Presentation

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Problem Statement

Solve the differential equation given below with initial conditions $x = 2$ and $y = 0$ and plot a graph.

$$x(x^2 - 1)\frac{dy}{dx} = 1 \tag{2.1}$$

Solution

Rearranging the Equation,

$$dy = \frac{dx}{x(x^2 - 1)} \tag{3.1}$$

Integration: Integrating on both sides.

$$\int dy = \int \frac{dx}{x(x^2 - 1)} \tag{3.2}$$

$$\int dy = \int \frac{dx}{x^3(1 - \frac{1}{x^2})} \tag{3.3}$$

Subsituting ,

$$1 - \frac{1}{x^2} = t \tag{3.4}$$

Solution

Differentiating on both side ,

$$\frac{dx}{x^3} = \frac{dt}{2} \tag{4.1}$$

Now integrating,

$$\int dy = \int \frac{dt}{2t} \tag{4.2}$$

$$y = \frac{1}{2}\ln t + c \tag{4.3}$$

substituting t,

$$y = \frac{1}{2}\left(\ln\left(1 - \frac{1}{x^2}\right)\right) + c \tag{4.4}$$

finding constant by putting x=2 and y=0

$$c = \frac{1}{2} ln \frac{4}{3} \tag{4.5}$$

Solution

This leads to:

$$y = \frac{1}{2}\left(\ln\frac{4}{3}\left(1 - \frac{1}{x^2}\right)\right) \tag{5.1}$$

Finite difference Method

Initial point of curve $(1.0001, -4.10)$

$$h = 0.001 \tag{6.1}$$

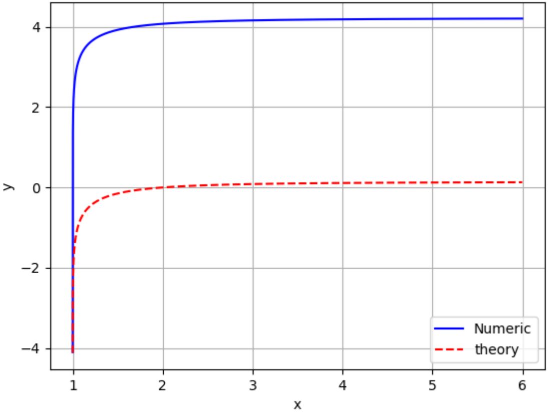
$$y_{n+1} = y_n + h \cdot \left(\frac{dy}{dx}\right) \tag{6.2}$$

$$x_{n+1} = x_n + h \tag{6.3}$$

Substituting $\frac{dy}{dx}$,

$$y_{n+1} = y_n + h \cdot \left(\frac{1}{x(x^2 - 1)}\right) \tag{6.4}$$

Plot by Finite Difference Method



Note:- Finite difference method fails here because $\frac{dy}{dx}$ is too large near x=1.Which creates a significant error in calculating y_{n+1}

Runga-kutta Method

Let ,

$$h = 0.001 \tag{8.1}$$

$$\frac{dy}{dx} = f(x_n) \tag{8.2}$$

$$\tag{8.3}$$

Then ,

$$k_1 = hf(x_n) \tag{8.4}$$

$$k_2 = hf(x_n + h/2) \tag{8.5}$$

$$k_3 = hf(x_n + h/2) \tag{8.6}$$

$$k_4 = hf(x_n + h) \tag{8.7}$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{8.8}$$

$$y_{n+1} = y_n + k \tag{8.9}$$

$$x_{n+1} = x_n + h \tag{8.10}$$

Plot by Runga-Kutta Method