JEE PYQ 1

EE24Btech11041 - Mohit

1) Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real function. If α and β are respectively the minimum and the maximum values of f , then $\alpha^2+2\beta^2$ is equal to
   a) 44
   b) 42
   c) 24
   d) 31

2) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = I + adj(A) + (adj(A))^2 + \cdots + (adj(A))^{10}$.
   Then,the sum of all the elements of the matrix B is:
   a) -110
   b) -88
   c) 22
   d) -124

3) Let three real numbers a, b, c be in arithmetic progression $a+1, b, c+3$ be in geometric progession.If $a > 10$ and the arithmetic mean of a, b and c is 8,then the cube of the geometric mean of a, b and c is
   a) 316
   b) 120
   c) 128
   d) 312

4) Let a relation R on $N \times N$ be defined as:
   (x_1, y_1) R (x_2, y_2) if and only if $x_1 \leq x_2$ or $y_1 \leq y_2$.
   Consider the two statments: (1)R is reflexive but not symmetric.
   (2)R is transitive
   Then which one of the following is true?

   a) Neither (1) nor (2) is correct.
   b) Only (2) is correct.
   c) Only (1) is correct.
   d) Both (1) and (2) are correct.

5) Given that the inverse trigonometric function assumes principal values only.Let x,y be any two real numbers in $[-1, 1]$ such that $\cos^{-1}(x) - \sin^{-1}(y) = \alpha, -\frac{\pi}{2} < \alpha < \pi$. Then,the minimum value of $x^2 - y^2 + 2xy\sin\alpha$ is
   a) -1
   b) 0
   c) $\frac{1}{2}$
   d) $-\frac{1}{2}$

6) If the function

$$f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ a \log 2 \log 3, & x = 0 \end{cases}$$

   is continious at $x = 0$, then the value of a^2 is equal to

a) 746
b) 968
c) 1250
d) 1152

7) Let C be a circle with radius $\sqrt{10}$ units and centre at the origin.Let the line $x + y = 2$ intersets the circle C at the points P and Q.Let MN be a chord of C of length 2 unit slope -1.Then,a distance (in units) between the chords PQ and the chords MN is

a) 2- $\sqrt{3}$
b) $\sqrt{2}$+1
c) $\sqrt{2}$-1
d) 3- $\sqrt{2}$

8) If the mean of the following probability distribution of a radian variable X:

X	0	2	4	6	8
P(X)	a	$2a$	$a+b$	$2b$	$3b$

is $\frac{46}{9}$,then the variance of the distribution is

a) $\frac{566}{81}$
b) $\frac{173}{27}$
c) $\frac{581}{81}$
d) $\frac{151}{81}$

9) The area (in sq. units) of the region
S={ $\mathbf{z} \in C : |z-1| \leq 2 \; ; \; (z+\bar{z}) + i\,(z-\bar{z}) \leq 2, \; Im(z) \geq 0\}$ is

a) $\frac{7\pi}{3}$
b) $\frac{7\pi}{4}$
c) $\frac{17\pi}{8}$
d) $\frac{3\pi}{2}$

10) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, and $\mathbf{c} = x\hat{i} + 2\hat{j} + 3\hat{k}$, $x \in \mathbb{R}$.
If $\mathbf{d}$ is the unit vector in the direction of $\mathbf{b} + \mathbf{c}$ such that $\mathbf{a}.\mathbf{d} = 1$,then $(\mathbf{a} \times \mathbf{b}).\mathbf{c}$ is equal to

a) 3
b) 6
c) 11
d) 9

11) Let P be the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$.Then , the shortest distance of P from the line $4x = 2y = z$ is

a) $\frac{\sqrt{14}}{7}$
b) $\frac{6\sqrt{14}}{7}$
c) $\frac{5\sqrt{14}}{7}$
d) $\frac{3\sqrt{14}}{7}$

12) Let $y = y(x)$ be the solution of the differential equation
$\left(x^2+4\right)^2 dy + \left(2x^3y + 8xy - 2\right)dx$=0.If $y(0) - 0$,Then $y(2)$ is equal to

a) $\frac{\pi}{32}$
b) $\frac{\pi}{8}$
c) $\frac{\pi}{16}$
d) 2π

13) For $\lambda > 0$, let θ be the angle between the vectors $\mathbf{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.
If the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to:

a) 40
b) 25
c) 50
d) 20

14) The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x$,and $y \geq 4x - 1\}$ is
a) $\frac{11}{32}$
b) $\frac{11}{12}$
c) $\frac{9}{32}$
d) $\frac{8}{9}$

15) Let PQ be a chord of parabola $y^2 = 12x$ and the midpoint of PQ be at $(4, 1)$.Then ,which of the following points lies on the line on the line passing through the points P and Q ?
a) $(3, -3)$
b) $\left(\frac{3}{2}, -16\right)$
c) $\left(\frac{1}{2}, -20\right)$
d) $(2, -9)$