Question 9-9.3-11

EE24BTECH11041 - Mohit

1) If the area of the region bounded by the line $y = mx + c$ and the curve $x^2 = y$ is $\frac{32}{7}$sq.unit then find the positive value of m.

Variable	Description
h	Constant in equation of line
m	Direction vector of given line
A	Area of the region

TABLE 1: Variables Used

The parameters of the given conic are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f = 0. \tag{1.1}$$

For the line, the parameters are

$$\mathbf{h} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1.2}$$

x-coordinate of point of intersection are

$$x_1 = \frac{m - \sqrt{m^2 + 4c}}{2}, x_2 = \frac{m + \sqrt{m^2 + 4c}}{2} \tag{1.3}$$

From the desired area is

$$\mathbf{A} = \int_{x_1}^{x_2} (mx + c - x^2)\,dx = \frac{m}{2}(x_2^2 - x_1^2) + c(x_2 - x_1) - \frac{1}{3}(x_2^3 - x_1^3) = 32/7 \tag{1.4}$$

On solving,we get m=2.86 and c=2

Hence, the slop line is 2.86.

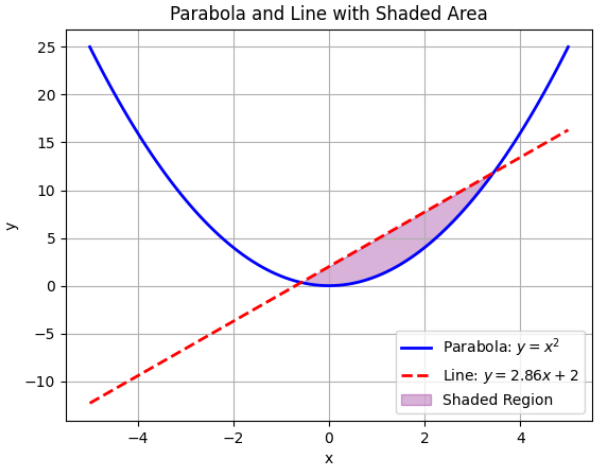

Fig. 1.1: Plot of curves