

NCERT 9.4.12

EE24BTECH11041 - Mohit

- 1) Solve the differential equation given below with initial conditions $x = 2$ and $y = 0$ and plot a graph.

$$x(x^2 - 1) \frac{dy}{dx} = 1 \quad (1)$$

Solution:-

- a) Rearranging the Equation,

$$dy = \frac{dx}{x(x^2 - 1)} \quad (2)$$

- b) **Integration:** Integrating on both sides.

$$\int dy = \int \frac{dx}{x(x^2 - 1)} \quad (3)$$

$$\int dy = \int \frac{dx}{x^3(1 - \frac{1}{x^2})} \quad (4)$$

- c) Substituting ,

$$1 - \frac{1}{x^2} = t \quad (5)$$

- d) Differentiating on both side ,

$$\frac{dx}{x^3} = \frac{dt}{2} \quad (6)$$

- e) Now integrating,

$$\int dy = \int \frac{dt}{2t} \quad (7)$$

$$y = \frac{1}{2} \ln t + c \quad (8)$$

- f) substituting t,

$$y = \frac{1}{2} \left(\ln \left(1 - \frac{1}{x^2} \right) \right) + c \quad (9)$$

g) finding constant by putting $x=2$ and $y=0$

$$c = \frac{1}{2} \ln \frac{4}{3} \quad (10)$$

h) This leads to:

$$y = \frac{1}{2} \left(\ln \frac{4}{3} \left(1 - \frac{1}{x^2} \right) \right) \quad (11)$$

- i) **NOTE:-** We are not using finite difference because the graph is discontinues at $x=1$. So, when we find $\frac{dy}{dx}$ its value is becoming too large that we are getting a significant error in calculating $\frac{dy}{dx}$. So, We have to use another method.
- j) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic : **Runga-kutta Method**

$$h = 0.001 \quad (12)$$

$$(13)$$

k) Let ,

$$\frac{dy}{dx} = f(x_n) \quad (14)$$

$$k_1 = hf(x_n) \quad (15)$$

$$k_2 = hf(x_n + h/2) \quad (16)$$

$$k_3 = hf(x_n + h/2) \quad (17)$$

$$k_4 = hf(x_n + h) \quad (18)$$

$$k = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \quad (19)$$

$$y_{n+1} = y_n + k \quad (20)$$

