Question 1-1.4-9p

EE24BTECH11041 - Mohit

1) Let $\mathbf{A}\begin{pmatrix}4\\2\end{pmatrix}$,$\mathbf{B}\begin{pmatrix}6\\5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\4\end{pmatrix}$ be the vertices of $\Delta\ ABC$.Find the coordinates of points **Q** and **R** on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$.

Variable	Description	Values
A	Points on triangle ΔABC	$\begin{pmatrix}4\\2\end{pmatrix}$
B	Points on triangle ΔABC	$\begin{pmatrix}6\\5\end{pmatrix}$
C	Points on triangle ΔABC	$\begin{pmatrix}1\\4\end{pmatrix}$
F	Mid point of AC	
E	Mid point of AB	

TABLE 1: Variables Used

Solution:-
F is the mid point of AB

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}6\\5\end{pmatrix}}{2} = \begin{pmatrix}5\\\frac{7}{2}\end{pmatrix} \tag{1.1}$$

E is the mid point of AC

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{C}}{2} = \frac{\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}1\\4\end{pmatrix}}{2} = \begin{pmatrix}\frac{5}{2}\\3\end{pmatrix} \tag{1.2}$$

By section formula,

$$\mathbf{R} = \frac{\mathbf{B}+K\mathbf{A}}{1+K} \tag{1.3}$$

It is given that $\frac{BQ}{QE} = \frac{2}{1}$
So,

$$\mathbf{Q} = \frac{\mathbf{B} + 2\mathbf{E}}{1+2} = \frac{\begin{pmatrix} 6 \\ 5 \end{pmatrix} + 2\begin{pmatrix} \frac{5}{2} \\ 3 \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{1.4}$$

It is given that $\frac{CR}{RF} = \frac{2}{1}$
So,

$$\mathbf{R} = \frac{\mathbf{C} + 2\mathbf{F}}{1+2} = \frac{\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{1.5}$$

Hence, Co-ordinates of $\mathbf{Q}$ and $\mathbf{R}$ are

$$\mathbf{Q}\begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \text{and } \mathbf{R}\begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{1.6}$$

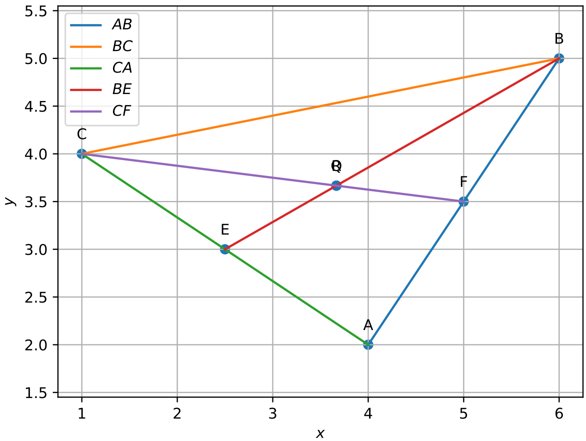

Fig. 1.1: Plot of Triangle ABC