10.3.2.2.2

EE24BTECH11041 - Mohit

1) On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pair of linear equation intersect at a point , are parallel or coincident:

$$9x + 3y + 12 = 0 \tag{1.1}$$
$$18x + 6y + 24 = 0 \tag{1.2}$$

Solution:-
Given,

$$a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18 = b_2 = 6, c_2 = 24 \tag{1.3}$$
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2} \tag{1.4}$$

2) Hence,no point of intersect because both lines are same.
CODING LOGIC

The set of linear equations $9x+3y+12 = 0$ and $18x+6y+24 = 0$ can be represented by the following equation

$$\begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -12 \\ -24 \end{pmatrix} \tag{2.1}$$

Any non-sigular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{2.2}$$

The upper triangular matrix U is found by row reducing A,

$$\begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \xrightarrow{R_2 -> R_2 - 2R_1} \begin{pmatrix} 9 & 3 \\ 0 & 0 \end{pmatrix} \tag{2.3}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{2.4}$$

$$l_{21} = A[1][0] = \frac{18}{9} = 2 \tag{2.5}$$

Now,

$$A = \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -12 \\ -24 \end{pmatrix} \tag{2.6}$$

Now we can get the solution to our problem by the two step process,

$$L\mathbf{y} = \mathbf{b} \tag{2.7}$$

$$U\mathbf{x} = \mathbf{y} \tag{2.8}$$

Using forward substitution to solve the first equation,

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -12 \\ -24 \end{pmatrix} \tag{2.9}$$

$$\implies \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -12 \\ 0 \end{pmatrix} \tag{2.10}$$

$$\tag{2.11}$$

Now using back-substitution for the second equation,

$$\begin{pmatrix} 9 & 3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -12 \\ 0 \end{pmatrix} \tag{2.12}$$

But U is singular, so there is no unique solution.
$\therefore$ The lines $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$ doesn't intersect.