Presentation

Mohit
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Problem Statement

Find the roots of the following quadratic equations by factorisation.

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \tag{2.1}$$

Solution

Theoritical Solution-

Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \tag{3.1}$$

$$= 49 - 4(\sqrt{2})\left(5\sqrt{2}\right) \tag{3.2}$$

$$= 9 \tag{3.3}$$

This means roots of equation exist.

And its roots are given by

$$x = \frac{b - \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a} \tag{3.4}$$

$$x = -\sqrt{2}, -\frac{5}{\sqrt{2}} \tag{3.5}$$

NEWTON-RAPHSON METHOD

Update Equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{4.1}$$

Steps:

1. Start with an initial guess x_0.
2. Define the function $f(x)$ and its derivative $f'(x)$.
3. Iterate using:

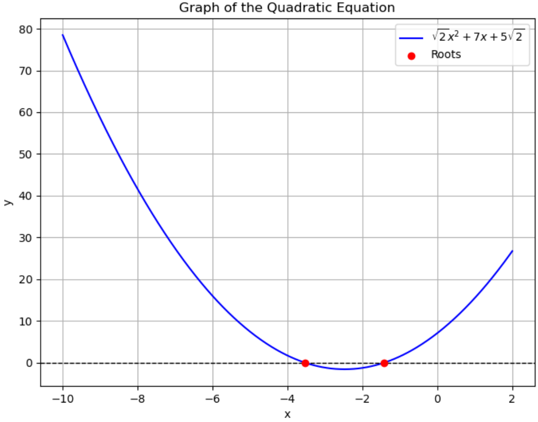
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{4.2}$$

until convergence, i.e.,

$$|x_{n+1} - x_n| < \text{tolerance} \tag{4.3}$$

4. Stop if $f'(x_n)$ is close to zero to avoid division by zero.

Plot by Finite Difference Method



FINDING ROOTS USING :-EIGENVALUES

The quadratic equation

$$ax^2 + bx + c = 0 \tag{6.1}$$

Its Companion Matrix

$$\mathrm{A} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \tag{6.2}$$

Companion matrix for $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\mathrm{A} = \begin{pmatrix} 0 & -5 \\ 1 & -\frac{7}{\sqrt{2}} \end{pmatrix} \tag{6.3}$$

QR-DECOMPOSITION:-GRAM-SCHMIDT METHOD

Let,

$$A = QR \tag{7.1}$$

Q is an $m \times n$ orthogonal matrix
R is an $n \times n$ upper triangular matrix.
Given a matrix $A = [\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_n}]$, where each $\mathbf{a_i}$ is a column vector of size $m \times 1$.
Normalize the first column of A:

$$\mathbf{q_1} = \frac{\mathbf{a_1}}{\|\mathbf{a_1}\|} \tag{7.2}$$

For obtaining each column $\mathbf{a_i}$, subtract the projections of the previously obtained orthonormal vectors from $\mathbf{a_i}$:

$$\mathbf{a_{i+1}} = \mathbf{a_i} - \sum_{k=1}^{i-1} \langle \mathbf{a_i}, \mathbf{q_k} \rangle \mathbf{q_k} \tag{7.3}$$

QR-DECOMPOSITION:-GRAM-SCHMIDT METHOD

Normalize the result to obtain the next column of Q:

$$\mathbf{q_i} = \frac{\mathbf{a_i}}{\|\mathbf{a_i}\|} \tag{8.1}$$

Repeat this process for all columns of A.

We can compute the elements of R by taking the dot product of the original columns of A with the columns of Q:

$$A = QR \tag{8.2}$$

$$Q^\top A = Q^\top QR \tag{8.3}$$

$$Q^\top A = R \quad Q^\top Q = I \tag{8.4}$$

After, this method we will get the matrix A in form of QR.

QR-ALGORITHM

Let $A_0 = A$.
For each iteration $k = 0, 1, 2, \ldots$:- We got the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{9.1}$$

Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).
R_k is an upper triangular matrix.
Then,form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{9.2}$$

Note:-The Eigenvalues of Matrix doesn't change in QR iteration because the orthogonal matrix Q_k preserves the lengths and angles of vectors.

$$A_0 = A_1 = A_2 = \cdots = A_k \tag{9.3}$$

Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.The eigenvalues will be the roots of the Equation.

Plot