Question 7-7.2-18

EE24BTECH11041 - Mohit

1) Equation of the circle with centre on the Y axis and passing through the orgin and the point (2, 3) is
   a) $x^2 + y^2 + 6x + 6y + 3 = 0$
   b) $x^2 + y^2 - 6x - 6y - 9 = 0$
   c) $x^2 + y^2 - 6x - 6y + 9 = 0$
   d) none of these

Variable	Description
$\mathbf{x_1}$	Point on circle
$\mathbf{x_2}$	Point on circle
$\mathbf{n}$	Equation of line on centre of circle lies
$\mathbf{u}$	Minus times the coordinate of centre
r	Radius of the circle
c	Constant in equation of line
f	$\|\mathbf{u}\|^2$ - r^2

TABLE 1: Variables Used

Solution:-

$$\mathbf{x}_1 = \begin{pmatrix}2\\3\end{pmatrix},\ \mathbf{x}_2 = \begin{pmatrix}0\\0\end{pmatrix},\ \mathbf{n} = \begin{pmatrix}1\\0\end{pmatrix},\ c = 0. \tag{1.1}$$

The centre is given by

$$\begin{pmatrix}2\mathbf{x}_1 & 2\mathbf{x}_2 & \mathbf{n}\\ 1 & 1 & 0\end{pmatrix}^{\top}\begin{pmatrix}\mathbf{u}\\ f\end{pmatrix} = -\begin{pmatrix}\|\mathbf{x}_1\|^2\\ \|\mathbf{x}_2\|^2\\ c\end{pmatrix} \tag{1.2}$$

Substituting values of $\mathbf{x}_1$,$\mathbf{x}_2$ and n

$$\begin{pmatrix}4 & 6 & 1\\ 0 & 0 & 1\\ -1 & 0 & 0\end{pmatrix}\begin{pmatrix}\mathbf{u}\\ f\end{pmatrix} = \begin{pmatrix}-13\\ 0\\ 0\end{pmatrix} \tag{1.3}$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c}4 & 6 & 1 & -13\\ 0 & 0 & 1 & 0\\ -1 & 0 & 0 & 0\end{array}\right) \tag{1.4}$$

Performing a sequence of row operations to transform into an Echelon form

$$\overset{R_1\leftrightarrow R_3}{\longleftrightarrow}\left(\begin{array}{ccc|c}1&0&0&0\\0&0&1&0\\4&6&1&-13\end{array}\right)\overset{R_3\to R_3-4R_1}{\longleftrightarrow}\left(\begin{array}{ccc|c}1&0&0&0\\0&0&1&0\\0&6&1&-13\end{array}\right)$$
$$\leftrightarrow\left(\begin{array}{ccc|c}1&0&0&0\\0&1&\frac{1}{6}&-\frac{13}{6}\\0&0&1&0\end{array}\right)\overset{R_2\to R_2-\frac{1}{6}R_3}{\longleftrightarrow}\left(\begin{array}{ccc|c}1&0&0&0\\0&1&0&-\frac{13}{6}\\0&0&1&0\end{array}\right)$$

So,the values of $\mathbf{u}$ and f are

$$\mathbf{u}=-\begin{pmatrix}0\\\frac{13}{6}\end{pmatrix},\ f=0. \tag{1.5}$$

The radius of circle is

$$r=\sqrt{(\|\mathbf{u}\|^2-f)}=\frac{13}{6} \tag{1.6}$$

The equation of circle is

$$\|\mathbf{x}\|^2-2\begin{pmatrix}0&\frac{13}{6}\end{pmatrix}\mathbf{x}=0 \tag{1.7}$$

OR

$$x^2+y^2-\frac{13}{3}y=0 \tag{1.8}$$

Hence,option (d) is correct

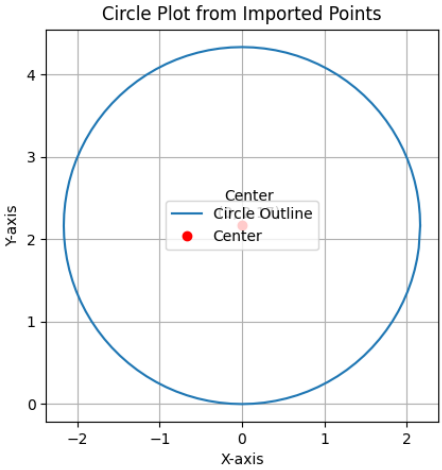

Fig. 1.1: Plot of circle