

Question-Circle

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Problem Statement

Equation of the circle with centre on the Y axis and passing through the origin and the point $(2, 3)$ is

a) $x^2 + y^2 + 6x + 6y + 3 = 0$

b) $x^2 + y^2 - 6x - 6y - 9 = 0$

c) $x^2 + y^2 - 6x - 6y + 9 = 0$

d) none of these

Solution

Variable	Description
\mathbf{x}_1	Point on circle
\mathbf{x}_2	Point on circle
\mathbf{n}	Normal vector of line at which centre of circle lies
\mathbf{u}	Minus times the coordinate of centre
r	Radius of the circle
c	Constant in equation of line
f	$\ \mathbf{u}\ ^2 - r^2$

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 0. \quad (3.1)$$

The centre is given by

$$\begin{pmatrix} 2\mathbf{x}_1 & 2\mathbf{x}_2 & \mathbf{n} \\ 1 & 1 & 0 \end{pmatrix}^\top \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ c \end{pmatrix} \quad (3.2)$$

Matrix Equation

Substituting values of $\mathbf{x}_1, \mathbf{x}_2$ and \mathbf{n}

$$\begin{pmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ 0 \end{pmatrix} \quad (3.3)$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right) \quad (3.4)$$

Row Reduction

Performing a sequence of row operations to transform into an Echelon form

$$\begin{aligned} &\xleftrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 6 & 1 & -13 \end{array} \right) \xleftrightarrow{R_3 \rightarrow R_3 - 4R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right) \\ &\quad \leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{6} & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{array} \right) \xleftrightarrow{R_2 \rightarrow R_2 - \frac{1}{6}R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{13}{6} \\ 0 & 0 & 1 & 0 \end{array} \right) \end{aligned}$$

Results

Thus, the values of \mathbf{u} and f are

$$\mathbf{u} = - \left(\begin{array}{c} 0 \\ \frac{13}{6} \end{array} \right), \quad f = 0. \quad (3.5)$$

The radius of circle is

$$r = \sqrt{(\|\mathbf{u}\|^2 - f)} = \frac{13}{6} \quad (3.6)$$

The equation of circle is

$$\|\mathbf{x}\|^2 - 2 \left(0 \quad \frac{13}{6} \right) \mathbf{x} = 0 \quad (3.7)$$

OR

$$x^2 + y^2 - \frac{13}{3}y = 0 \quad (3.8)$$

Hence, option (d) is correct

Figure

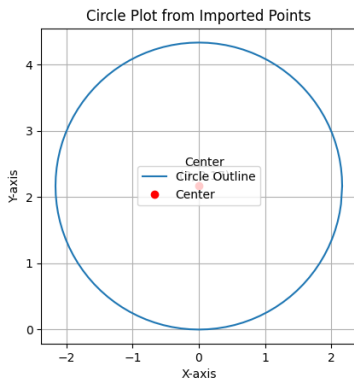


Figure: Circle

C-Code

```
#include <stdio.h>
#include <math.h>

#define PI 3.141592653589793

int main(void) {
    int num_points = 100; // Change number of points to 100
    double r = 13.0 / 6.0; // Radius of the circle
    double center_y = 13.0 / 6.0; // y-coordinate of the
        center of the circle
    double theta; // Angle variable
    double x, y; // Coordinates of the points

    // Open file to write points
    FILE *file = fopen("circle_points.dat", "w");
    if (file == NULL) {
        printf("Error opening file!\n");
    }
}
```

C-Code

```
    return 1;
}

// Calculate 100 points on the circle
for (int i = 0; i < num_points; i++) {
    // Calculate angle (theta) for current point
    theta = 2 * PI * i / num_points;

    // Parametric equations of a circle
    x = r * cos(theta);
    y = r * sin(theta) + center_y;

    // Print the points to the file
    fprintf(file, "%lf %lf\n", x, y);
}

// Close the file
```

C-Code

```
fclose(file);

printf("100 points on the circle have been saved to
      circle_points.dat.\n");
return 0;
}
```

Python-Code

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 def load_points_from_file(filename):
5     with open(filename, "r") as file:
6         points = np.loadtxt(file)
7     return points
8
9 # Load points from the circle_points.dat file
0 points = load_points_from_file("circle_points.dat") #
1     Ensure this file contains 100 points
2
3 # Extract x and y coordinates
4 x_coords = points[:, 0]
5 y_coords = points[:, 1]
6
7 # Define the center of the circle
```

Python-Code

```
center_x = 0
center_y = 13.0 / 6.0

# Plot the circle using the imported points without showing
  them
fig, ax = plt.subplots()
ax.plot(x_coords, y_coords, label="Circle Outline") # Just
  plot the outline

# Plot the center of the circle
ax.plot(center_x, center_y, 'ro', label="Center") # 'ro'
  for red dot
ax.annotate(f'Center\n({center_x}, {center_y:.2f})',
            (center_x, center_y), textcoords="offset points",
            xytext=(5,5), ha='center')
```

Python-Code

```
# Plot configurations
plt.grid(True)
plt.title("Circle Plot from Imported Points")
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.legend()

# Save and show plot
plt.savefig("circle_plot_with_center.png")
plt.show()
```