12.9.7.3

EE24BTECH11041 - Mohit

1) From the differential equation representing the family of curves given by $(x-a)^2 + y^2 = a^2$, where a is an arbitrary conatant. **Solution:-**

a) Simplyfying the equation to,

$$x^2 - 2ax + 2y^2 = 0 \tag{1.1}$$

$$a = \frac{x^2 + 2y^2}{2x} \tag{1.2}$$

b) Differtiating the Equation 1 with respect to x

$$2x - 2a + 4y\frac{dy}{dx} = 0 \tag{1.3}$$

c) Substituing 2nd equation in 3rd equation

$$\frac{x^2 - 2y}{x} + 4y\frac{dy}{dx} = 0 \tag{1.4}$$

The given differential equation is:

$$\frac{x^2 - y}{x} + 4y\frac{dy}{dx} = 0 \tag{1.5}$$

d) **Simplify the equation:**
Rewrite the equation:

$$\frac{x^2}{x} - \frac{2y}{x} + 4y\frac{dy}{dx} = 0 \tag{1.6}$$

$$x - \frac{2y}{x} + 4y\frac{dy}{dx} = 0 \tag{1.7}$$

$$x + 4y\frac{dy}{dx} = \frac{2y}{x}. \tag{1.8}$$

Rearranging gives:

$$4y\frac{dy}{dx} = \frac{2y}{x} - x. \tag{1.9}$$

e) **Separate the variables:**
Divide through by y (assuming $y \neq 0$):

$$\frac{dy}{dx} = \frac{1}{4}\left(\frac{2}{x} - \frac{x}{y}\right) \tag{1.10}$$

f) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$x_0 = 0.001 \tag{1.11}$$
$$y_0 = 0.031 \tag{1.12}$$
$$h = 0.01 \tag{1.13}$$
$$y_{n+1} = y_n + h \cdot \left(\frac{dy}{dx}\right) \tag{1.14}$$
$$y_{n+1} = y_n + h \cdot \left(\frac{1}{4}\left(\frac{2}{x} - \frac{x}{y}\right)\right) \tag{1.15}$$
$$x_{n+1} = x_n + h \tag{1.16}$$