Question 9-9.3-11

EE24BTECH11041 - Mohit

1) If the area of the region bounded by the line $y = mx + c$ and the curve $x^2 = y$ is $\frac{32}{7}$sq.unit then find the positive value of m.

Variable	Description
$\mathbf{x_1}$	First intersection point
$\mathbf{x_2}$	Second intersection point
x_1	x co-ordinate of First intersection point
x_2	x co-ordinate of Second intersection point
$\mathbf{h}$	Constant in equation of line
$\mathbf{m}$	Direction vector of given line
A	Area of the region

TABLE 1: Variables Used

The parameters of the given conic are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f = 0. \tag{1.1}$$

For the line, the parameters are

$$\mathbf{h} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1.2}$$

Substituing the line equation in parabola equation gives the values of κ

$$(\mathbf{h} + \kappa\mathbf{m})^\top \mathbf{V} (\mathbf{h} + \kappa\mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + \kappa\mathbf{m}) + f = 0 \tag{1.3}$$

$$\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix}\right)^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix}\right) + 2 \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}^\top \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ m \end{pmatrix}\right) + 0 = 0 \tag{1.4}$$

$$\begin{pmatrix} 1 & c + km \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ c + km \end{pmatrix} + \begin{pmatrix} 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ c + km \end{pmatrix} = 0 \tag{1.5}$$

$$\begin{pmatrix} 0 & c + km \end{pmatrix} \begin{pmatrix} 1 \\ c + km \end{pmatrix} - (c + km) = 0 \tag{1.6}$$

$$(c + km)((c + km) - 1) = 0 \tag{1.7}$$

$$k = -\frac{c}{m}, \frac{1 - c}{m} \tag{1.8}$$

$$\tag{1.9}$$

Putting in equation of line

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{1.10}$$

$$\tag{1.11}$$

We will get x-coordinate of point of intersection are

$$x_1 = \frac{m - \sqrt{m^2 + 4c}}{2}, x_2 = \frac{m + \sqrt{m^2 + 4c}}{2} \tag{1.12}$$

From the desired area is

$$\mathbf{A} = \int_{x_1}^{x_2} (mx + c - x^2)\, dx = \frac{m}{2}(x_2^2 - x_1^2) + c(x_2 - x_1) - \frac{1}{3}(x_2^3 - x_1^3) = 32/7 \tag{1.13}$$

On solving,we get m=2.86 and c=2

Hence, the slop line is 2.86.

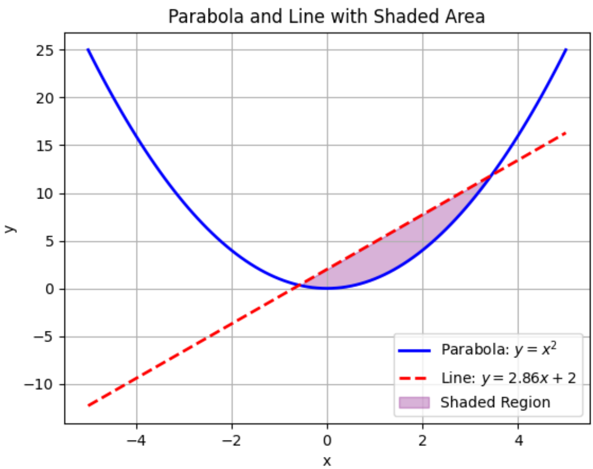

Fig. 1.1: Plot of curves