10.3.2.2.2

EE24BTECH11041 - Mohit

Question:- A fair coin is tossed four times, and a person wins Rs.1 for each head and loses Rs.1.50 for each tail. From the sample space, calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution

Step 1: Net Money Calculation

Let:

$$H = +1 \quad \text{(gain Rs.1 for Head)}, \quad T = -1.50 \quad \text{(lose Rs.1.50 for Tail)}.$$

For x, the number of heads in 4 tosses, the total net money can be calculated using the formula:

$$\text{Net Money} = x(1) + (4 - x)(-1.5)$$

$$\text{Net Money} = x - 1.5(4 - x)$$

$$\text{Net Money} = 2.5x - 6,$$

where $x = 0, 1, 2, 3, 4$.

Step 2: Possible Outcomes and Net Money

- $x = 0$: All tails ($TTTT$):

$$\text{Net Money} = 2.5(0) - 6 = -6$$

- $x = 1$: One head, three tails ($HTTT, THTT, TTHT, TTTH$, etc.):

$$\text{Net Money} = 2.5(1) - 6 = -3.5$$

- $x = 2$: Two heads, two tails ($HHTT, HTHT, HTTH, \ldots$):

$$\text{Net Money} = 2.5(2) - 6 = -1$$

- $x = 3$: Three heads, one tail ($HHHT, HHTH, HTHH, THHH$):

$$\text{Net Money} = 2.5(3) - 6 = 1.5$$

- $x = 4$: All heads ($HHHH$):

$$\text{Net Money} = 2.5(4) - 6 = 4$$

Step 3: Number of Outcomes for Each Case

The number of outcomes for each x is given by the binomial coefficient $\binom{4}{x}$:

$$x = 0 : \binom{4}{0} = 1,$$
$$x = 1 : \binom{4}{1} = 4,$$
$$x = 2 : \binom{4}{2} = 6,$$
$$x = 3 : \binom{4}{3} = 4,$$
$$x = 4 : \binom{4}{4} = 1.$$

Step 4: Probabilities of Each Case

Since the coin is fair, the probability of each outcome is $\frac{1}{16}$. The probabilities for each x are:

$$x = 0 : \text{Probability} = \frac{\binom{4}{0}}{16} = \frac{1}{16},$$
$$x = 1 : \text{Probability} = \frac{\binom{4}{1}}{16} = \frac{4}{16} = \frac{1}{4},$$
$$x = 2 : \text{Probability} = \frac{\binom{4}{2}}{16} = \frac{6}{16} = \frac{3}{8},$$
$$x = 3 : \text{Probability} = \frac{\binom{4}{3}}{16} = \frac{4}{16} = \frac{1}{4},$$
$$x = 4 : \text{Probability} = \frac{\binom{4}{4}}{16} = \frac{1}{16}.$$

Step 5: Final Answer

The different amounts of money and their probabilities are summarized below:

Net Money (Rs)	Probability
-6	$\frac{1}{16}$
-3.5	$\frac{1}{4}$
-1	$\frac{3}{8}$
1.5	$\frac{1}{4}$
4	$\frac{1}{16}$

CODING LOGIC:-

1.C Code Description

The C program computes the net money for a given sequence of coin tosses. The outcomes are represented as a string, where:

1) Each **H** (head) contributes Rs.1 to the net money.
2) Each **T** (tail) deducts Rs.1.50 from the net money.

The program performs the following steps:

1) Accepts a string of coin toss outcomes (e.g., `"HHTT"`).
2) Iterates through the string, updating the net money:
   - Add Rs.1 for each `H`.
   - Subtract Rs.1.50 for each `T`.
3) Returns the final net money.

2.Python Code Description

The Python code performs the following:

1) Simulates a specified number of random coin tosses (e.g., 100,000 trials).
2) Calls the C function for each outcome to compute the net money.
3) Counts the occurrences of each net money value and calculates their probabilities using:
$$P(\text{Net Money} = x) = \frac{\text{Frequency of } x}{\text{Total Simulations}}$$
4) Plots the probability distribution using `matplotlib`.

3.Graphical Output

The Python code generates a bar chart where:

- The x-axis represents the possible net money values $(-6, -3.5, -1, 1.5, 4)$.
- The y-axis represents the probabilities, ranging from 0 to 1.
- Each bar corresponds to the probability of a specific net money value.

4.Probability Mass Function (PMF)

The PMF represents the probability of each possible net money value. For 4 coin tosses, the possible net money values and their probabilities are:

$$S = \{-6, -3.5, -1, 1.5, 4\}$$

$$P(\text{Net Money} = x) = \begin{cases} \frac{1}{16}, & x = -6 \\ \frac{1}{4}, & x = -3.5 \\ \frac{3}{8}, & x = -1 \\ \frac{1}{4}, & x = 1.5 \\ \frac{1}{16}, & x = 4 \\ 0, & x \notin S \end{cases}$$

5.Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(\text{Net Money} \leq x) = \sum_{k \leq x} P(\text{Net Money} = k)$$

For the coin toss:

$$F(x) = \begin{cases} 0, & x < -6 \\ \frac{1}{16}, & -6 \leq x < -3.5 \\ \frac{5}{16}, & -3.5 \leq x < -1 \\ \frac{7}{16}, & -1 \leq x < 1.5 \\ \frac{15}{16}, & 1.5 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

6.Simulation Process

The Python simulation performs the following steps:

1) Simulates 4 coin tosses for each trial using random choices of `'H'` and `'T'`.
2) Calls the C function to compute the net money for each simulated outcome.
3) Tracks the frequency of each net money value over all trials.
4) Calculates probabilities from the frequencies:

$$P(x) = \frac{\text{Frequency of } x}{\text{Total Trials}}$$

5) Plots the PMF and calculates the CDF.