Presentation

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Problem Statement

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pair of linear equation intersect at a point , are parallel or coincident:

$$9x + 3y + 12 = 0 \tag{2.1}$$

$$18x + 6y + 24 = 0 \tag{2.2}$$

Solution

Given,

$$a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18 = b_2 = 6, c_2 = 24 \tag{3.1}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2} \tag{3.2}$$

Hence,both lines are same.

LU-Decomposition

We rewrite the equations as:

$$x = x_1, \tag{3.3}$$

$$y = x_2, \tag{3.4}$$

giving the system:

$$9x_1 + 3x_2 = -12, \tag{3.5}$$

$$18x_1 + 6x_2 = -24. \tag{3.6}$$

Step 1: Convert to Matrix Form We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{3.7}$$

where:

LU-Decomposition

$$A = \begin{bmatrix} 9 & 3 \\ 18 & 6 \end{bmatrix}, \tag{3.8}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{3.9}$$

$$\mathbf{b} = \begin{bmatrix} -12 \\ -24 \end{bmatrix}. \tag{3.10}$$

Step-by-Step Procedure:

1.Let

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & 0 \\ l_{21} & 0 \end{bmatrix} \tag{3.11}$$

LU-Decomposition

2. Result:

- After completing the iterations, the matrix **A** is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

LU-Decomposition

2. Update for $L_{i,k}$ (Entries of L)
For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} U = \begin{bmatrix} 9 & 3 \\ 0 & 0 \end{bmatrix} \tag{3.12}$$

LU-Decomposition

Step 3: Solve $L\mathbf{y} = \mathbf{b}$ (Forward Substitution) We solve:

$$L\mathbf{y} = \mathbf{b} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -12 \\ -24 \end{bmatrix}. \tag{3.13}$$

Thus:

$$\mathbf{y} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{3.14}$$

Step 4: Solve $U\mathbf{x} = \mathbf{y}$ (Backward Substitution) We solve:

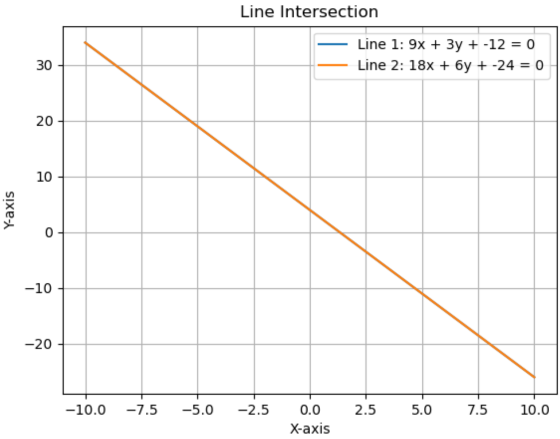
$$U\mathbf{x} = \mathbf{y} \quad \text{or} \quad \begin{bmatrix} 9 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{3.15}$$

$$\begin{bmatrix} 9x_1 + 3x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{3.16}$$

$$\tag{3.17}$$

Hence ,there exist infinity many values of x_1 and x_2. So, both lines are same.

Plot



Line Intersection
Line 1: 9x + 3y + -12 = 0
Line 2: 18x + 6y + -24 = 0
30
20
10
0
−10
−20
Y-axis
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
X-axis