx \tag{1.4}$$

Now, solve $\int e^x \cos x dx$ using integration by parts again:

$$\int e^x \cos x dx = e^x \cos x - \int e^x (-\sin x) dx = e^x \cos x + \int e^x \sin x dx \tag{1.5}$$

Substituting back, we get:

$$I = e^x \sin x - (e^x \cos x + I) \tag{1.6}$$

$$I + I = e^x (\sin x - \cos x) \tag{1.7}$$

$$2I = e^x (\sin x - \cos x) \tag{1.8}$$

$$I = \frac{e^x (\sin x - \cos x)}{2} \tag{1.9}$$

Thus, the solution to the differential equation is:

$$y = \frac{e^x (\sin x - \cos x)}{2} + C \tag{1.10}$$

Using the initial condition $y(0) = 0$:

$$y(0) = \frac{e^0(\sin 0 - \cos 0)}{2} + C \tag{1.11}$$
$$0 = \frac{(0-1)}{2} + C \tag{1.12}$$
$$C = \frac{1}{2} \tag{1.13}$$

The final solution is:

$$y = \frac{e^x(\sin x - \cos x)}{2} + \frac{1}{2} \tag{1.14}$$

CODING LOGIC:-

The trapezoidal rule is as follows.

$$A = \int_a^b f(x)\,dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2)\cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{1.15}$$
$$h = \frac{b-a}{n} \tag{1.16}$$
$$A = j_n, \text{ where, } j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{1.17}$$
$$\rightarrow j_{i+1} = j_i + h\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{1.18}$$
$$x_{i+1} = x_i + h \tag{1.19}$$
$$\tag{1.20}$$