12.9.7.3

EE24BTECH11041 - Mohit

1) From the differential equation representing the family of curves given by $(x-a)^2+y^2=a^2$, where a is an arbitrary conatant. **Solution:-**

   a) Simplyfying the equation to,

   $$x^2-2ax+2y^2=0 \tag{1.1}$$

   $$a=\frac{x^2+2y^2}{2x} \tag{1.2}$$

   b) Differtiating the Equation 1 with respect to x

   $$2x-2a+4y\frac{dy}{dx}=0 \tag{1.3}$$

   c) Substituing 2nd equation in 3rd equation

   $$\frac{x^2-2y}{x}+4y\frac{dy}{dx}=0 \tag{1.4}$$

   The given differential equation is:

   $$\frac{x^2-y}{x}+4y\frac{dy}{dx}=0 \tag{1.5}$$

   d) **Simplify the equation:**
   Rewrite the equation:

   $$\frac{x^2}{x}-\frac{2y}{x}+4y\frac{dy}{dx}=0 \tag{1.6}$$

   $$x-\frac{2y}{x}+4y\frac{dy}{dx}=0 \tag{1.7}$$

   $$x+4y\frac{dy}{dx}=\frac{2y}{x}. \tag{1.8}$$

   Rearranging gives:

   $$4y\frac{dy}{dx}=\frac{2y}{x}-x. \tag{1.9}$$

   e) **Separate the variables:**
   Divide through by y (assuming $y \neq 0$):

$$\frac{dy}{dx} = \frac{1}{4}\left(\frac{2}{x} - \frac{x}{y}\right) \tag{1.10}$$

f) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$x_0 = 0.001 \tag{1.11}$$
$$y_0 = 0.031 \tag{1.12}$$
$$h = 0.01 \tag{1.13}$$
$$y_{n+1} = y_n + h \cdot \left(\frac{dy}{dx}\right) \tag{1.14}$$
$$y_{n+1} = y_n + h \cdot \left(\frac{1}{4}\left(\frac{2}{x} - \frac{x}{y}\right)\right) \tag{1.15}$$
$$x_{n+1} = x_n + h \tag{1.16}$$

Solving Question for a=1