10.3.2.2.2

EE24BTECH11041 - Mohit

1) On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pair of linear equation intersect at a point , are parallel or coincident:

$$9x + 3y + 12 = 0 \tag{1.1}$$
$$18x + 6y + 24 = 0 \tag{1.2}$$

Solution:-
Given,

$$a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18 = b_2 = 6, c_2 = 24 \tag{1.3}$$
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2} \tag{1.4}$$

2) Hence,both lines are same.
CODING LOGIC

Let us assume the given system of equations are consistent and we will try solving using LU decomposition
Given the system of linear equations:

$$9x + 3y + 12 = 0 \tag{2.1}$$
$$18x + 6y + 24 = 0 \tag{2.2}$$

We rewrite the equations as:

$$x_1 = x, \tag{2.3}$$
$$x_2 = y, \tag{2.4}$$

giving the system:

$$9x_1 - 3x_2 = -12, \tag{2.5}$$
$$18x_1 - 6x_2 = -24. \tag{2.6}$$

Step 1: Convert to Matrix Form

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{2.7}$$

where:

$$A = \begin{bmatrix} 9 & 3 \\ 18 & 6 \end{bmatrix}, \tag{2.8}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{2.9}$$

$$\mathbf{b} = \begin{bmatrix} -12 \\ -24 \end{bmatrix}. \tag{2.10}$$

Step 2: LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:
Step-by-Step Procedure:
1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} U = \begin{bmatrix} 9 & 3 \\ 0 & 0 \end{bmatrix} \tag{2.11}$$

Step 3: Solve $L\mathbf{y} = \mathbf{b}$ (Forward Substitution)

We solve:

$$L\mathbf{y} = \mathbf{b} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -12 \\ -24 \end{bmatrix}. \tag{2.12}$$

Thus:

$$\mathbf{y} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{2.13}$$

Step 4: Solve $U\mathbf{x} = \mathbf{y}$ (Backward Substitution)

We solve:

$$U\mathbf{x} = \mathbf{y} \quad \text{or} \quad \begin{bmatrix} 9 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{2.14}$$

$$\begin{bmatrix} 9x_1 + 3x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{2.15}$$

$$\tag{2.16}$$

Hence ,there exist infinity many values of x_1 and x_2. So, both lines are same.