10.4.2.1.3

EE24BTECH11041 - Mohit

1) Find the roots of the following quadratic equations by factorisation.

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \tag{1.1}$$

Theoritical Solution-
Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \tag{1.2}$$

$$= 49 - 4(\sqrt{2})\left(5\sqrt{2}\right) \tag{1.3}$$

$$= 9 \tag{1.4}$$

This means roots of equation exist.
And its roots are given by

$$x = \frac{b - \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a} \tag{1.5}$$

$$x = -\sqrt{2}, -\frac{5}{\sqrt{2}} \tag{1.6}$$

CODING LOGIC:-
Newton-Raphson Method

a) Update Equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1.7}$$

b) Steps:
1. Start with an initial guess x_0.
2. Define the function $f(x)$ and its derivative $f'(x)$.
3. Iterate using:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1.8}$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance} \tag{1.9}$$

4. Stop if $f'(x_n)$ is close to zero to avoid division by zero.

c) Convergence Criteria: The method converges quadratically if the initial guess is sufficiently close to the root and $f'(x) \neq 0$.

Secant Method

a) Update Formula:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{1.10}$$

b) Steps:
1. Start with two initial guesses x_0 and x_1.
2. Define the function $f(x)$.
3. Iterate using:

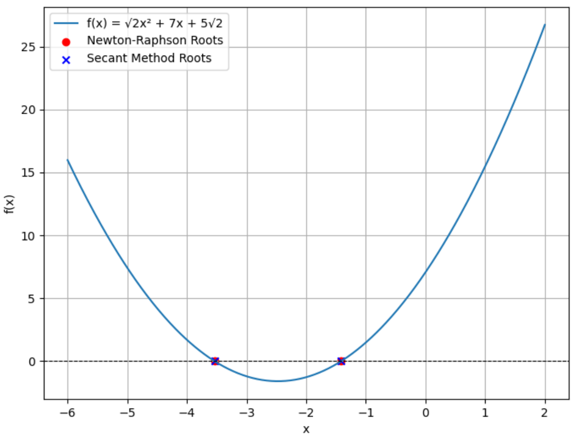
$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{1.11}$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance}. \tag{1.12}$$

4. Stop if $f(x_n) - f(x_{n-1})$ is close to zero to avoid division by zero.

c) Convergence Criteria: The method converges superlinearly and does not require the derivative $f'(x)$.



Another Method

Finding Roots using Eigenvalues

a) Matrix Representation
The quadratic equation

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \tag{1.13}$$

is rewritten in matrix form:

$$\text{Matrix} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \tag{1.14}$$

$$a = \sqrt{2}, \quad b = 7, \quad c = 5\sqrt{2}. \tag{1.15}$$

Substituting the values of a, b and c, the matrix becomes:

$$\text{Matrix} = \begin{pmatrix} 0 & -5 \\ 1 & -\frac{7}{\sqrt{2}} \end{pmatrix} \tag{1.16}$$

b) Eigenvalue Calculation

$$\left| \lambda I - \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \right| = 0 \tag{1.17}$$

where λ represents the eigenvalues.

c) Characteristic Equation

The determinant is expanded as follows:

$$\left| \begin{pmatrix} \lambda & \frac{c}{a} \\ -1 & \lambda + \frac{b}{a} \end{pmatrix} \right| = 0. \tag{1.18}$$

Simplifying:

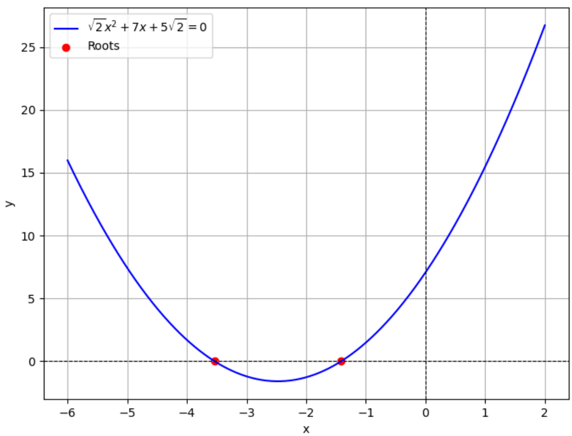
$$\lambda^2 + \frac{b}{a}\lambda + \frac{c}{a} = 0. \tag{1.19}$$

Substitute $a = \sqrt{2}$, $b = 7$, and $c = 5\sqrt{2}$:

$$\lambda^2 + \frac{7}{\sqrt{2}}\lambda + 5 = 0. \tag{1.20}$$

d) Final Roots

$$\lambda_1 = -\sqrt{2}, \quad \lambda_2 = \frac{-5}{\sqrt{2}}. \tag{1.21}$$


$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
Roots
25
20
15
10
5
0
y
−6
−5
−4
−3
−2
−1
0
1
2
x