Question-Circle

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Problem Statement

Equation of the circle with centre on the Y axis and passing through the origin and the point (2,3) is

a)
$$x^2 + y^2 + 6x + 6y + 3 = 0$$

b)
$$x^2 + y^2 - 6x - 6y - 9 = 0$$

c)
$$x^2 + y^2 - 6x - 6y + 9 = 0$$

d) none of these

Solution

Variable	Description
x ₁	Point on circle
x ₂	Point on circle
n	Normal vector of line at which centre of circle lies
u	Minus times the coordinate of centre
r	Radius of the circle
С	Constant in equation of line
f	$\ \mathbf{u}\ ^2$ - r^2

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ \mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ c = 0.$$
 (3.1)

The centre is given by

$$\begin{pmatrix} 2\mathbf{x}_1 & 2\mathbf{x}_2 & \mathbf{n} \\ 1 & 1 & 0 \end{pmatrix}^{\top} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ c \end{pmatrix}$$
(3.2)

Matrix Equation

Substituting values of $\mathbf{x}_1, \mathbf{x}_2$ and \mathbf{n}

$$\begin{pmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ 0 \end{pmatrix}$$
 (3.3)

The augmented matrix is expressed as

$$\begin{pmatrix}
4 & 6 & 1 & | & -13 \\
0 & 0 & 1 & | & 0 \\
-1 & 0 & 0 & | & 0
\end{pmatrix}$$
(3.4)

Row Reduction

Performing a sequence of row operations to transform into an Echelon form

$$\stackrel{R_1 \leftrightarrow R_3}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 4 & 6 & 1 & -13 \end{pmatrix} \stackrel{R_3 \to R_3 - 4R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 6 & 1 & -13 \end{pmatrix}$$

$$\longleftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{6} & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{pmatrix} \stackrel{R_2 \to R_2 - \frac{1}{6}R_3}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Results

Thus, the values of \mathbf{u} and f are

$$\mathbf{u} = -\begin{pmatrix} 0 \\ \frac{13}{6} \end{pmatrix}, \ f = 0. \tag{3.5}$$

The radius of circle is

$$r = \sqrt{(\|\mathbf{u}\|^2 - f)} = \frac{13}{6} \tag{3.6}$$

The equation of circle is

$$\|\mathbf{x}\|^2 - 2 \begin{pmatrix} 0 & \frac{13}{6} \end{pmatrix} \mathbf{x} = 0$$
 (3.7)

OR

$$x^2 + y^2 - \frac{13}{3}y = 0 (3.8)$$

Hence, option (d) is correct

Figure

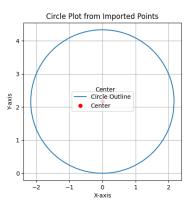


Figure: Circle

C-Code

```
#include <stdio.h>
#include <math.h>
#define PI 3.141592653589793
int main(void) {
    int num_points = 100; // Change number of points to 100
    double r = 13.0 / 6.0; // Radius of the circle
    double center_y = 13.0 / 6.0; // y-coordinate of the center of the
        circle
    double theta; // Angle variable
    double x, y; // Coordinates of the points
    // Open file to write points
    FILE *file = fopen("circle_points.dat", "w");
    if (file == NULL) {
        printf("Error_opening_file!\n");
```

C-Code

```
return 1;
// Calculate 100 points on the circle
for (int i = 0; i < num\_points; i++) {
    // Calculate angle (theta) for current point
    theta = 2 * PI * i / num_points;
    // Parametric equations of a circle
    x = r * cos(theta);
    y = r * sin(theta) + center_y;
    // Print the points to the file
    fprintf(file, "%lf_{n}", x, y);
   Close the file
```

C-Code

Python-Code

```
import numpy as np
import matplotlib.pyplot as plt
def load_points_from_file(filename):
    with open(filename, "r") as file:
         points = np.loadtxt(file)
    return points
# Load points from the circle_points.dat file
points = load_points_from_file("circle_points.dat") # Ensure this file
    contains 100 points
# Extract x and y coordinates
x_{coords} = points[:, 0]
y_{coords} = points[:, 1]
  Define the center of the circle
```

Python-Code

```
center x = 0
center_y = 13.0 / 6.0
# Plot the circle using the imported points without showing them
fig, ax = plt.subplots()
ax.plot(x_coords, y_coords, label="Circle Outline") # Just plot the
    outline
# Plot the center of the circle
ax.plot(center_x, center_y, 'ro', label="Center") # 'ro' for red dot
ax.annotate(f'Center\n(\{center\_x\}, \{center\_y:.2f\})),
              (center_x, center_y), textcoords="offset points",
              xytext=(5,5), ha='center')
# Set equal scaling to ensure the plot is circular
ax.set_aspect('equal', 'box')
```

Python-Code

```
# Plot configurations
plt.grid(True)
plt.title("Circle Plot from Imported Points")
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.legend()
# Save and show plot
plt.savefig("circle_plot_with_center.png")
plt.show()
```