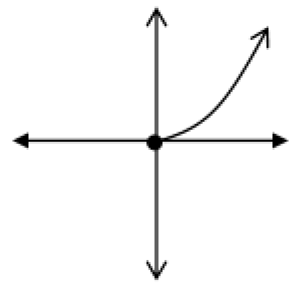
Functions and their graphs

Introduction: A function is an activity (work) but the graph is its reflection. A function is, so to say, completely observed only through its graph as we see that a man's image is clearly reflected by a mirror. In mathematics the graph of a function is the geometrical representation (visual form) of its equation. In physics the same thing is called the wave which as for the musician is the representation of a sound that a sound source makes.

Image of a man into a Mirror; it helps him to observe himself



Geometrical shape of the function $y = x^2, x \geq 0$

Function: If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then y is called a function of x and it is denoted by the following symbol,

$$y = f(x)$$

where x is independent variable and y is dependent variable. The inverse of this function is denoted by $f^{-1}(y) = x$.

Example: $y = x^2 + x + 1$; $y = sinx$; $y = e^x$; $y = lnx$ etc.

Alternatively, let A and B be two non empty sets. A mapping $f : A \rightarrow B$ is called function if each element in A is assigned to unique element in B.

Types of functions: There are many types of functions. These have been discussed as:

Single valued function: A function $y = f(x)$ is called a single valued function if there exist only one value of y for each value of x.

Example: $y = x^2 + 5$; $y = cosx$; $y = e^x + 2$; $y = lnx$ etc.

Many valued function: A function $y = f(x)$ is called a many valued function or multiple valued function if there exist more than one value of y for each value of x.

Example: $y^2 = 4ax$; $y = cos^{-1}x$; $y = sin^{-1}x$ etc.

Algebraic function: A function $y = f(x)$ which consists of a finite number of terms involving powers and roots of x is defined as an algebraic function.

Example: $y = 3x^2 + 4x + 1$ is an algebraic function.

Polynomial Function: A polynomial is an expression containing multiple terms with the operations of addition, subtraction, multiplication and degree of the each term is non-negative. A Function that consist with polynomial is called polynomial function.

Example: The function $f(x) = a_0 x^n + a_1 x^{n-1} + \cdots\cdots\cdots\cdots\cdots + a_n$ is a polynomial function of order n where $a_0,\ a_1,\ \cdots\cdots\cdots\cdots\cdots a_n \in R$ and $a_0 \neq 0$.

Note:

1. When $a_0 \neq 0$ then $f(\mathrm{x})$ is called polynomial function of order n.
2. When $a_0 = 1$ then $f(\mathrm{x})$ is called monic polynomial function of order n.
3. When $n = 0$ then $f(\mathrm{x})$ is called polynomial function of order zero means constant polynomial function.
4. When $n = 1$ then $f(\mathrm{x})$ is called polynomial function of order one (1) means linear polynomial function.
5. When $n = 2$ then $f(\mathrm{x})$ is called polynomial function of order two means polynomial function of degree 2 or Quadratic polynomial function. The graph of a quadratic polynomial is a parabola.
6. When $n = 3$ then $f(\mathrm{x})$ is called polynomial function of order three means polynomial function of degree 3 or Cubic polynomial function.
7. When $n = 4$ then $f(\mathrm{x})$ is called polynomial function of order four means polynomial function of degree 4 or by-quadratic polynomial function.
8. When $f(\mathrm{x}) = 0$ then this types of polynomial is called zero polynomial with explicitly undefined degree. The graph of a zero polynomial $f(\mathrm{x}) = 0$ is the x-axis.

Polynomials can be classified by the number of terms with nonzero coefficients, so that a one-term polynomial is called a monomial, a two-term polynomial is called a binomial, and a three-term polynomial is called a *trinomial*. The term "quadrinomial" is occasionally used for a four-term polynomial. A polynomial in one variable is called a *univariate polynomial*, a polynomial in more than one variable is called a multivariate polynomial. A polynomial with two variables is called a bivariate polynomial.

Linear polynomial function: A polynomial function in which degree/ order of the leading term is exactly one is called linear polynomial function.

Example: $f(x) = 3x + 5$ is a linear polynomial function with single variable x.

Quadratic polynomial function: **Case 01:** A polynomial function of the form $y = ax^2 + bx + c \ \textit{with}\ a \neq 0$ is called quadratic polynomial function which represents a parabola. When the value of "a "is positive then the parabola is concave up/open upward and otherwise concave down/open downward. The vertex of the parabola $y = ax^2 + bx + c \ \textit{with}\ a \neq 0$ is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$. In another way we get value of the ordinate of vertex by putting the value of abscissa $x = -\frac{b}{2a}$ in the equation $y = ax^2 + bx + c \ \textit{with}\ a \neq 0$.

Case 02: A polynomial function of the form $x = ay^2 + by + c \ \textit{with}\ a \neq 0$ is called quadratic polynomial function that represents geometrically a parabola. When the value of "a "is positive then the parabola is open right parabola and otherwise it is open left parabola. The vertex of the parabola $x = ay^2 + by + c \ \textit{with}\ a \neq 0$ is $\left(\frac{4ac - b^2}{4a}, -\frac{b}{2a}\right)$. In another way we get value of the abscissa of vertex by putting the value of ordinate $y = -\frac{b}{2a}$ in the equation $x = ay^2 + by + c \ \textit{with}\ a \neq 0$.

Rational function: A function of the form $f(x)=\dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ both are the function of x and also $q(x)\neq 0,$ is called a rational function.

Example: The function $f(x)=\dfrac{x^2-4x+3}{x^2+4x+7}$ is a rational function in single variable x.

Transcendental function: Functions that can't be expressed as algebraic functions are called transcendental functions. These functions are of the following types:

a) **Exponential function:** A function of the form $y=b^x$ $,where\ b>0$, is called an exponential function with base *b*.

Examples: $y=e^x$, $y=\pi^x$, $y=\left(\dfrac{1}{2}\right)^x$, etc.

b) **Logarithmic function:** A function of the form $y=\log_b x$ $,where\ x>0\ ,b>0\ and\ b\neq 1$ is called a logarithmic function with base *b*.

Examples: $y=\log x$, $y=\ln(x+1)$, etc.

c) **Trigonometric function:** Functions of the types $\sin x, \cos x, \tan x, \cot x$ *etc.* are called trigonometric functions.

d) **Inverse trigonometric functions:** Functions of the types $cos^{-1}x, sin^{-1}x,\ etc.$ are called inverse trigonometric functions.

Hyperbolic function:

Explicit function: When a relation of two variables x and $y$is expressed as$y=f(x)$ where$y$ can be expressed directly in terms of x, then y is called an explicit function of x.

Example:$y=ax^2+bx+c$ is an explicit function of x.

Implicit function: When a relation of two variables x and y is expressed as$f(x,y)=0$, where$x$and$y$ cannot be expressed directly in terms of the other, then either variable is called an implicit function of the other.

Example:$ax^2+2hxy+by^2+2gx+2fy+c=0$ is an implicit function.

Even function: A function $y=f(x)$ is called an even function if it satisfies the condition

$f(-x)=f(x)$.

Example:$y=cosx,\ \ y=x^4, etc.$ are even functions.

Odd function: A function $y=f(x)$is called an odd function if it satisfies the condition

$f(-x)=-f(x)$.

Example:$y=sinx,\ \ y=x^3, etc.$ are odd functions.

Periodic function: A function $y=f(x)$ is called a periodic function of period $\boldsymbol{T}$ if it satisfies the condition $f(x+\boldsymbol{T})=f(x)$.

Example: (1).$sinx\ and\ cosx$ are periodic function of period 2π.

(2). $tanx\ and\ cotx$ are periodic function of period π.

Absolute value function: A function $y = |f(x)|$ is called an absolute value function.

Example: $y = |x|$is an absolute value function.

Bounded function: A function $y = f(x)$ defined on an interval (a, b), is called a bounded function if there exists a number M such that $|f(x)| < M \ \forall\, x \in (a, b)$.

Or, A function $y = f(x)$ is called a bounded function if its range is a bounded set.

Example: $y = sinx$ is a bounded function.

Increasing function: A function $y = f(x)$ defined on an interval $(a, b)$where $a < b$, is called an increasing function over the interval if $f(a) < f(b)$.

Example: $y = x^2,\ 0 \le x \le 5$is an increasing function.

Decreasing function: A function $y = f(x)$ defined on an interval $(a, b)$where $a < b$, is called a decreasing function over the interval if $f(a) > f(b)$.

Example: $y = \frac{1}{x},\ 1 \le x \le 5$is a decreasing function.

One-one function:

Onto function:

Constant function:

Compositions of functions:

Inverse function:

Domain: The set of all values of$x$ for which the function $y = f(x)$ is defined, is called domain of the function. Simply domain is the set of all allowable x-values.

Mathematically, $D_f = \{x : y = f(\mathrm{x}) \in \mathrm{R} \text{ and } \mathrm{x} \in \mathrm{R}\}$.

Range: The set of all values of y corresponding to the x values for which the function $y = f(x)$ is defined, is called range of the function. Simply range is the set of all possible y-values.

Mathematically, $R_f = \{\mathrm{y} : y = f(\mathrm{x}) \in \mathrm{R} \text{ and } \mathrm{x} \in \mathrm{R}\}$.

Interval: If the set of all real numbers lie between two real numbers a and b, where $a < b$ then the set of all real numbers is called an interval.

Intervals are four kinds:

a) The set $\{x \epsilon \mathcal{R}: a \le x \le b\}$ is called a closed interval, denoted by $[a, b]$.
b) The set $\{x \epsilon \mathcal{R}: a < x < b\}$ is called an open interval, denoted by (a, b).
c) The set $\{x \epsilon \mathcal{R}: a < x \le b\}$ is called a left half open interval, denoted by $(a, b]$.
d) The set $\{x \epsilon \mathcal{R}: a \le x < b\}$ is called a right half open interval, denoted by $[a, b)$.

Problem 01: Find the domain and range of the function $y = 2x + 5$.

Solution: Given function is,

$$y = 2x + 5$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = R$$

Again, we have,

$$y = 2x + 5$$
$$or,\ 2x = y - 5$$
$$or,\ x = \frac{y-5}{2}$$

Here, x gives real values for all real values of y.
So, the range of the given function is,

$$R_f = R \text{ (Ans)}$$

H.W:
Find the domain and range of the following functions

1. $y = 3x + 5$ Ans: $D_f = R$ and $R_f = R$
2. $y = 4x - 3$ Ans: $D_f = R$ and $R_f = R$
3. $y = ax + b$ Ans: $D_f = R$ and $R_f = R$

Problem 02: Find the domain and range of the function $y = x^2 + 3x + 2$.

Solution: Given function is,

$$y = x^2 + 3x + 2$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = R$$

Again, we have

$$y = x^2 + 3x + 2$$
$$or,\ x^2 + 3x + (2 - y) = 0$$

In the above equation the values of x will be real if and only if its $Discriminant \geq 0$.

$$i.e,\ 3^2 - 4.1.(2 - y) \geq 0 \qquad ;\ [b^2 - 4ac \geq 0]$$
$$or,\ 9 - 4(2 - y) \geq 0$$
$$or,\ 9 - 8 + 4y \geq 0$$
$$or,\ 1 + 4y \geq 0$$
$$or,\ 4y \geq -1$$
$$or,\ y \geq -\frac{1}{4}$$

Therefore the range of the given function is,

$$R_f = [-\frac{1}{4}, \infty) \text{ (Ans)}$$

Alternative way, For range we have

$$y = x^2 + 3x + 2$$

$$or,\ x^2+3x+2=y$$

$$or,\ x^2+2.x.\frac{3}{2}+\left(\frac{3}{2}\right)^2+2-\frac{9}{4}=y$$

$$or,\ \left(x+\frac{3}{2}\right)^2-\frac{1}{4}=y$$

$$or,\ \left(x+\frac{3}{2}\right)^2=y+\frac{1}{4}$$

$$or,\ x+\frac{3}{2}=\pm\sqrt{y+\frac{1}{4}}$$

$$or,\ x=\pm\sqrt{y+\frac{1}{4}}-\frac{3}{2}$$

Here, x is defined if

$$y+\frac{1}{4}\geq 0$$

$$or,\ y\geq -\frac{1}{4}$$

Therefore the range of the given function is,

$$R_f=[-\frac{1}{4},\infty)\ (\text{Ans})$$

H.W:
Find the domain and range of the following quadratic functions

1. $y=x^2+5x+6$ Ans: $D_f=R$ and $R_f=[-\frac{1}{4},\infty)$
2. $y=-x^2+5x-6$ Ans: $D_f=R$ and $R_f=(-\infty,\frac{1}{4}]$
4. $y=-x^2+1$ Ans: $D_f=R$ and $R_f=(-\infty,1]$
5. $y=x^2+4x+7$ Ans: $D_f=R$ and $R_f=[3,\infty)$
6. $y=x^2-4x+3$ Ans: $D_f=R$ and $R_f=[-1,\infty)$
7. $y=(x+2)^2+3$ Ans: $D_f=R$ and $R_f=[3,\infty)$

Problem 03: Find the domain and range of the function $y=\frac{x-3}{2x+1}$.

Solution: Given function is,

$$y=\frac{x-3}{2x+1}$$

Here, y is undefined if

$$2x+1\ =\ 0$$

$$or,\ x=-\frac{1}{2}$$

So, y gives real values for all real values of x except $x=-\frac{1}{2}$.

Therefore, the domain of the given function is

$$D_f = R - \left\{ -\frac{1}{2} \right\}.$$

Again we have,

$$y = \frac{x-3}{2x+1}$$
$$or,\ 2xy + y = x - 3$$
$$or,\ x - 2xy = y + 3$$
$$or,\ x(1-2y) = y + 3$$
$$or,\ x = \frac{y+3}{1-2y}$$

Here, x is undefined if

$$1 - 2y = 0$$
$$or,\ y = \frac{1}{2}$$

So, x gives real values for all real values of y except $y = \frac{1}{2}$.

Therefore, the range of the given function is

$$R_f = R - \left\{ \frac{1}{2} \right\} \qquad \text{(Ans)}$$

Problem 04: Find the domain and range of the function $y = \frac{x^2-4}{x-2}$.

Solution: Given function is,

$$y = \frac{x^2-4}{x-2}$$

Here, y is undefined if

$$x - 2 = 0$$
$$or,\ x = 2$$

So, y gives real values for all real values of x except $x = 2$.
Therefore, the domain of the given function is

$$D_f = R - \{2\}.$$

Again we have,

$$y = \frac{x^2-4}{x-2}$$
$$or,\ y = \frac{(x+2)(x-2)}{x-2} \quad ;\ x \neq 2$$
$$or,\ y = x + 2 \quad ;\ x \neq 2$$
$$or,\ x = y - 2 \quad ;\ x \neq 2$$

Here, x is defined for all real values of y except $y = 4$

Therefore, the range of the given function is

$$R_f = R - \{4\} \qquad \text{(Ans)}$$

H.W:
Find the domain and range of the following quadratic functions

1. $y=\dfrac{x}{x+1}$ Ans: $D_f=R-\{-1\}$ and $R_f=R-\{1\}$
2. $y=\dfrac{1+x}{5-x}$ Ans: $D_f=R-\{5\}$ and $R_f=R-\{-1\}$
3. $y=\dfrac{2}{x+3}$ Ans: $D_f=R-\{-3\}$ and $R_f=R-\{0\}$
4. $y=\dfrac{x-3}{x^2-9}$ Ans: $D_f=R-\{-3,3\}$ and $R_f=R-\left\{0,\dfrac{1}{6}\right\}$
5. $y=\dfrac{4x+3}{x^2+1}$ Ans: $D_f=R$ and $R_f=[-1,4]$

Problem 05: Find the domain and range of the function $y=\sqrt{2x+5}$.
Solution: Given function is,

$$y=\sqrt{2x+5}$$

Here, y gives real values iff

$$2x+5 \ge 0$$
$$or,\ 2x \ge -5$$
$$or,\ x \ge -\frac{5}{2}$$

Therefore, the domain of the given function is

$$D_f=\left[-\frac{5}{2},\infty\right).$$

Again,

$$y=\sqrt{2x+5} \quad \cdots\cdots(1)$$

The values of y in (1) are positive or zero, $i.e, y \nless 0$.

Now $y^2=2x+5$; $y \nless 0$. [Squaring both sides]

$$2x+5=y^2 \quad ;y \nless 0.$$
$$2x=y^2-5 \quad ;y \nless 0.$$
$$x=\frac{y^2-5}{2} \quad ;y \nless 0.$$

Here, x is defined for $y \ge 0$.
Therefore, the range of the given function is

$$R_f=\{y : y \ge 0\}$$
$$=[0,\infty)$$ ***(Ans).***

Problem 06: Find the domain and range of the function $y=-\sqrt{1-2x}$.
Solution: Given function is,

$$y=-\sqrt{1-2x}$$

Here, y gives real values iff

$$1-2x \ge 0$$

$$or,\ -2x \ge -1$$
$$or,\ 2x \le 1$$
$$or,\ x \le \frac{1}{2}$$

Therefore, the domain of the given function is

$$D_f = \left(-\infty, \frac{1}{2}\right].$$

Again, we have,

$$y = -\sqrt{1-2x} \quad \cdots\cdots(1)$$

The values of y in (1) are negative or zero, *i.e*, $y \ngtr 0$.

Now $y^2 = 1-2x;\ y \ngtr 0$ [Squaring both sides]

$$1-2x = y^2;\ y \ngtr 0$$
$$2x = 1-y^2;\ y \ngtr 0$$
$$x = \frac{1-y^2}{2};\ y \ngtr 0$$

Here, x is defined for $y \le 0$.

Therefore, the range of the given function is

$$R_f = \{y : y \le 0\}$$
$$= (-\infty, 0]$$ **(Ans).**

H.W:

Find the domain and range of the following functions

1. $y=\sqrt{2x-1}$ Ans: $D_f = \left[\frac{1}{2}, \infty\right)$ and $R_f = [0, \infty)$
2. $y=\sqrt{1-5x}$ Ans: $D_f = \left(-\infty, \frac{1}{5}\right]$ and $R_f = [0, \infty)$
3. $y=\sqrt{2x-1}+5$ Ans: $D_f = \left[\frac{1}{2}, \infty\right)$ and $R_f = [5, \infty)$
4. $y=\sqrt{x+6}-3$ Ans: $D_f = [-6, \infty)$ and $R_f = [-3, \infty)$
5. $y=5-\sqrt{8-2x}$ Ans: $D_f = (-\infty, 4]$ and $R_f = [5, -\infty)$
6. $y=-\sqrt{x-1}$ Ans: $D_f = [1, \infty)$ and $R_f = (-\infty, 0]$
7. $y=-\sqrt{1-4x}$ Ans: $D_f = \left(-\infty, \frac{1}{4}\right]$ and $R_f = (-\infty, 0]$

Problem 07: Find the domain and range of the function $y=\sqrt{x^2-4x+3}$.

Solution: Given function is,

$$y = \sqrt{x^2-4x+3}$$

Here, y gives real values iff,

$$x^2-4x+3 \ge 0$$

$$or,\ x^2-3x-x+3\geq 0$$

$$or,\ x(x-3)-1(x-3)\geq 0$$

$$or,\ (x-3)(x-1)\geq 0$$

This inequality is satisfied if

$$x\leq 1\ or\ x\geq 3$$

Therefore, the domain of the given function is,

$$D_f=\{x:x\leq 1\}\cup\{x:x\geq 3\}$$

$$=(-\infty,1]\cup[3,\infty)$$

$$=R-(1,3)$$

Again, we have,

$$y=\sqrt{x^2-4x+3}\ \cdots\cdots (1)$$

The values of y in (1) are positive or zero *i.e,* ,$y\nless 0$.

Now, $y^2=x^2-4x+3\,;y\nless 0$ [Squaring both sides]

$$x^2-4x+3-y^2=0\,;y\nless 0$$

$$x^2-4x+\left(3-y^2\right)=0\,;y\nless 0$$

In the above equation the values of x will be real if and only if it's $Discriminant\geq 0$.

$$i.e,(-4)^2-4\times 1.\left(3-y^2\right)\geq 0\,;y\nless 0[\,b^2-4ac\geq 0\,]$$

$$or,\ 16-4\left(3-y^2\right)\geq 0\,;y\nless 0$$

$$or,\ 16-12+4y^2\geq 0\,;y\nless 0$$

$$or,\ 4+4y^2\geq 0\,;y\nless 0$$

$$or,\ 1+y^2\geq 0\,;y\nless 0$$

Here, x is defined for $y\geq 0$.

So the range of the given function is

$$R_f=\{y:y\geq 0\}$$

$=[0,\infty)$ **(Ans).**

Problem 08: Find the domain and range of the function $y=\sqrt{x^2+1}$.

Solution: Given function is,

$$y=\sqrt{x^2+1}$$

Here, y gives real values iff,

$$x^2+1\geq 0$$

This inequality is satisfied for all real values of x.

Therefore the domain of the given function is,

$$D_f=R.$$

Again, we have,

$$y=\sqrt{x^2+1} \quad \dots \dots (1)$$

The values of y in (1) are positive and lowest value is 1,*i.e*,$y \nless 1$.

$$\text{Now} \quad y^2 = x^2+1 \qquad ;y \nless 1 \qquad \text{[Squaring both sides]}$$

$$\Rightarrow x^2+1-y^2=0 \qquad ;y \nless 1$$

$$\Rightarrow x^2+0.x+\left(1-y^2\right)=0 \;; y \nless 1$$

In the above equation the values of x will be real if and only if its $Discriminant \geq 0$.

$$\text{i.e,} \; 0^2-4.1.\left(1-y^2\right) \geq 0 ;y \nless 1 [\, b^2-4ac \geq 0 \,]$$

$$or, \; -4\left(1-y^2\right) \geq 0 \,; y \nless 1$$

$$or, \; 4y^2-4 \geq 0 \,; y \nless 1$$

$$or, \; y^2-1 \geq 0 \,; y \nless 1$$

Here, x is defined for all $y \geq 1$.

$$R_f = \{\text{y} : \text{y} \geq 1\}$$

$$= [1, \infty) \; \textbf{(Ans).}$$

Problem 09: Find the domain and range of the function $y=\sqrt{4-x^2}$.

Solution: Given function is,

$$y=\sqrt{4-x^2}$$

Here, y gives real values iff,

$$4-x^2 \geq 0$$

$$or, \; (2+x)(2-x) \geq 0$$

This inequality is satisfied if,

$$-2 \leq x \leq 2$$

Therefore, the domain of the given function is,

$$D_f = \{x : -2 \leq \; x \leq 2\}$$

$$= [-2, \, 2]$$

Again, we have,

$$y=\sqrt{4-x^2} \quad \dots \dots (1)$$

The values of y in (1) are positive and lowest value is zero, *i.e*,$y \nless 0$.

$$\text{Now} \qquad y^2 = 4-x^2 \qquad ; \; y \nless 0 \quad \text{[Squaring both sides]}$$

$$\Rightarrow x^2+y^2-4=0 \qquad ;y \nless 0$$

$$\Rightarrow x^2+0.x+\left(y^2-4\right)=0 \qquad ;y \nless 0$$

In the above equation the values of x will be real if and only if it's $Discriminant \geq 0$.

$$\text{i.e,} \;\; 0^2-4.1.\left(y^2-4\right) \geq 0 \qquad ;y \nless 0 [\, b^2-4ac \geq 0 \,]$$

$$or, \; -4y^2+16 \geq 0 \qquad ;y \nless 0$$

$$or, \; y^2-4 \leq 0 \qquad ;y \nless 0 \text{ [Dividing by -4]}$$

Here, x is defined for all $0 \le y \le 2$.

Therefore the range of the given function is,

$$R_f = \{y : 0 \le y \le 2\}$$
$$= [0, 2] \qquad \text{(Ans.)}$$

H.W:

Find the domain and range of the following functions

1. $y = \sqrt{x^2 - 3}$ Ans: $D_f = R - \left(-\sqrt{3}, \sqrt{3}\right)$ and $R_f = [0, \propto)$
2. $y = \sqrt{x^2 - 25}$ Ans: $D_f = R - (-5, 5)$ and $R_f = [0, \propto)$
3. $y = \sqrt{x^2 + 3x}$ Ans: $D_f = R - (-3, 0)$ and $R_f = [0, \propto)$
4. $y = \sqrt{x^2 - 2x}$ Ans: $D_f = R - (0, 2)$ and $R_f = [0, \propto)$
5. $y = \sqrt{x^2 + 3}$ Ans: $D_f = R$ and $R_f = [3, \propto)$
6. $y = \sqrt{x^2 + 25}$ Ans: $D_f = R$ and $R_f = [25, \propto)$
7. $y = \sqrt{16 - x^2}$ Ans: $D_f = [-4, 4]$ and $R_f = [0, 4]$
8. $y = \sqrt{x^2 - 2x + 2}$ Ans: $D_f = R$ and $R_f = [1, \infty)$

Problem 10: Find the domain and range of the function $y = \dfrac{1}{\sqrt{2x+3}}$.

Solution: Given function is,

$$y = \frac{1}{\sqrt{2x+3}}$$

Here, y gives real values iff,

$$2x + 3 > 0$$
$$or,\ 2x > -3$$
$$or,\ x > -\frac{3}{2}$$

Therefore the domain of the given function is $D_f = \{x : x > -\frac{3}{2}\}$.

$$D_f = \left(-\frac{3}{2}, \infty\right)$$

Again, we have,

$$y = \frac{1}{\sqrt{2x+3}} \quad \cdots\cdots (1)$$

The values of y in (1) are positive and lowest value is near to 0,*i.e*, $y > 0$.

Now, $y^2 = \dfrac{1}{2x+3} \qquad ; y > 0$

$$or,\ 2x + 3 = \frac{1}{y^2} \qquad ; y > 0$$

$$or,\ 2x = \frac{1}{y^2} - 3 \qquad ; y > 0$$

$$or,\ x=\frac{1}{2}\left(\frac{1}{y^2}-3\right) \qquad ;y>0$$

Here, x is defined for all $y>0$.

Therefore the range of the given function is

$$R_f=\{y:y>0\}$$
$$=(0,\infty)$$ **(Ans)**

Problem 11: Find the domain and range of the function $f(x)=\sqrt{\frac{2x+3}{x-5}}$.

Solution: Given function is,

$$y=\sqrt{\frac{2x+3}{x-5}}$$

Here, y gives real values iff,

$$\frac{2x+3}{x-5}\geq 0$$

This inequality is satisfied if $x\leq -\frac{3}{2}$ or $x>5$.

Therefore the domain of the given function is,

$$D_f=\{x:x\leq -\frac{3}{2}\}\cup\{x:x>5\}.$$
$$=(-\infty,-\frac{3}{2}]\cup(5,\infty).$$

Again, we have,

$$y=\sqrt{\frac{2x+3}{x-5}} \quad \cdots\cdots\cdots (1)$$

The values of y in (1) are positive or zero,*i.e*,$y\not< 0$.

Now, $y^2=\frac{2x+3}{x-5}$ $\quad ;y\not<0 \qquad$ [Squaring both-sides]

$$or,\ xy^2-5y^2=2x+3 \quad ;y\not<0$$
$$or,\ xy^2-2x=5y^2+3 \quad ;y\not<0$$
$$or,\ x\left(y^2-2\right)=5y^2+3 \quad ;y\not<0$$
$$or,\ x=\frac{5y^2+3}{y^2-2} \qquad ;y\not<0$$
$$or,\ x=\frac{5y^2+3}{y^2-\left(\sqrt{2}\right)^2} \qquad ;y\not<0$$
$$or,\ x=\frac{5y^2+3}{\left(y-\sqrt{2}\right)\left(y+\sqrt{2}\right)} \qquad ;y\not<0$$

Here, $x$is defined for all $y\geq 0$ except $y=\sqrt{2}$.

So, the range of the given function is

$$R_f=\left\{y:y\geq 0\ ;\ y\neq\sqrt{2}\right\}$$
$$=[0,\infty)-\sqrt{2}$$ **(Ans.)**

Problem 12: Find the domain and range of $y=e^x$.

Solution: Given function is,

$$y=e^x$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f=R$$

Again, we have,

$$y=e^x$$
$$or,\ \ln y=x$$
$$or,\ x=\ln y$$

Here, x gives real values iff $y>0$.
So, the range of the given function is,

$$R_f=\{y: y>0\}$$
$$=(0,\infty)$$ **(Ans).**

Problem 13: Find the domain and range of $y=\ln\left(\frac{1+x}{1-x}\right)$.

Solution: Given function is,

$$y=\ln\left(\frac{1+x}{1-x}\right)$$

Here, y gives real values iff

$$\frac{1+x}{1-x}>0$$

This inequality is satisfied if $-1<x<1$.
So, the domain of the given function is,

$$D_f=\{x:-1<x<1\}$$
$$=(-1,1)$$

Again, we have,

$$y=\ln\left(\frac{1+x}{1-x}\right)$$
$$or,\ \frac{1+x}{1-x}=e^y$$
$$or, 1+x=e^y-xe^y$$
$$or, xe^y+x=e^y-1$$
$$or, x\left(e^y+1\right)=e^y-1$$

$$or, x = \frac{e^y - 1}{e^y + 1}$$

Here, x gives real values for all real values of y.
So, the range of the given function is,

$$R_f = \{y : -\infty < y < \infty\}$$
$$= (-\infty, \infty)$$
$$= R$$ **(Ans).**

Problem 14: Find the domain and range of $y = \sin x$.

Solution: Given function is,

$$y = \sin x$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = \{x : -\infty < x < \infty\}$$
$$= (-\infty, \infty)$$
$$= R$$

Again, we have,

$$y = \sin x$$
$$or,\ x = \sin^{-1} y$$

Here, x gives real values for $-1 \le y \le 1$.
So, the range of the given function is,

$$R_f = \{y : -1 < y < 1\}$$
$$= [-1, 1]$$ **(Ans).**

Problem 15: Find the domain and range of $y = \tan x$.

Solution: Given function is,

$$y = \tan x$$

Here, y gives real values for all real values of x except $x = (2n+1)\frac{\pi}{2}$; $where, n = 0, \pm 1, \pm 2, \cdots\cdots\cdots$

So, the domain of the given function is,

$$D_f = R - \left\{\cdots\cdots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots\cdots\right\}$$

Again, we have,

$$y = \tan x$$
$$or,\ x = \tan^{-1} y$$

Here, x gives real values for all real values of y.
So, the range of the given function is,

$$R_f = \{y : -\infty < y < \infty\}$$

$$=(-\propto, \propto)$$

$$= R$$ **(Ans).**

Problem 16: Find the domain and range of $y = \cot x$.

Solution: Given function is,

$$y = \cot x$$

Here, y gives real values for all real values of x except $x = n\pi \;\; ; where, n = 0, \pm 1, \pm 2, \cdots \cdots \cdots$

So, the domain of the given function is,

$$D_f = R - \{\cdots \cdots -2\pi, -\pi, 0, \pi, 2\pi, \cdots \cdots\}$$

Again, we have,

$$y = \cot x$$

$$or, \; x = \cot^{-1} y$$

Here, x gives real values for all real values of y.

So, the range of the given function is,

$$R_f = \{y : -\propto < y < \propto\}$$

$$=(-\propto, \propto)$$

$$= R$$ **(Ans).**

Problem 17: Find the domain and range of $y = \sec x$.

Solution: Given function is,

$$y = \sec x$$

Here, y gives real values for all real values of x except $x = (2n+1)\frac{\pi}{2} \;\; ; where, n = 0, \pm 1, \pm 2, \cdots \cdots \cdots$

So, the domain of the given function is,

$$D_f = R - \left\{\cdots \cdots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots \cdots\right\}$$

Again, we have,

$$y = \sec x$$

$$or, \; x = \sec^{-1} y$$

Here, x gives real values for all real values of y except $-1 < y < 1$.

So, the range of the given function is,

$$R_f = (-\propto, -1] \cup [1, \propto)$$

$$= R - (-1, 1)$$ **(Ans).**

Problem 18: Find the domain and range of $y = \cos ecx$.

Solution: Given function is,

$$y = \cos ecx$$

Here, y gives real values for all real values of x except $x = n\pi \;\; ; where, n = 0, \pm 1, \pm 2, \cdots \cdots \cdots$

So, the domain of the given function is,

$$D_f = R - \{\cdots\cdots -2\pi, -\pi, 0, \pi, 2\pi, \cdots\cdots\}$$

Again, we have,

$$y = \cos ecx$$

$$or,\ x = \cos ec^{-1} y$$

Here, x gives real values for all real values of y except $-1 < y < 1$.

So, the range of the given function is,

$$R_f = (-\propto, -1] \cup [1, \propto)$$

$$= R - (-1, 1)$$ **(Ans).**

H.W:

Find the domain and range of the following functions:

1. $y = e^{(x-2)}\ Ans: D_f = R\ \ and\ \ R_f = (0, \propto)$
2. $y = \ln(x-2)\ Ans: D_f = (2, \propto)\ \ and\ \ R_f = R$
3. $y = \cos x\ Ans: D_f = R\ \ and\ \ R_f = [-1, 1]$

Graph of Functions

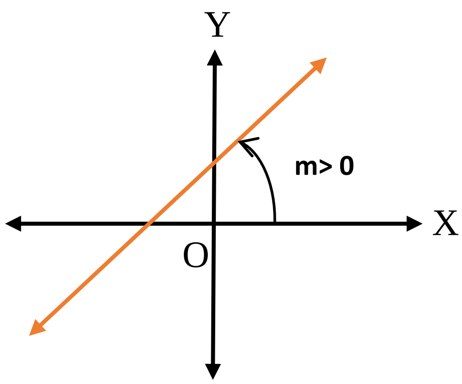
If $f: A \rightarrow B$ denotes a function, then the graph of the function $f(x)$ is the set of all ordered pairs $(x, f(x))$ for all values of x in the domain A.

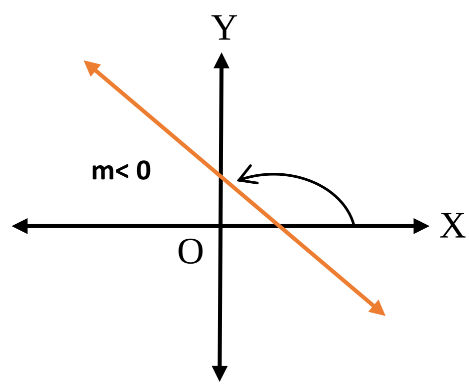
$\therefore$ Graph of $f(x) = \{(x, y): x \epsilon A, y = f(x) \epsilon B\}$

Therefore, Graph is the geometrical/Pictorial representation of a function or visualization of a function.

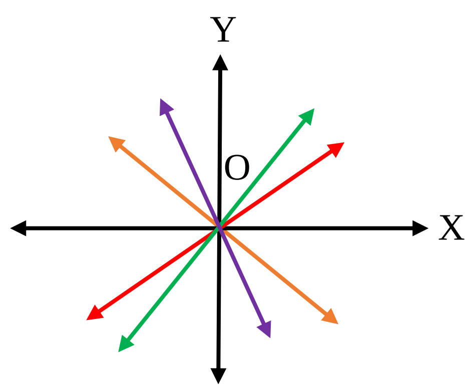
Graph of some elementary functions:

❖ ***Graph of*** $\boldsymbol{y = mx + c}$

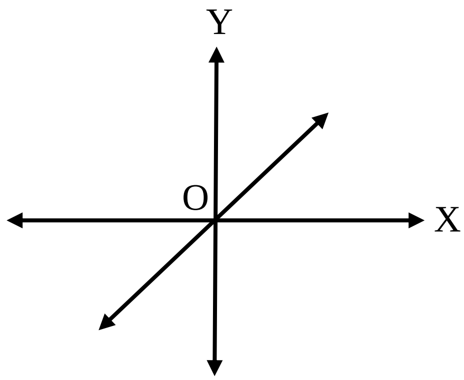


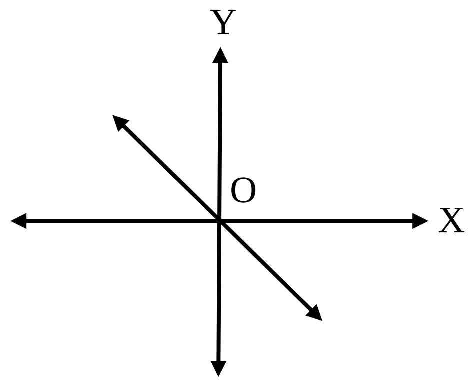


❖ ***Graph of*** $\boldsymbol{y = mx}$

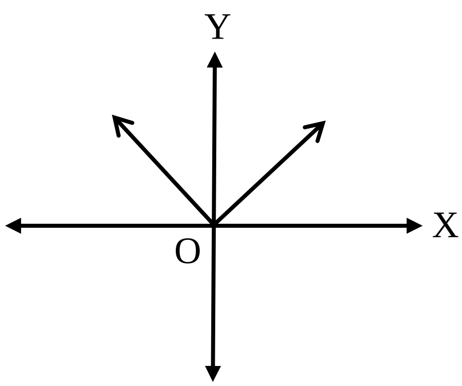


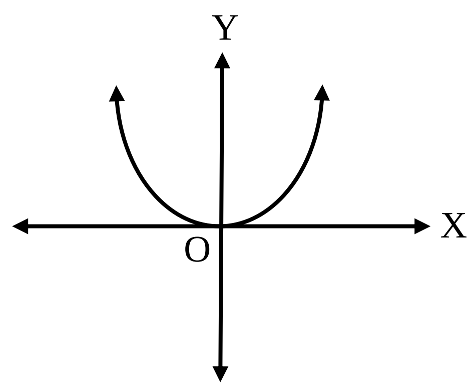
❖ ***Graph of*** $\boldsymbol{y = x}$ ⬟***Graph of*** $\boldsymbol{y = -x}$



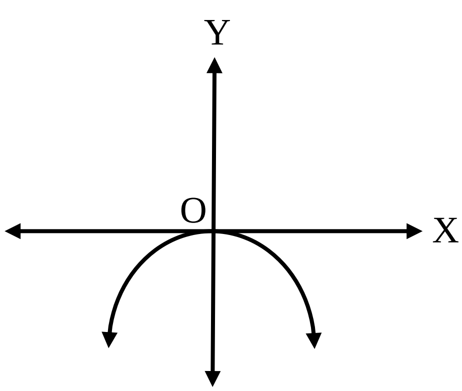


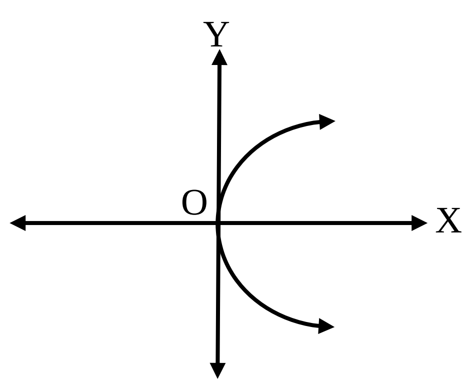
❖ ***Graph of*** $\boldsymbol{y = |x|}$ ⬟***Graph of*** $\boldsymbol{y = x^2}$





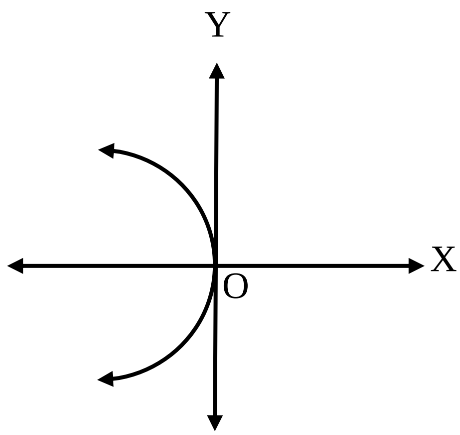
❖ ***Graph of*** $\boldsymbol{y = -x^2}$ ⬟***Graph of*** $\boldsymbol{x = y^2}$

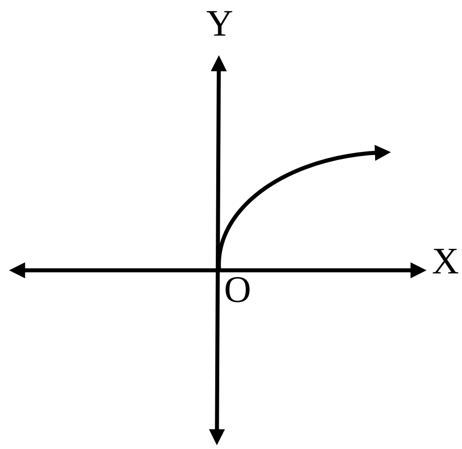




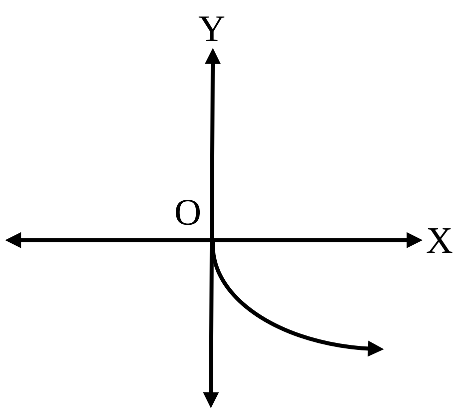
Note: *when power of the variable increases then graph will be wider.*

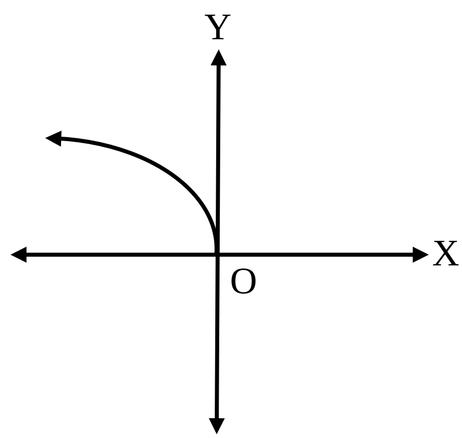
❖ ***Graph of*** $x = -y^2$ ⬟***Graph of*** $y = \sqrt{x}$



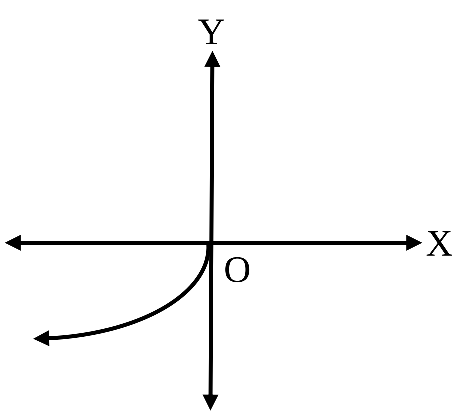


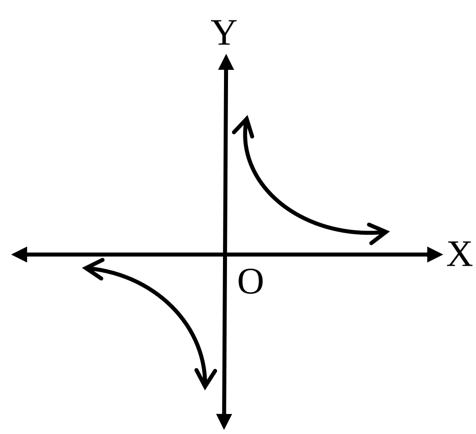
❖ ***Graph of*** $y = -\sqrt{x}$ ⬟***Graph of*** $y = \sqrt{-x}$



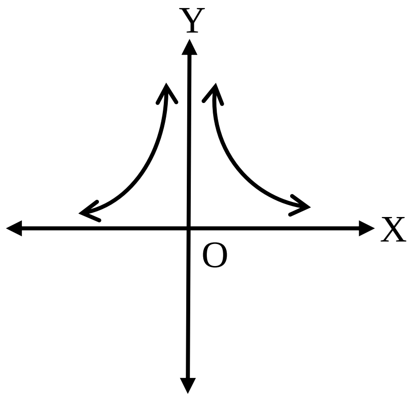


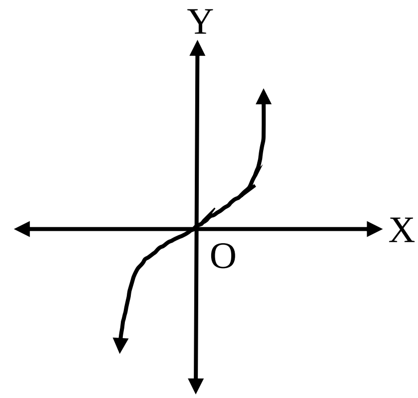
❖ ***Graph of*** $y = -\sqrt{-x}$ ⬟***Graph of*** $y = \frac{1}{x}$



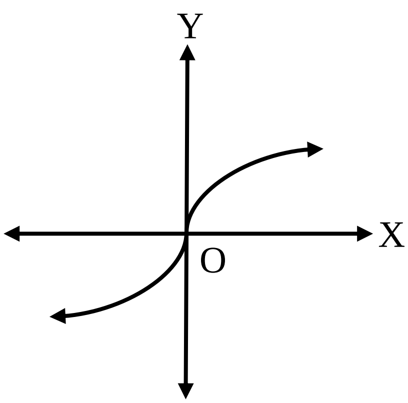


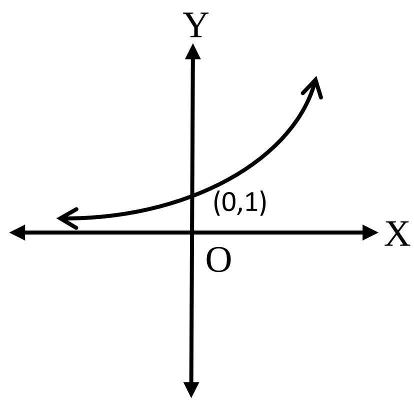
❖ ***Graph of*** $\boldsymbol{y = \frac{1}{x^2}}$ ⬟***Graph of*** $\boldsymbol{y = x^3}$



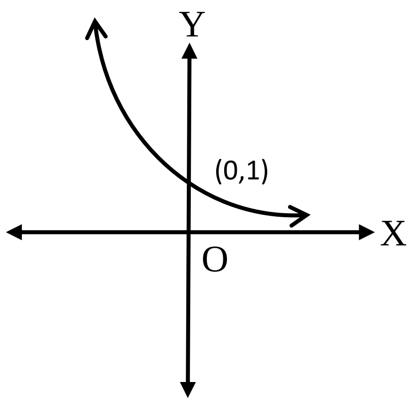


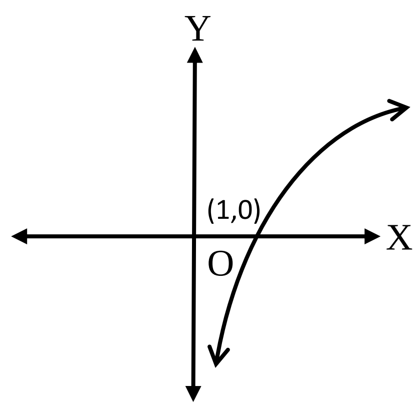
❖ ***Graph of*** $\boldsymbol{y = \sqrt[3]{x}}$ ⬟***Graph of*** $\boldsymbol{y = e^x}$



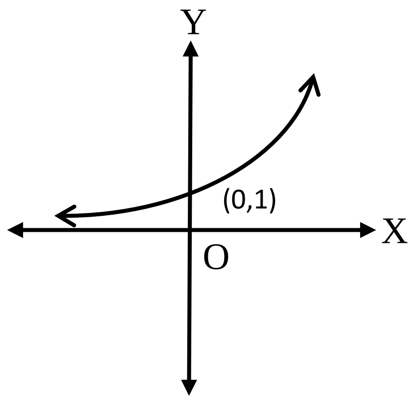


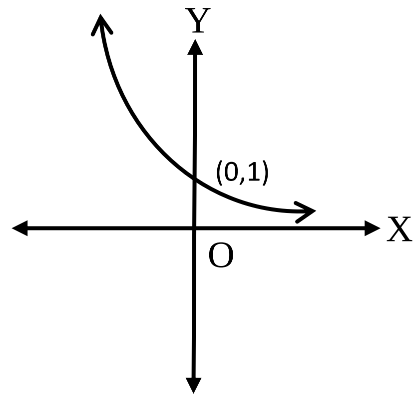
❖ ***Graph of*** $\boldsymbol{y = e^{-x}}$ ⬟***Graph of*** $\boldsymbol{y = ln|x|}$



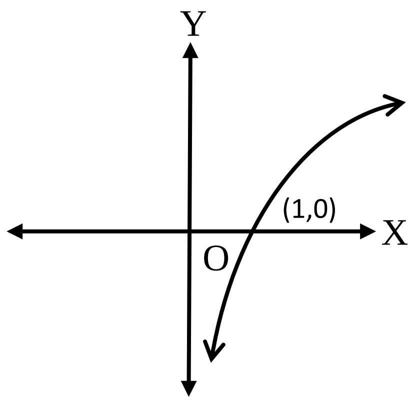


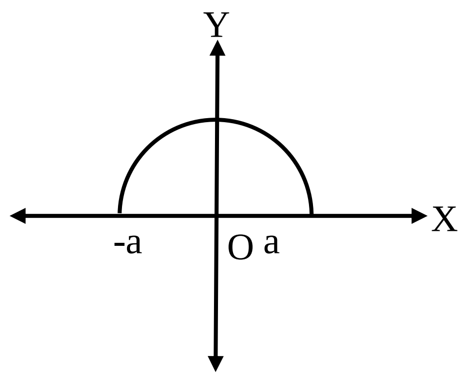
❖ ***Graph of*** $y = a^x$, $a > 1$ ⬟ ***Graph of*** $y = a^{-x}$



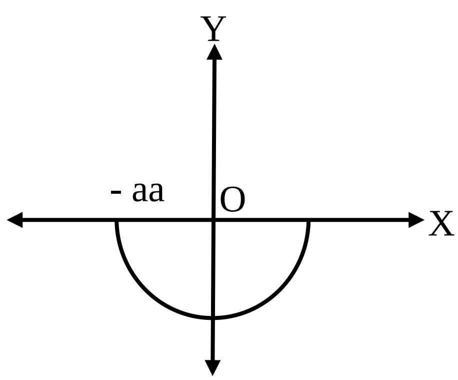


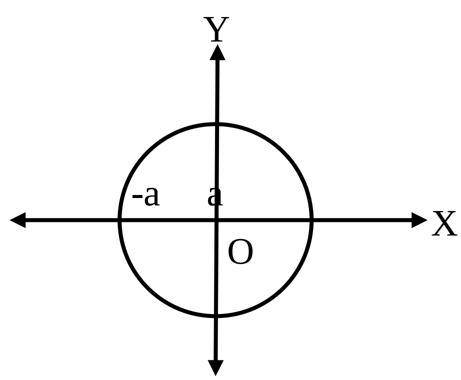
❖ ***Graph of*** $y = log_a|x|$, $a > 1$ ⬟ ***Graph of*** $y = \sqrt{a^2 - x^2}$



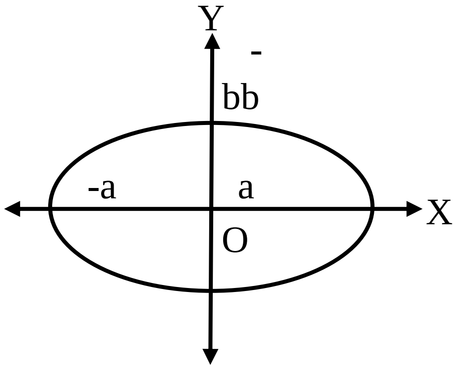


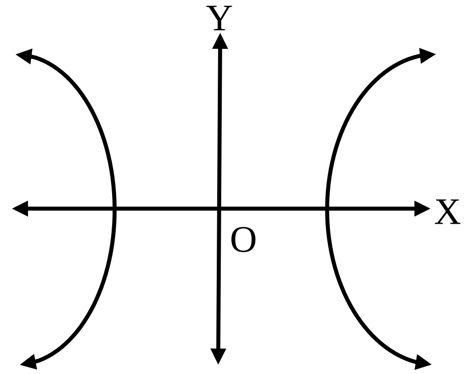
❖ ***Graph of*** $y = -\sqrt{a^2 - x^2}$ ⬟ ***Graph of*** $x^2 + y^2 = a^2$



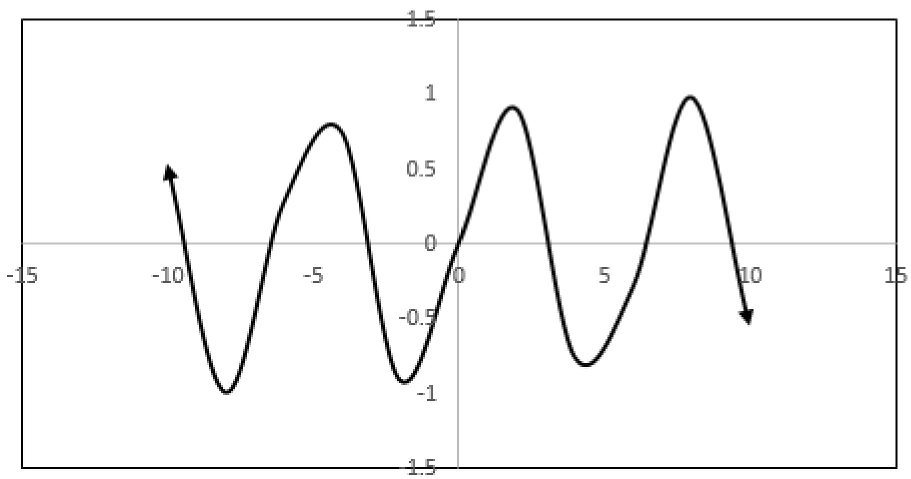


❖ ***Graph of*** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ⬟***Graph of*** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

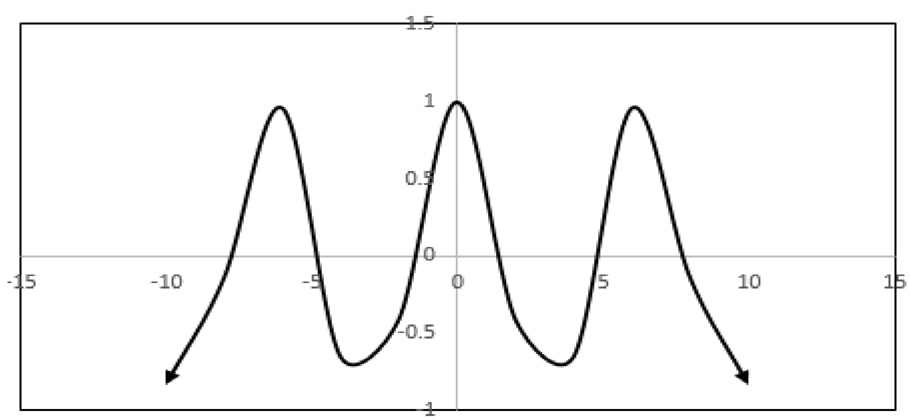




❖ ***Graph of*** $y = sinx$



❖ ***Graph of*** $y = cosx$



Transformation of Function

Transformation of a function is any kind of change in the function such as move or resize the graphs of functions. There are two types of transformation of thefunctions such as,

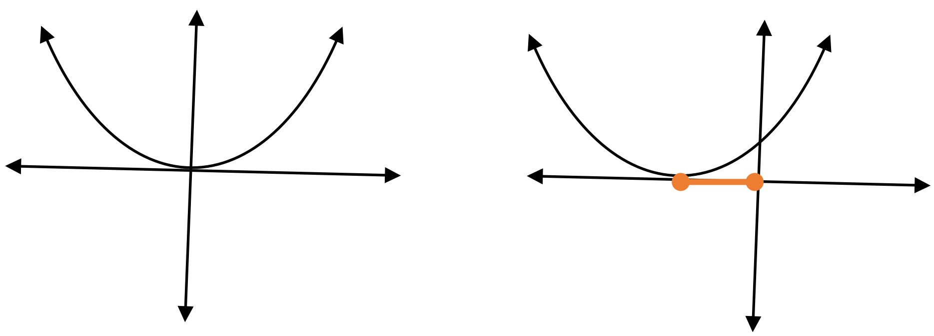
1. ***Translation/Shifting:***Any kind of shifting of the graph of a function is called translationof the function that means changing the location of the graph without changing its size and shape is called translation.
2. ***Scaling:***Scaling of a graph of a function is a transformation in which the size and shape of the graph is changed.

➢ *Translation*

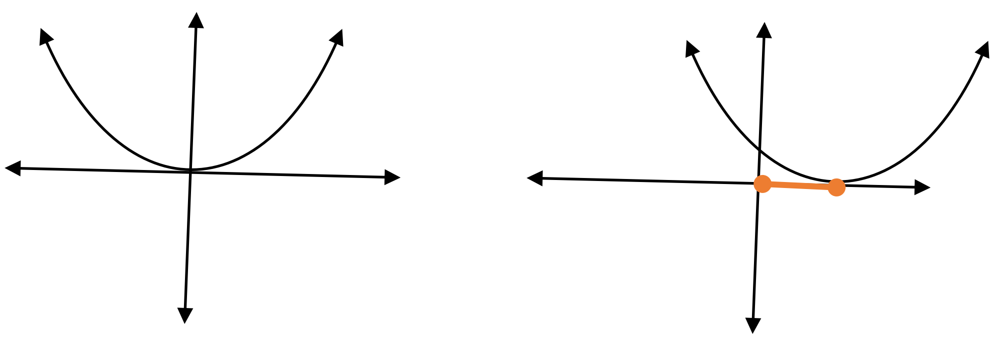
Horizontal translation:

Function: $g(x) = f(x + c)$

For$c > 0$*the graph is translated c units to the left.*

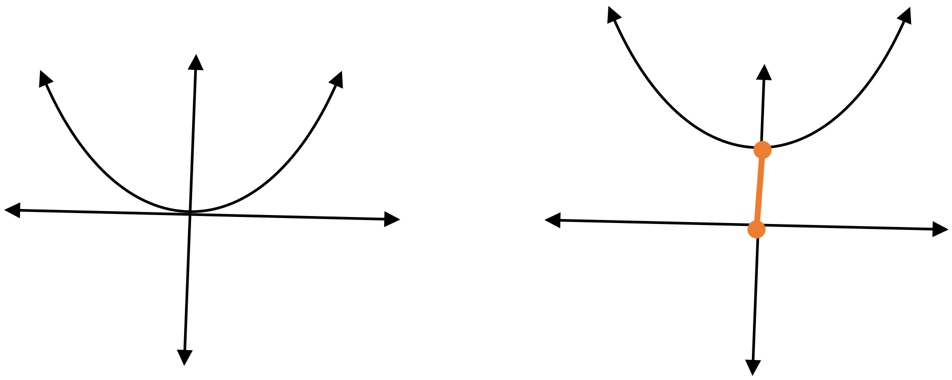


For$c < 0$*the graph is translated c units to the right.*

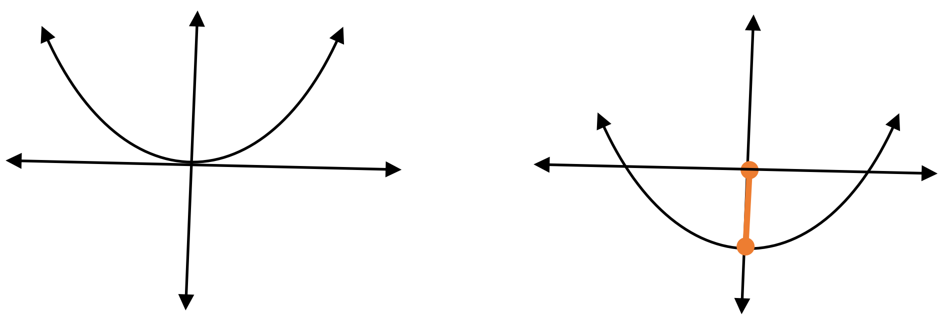


Vertical Translation:

Function: $g(x) = f(x) + c$
For $c > 0$ *the graph is translated c units upward.*

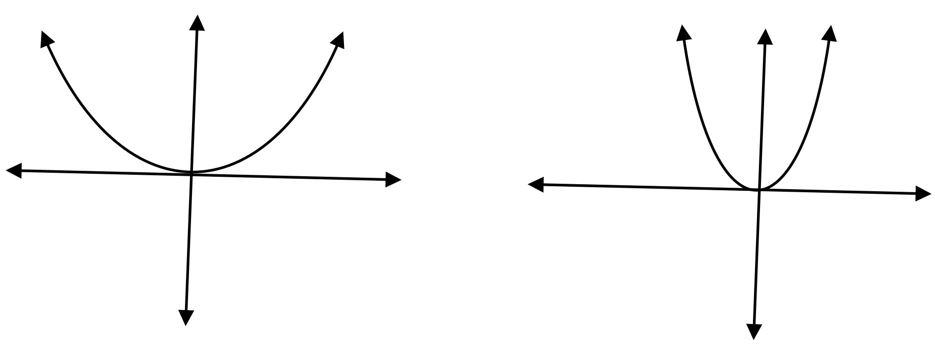


For $c < 0$ *the graph is translated c units downward.*



➢ ***Scaling***

Function: $g(x) = cf(x)$
For $|c| > 1$ $(integer)$ *the graph is compressed.*



For $|c| < 1$ $(integer)$ *the graph is stretched.*

Problem- 01: Sketch the graph of the function $y = x^2 + 6x + 10$.

Solution: The equation of the given function is,

$y = x^2 + 6x + 10$

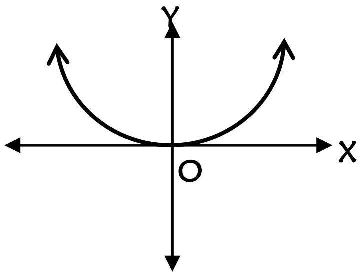
Completing the given equation in a square form it becomes as

$y = x^2 + 6x + 10$

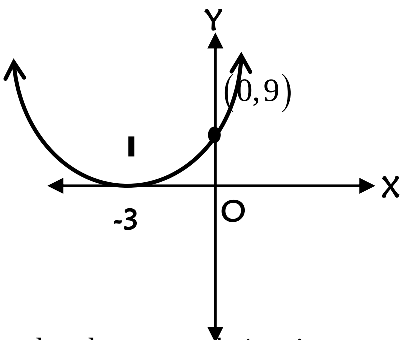
$= x^2 + 2.x.3 + 3^2 - 3^2 + 10$

$= (x+3)^2 + 1$

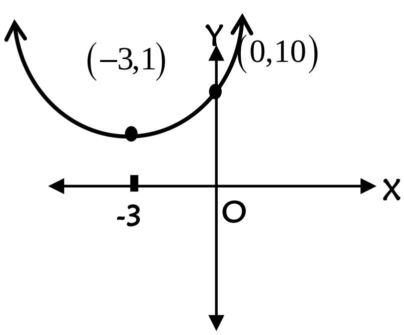
The graph of the standard function $y = x^2$ is as follows



Translating or shifting the above graph 3 units to the left, we get the graph of the function $y = (x+3)^2$.



Translating or shifting the above graph 1 units upward, we get the graph of the function $y = (x+3)^2 + 1$.



(Desired Graph)

H.W:

Sketch the graph of the following functions

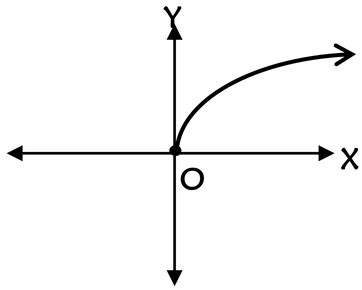
1. $y = x^2 + 4x + 10$ **4.** $y = 2x^2 - 5$
2. $y = 2x^2 + 5x + 10$ **5.** $y = 2x^2 + 5$
3. $y = x^2 - 4x + 5$ **6.** $y = -2(x+1)^2 - 3$

Problem -02: Sketch the graph of the function $y=\sqrt{x-2}+5$.

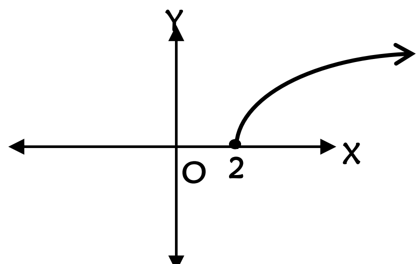
Solution: The equation of the given function is,

$$y=\sqrt{x-2}+5$$

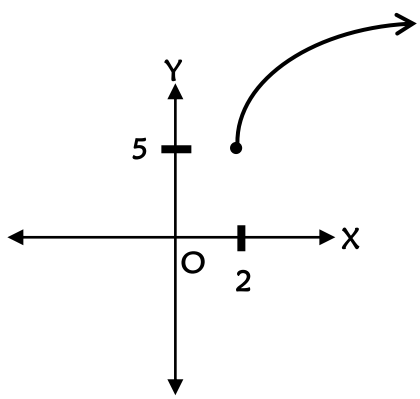
The graph of the standard positive square root function $y=\sqrt{x}$ is as follows



Translating or shifting the above graph 2 units to the right, we get the graph of the function $y=\sqrt{x-2}$.



Translating or shifting the above graph 5 units upward, we get the graph of the function $y=\sqrt{x-2}+5$.



(Desired Graph)

H.W:

Sketch the graph of the following functions

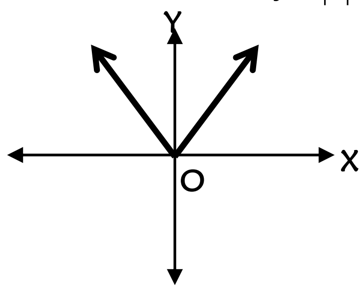
1. $y=\sqrt{x+2}$ **5.** $y=\sqrt{5-x^2}+6$
2. $y=\sqrt{2x+5}$ **6.** $y=\dfrac{1}{(x-3)^5}$
3. $y=2-\sqrt{x+5}$ **7.** $y=\sqrt{x^2-4x+4}$
4. $y=1-\sqrt[3]{x+2}$ **8.** $y=2-\dfrac{1}{x+1}$

Problem -03: Sketch the graph of the function $y=2-|x+2|$.

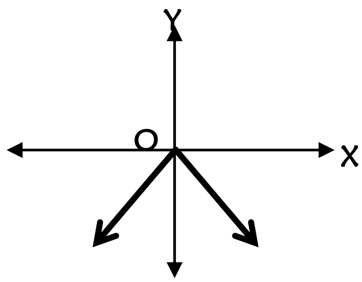
Solution: The equation of the given function is,

$$y = 2 - |x+2|$$

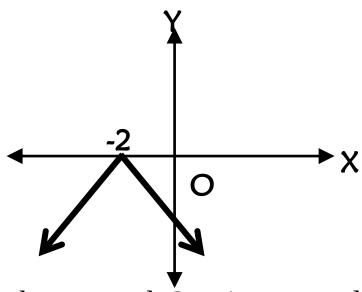
The graph of the standard absolute value function $y = |x|$ is as follows



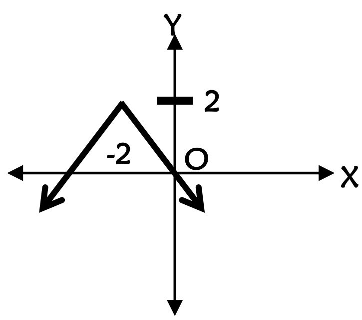
Therefore the graph of the standard absolute value function $y = -|x|$ is as follows



Translating or shifting the above graph 2 units to the left, we get the graph of the function $y = -|x+2|$.



Translating or shifting the above graph 2 units upward, we get the graph of the function $y = -|x+2| + 2$ or $y = 2 - |x+2|$.



(Desired Graph)

H.W: Sketch the graph of the following functions:

1. $y = |x+2| - 2$
2. $y = |x-2| + 3$
3. $y = 1 - |x-3|$

Piecewise function: A **piecewise-defined function** (also called a **piecewise function** or a **hybrid function**) is a function which is defined by multiple sub-functions, each sub-function applying to a certain interval of the main function's domain (a sub-domain).

For example: The following function is the piecewise function

$$y = f(x) = \begin{cases} f_1(x) & , \quad x < a \\ f_2(x) & , \quad a \le x < b \\ f_3(x) & , \quad x \ge b \end{cases}$$

Note that:

1. The function $f_1(x)$ is defined on the interval $(-\infty, a)$.
2. The function $f_2(x)$ is defined on the interval $[a, b)$.
3. The function $f_3(x)$ is defined on the interval $[b, \infty)$.

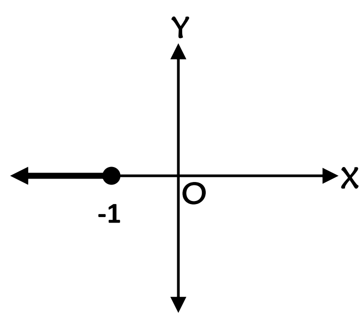
Mathematical Problem

Problem -01: Sketch the graph of the function $f(x) = \begin{cases} 0 & ; x \le -1 \\ \sqrt{1-x^2} & ; -1 < x < 1 \\ x & ; x \ge 1 \end{cases}$. Also find domain and range of the function.

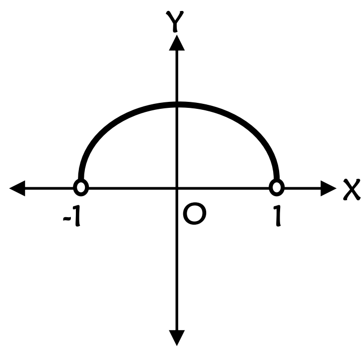
Solution: Given function is

$$y = f(x) = \begin{cases} 0 & ; x \le -1 \\ \sqrt{1-x^2} & ; -1 < x < 1 \quad [say] \\ x & ; x \ge 1 \end{cases}$$

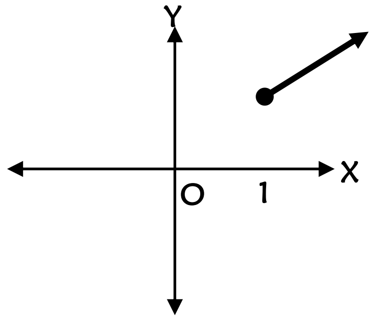
In the interval $x \le -1$ or $(-\infty, -1]$, the graph of the function $y = 0$ is,



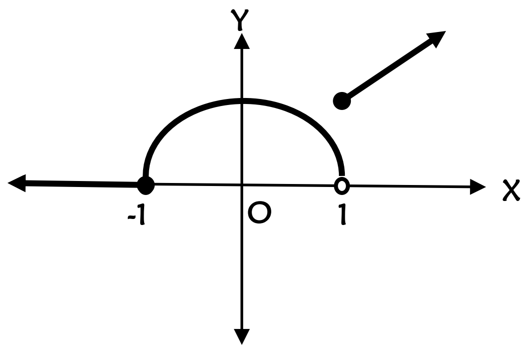
In the interval $-1 < x < +1$ or $(-1, 1)$, the graph of the function $y = \sqrt{1-x^2}$ which is an upper semi-circleof radius 1 units and Centre at origin is,



In the interval $x \geq 1$ or $[1, \infty)$, the graph of the function $y = x$ is,



Therefore, the graph of the given function is as follows:



(Desired Graph)

Again, the domain is,

$D_f = (-\infty, -1] \cup (-1, 1) \cup [1, \infty)$

$= (-\infty, \infty)$

$= R$

And the range is,

$R_f = \{0\} \cup (0, 1] \cup [1, \infty)$

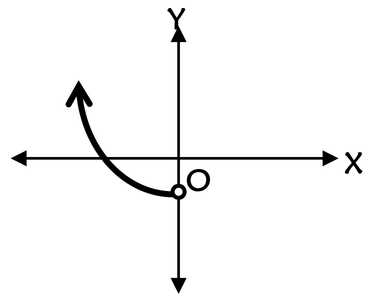
$= [0, \infty)$ **(Ans.)**

<u>**Problem -02:**</u>Sketch the graph of the function $f(x) = \begin{cases} x^2 - 1 & ; x < 0 \\ x & ; 0 \leq x \leq 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$. Also find domain and range of the function.

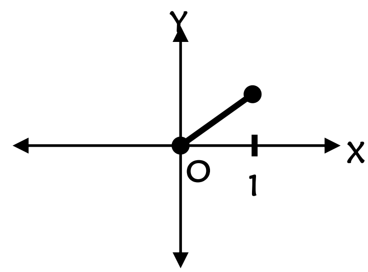
Solution: Given function is,

$$y = f(x) = \begin{cases} x^2 - 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases} \qquad [say]$$

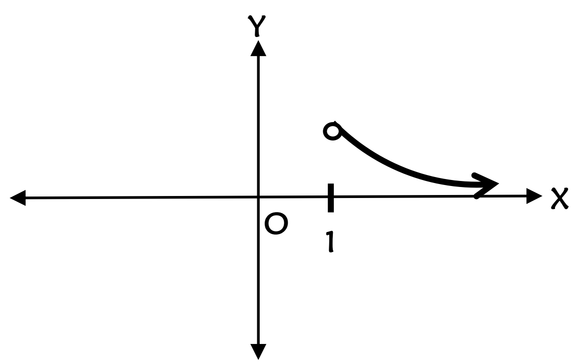
In the interval $x < 0$ or $(-\infty, 0)$, the graph of the function $y = x^2 - 1$ is,



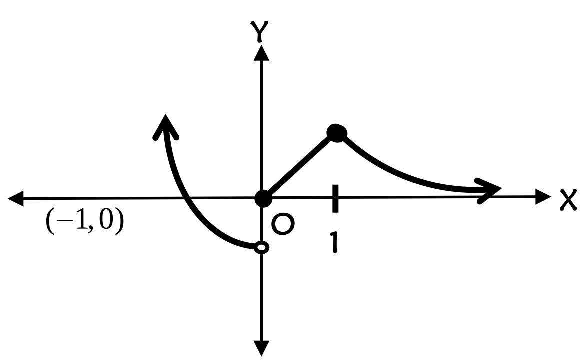
In the interval $0 \le x \le 1$ or $[0,1]$, the graph of the function $y = x$ is,



In the interval $x \ge 1$ or $[1, \infty)$, the graph of the function $y = \dfrac{1}{x}$ is,



Finally, the graph of the given function is as follows,



(Desired Graph)

Again, the domain is, $D_f = (-\infty, 0) \cup [0,1] \cup (1, \infty)$

$=(-\infty,\infty)$

$=R$

And the range is, $R_f=[-1,\infty)\cup[0,1]\cup(0,1)$

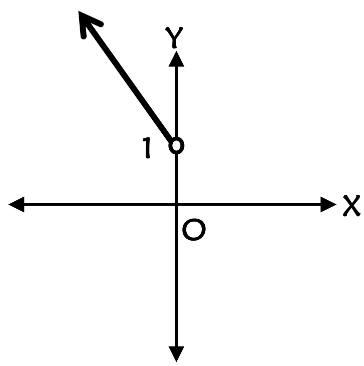
$=(-1,\infty)$ **(Ans.)**

<u>Problem -03:</u>Sketch the graph of the function $f(x)=\begin{cases}-2x+1 & ;x<0\\ 1 & ;0\le x<1\\ 2x-1 & ;x\ge 1\end{cases}$. Also find domain and range of the function.

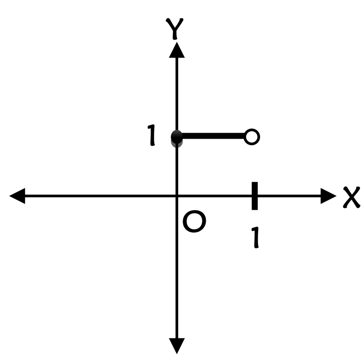
Solution: Given function is,

$$y=f(x)=\begin{cases}-2x+1 & ;x<0\\ 1 & ;0\le x<1\\ 2x-1 & ;x\ge 1\end{cases} \qquad [Say]$$

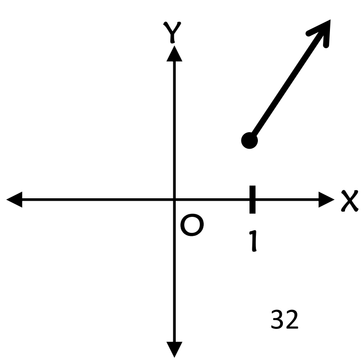
In the interval $x<0$ or $(-\infty,0)$, the graph of the function $y=-2x+1$ is,



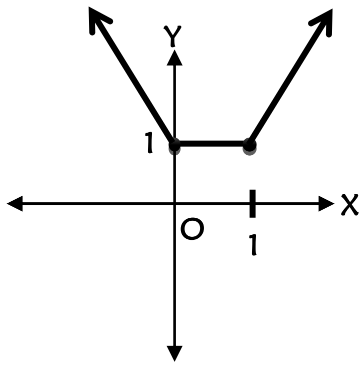
In the interval $0\le x<1$ or $[0,1)$, the graph of the function $y=1$ is,



In the interval $x\ge 1$ or $[1,\infty)$, the graph of the function $y=2x-1$ is,



Finally, the graph of the given function is as follows:



(Desired Graph)

Again, the domain is,

$D_f = (-\infty, 0) \cup [0,1) \cup [1, \infty)$

$= (-\infty, \infty)$

$= R$

And the range is,

$R_f = (1, \infty) \cup \{1\} \cup [1, \infty)$

$= [1, \infty)$ **(Ans.)**

H.W: Sketch the graph of the following piecewise functions:

1. $f(x) = \begin{cases} x^2 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$

2. $f(x) = \begin{cases} x^2 + 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$

3. $f(x) = \begin{cases} 1 - x & ; -1 \le x < 1 \\ 0 & ; 1 \le x \le 2 \\ x^2 - 4 & ; x > 2 \end{cases}$

4. $f(x)=\begin{cases} x^2+1 & ;x<0 \\ x & ;0\le x\le 1 \\ 0 & ;x>1 \end{cases}$

5. $f(x)=\begin{cases} 0 & ;1<x \\ 1+x & ;-1\le x<0 \\ 1-x & ;0\le x\le 1 \end{cases}$

6. $f(x)=\begin{cases} 2-x & ;x\ge 1 \\ x & ;0<x\le 1 \\ -x & ;x\le 0 \end{cases}$

7. $f(x)=\begin{cases} 0 & ;|x|>1 \\ 1+x & ;-1\le x\le 0 \\ 1-x & ;0<x<1 \end{cases}$

8. $f(x)=\begin{cases} \dfrac{x}{|x|} & ;x\ne 0 \\ 0 & ;x=0 \end{cases}$

Modulus/absolute function: The modulus or absolute value of x is denoted by the symbol $|x|$ and is defined as follows,

$$|x|=\begin{cases} x & ;\ x\ge 0 \\ -x & ;x<0 \end{cases}$$

Geometrically the modulus or absolute value of a number represents the distance of that number from the origin. The absolute value of x is always positive or zero.
A function together with modulus or absolute value sign is called modulus function.

For example: The function $f(x)=5|x+3|+2|x-2|$ is an absolute value function or Modulus function.

Breaking point of a function: Breaking point of a function is a point at which the function changes.

For example: The function $f(x)=5|x+3|+2|x-2|$ has two breaking points are $x=-3\ \&\ x=2$.

Procedure of Graphing Absolute value function:
1. At first convert the modulus function into piecewise function according to its number of breaking points.
2. After that sketch the graph as piecewise function.

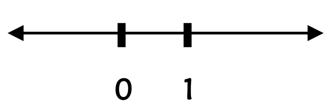
Mathematical Problem

Problem -01: Sketch the graph of the function $f(x)=|x|+|x-1|$.

Solution: Given absolute value function is,

$$y=f(x)=|x|+|x-1| \qquad [Say]$$

For breaking points $x=0$ and $x-1=0 \ \Rightarrow x=1$.



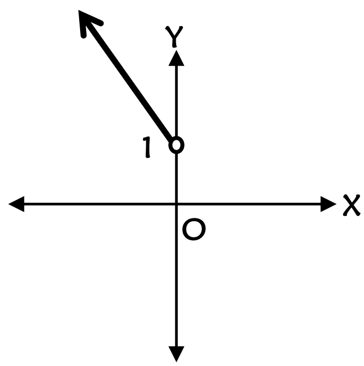
There are two breaking points in this mathematical problem such as $x=0$ & $x=1$ and these points divide real number line into three intervals. Therefore, we define this absolute value function section-ally by three parts.

Now, $y=|x|+|x-1|$

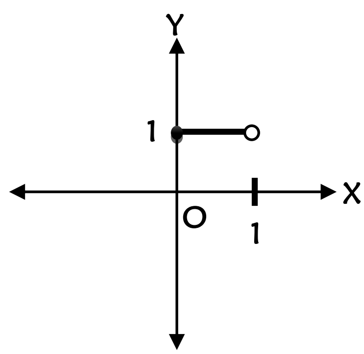
$$=\begin{cases} x+(x-1) & ;x\geq 1 \\ x+(-(x-1)) & ;0\leq x<1 \\ (-x)+(-(x-1)) & ;x<0 \end{cases}$$

$$=\begin{cases} 2x+1 & ;x\geq 1 \\ 1 & ;0\leq x<1 \\ -2x+1 & ;x<0 \end{cases}$$

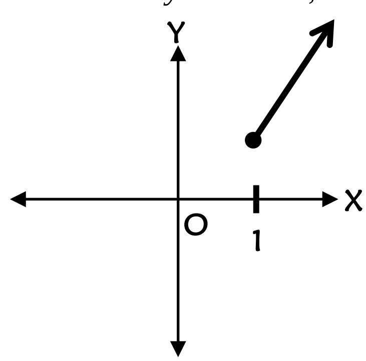
Graph: In the interval $x<0$ or $(-\infty,0)$, the graph of the function $y=-2x+1$ is,



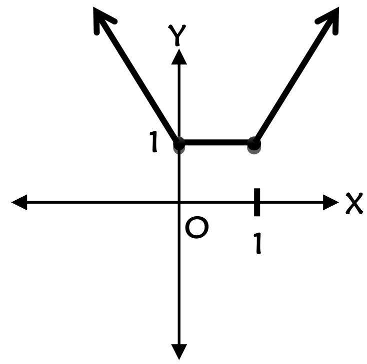
In the interval $0\leq x<1$ or $[0,1)$, the graph of the function $y=1$ is,



In the interval $x\geq 1$ or $[1,\infty)$, the graph of the function $y=2x-1$ is,



Finally, the graph of the given function is as follows:



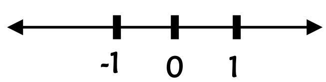
(Desired Graph)

<u>Problem -02:</u>Sketch the graph of the function $f(x)=|x-1|+|x|+|x+1|$.

Solution: Given absolute value function is,

$y=f(x)=|x-1|+|x|+|x+1|$ [*Say*]

For breaking points $x-1=0 \Rightarrow x=1$ and $x=0$ an also $x+1=0 \Rightarrow x=-1$



There are three breaking points in this mathematical problem such as $x=-1$, $x=0$ & $x=1$ and those points divide real number line into four intervals.Therefore, we define this absolute value function section-ally by four parts.

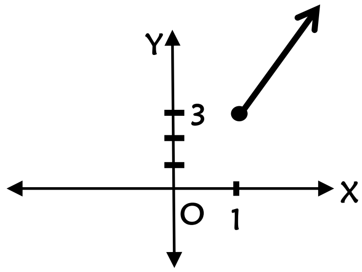
Now,

$y=|x-1|+|x|+|x+1|$

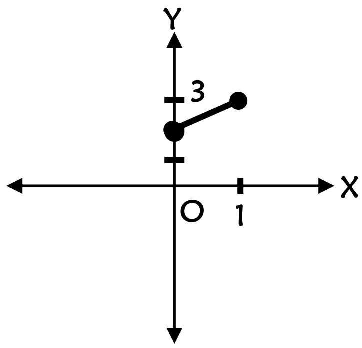
$$=\begin{cases} (x-1)+x+(x+1) & ;x\geq 1 \\ -(x-1)+x+(x+1) & ;0\leq x<1 \\ -(x-1)+(-x)+(x+1) & ;-1\leq x<0 \\ -(x-1)+(-x)+(-(x+1)) & ;x<-1 \end{cases}$$

$$=\begin{cases} 3x & ;x\geq 1 \\ x+2 & ;0\leq x<1 \\ -x+2 & ;-1\leq x<0 \\ -3x & ;x<-1 \end{cases}$$

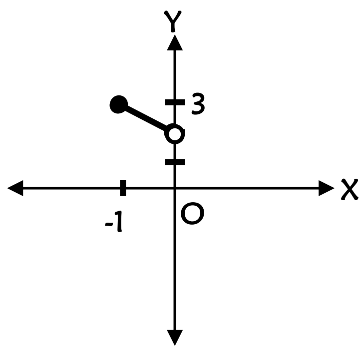
Graph: In the interval $x \geq 1$ or $[1, \infty)$, the graph of the function $y = 3x$ is,



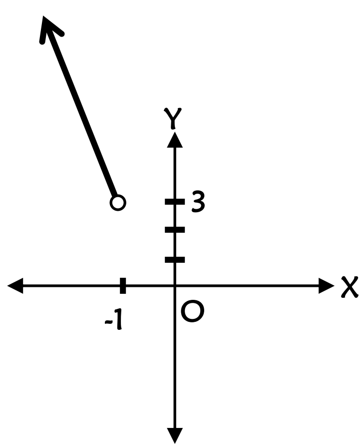
In the interval $0 \leq x < 1$ or $[0,1)$, the graph of the function $y = x + 2$ is,



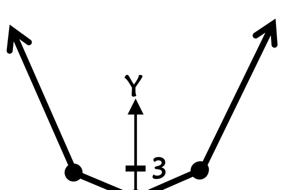
In the interval $-1 \leq x < 0$ or $[-1, 0)$, the graph of the function $y = -x + 2$ is,



In the interval $x < -1$ or $[-1, 0)$, the graph of the function $y = -3x$ is,



Finally, the graph of the given function is as follows:



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(Desired Graph)

H.W:

Sketch the graph of the following absolute value functions:

1. $f(x) = |x| - x$
2. $f(x) = |x| + |x+1|$
3. $f(x) = |x+1| + |x-1|$
4. $f(x) = |x+1| + |x-2|$
5. $f(x) = |x+2| + |x-2|$
6. $f(x) = |x| + |x-1| + |x-2|$