EE24BTECH11041 - Mohit

1) Find the equation of a curve passing through the point (0,0) and whose differential equation is $y' = e^x \sin x$

Solution:-

The given differential equation is:

$$y' = e^x \sin x \tag{1.1}$$

Integrating both sides:

$$y = \int e^x \sin x dx \tag{1.2}$$

Using integration by parts:

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx \tag{1.3}$$

Let $I = \int e^x \sin x dx$. Then:

$$I = e^x \sin x - \int e^x \cos x dx \tag{1.4}$$

Now, solve $\int e^x \cos x dx$ using integration by parts again:

$$\int e^x \cos x dx = e^x \cos x - \int e^x (-\sin x) dx = e^x \cos x + \int e^x \sin x dx$$
 (1.5)

Substituting back, we get:

$$I = e^x \sin x - (e^x \cos x + I) \tag{1.6}$$

$$I + I = e^x(\sin x - \cos x) \tag{1.7}$$

$$2I = e^x(\sin x - \cos x) \tag{1.8}$$

$$I = \frac{e^x(\sin x - \cos x)}{2} \tag{1.9}$$

Thus, the solution to the differential equation is:

$$y = \frac{e^x(\sin x - \cos x)}{2} + C \tag{1.10}$$

Using the initial condition y(0) = 0:

$$y(0) = \frac{e^0(\sin 0 - \cos 0)}{2} + C \tag{1.11}$$

$$0 = \frac{(0-1)}{2} + C \tag{1.12}$$

$$C = \frac{1}{2} \tag{1.13}$$

The final solution is:

$$y = \frac{e^x(\sin x - \cos x)}{2} + \frac{1}{2} \tag{1.14}$$

CODING LOGIC:-

The trapezoidal rule is as follows.

$$A = \int_{a}^{b} f(x) dx \approx h \left(\frac{1}{2} f(a) + f(x_{1}) + f(x_{2}) \dots + f(x_{n-1}) + \frac{1}{2} f(b) \right)$$
 (1.15)

$$h = \frac{b-a}{n} \tag{1.16}$$

$$A = j_n$$
, where, $j_{i+1} = j_i + h \frac{f(x_{i+1}) + f(x_i)}{2}$ (1.17)

$$\rightarrow j_{i+1} = j_i + h\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{1.18}$$

$$x_{i+1} = x_i + h (1.19)$$

(1.20)

