

10.4.2.1.3

EE24BTECH11041 - Mohit

- 1) Find the roots of the following quadratic equations by factorisation.

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \quad (1.1)$$

Theoretical Solution-

Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \quad (1.2)$$

$$= 49 - 4(\sqrt{2})(5\sqrt{2}) \quad (1.3)$$

$$= 9 \quad (1.4)$$

This means roots of equation exist.

And its roots are given by

$$x = \frac{b - \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a} \quad (1.5)$$

$$x = -\sqrt{2}, -\frac{5}{\sqrt{2}} \quad (1.6)$$

CODING LOGIC:-

Newton-Raphson Method

- a) Update Equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1.7)$$

- b) Steps:

1. Start with an initial guess x_0 .
2. Define the function $f(x)$ and its derivative $f'(x)$.
3. Iterate using:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1.8)$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance} \quad (1.9)$$

4. Stop if $f'(x_n)$ is close to zero to avoid division by zero.

- c) Convergence Criteria: The method converges quadratically if the initial guess is sufficiently close to the root and $f'(x) \neq 0$.

Secant Method

a) Update Formula:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad (1.10)$$

b) Steps:

1. Start with two initial guesses x_0 and x_1 .
2. Define the function $f(x)$.
3. Iterate using:

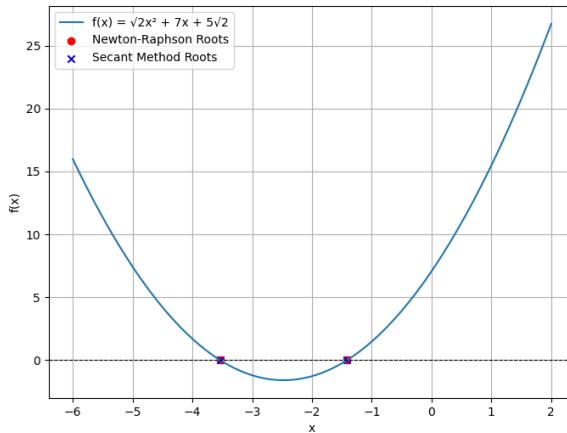
$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad (1.11)$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance}. \quad (1.12)$$

4. Stop if $f(x_n) - f(x_{n-1})$ is close to zero to avoid division by zero.

c) Convergence Criteria: The method converges superlinearly and does not require the derivative $f'(x)$.



CODING LOGIC FOR FINDING EIGENVALUES :-

The quadratic equation

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \quad (1.13)$$

is rewritten in matrix form:

$$\text{Matrix} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \quad (1.14)$$

$$a = \sqrt{2}, \quad b = 7, \quad c = 5\sqrt{2}. \quad (1.15)$$

Substituting the values of a, b and c , the matrix becomes:
Let

$$A = \begin{pmatrix} 0 & -5 \\ 1 & -\frac{7}{\sqrt{2}} \end{pmatrix} \quad (1.16)$$

QR-Algorithm

The QR method is an iterative algorithm used to compute the eigenvalues of a square matrix A . The algorithm works as follows:

a) Initialization

Let $A_0 = A$, where A is the given matrix.

b) QR Decomposition

For each iteration $k = 0, 1, 2, \dots$:

i) Compute the QR decomposition of A_k , such that:

$$A_k = Q_k R_k \quad (1.17)$$

where:

A) Q_k is an orthogonal matrix ($Q_k^T Q_k = I$).

B) R_k is an upper triangular matrix.

The decomposition ensures $A_k = Q_k R_k$.

ii) Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \quad (1.18)$$

c) Convergence

Repeat Step 2 until A_k converges to an upper triangular matrix T . The diagonal entries of T are the eigenvalues of A .

d) The eigenvalues of matrix will be the roots of the equation.

