

Question 9-9.3-11

EE24BTECH11041 - Mohit

- 1) If the area of the region bounded by the line $y = mx + c$ and the curve $x^2 = y$ is $\frac{32}{7}$ sq.unit then find the positive value of m .

Variable	Description
\mathbf{x}_1	First intersection point
\mathbf{x}_2	Second intersection point
x_1	x co-ordinate of First intersection point
x_2	x co-ordinate of Second intersection point
\mathbf{h}	Constant in equation of line
\mathbf{m}	Direction vector of given line
A	Area of the region

TABLE 1: Variables Used

The parameters of the given conic are

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f = 0. \quad (1.1)$$

For the line, the parameters are

$$\mathbf{h} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \quad (1.2)$$

Substituting the line equation in parabola equation gives the values of κ

$$(\mathbf{h} + \kappa \mathbf{m})^\top V (\mathbf{h} + \kappa \mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + \kappa \mathbf{m}) + f = 0 \quad (1.3)$$

$$\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix} \right)^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix} \right) + 2 \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}^\top \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix} \right) + 0 = 0 \quad (1.4)$$

$$(1 \quad c + km) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ c + km \end{pmatrix} + (0 \quad -1) \begin{pmatrix} 1 \\ c + km \end{pmatrix} = 0 \quad (1.5)$$

$$(0 \quad c + km) \begin{pmatrix} 1 \\ c + km \end{pmatrix} - (c + km) = 0 \quad (1.6)$$

$$(c + km)((c + km) - 1) = 0 \quad (1.7)$$

$$k = -\frac{c}{m}, \frac{1-c}{m} \quad (1.8)$$

$$(1.9)$$

Putting in equation of line

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (1.10)$$

$$\mathbf{x}_1 = \left(m \left(\frac{m - \sqrt{m^2 + 4c}}{2} + c \right) \right) \quad (1.11)$$

$$\mathbf{x}_2 = \left(m \left(\frac{m + \sqrt{m^2 + 4c}}{2} + c \right) \right) \quad (1.12)$$

We will get x-coordinate of point of intersection are

$$x_1 = \frac{m - \sqrt{m^2 + 4c}}{2}, x_2 = \frac{m + \sqrt{m^2 + 4c}}{2} \quad (1.13)$$

From the desired area is

$$A = \int_{x_1}^{x_2} (mx + c - x^2) dx = \frac{m}{2}(x_2^2 - x_1^2) + c(x_2 - x_1) - \frac{1}{3}(x_2^3 - x_1^3) = 32/7 \quad (1.14)$$

On solving, we get $m=2.86$ and $c=2$

Hence, the slop line is 2.86 .

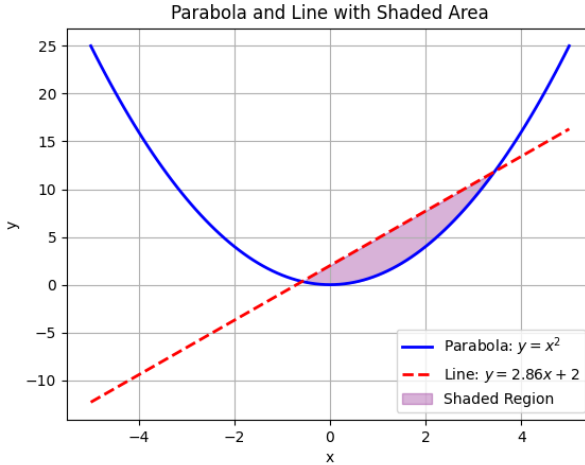


Fig. 1.1: Plot of curves