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The Straight line

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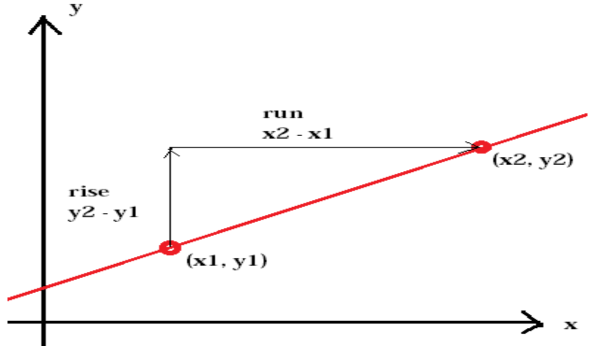
Straight line

Locus of a Point: A locus of a point is a path in which it moves in a plane or in a space by following the certain rules/conditions.

Straight line: A straight line is a straight one-dimensional figure having no thickness and extending infinitely in both directions.

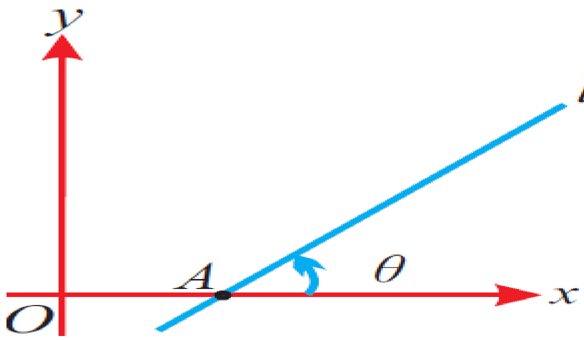
Slope: The slope is defined as the ratio of the vertical change between two points to the horizontal change between the same two points. The slope of a line is usually represented by the letter m .If the points are (x_1, y_1) and (x_2, y_2), then the slope is defined as

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$



Again, if a line makes an angle θ with the positive direction of the x- axis, then θ is called the inclination of the line and the slope is defined as,

$$m = \tan\theta.$$



Horizontal straight line: The equation of the horizontal line passing through the point (x_1, y_1) is $y = y_1$.

Note: 1. The equation of the $x-$axis is $y = 0$.

2. The equation of any line parallel to $x-$axis is $y = k$, where k is an unknown constant.

Vertical straight line: The equation of the vertical line passing through the point (x_1, y_1) is $x = x_1$.

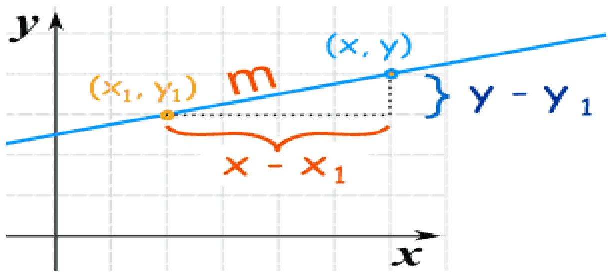
Note: 1. The equation of the $y-$axis is $x = 0$.

2. The equation of any line parallel to $y-$axis is $x = k$, where k is an unknown constant.

Straight line in point-slope form: Suppose that (x_1, y_1) is a fixed point on a non-vertical line, whose slope is m. Let (x, y) be an arbitrary point on the line. Then by the definition, the slope of the line is given by

$$m = \frac{y - y_1}{x - x_1}$$

$$\therefore\ y - y_1 = m(x - x_1).$$



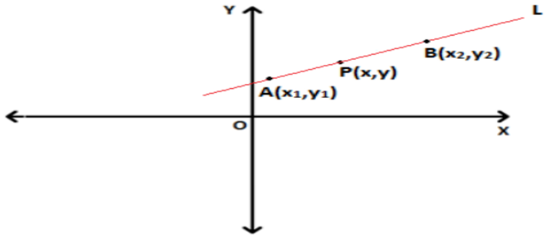
This is the equation of the straight line in point-slope form.

Note: The equation of the line passing through the origin and having slope m is $y = mx$.

Straight line in two-point form: Let $P(x, y)$ be any point on the line other than $A(x_1, y_1)$ and $B(x_2, y_2)$. Since AP and AB are on the same line, so the slope of the line segment AP is equal to the slope of the line segment AB.

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$i.e.\ \ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

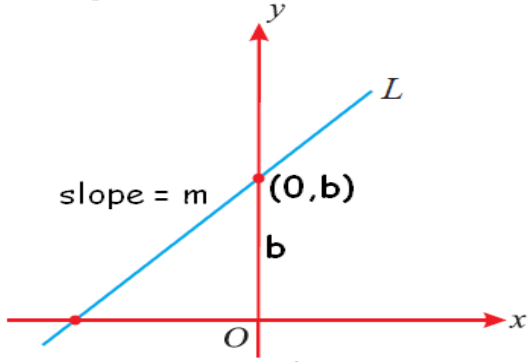


This is the equation of the straight line in two-point form.

Straight line in slope-intercept form: Suppose a line L with slope m cuts the axis at a distance b from the origin. The distance b is called the y-intercept of the line L. Obviously, the coordinate of the point where the line meet the y-axis is $(0, b)$. Thus, the line L has slope m and passes through a fixed point $(0, b)$. Therefore, by the point- slope form, the equation of the line L is,

$$m = \frac{y - b}{x - 0}$$

$$\therefore\ \ y = mx + b.$$

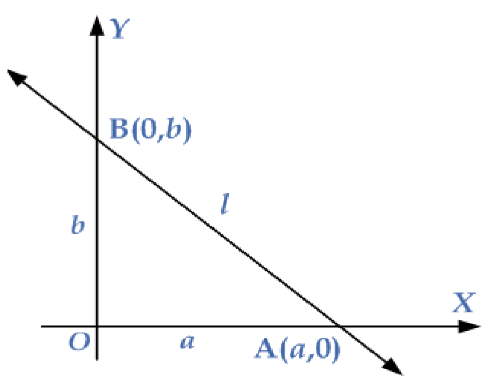


This is the equation of the straight line in slope-intercept form.

Straight line in intercept form: Let, the x-intercept of the line is a and the y-intercept of the line is b. So, the line cuts the x-axis at $A(0, a)$ and the y-axis at $B(0, b)$. Therefore, the equation of the line passing through $A(0, a)$ and $B(0, b)$ is,

$$\frac{y - 0}{0 - b} = \frac{x - a}{a - 0}$$

$$\therefore \frac{x}{a} + \frac{y}{b} = 1.$$



This is the equation of the straight line in intercept form.

Straight line in normal form: Let, the straight line cut the x-axis at A and the y-axis at B. Then the equation of the line in the intercept form is,

$$\frac{x}{OA} + \frac{y}{OB} = 1 \qquad \cdots(1)$$

Let p be the length of the perpendicular OD from the origin $O(0, 0)$ on the line (1) and let $\angle AOD = \alpha$.

$\therefore \ \angle BOD = 90^0 - \alpha$.

Then from the right angled ΔOAD, we have

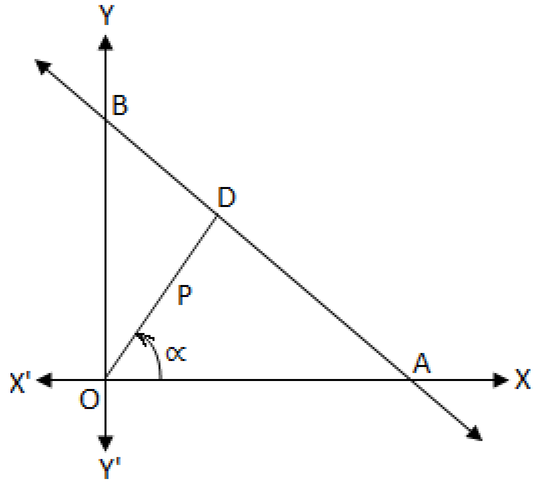
$$\frac{OD}{OA} = \cos\alpha \Rightarrow OA = \frac{p}{\cos\alpha}$$

and $$\frac{OD}{OB} = \cos\left(90^0 - \alpha\right) \Rightarrow OB = \frac{p}{\sin\alpha}.$$

So from (1), the required equation of the line is

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$$

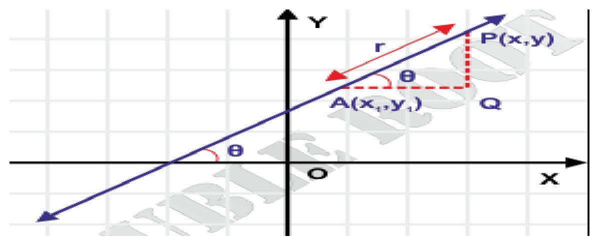
$$\therefore \ x\cos\alpha + y\sin\alpha = p\ .$$



This is the equation of the straight line in normal form.

Straight line in general form: A first degree equation in x and y represents a straight line. The equation $ax + by + c = 0$ is called the general equation of a line.

Straight line in parametric form: If $P(x, y)$ be any point on the line passing through $A(x_1, y_1)$ and having inclination θ, then parametric equation of such line is $x = x_1 + r\cos\theta$, $y = y_1 + r\sin\theta$, where r (r is a real parameter) is the distance from A to P .



Angle between two straight lines: Let, the two straight lines be,

$$y = m_1x + c_1 \qquad \cdots(1)$$

and $$y = m_2x + c_2 \qquad \cdots(2)$$

Let the lines (1) and (2) make the angles θ_1 and θ_2 with x-axis respectively. So the slopes of the lines are $\tan\theta_1$ and $\tan\theta_2$ respectively. But m_1 and m_2 are the slopes of (1) and (2) respectively, so that we have $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$.

Let θ be the angle between two straight lines, so from the figure we have

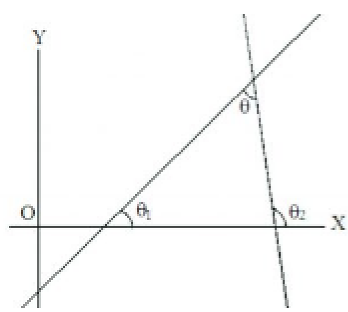
$$\theta = \theta_2 - \theta_1$$

$$or,\ \tan\theta = \tan\left(\theta_2 - \theta_1\right)$$

$$or,\ \tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2}$$

$$or,\ \tan\theta = \frac{m_2 - m_1}{1 + m_1m_2}$$

$$\therefore \ \theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1m_2}\right).$$



This is the required angle between the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$.

Note: 1. The lines are parallel if $\theta = 0$ i.e. $m_1 = m_2$.

2. The lines are perpendicular if $\theta = \frac{\pi}{2}$ i.e. $m_1 m_2 = -1$.

If the equations of the lines are $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, then the slopes of these equations are $\tan\theta_1 = -\frac{a_1}{b_1}$ and $\tan\theta_2 = -\frac{a_2}{b_2}$ respectively. In this case, the angle between the lines is

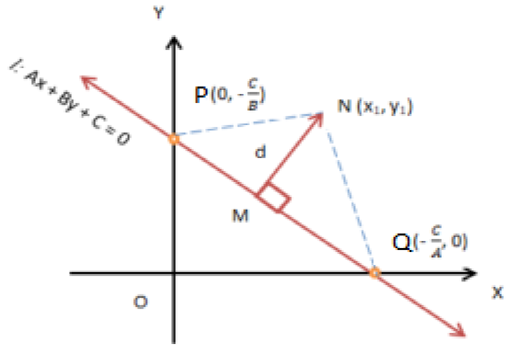
$$\theta = \tan^{-1}\left(\frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2}\right).$$

These lines are parallel if $a_2 b_1 - a_1 b_2 = 0$ and perpendicular if $a_1 a_2 + b_1 b_2 = 0$.

Distance of a point from a line: Let, the given point be $N(x_1, y_1)$ and the equation of the line l be

$$Ax + By + C = 0. \qquad \cdots(1)$$

Let the distance from the point $N(x_1, y_1)$ to the line l be d. Draw a perpendicular MN from the point to the line l. If the line meets the x-axis and y-axis at the points Q and P respectively, then the coordinates of the points are $Q\left(-\frac{C}{A}, 0\right)$ and $P\left(0, -\frac{C}{B}\right)$.



Thus, the area of the triangle NQP is given by,

Area of

$$\Delta NQP = \frac{1}{2} MN.PQ$$

$$\therefore MN = \frac{2(Area\ of\ \Delta NQP)}{PQ} \qquad \cdots(2)$$

Now, $$PQ = \sqrt{\left(-\frac{C}{A} - 0\right)^2 + \left(0 + \frac{C}{A}\right)^2}$$

$$= \frac{C\sqrt{A^2 + B^2}}{AB}$$

and area of $$\Delta NQP = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ -\frac{C}{A} & 0 & 1 \\ 0 & -\frac{C}{B} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[x_1\left(0 + \frac{C}{B}\right) - y_1\left(-\frac{C}{A} - 0\right) + 1\left(\frac{C^2}{AB} - 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{Cx_1}{B} + \frac{Cy_1}{A} + \frac{C^2}{AB}\right].$$

From (2), we get

$$MN=\frac{2.\frac{1}{2}\left[\frac{Cx_1}{B}+\frac{Cy_1}{A}+\frac{C^2}{AB}\right]}{\frac{C\sqrt{A^2+B^2}}{AB}}$$

$$or,\ d=\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$$

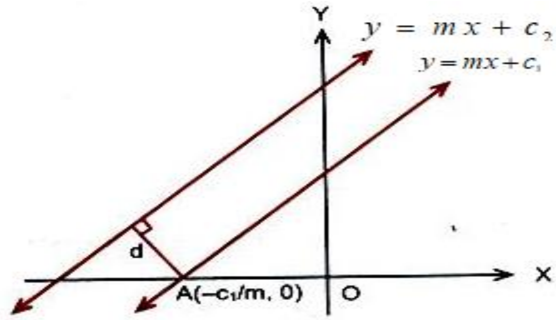
$$\therefore\ d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}.$$

This is the required distance/ length of perpendicular from the point $N(x_1, y_1)$ to the line $Ax+By+C=0$.

Distance between two parallel lines: Let, the equations of two parallel lines are,

$$y=mx+c_1 \quad \cdots(1)$$

and $$y=mx+c_2 \quad \cdots(2)$$



Line (1) will intersect x-axis at the point $A\left(-\frac{c_1}{m},0\right)$.

The distance between two lines is equal to the length of the perpendicular from the point $A\left(-\frac{c_1}{m},0\right)$ to the line (2).

i.e. $$d=\frac{\left|0+(-m)\left(-\frac{c_1}{m}\right)+(-c_2)\right|}{\sqrt{1+m^2}}=\frac{|c_1-c_2|}{\sqrt{1+m^2}}.$$

Note: The distance between two parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is $d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$.

Point of intersection of two straight lines: Let us assume the equations of two given straight lines are

$$a_1x+b_1y+c_1=0 \quad \cdots(1)$$

and $$a_2x+b_2y+c_2=0 \quad \cdots(2)$$

Solving these for x and y, we have

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\therefore\ x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\quad y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

So the point of intersection of two straight lines (1) and (2) is $\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$.

Equations of the line passing through the intersection of two lines: Let us consider two straight lines P and Q whose equations are

$$a_1x+b_1y+c_1=0 \quad \cdots(1)$$

and $$a_2x+b_2y+c_2=0 \quad \cdots(2)$$

respectively. The equation of the line passing through these two lines is

$$(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0$$

$$or,\ P+\lambda Q=0$$

where λ is an arbitrary constant.

Area of a triangle formed by three given lines: Let us consider the equations of three given lines are

$$a_1x+b_1y+c_1=0 \quad \cdots(1)$$

$$a_2x+b_2y+c_2=0 \quad \cdots(2)$$

and $$a_3x+b_3y+c_3=0 \quad \cdots(3)$$

The area of these three lines is given by

$$Area=\frac{\Delta^2}{2C_1C_2C_3}$$

where $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $C_1=\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$, $C_2=\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$ and $C_3=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$.

Theorem-01: Prove that the equation of a straight line is always of the first degree in x, y. Also its converse is true.

Proof: We know that through two given points, one and only one straight line can be drawn. Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be two given points on the straight line and $P(x,y)$ be any point on it. Then if P is a point on the straight line, the area of the triangle PAB must be zero.

i.e. $$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$$

$$or,\ x(y_1-y_2)-y(x_1-x_2)+1(x_1y_2-x_2y_1)=0$$

$$or,\ x(y_1-y_2)+y(x_2-x_1)+1(x_1y_2-x_2y_1)=0$$

$$\therefore\ ax+by+c=0,$$

where $a=y_1-y_2$, $b=x_2-x_1$, $c=x_1y_2-x_2y_1$.
This is the first degree equation in x, y.
Hence the equation of a straight line is always of the first degree in x, y.

Conversely: Every first degree equation in x, y represents a straight line. Let $ax+by+c=0$ be any first degree equation. Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be three points on it. Then their co-ordinates must satisfy this equation, i.e. we have

$$\left.\begin{aligned} ax_1+by_1+c=0 \\ ax_2+by_2+c=0 \\ ax_3+by_3+c=0 \end{aligned}\right\}$$

Now eliminating a, b, c from the above relations, we have

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0,$$

This is the condition for the points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ to be collinear. So these three points lie on a straight line, i.e. the equation $ax+by+c=0$ represents a straight line.
Hence the general equation of a straight line is of the form $ax+by+c=0$ and every first degree equation of the form $ax+by+c=0$ represents a straight line. (**Proved**)

Question-01: Find the equations of the bisectors of angles between two lines.

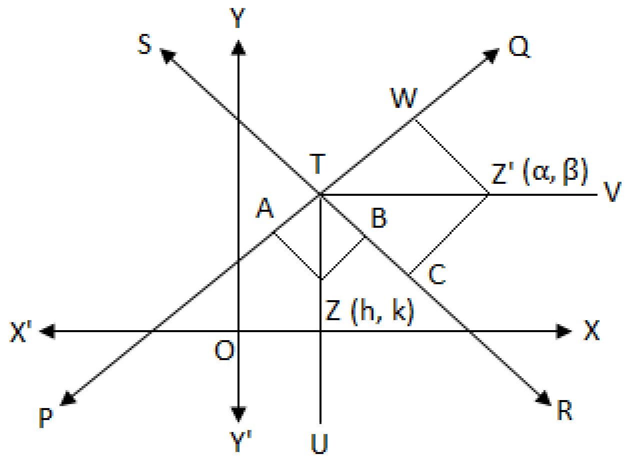
Answer: Let us assume the two given straight lines be PQ and RS whose equations are

$$a_1x+b_1y+c_1=0 \quad \cdots(1)$$

and $$a_2x+b_2y+c_2=0 \quad \cdots(2)$$

respectively, where c_1 and c_2 are of the same symbols.

Let the two straight lines PQ and RS intersect at T and $\angle PTR$ contains origin O. Again, let TV is the bisector of $\angle PTR$ and $Z(h,k)$ is any point on TU. Then the origin O and the point Z are on the same side of both the lines PQ and RS.



Again, let us assume that TU is the bisector of $\angle PTR$ and $Z(h,k)$ is any point on TU. Then the origin O and the point Z are on the same side of both the lines PQ and RS.

Therefore, c_1 and $a_1h+b_1k+c_1$ are of the same symbols as well as c_2 and $a_2h+b_2k+c_2$ are also of the same symbols. Since we already assumed that c_1 and c_2 are of the same symbols so $a_1h+b_1k+c_1$ and $a_2h+b_2k+c_2$ shall be of the same symbols.

Therefore, the lengths of the perpendiculars from Z upon PQ and RS are of the same symbols. Now, if $ZA \perp PQ$ and $ZB \perp RS$ then it implies that $ZA = ZB$.

i.e. $\dfrac{a_1h+b_1k+c_1}{\sqrt{a_1^2+b_1^2}} = \dfrac{a_2h+b_2k+c_2}{\sqrt{a_2^2+b_2^2}}$.

Therefore, the equation to the locus of $Z(h,k)$ is,

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \qquad \cdots(3)$$

This is the equation of the bisector of angle containing the origin.

Again, suppose that TU is the bisector of $\angle QTR$ which does not contain the origin O and $Z'(\alpha,\beta)$ is any point on TV. Then the origin O and the point Z' are on the same side of the line PQ but they are on opposite sides of the straight RS.

Therefore, c_1 and $a_1\alpha+b_1\beta+c_1$ are of the same symbols but c_2 and $a_2\alpha+b_2\beta+c_2$ are of opposite symbols. Since we already assumed that c_1 and c_2 are of the same symbols so $a_1\alpha+b_1\beta+c_1$ and $a_2\alpha+b_2\beta+c_2$ shall be of opposite symbols.

Therefore, the lengths of the perpendiculars from Z' upon PQ and RS are of opposite symbols. Now, if $Z'W \perp PQ$ and $Z'C \perp RS$ then it implies that $Z'W = -Z'C$.

i.e. $\dfrac{a_1\alpha+b_1\beta+c_1}{\sqrt{a_1^2+b_1^2}} = -\dfrac{a_2\alpha+b_2\beta+c_2}{\sqrt{a_2^2+b_2^2}}$.

Therefore, the equation to the locus of $Z'(\alpha,\beta)$ is,

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = -\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \qquad \cdots(4)$$

This is the equation of the bisector of angle not containing the origin.

From (3) and (4) it is seen that the equations of bisectors of the angles between the lines (1) and (2) are

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} .$$ (**Solved**)

Problem-01: Find the equations to lines passing through $(-5,6)$ and (a) parallel (b) perpendicular to $7x-8y=9$.

Solution: The given line is

$$7x-8y-9=0 \qquad \cdots(1)$$

First part: The equation parallel to (1) is

$$7x-8y+k=0 \qquad \cdots(2)$$

Since (2) passes through $(-5,6)$ so we have

$$7(-5)-8(6)+k=0$$

$$\therefore\ k=-83$$

Putting the value of k in (2), we have

$$7x-8y-83=0.$$

This is the required equation of line passing through $(-5,6)$ and parallel to $7x-8y=9$.

Second part: The equation perpendicular to (1) is

$$-8x-7y+k=0 \qquad \cdots(3)$$

Since (3) passes through $(-5,6)$ so we have

$$-8(-5)-7(6)+k=0$$

$$\therefore\ k=2$$

Putting the value of k in (3), we have

$$8x+7y-2=0.$$

This is the required equation of line passing through $(-5,6)$ and perpendicular to $7x-8y=9$.

Problem-02: Determine the equations to the bisectors of the angle between the lines $3x-4y+12=0$ and $12x+5y-30=0$.

Solution: The given lines are

$$3x-4y+12=0 \qquad \cdots(1)$$

and $$12x+5y-30=0 \qquad \cdots(2)$$

If (x, y) be any point on any one of the bisectors, then the equations of the bisectors are given by

$$\frac{3x-4y+12}{\sqrt{3^2+(-4)^2}}=\pm\frac{12x+5y-30}{\sqrt{12^2+5^2}}$$

$$or,\ \frac{3x-4y+12}{5}=\pm\frac{12x+5y-30}{13} \qquad \cdots(3)$$

Taking (+) sign, we have

$$21x+77y-306=0.$$

Taking (-) sign, we have

$$33x-9y+2=0.$$

Problem-03: Find the area of the triangle formed by the lines $2x+y-3=0$, $3x+2y-1=0$ and $2x+3y+4=0$. Also comment about the concurrency of these lines.

Solution: The given lines are

$$2x+y-3=0 \qquad \cdots(1)$$

$$3x+2y-1=0 \qquad \cdots(2)$$

and $$2x+3y+4=0 \qquad \cdots(3)$$

First part: The determinant formed by the equations (1), (2) and (3) is defined as

$$Area=\frac{\Delta^2}{2C_1C_2C_3} \qquad \cdots(4).$$

Here $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=\begin{vmatrix} 2 & 1 & -3 \\ 3 & 2 & -1 \\ 2 & 3 & 4 \end{vmatrix}=-7$

$$C_1=\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}=\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}=5$$

$$C_2=\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}=\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}=4$$

and $C_3=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}=1$

Putting these values in (4), we get

$$Area = \frac{(-7)^2}{2\times5\times4\times1} = \frac{49}{40}.$$

Second part: Since $\Delta \neq 0$, so the lines do not meet at a point i.e. they are not concurrent.

Problem-04: Find the equation of the line which passes through the point of intersection of the lines $7x-6y+6=0$, $2x+9y-5=0$ and perpendicular to $x-3y+19=0$.

Solution: The given lines are

$$7x-6y+6=0 \qquad \cdots(1)$$

$$2x+9y-5=0 \qquad \cdots(2)$$

and $$x-3y+19=0 \qquad \cdots(3)$$

Any line through the point of intersection of lines (1) and (2) is defined as

$$(7x-6y+6)+\lambda(2x+9y-5)=0 \quad ; (\lambda \neq 0)$$

$$or,\ (7+2\lambda)x+(-6+9\lambda)y+(6-5\lambda)=0 \qquad \cdots(4)$$

If (4) is perpendicular to (3), then we have

$$(7+2\lambda).1+(-6+9\lambda).(-3)=0$$

$$or, 7+2\lambda+18-27\lambda=0$$
$$\therefore \lambda=1$$

Putting the value of λ in (4), we get

$$9x+3y+1=0.$$

This is the required equation of the line.

Problem-05: Find the equation of the line which passes through the point of intersection of the lines $2x+5y-1=0$, $x-3y+2=0$ and which makes equal intercepts on the axes.

Solution: The given lines are

$$2x+5y-1=0 \qquad \cdots(1)$$

and $$x-3y+2=0 \qquad \cdots(2)$$

Any line through the point of intersection of lines (1) and (2) is defined as

$$(2x+5y-1)+\lambda(x-3y+2)=0 \quad ; (\lambda \neq 0)$$

$$or, (2+\lambda)x+(5-3\lambda)y+(-1+2\lambda)=0 \qquad \cdots(3)$$

The intercept form of (3) is

$$\frac{x}{\frac{1-2\lambda}{2+\lambda}}+\frac{y}{\frac{1-2\lambda}{5-3\lambda}}=1 \qquad \cdots(4)$$

Since the intercepts of (4) on the axes are equal so

$$\frac{1-2\lambda}{2+\lambda}=\frac{1-2\lambda}{5-3\lambda}$$

$$\therefore \lambda=\frac{3}{4}$$

Putting the value of λ in (3), we get

$$11x+11y+2=0.$$

This is the required equation of the line.

Problem-06: Find the equation of the bisector of that angle between the lines $4x-3y+1=0$ and $12x+5y+13=0$ in which the origin lies.

Solution: The given lines are

$$4x-3y+1=0 \qquad \cdots(1)$$

and $$12x+5y+13=0 \qquad \cdots(2)$$

If (x,y) be any point on the bisector in which the origin lies, then the equations of the bisector is given by

$$\frac{4x-3y+1}{\sqrt{4^2+(-3)^2}}=\frac{12x+5y+13}{\sqrt{12^2+5^2}}$$

$$or, \frac{4x-3y+1}{5}=\frac{12x+5y+13}{13}$$

$$or, 52x-39y+13=60x+25y+65$$

$$or, 8x+64y+52=0$$

$$\therefore \quad x+8y+13/2=0.$$

This is the required equation of the bisector of that angle between the given lines in which the origin lies.

Problem-07: Find the equations of the two straight lines passing through the point (2, 1) and inclined at angle of 45^0 with the line $5x+y-1=0$.

Solution: The given line is

$$5x+y-1=0 \qquad \cdots(1)$$

The equation can be written as

$$y=-5x+1 \qquad \cdots(2)$$

The of equation (2) is $m=-5$. The equations of the lines, which pass through (2, 1) and make angle $\theta=45^0$ with (2), are

$$y-1=\frac{m+\tan\theta}{1-m\tan\theta}(x-2) \qquad \cdots(3)$$

and
$$y-1=\frac{m-\tan\theta}{1+m\tan\theta}(x-2) \qquad \cdots(4)$$

From equation (3), we have

$$y-1=\frac{-5+1}{1+5}(x-2)$$

$$or,\ y-1=\frac{-2}{3}(x-2)$$

$$\therefore\ 2x+3y-7=0.$$

From equation (4), we have

$$y-1=\frac{-5-1}{1-5}(x-2)$$

$$or,\ y-1=\frac{3}{2}(x-2)$$

$$\therefore\ 3x-2y-4=0.$$

These are the required equations.

Exercise:

Problem-01: Find the equation of the line which passes through the point of intersection of the lines

$3x-5y+9=0$, $4x+7y-28=0$ and satisfies the following conditions.

(a). Passes through (4,2) Ans: $38x+87y-326=0$.

(b). Is parallel to $2x+3y-5=0$ Ans: $82x+123y-514=0$.

(c). Is perpendicular to $4x+5y-20=0$ Ans: $175x-114y+205=0$.

(d). Whose intercepts are equal Ans: $41x+41y-197=0$.

Problem-02: Prove that the three straight lines $2x-7y+10=0$, $3x-2y-1=0$ and $x-12y+21=0$ concur/meet at a point.

Problem-03: Show that the area of the triangle formed by the straight lines $y-2x=0$, $y-3x=0$ and $y=5x+4$ is $\frac{4}{3}$.

Problem-04: Find the equations of the two straight lines passing through the point (1, -3) and inclined at angle of $\tan^{-1}3$ with the line $2x-y+2=0$. Ans: $x+y+2=0,\ x+7y+20=0$.