10.4.2.1.3

EE24BTECH11041 - Mohit

1) Find the roots of the following quadratic equations by factorisation.

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \tag{1.1}$$

Theoritical Solution-
Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \tag{1.2}$$
$$= 49 - 4(\sqrt{2})\left(5\sqrt{2}\right) \tag{1.3}$$
$$= 9 \tag{1.4}$$

This means roots of equation exist.
And its roots are given by

$$x = \frac{b - \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a} \tag{1.5}$$
$$x = -\sqrt{2}, -\frac{5}{\sqrt{2}} \tag{1.6}$$

CODING LOGIC:-
Newton-Raphson Method

a) Update Equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1.7}$$

b) Steps:
   1. Start with an initial guess x_0.
   2. Define the function $f(x)$ and its derivative $f'(x)$.
   3. Iterate using:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1.8}$$

   until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance} \tag{1.9}$$

   4. Stop if $f'(x_n)$ is close to zero to avoid division by zero.

c) Convergence Criteria: The method converges quadratically if the initial guess is sufficiently close to the root and $f'(x) \neq 0$.

Secant Method

a) Update Formula:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{1.10}$$

b) Steps:
1. Start with two initial guesses x_0 and x_1.
2. Define the function $f(x)$.
3. Iterate using:

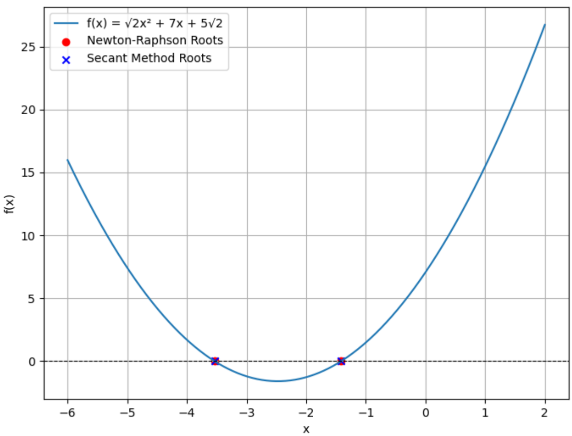
$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{1.11}$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance}. \tag{1.12}$$

4. Stop if $f(x_n) - f(x_{n-1})$ is close to zero to avoid division by zero.

c) Convergence Criteria: The method converges superlinearly and does not require the derivative $f'(x)$.



CODING LOGIC FOR FINDING EIGENVALUES :-

The quadratic equation

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \tag{1.13}$$

is rewritten in matrix form:

$$\text{Matrix} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \tag{1.14}$$

$$a = \sqrt{2}, \quad b = 7, \quad c = 5\sqrt{2}. \tag{1.15}$$

Substituting the values of a, b and c, the matrix becomes:
Let

$$\mathrm{A} = \begin{pmatrix} 0 & -5 \\ 1 & -\frac{7}{\sqrt{2}} \end{pmatrix} \tag{1.16}$$

QR-DECOMPOSITION:-GRAM-SCHMIDT METHOD

a) QR decomposition

$$A = QR \tag{1.17}$$

i) Q is an $m \times n$ orthogonal matrix
ii) R is an $n \times n$ upper triangular matrix.

Given a matrix $A = [a_1, a_2, \ldots, a_n]$, where each a_i is a column vector of size $m \times 1$.

b) Normalize the first column of A:

$$q_1 = \frac{a_1}{||a_1||} \tag{1.18}$$

c) For each subsequent column a_i, subtract the projections of the previously obtained orthonormal vectors from a_i :

$$a_i' = a_i - \sum_{k=1}^{i-1} \langle a_i, q_k \rangle q_k \tag{1.19}$$

Normalize the result to obtain the next column of Q:

$$q_i = \frac{a_i'}{\left\| a_i' \right\|} \tag{1.20}$$

Repeat this process for all columns of A.

d) Finding R:-
After constructing the ortho-normal columns $q_1, q_2, \ldots, q_n$ of Q, we can compute the elements of R by taking the dot product of the original columns of A with the columns of Q:

$$r_{ij} = \langle a_j, q_i \rangle \text{ , for } i \leq j \tag{1.21}$$

QR-Algorithm

a) Initialization
Let $A_0 = A$, where A is the given matrix.

b) QR Decomposition
For each iteration $k = 0, 1, 2, \ldots$:

i) Compute the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{1.22}$$

where:

A) Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).
B) R_k is an upper triangular matrix.

The decomposition ensures $A_k = Q_k R_k$.

ii) Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{1.23}$$

c) Convergence
Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.

d) The eigenvalues of matrix will be the roots of the equation.