

Question 7-7.2-18

EE24BTECH11041 - Mohit

- 1) Equation of the circle with centre on the Y axis and passing through the origin and the point $(2, 3)$ is
- $x^2 + y^2 + 6x + 6y + 3 = 0$
 - $x^2 + y^2 - 6x - 6y - 9 = 0$
 - $x^2 + y^2 - 6x - 6y + 9 = 0$
 - none of these

Variable	Description
\mathbf{x}_1	Point on circle
\mathbf{x}_2	Point on circle
\mathbf{n}	Equation of line on centre of circle lies

TABLE 1: Variables Used

Solution:-

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, c = 0. \quad (1.1)$$

The centre is given by

$$\begin{pmatrix} 2\mathbf{x}_1 & 2\mathbf{x}_2 & \mathbf{n} \\ 1 & 1 & 0 \end{pmatrix}^\top \begin{pmatrix} \mathbf{u} \\ f \\ c \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ c \end{pmatrix} \quad (1.2)$$

Substituting values of $\mathbf{x}_1, \mathbf{x}_2$ and n

$$\begin{pmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \\ c \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ 0 \end{pmatrix} \quad (1.3)$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right) \quad (1.4)$$

Performing a sequence of row operations to transform into an Echelon form

$$\begin{aligned} & \xleftrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 6 & 1 & -13 \end{array} \right) \xleftrightarrow{R_3 \rightarrow R_3 - 4R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right) \\ & \leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{6} & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{array} \right) \xleftrightarrow{R_2 \rightarrow R_2 - \frac{1}{6}R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{array} \right) \end{aligned}$$

So, the values of \mathbf{u} and f are

$$\mathbf{u} = -\begin{pmatrix} 0 \\ \frac{13}{6} \end{pmatrix}, \quad f = 0. \quad (1.5)$$

The radius of circle is

$$r = \sqrt{(\|\mathbf{u}\|^2 - f)} = \frac{13}{6} \quad (1.6)$$

The equation of circle is

$$\|x\|^2 - 2\left(0 \quad \frac{13}{6}\right)x = 0 \quad (1.7)$$

OR

$$x^2 + y^2 - \frac{13}{3}y = 0 \quad (1.8)$$

Hence, option (d) is correct

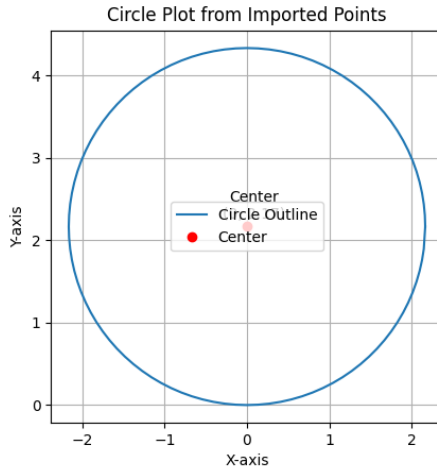


Fig. 1.1: Plot of circle