

12.6.5.12

EE24BTECH11041 - Mohit

- 1) Find the maximum and minimum values of $x + \sin 2x$ on $(0, 2\pi)$

Solution:-

- a) Theoretical Solution:-

$$y = x + \sin 2x \quad (1.1)$$

$$(1.2)$$

Differentiating on both side,

$$\frac{dy}{dx} = 1 + 2 \cos 2x \quad (1.3)$$

Putting $\frac{dy}{dx} = 0$

$$1 + 2 \cos 2x = 0 \quad (1.4)$$

$$\cos 2x = -\frac{1}{2} \quad (1.5)$$

$$x = \frac{1}{2} \cos\left(-\frac{1}{2}\right) \quad (1.6)$$

The value of x we get in interval in $(0, 2\pi)$,

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \quad (1.7)$$

Again differentaiting , Equation 1.6 ,

$$\frac{d^2y}{dx^2} = -4 \sin 2x \quad (1.8)$$

Putting the value of x in Equation 1.8 ,We will get $-4 \sin 2x$ negative for $x = \frac{\pi}{3}, \frac{4\pi}{3}$ and positive for $x = \frac{2\pi}{3}, \frac{5\pi}{3}$

Hence, we will get maxima for $x = \frac{\pi}{3}, \frac{4\pi}{3}$ and minima for $x = \frac{2\pi}{3}, \frac{5\pi}{3}$

Value of Function when maxima occur is 1.91 and 5.05 and when minima occur is 1.23 and 4.36 .

But we have to consider initial and final values of Function in interval $(0, 2\pi)$.

Value of function at $x = 0$ is 0 and at $x = 2\pi$ is 6.29 Therefore, minimum value of function is 0 and maximum value 2π .

CODING LOGIC:-

Define the function:

$$f(x) = x + \sin(2x) \quad (1.9)$$

Set parameters for gradient descent and ascent:

- a) Learning rate (η) = 0.01
- b) Maximum iterations = 10,000
- c) Tolerance = 1×10^{-6}

Initialize starting points

$$\text{start_points} = \text{linspace}(0, 2\pi, 50) \quad (1.10)$$

A range of 50 equally spaced starting points between 0 and 2π is chosen to search for critical points.

Use C functions for gradient descent and ascent

And Value of x changes as:-

$$\eta = 0.01 \quad (1.11)$$

$$x_{n+1} = x_n + \eta \frac{dy}{dx_n}, \text{when gradient is positive} \quad (1.12)$$

$$x_{n+1} = x_n - \eta \frac{dy}{dx_n}, \text{when gradient is negative} \quad (1.13)$$

Change in value of x depends on gradient ,

Find critical points - For each starting point, we use both gradient descent and ascent to find the corresponding critical points:

Classify critical points:

- a) Minima are identified by checking the slope:

$$f(x + 1 \times 10^{-3}) - f(x - 1 \times 10^{-3}) > 0 \quad (1.14)$$

- b) Maxima are identified by :-

$$f(x + 1 \times 10^{-3}) - f(x - 1 \times 10^{-3}) < 0 \quad (1.15)$$

