Question 1-1.4-9p

EE24BTECH11041 - Mohit

1) Let $\mathbf{A}(4,2)$,$\mathbf{B}(6,5)$ and $\mathbf{C}(1,4)$ be the vertices of $\Delta\ ABC$.Find the coordinates of points $\mathbf{Q}$ and $\mathbf{R}$ on medians BE and CF respectively such that $BQ:QE = 2:1$ and $CR:RF = 2:1$.

Variable	Description
Three points makes a triangle whose co-ordinates are	$\mathbf{A}(4,2)$,$\mathbf{B}(6,5)$,$\mathbf{C}(1,4)$
F	Mid point of AB
E	Mid point of AB
Centroid of triangle	G

TABLE 1: Variables Used

Solution:-
$\mathbf{F}$ is the mid point of AB

$$\mathbf{F} = \frac{A+B}{2} = \frac{\binom{4}{2}+\binom{6}{5}}{2} = \binom{5}{\frac{7}{2}} \tag{1.1}$$

$\mathbf{E}$ is the mid point of AC

$$\mathbf{E} = \frac{A+C}{2} == \frac{\binom{4}{2}+\binom{1}{4}}{2} = \binom{\frac{5}{2}}{3} \tag{1.2}$$

By section formula,

$$\mathbf{R} = \frac{B+KA}{1+K} \tag{1.3}$$

It is given that $\frac{BQ}{QE} = \frac{2}{1}$
So,

$$\mathbf{Q} = \frac{B+2E}{1+2} = \frac{\binom{6}{5}+2\binom{\frac{5}{2}}{3}}{3} = \binom{\frac{11}{3}}{\frac{11}{3}} \tag{1.4}$$

It is given that $\frac{CR}{RF} = \frac{2}{1}$
So,

$$\mathbf{R} = \frac{C + 2F}{1 + 2} = \frac{\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{1.5}$$

Hence, Co-ordinates of **Q** and **R** are

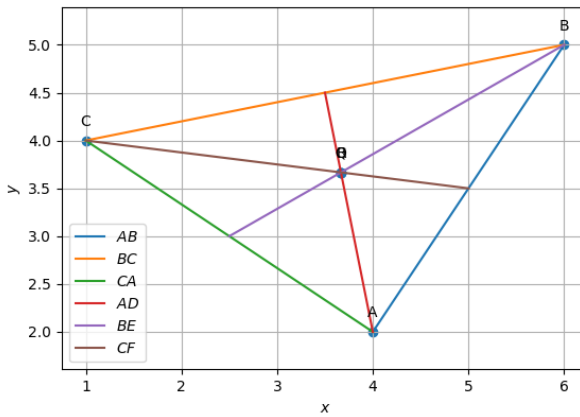$$\mathbf{Q}\left(\frac{11}{3}, \frac{11}{3}\right) \text{and } \mathbf{R}\left(\frac{11}{3}, \frac{11}{3}\right) \tag{1.6}$$

Fig. 1.1: Plot of Triangle *ABC*