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Function, Domain, Range & Graph

Function: If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then y is called a function of x and it is denoted by the following symbol,

$$y = f(x)$$

where x is independent variable and y is dependent variable.

Example: $y = x^2 + x + 1$; $y = sinx$; $y = e^x$; $y = lnx$ etc.

Domain: The set of all values of x for which the function $y = f(x)$ is defined, is called domain of the function. Simply domain is the set of all allowable x-values.

Mathematically, $D_f = \{x : y = f(\mathrm{x}) \in \mathrm{R} \text{ and } \mathrm{x} \in \mathrm{R}\}$.

Range: The set of all values of y corresponding to the x values for which the function $y = f(x)$ is defined, is called range of the function. Simply range is the set of all possible y-values.

Mathematically, $R_f = \{\mathrm{y} : y = f(\mathrm{x}) \in \mathrm{R} \text{ and } \mathrm{x} \in \mathrm{R}\}$.

Interval: If the set of all real numbers lie between two real numbers a and b, where $a < b$ then the set of all real numbers is called an interval.

Intervals are four kinds:

a) The set $\{x \epsilon \mathcal{R} : a \leq x \leq b\}$ is called a closed interval, denoted by $[a, b]$.
b) The set $\{x \epsilon \mathcal{R} : a < x < b\}$ is called an open interval, denoted by (a, b).
c) The set $\{x \epsilon \mathcal{R} : a < x \leq b\}$ is called a left half open interval, denoted by $(a, b]$.
d) The set $\{x \epsilon \mathcal{R} : a \leq x < b\}$ is called a right half open interval, denoted by $[a, b)$.

Single valued function: A function $y = f(x)$ is called a single valued function if there exist only one value of y for each value of x.

Example: $y = x^2 + 5$; $y = cosx$; $y = e^x + 2$; $y = lnx$ etc.

Many valued function: A function $y = f(x)$ is called a many valued function or multiple valued function if there exist more than one value of y for each value of x.

Example: $y^2 = 4ax$; $y = cos^{-1}x$; $y = sin^{-1}x$ etc.

Algebraic function: A function $y = f(x)$ which consists of a finite number of terms involving powers and roots of x is defined as an algebraic function.

Example: $y = 3x^2 + 4x + 1$ is an algebraic function.

Polynomial Function: A polynomial is an expression containing multiple terms with the operations of addition, subtraction, multiplication and degree of the each term is non-negative. A Function that consist with polynomial is called polynomial function.

Example: The function $f(x) = a_0x^n + a_1x^{n-1} + \cdots\cdots\cdots\cdots + a_n$ is a polynomial function of order n where $a_0,\ a_1,\ \cdots\cdots\cdots\cdots a_n \in R$ and $a_0 \neq 0$.

Note:

1. When $a_0 \neq 0$ then $f(\text{x})$ is called polynomial function of order n.
2. When $a_0 = 1$ then $f(\text{x})$ is called monic polynomial function of order n.
3. When $n = 0$ then $f(\text{x})$ is called polynomial function of order zero means constant polynomial function.
4. When $n = 1$ then $f(\text{x})$ is called polynomial function of order one (1) means linear polynomial function.
5. When $n = 2$ then $f(\text{x})$ is called polynomial function of order two means polynomial function of degree 2 or Quadratic polynomial function. The graph of a quadratic polynomial is a parabola.
6. When $n = 3$ then $f(\text{x})$ is called polynomial function of order three means polynomial function of degree 3 or Cubic polynomial function.
7. When $n = 4$ then $f(\text{x})$ is called polynomial function of order four means polynomial function of degree 4 or by-quadratic polynomial function.
8. When $f(\text{x}) = 0$ then this types of polynomial is called zero polynomial with explicitly undefined degree. The graph of a zero polynomial $f(\text{x}) = 0$ is the x-axis.

Polynomials can be classified by the number of terms with nonzero coefficients, so that a one-term polynomial is called a monomial, a two-term polynomial is called a binomial, and a three-term polynomial is called a *trinomial*. The term "quadrinomial" is occasionally used for a four-term polynomial. A polynomial in one variable is called a *univariate polynomial*, a polynomial in more than one variable is called a multivariate polynomial. A polynomial with two variables is called a bivariate polynomial.

Linear polynomial function: A polynomial function in which degree/ order of the leading term is exactly one is called linear polynomial function.

Example: $f(x) = 3x + 5$ is a linear polynomial function with single variable *x*.

Quadratic polynomial function: **Case 01:** A polynomial function of the form $y = ax^2 + bx + c\ with\ a \neq 0$ is called quadratic polynomial function which represents a parabola. When the value of "a "is positive then the parabola is concave up/open upward and otherwise concave down/open downward. The vertex of the parabola $y = ax^2 + bx + c\ with\ a \neq 0$ is $\left(-\dfrac{b}{2a}\ ,\ \dfrac{4ac - b^2}{4a}\right)$. In another way we get value of the ordinate of vertex by putting the value of abscissa $x = -\dfrac{b}{2a}$ in the equation $y = ax^2 + bx + c\ with\ a \neq 0$.

Case 02: A polynomial function of the form $x = ay^2 + by + c\ with\ a \neq 0$ is called quadratic polynomial function that represents geometrically a parabola. When the value of "a "is positive then the parabola is open right parabola and otherwise it is open left parabola. The vertex of the parabola $x = ay^2 + by + c \ with \ a \neq 0$ is $\left(\dfrac{4ac - b^2}{4a}, -\dfrac{b}{2a} \right)$. In another way we get value of the abscissa of vertex by putting the value of ordinate $y = -\dfrac{b}{2a}$ in the equation $x = ay^2 + by + c \ with \ a \neq 0$.

Rational function: A function of the form $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ both are the function of x and also $q(x) \neq 0,$ is called a rational function.

Example: The function $f(x) = \dfrac{x^2 - 4x + 3}{x^2 + 4x + 7}$ is a rational function in single variable x.

Transcendental function: Functions that can't be expressed as algebraic functions are called transcendental functions. These functions are of the following types:

a) **Exponential function:** A function of the form $y = b^x \ , where \ b > 0$, is called an exponential function with base b.

   Examples: $y = e^x, \ y = \pi^x, \ y = \left(\dfrac{1}{2}\right)^x$, etc.

b) **Logarithmic function:** A function of the form $y = \log_b x \ , where \ x > 0 \ , b > 0 \ and \ b \neq 1$ is called a logarithmic function with base b.

   Examples: $y = \log x, \ y = \ln(x+1),$ etc.

c) **Trigonometric function:** Functions of the types $\sin x, \cos x, \tan x, \cot x \ etc.$ are called trigonometric functions.

d) **Inverse trigonometric functions:** Functions of the types $cos^{-1}x$, $sin^{-1}x$, $etc.$ are called inverse trigonometric functions.

Explicit function: When a relation of two variables x and y is expressed as $y = f(x)$ where y can be expressed directly in terms of x, then y is called an explicit function of x.

Example: $y = ax^2 + bx + c$ is an explicit function of x.

Implicit function: When a relation of two variables x and y is expressed as $f(x, y) = 0$, where x and y cannot be expressed directly in terms of the other, then either variable is called an implicit function of the other.

Example: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is an implicit function.

Even function: A function $y = f(x)$ is called an even function if it satisfies the condition

$$f(-x) = f(x).$$

Example: $y = cosx, \ y = x^4, etc.$ are even functions.

Odd function: A function $y = f(x)$ is called an odd function if it satisfies the condition

$$f(-x) = -f(x).$$

Example: $y = sinx,\ y = x^3, etc.$ are odd functions.

Periodic function: A function $y = f(x)$ is called a periodic function of period $\boldsymbol{T}$ if it satisfies the condition $f(x + \boldsymbol{T}) = f(x)$.

Example: (1). $sinx\ and\ cosx$ are periodic function of period 2π.

(2). $tanx\ and\ cotx$ are periodic function of period π.

Absolute value function: A function $y = |f(x)|$ is called an absolute value function.

Example: $y = |x|$ is an absolute value function.

Bounded function: A function $y = f(x)$ defined on an interval (a, b), is called a bounded function if there exists a number M such that $|f(x)| < M\ \ \forall\, x \in (a, b)$.

Or, A function $y = f(x)$ is called a bounded function if its range is a bounded set.

Example: $y = sinx$ is a bounded function.

Increasing function: A function $y = f(x)$ defined on an interval (a, b) where $a < b$, is called an increasing function over the interval if $f(a) < f(b)$.

Example: $y = x^2,\ 0 \le x \le 5$ is an increasing function.

Decreasing function: A function $y = f(x)$ defined on an interval (a, b) where $a < b$, is called a decreasing function over the interval if $f(a) > f(b)$.

Example: $y = \frac{1}{x},\ 1 \le x \le 5$ is a decreasing function.

Problem 01: Find the domain and range of the function $y = 2x + 5$.

Solution: Given function is,

$$y = 2x + 5$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = R$$

Again, we have,

$$y = 2x + 5$$
$$or,\ 2x = y - 5$$
$$or,\ x = \frac{y-5}{2}$$

Here, x gives real values for all real values of y.
So, the range of the given function is,

$$R_f = R \quad \text{(Ans)}$$

H.W:
Find the domain and range of the following functions

1. $y = 3x + 5$ Ans: $D_f = R$ and $R_f = R$
2. $y = 4x - 3$ Ans: $D_f = R$ and $R_f = R$
3. $y = ax + b$ Ans: $D_f = R$ and $R_f = R$

Problem 02: Find the domain and range of the function $y = x^2 + 3x + 2$.

Solution: Given function is,

$$y = x^2 + 3x + 2$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = R$$

Again, we have

$$y = x^2 + 3x + 2$$
$$or,\ x^2 + 3x + (2 - y) = 0$$

In the above equation the values of x will be real if and only if its $Discriminant \geq 0$.

$$i.e,\ 3^2 - 4.1.(2 - y) \geq 0 \qquad ;\ [b^2 - 4ac \geq 0]$$
$$or,\ 9 - 4(2 - y) \geq 0$$
$$or,\ 9 - 8 + 4y \geq 0$$
$$or,\ 1 + 4y \geq 0$$
$$or,\ 4y \geq -1$$
$$or,\ y \geq -\frac{1}{4}$$

Therefore the range of the given function is,

$$R_f = [-\frac{1}{4}, \infty) \qquad \text{(Ans)}$$

H.W:
Find the domain and range of the following quadratic functions

1. $y = x^2 + 5x + 6$ Ans: $D_f = R$ and $R_f = [-\frac{1}{4}, \infty)$
2. $y = -x^2 + 5x - 6$ Ans: $D_f = R$ and $R_f = (-\infty, \frac{1}{4}]$
4. $y = -x^2 + 1$ Ans: $D_f = R$ and $R_f = (-\infty, 1]$
5. $y = x^2 + 4x + 7$ Ans: $D_f = R$ and $R_f = [3, \infty)$
6. $y = x^2 - 4x + 3$ Ans: $D_f = R$ and $R_f = [-1, \infty)$
7. $y = (x + 2)^2 + 3$ Ans: $D_f = R$ and $R_f = [3, \infty)$

Problem 03: Find the domain and range of the function $y = \frac{x-3}{2x+1}$.

Solution: Given function is,

$$y = \frac{x-3}{2x+1}$$

Here, y is undefined if

$$2x+1 = 0$$
$$or,\ x = -\frac{1}{2}$$

So, y gives real values for all real values of x except $x = -\frac{1}{2}$.

Therefore, the domain of the given function is

$$D_f = R - \left\{-\frac{1}{2}\right\}.$$

Again we have,

$$y = \frac{x-3}{2x+1}$$
$$or,\ 2xy + y = x - 3$$
$$or,\ x - 2xy = y + 3$$
$$or,\ x(1-2y) = y + 3$$
$$or,\ x = \frac{y+3}{1-2y}$$

Here, x is undefined if

$$1-2y = 0$$
$$or,\ y = \frac{1}{2}$$

So, x gives real values for all real values of y except $y = \frac{1}{2}$.

Therefore, the range of the given function is

$$R_f = R - \left\{\frac{1}{2}\right\} \qquad \text{(Ans)}$$

Problem 04: Find the domain and range of the function $y = \frac{x^2-4}{x-2}$.

Solution: Given function is,

$$y = \frac{x^2-4}{x-2}$$

Here, y is undefined if

$$x-2 = 0$$
$$or,\ x = 2$$

So, y gives real values for all real values of x except $x = 2$.

Therefore, the domain of the given function is

$$D_f = R - \{2\}.$$

Again we have,

$$y=\frac{x^2-4}{x-2}$$

$$or,\ \ y=\frac{(x+2)(x-2)}{x-2}\ \ ;\ x\neq 2$$

$$or,\ \ y=x+2\qquad ;\ x\neq 2$$

$$or,\ \ x=y-2\qquad ;\ x\neq 2$$

Here, x is defined for all real values of y except $y=4$

Therefore, the range of the given function is

$$R_f=R-\{4\}\qquad \text{(Ans)}$$

H.W:

Find the domain and range of the following quadratic functions

1. $y=\dfrac{x}{x+1}$ Ans: $D_f=R-\{-1\}$ and $R_f=R-\{1\}$
2. $y=\dfrac{1+x}{5-x}$ Ans: $D_f=R-\{5\}$ and $R_f=R-\{-1\}$
3. $y=\dfrac{2}{x+3}$ Ans: $D_f=R-\{-3\}$ and $R_f=R-\{0\}$
4. $y=\dfrac{x-3}{x^2-9}$ Ans: $D_f=R-\{-3,3\}$ and $R_f=R-\left\{0,\dfrac{1}{6}\right\}$
5. $y=\dfrac{4x+3}{x^2+1}$ Ans: $D_f=R\ and\ R_f=[-1,4]$

Problem 05: Find the domain and range of the function $y=\sqrt{2x+5}$.

Solution: Given function is,

$$y=\sqrt{2x+5}$$

Here, y gives real values iff

$$2x+5\ \geq\ 0$$

$$or,\ 2x\ \geq\ -5$$

$$or,\ x\ \geq\ -\frac{5}{2}$$

Therefore, the domain of the given function is

$$D_f=\left[-\frac{5}{2},\infty\right).$$

Again,

$$y=\sqrt{2x+5}\ \ \cdots\cdots(1)$$

The values of y in (1) are positive or zero, *i.e,* $y\nless 0$.

Now $y^2=2x+5\qquad ;\ y\nless 0.$ [Squaring both sides]

$$2x+5=y^2\qquad ;\ y\nless 0.$$

$$2x = y^2 - 5 \qquad ; y \nless 0.$$

$$x = \frac{y^2 - 5}{2} \qquad ; y \nless 0.$$

Here, x is defined for $y \geq 0$.

Therefore, the range of the given function is

$$R_f = \{y : y \geq 0\}$$

$$= [0, \infty) \qquad \textbf{\textit{(Ans).}}$$

Problem 06: Find the domain and range of the function $y = -\sqrt{1-2x}$.

Solution: Given function is,

$$y = -\sqrt{1-2x}$$

Here, y gives real values iff

$$1 - 2x \geq 0$$

$$or,\ -2x \geq -1$$

$$or,\ 2x \leq 1$$

$$or,\ x \leq \frac{1}{2}$$

Therefore, the domain of the given function is

$$D_f = \left(-\propto, \frac{1}{2}\right].$$

Again, we have,

$$y = -\sqrt{1-2x} \quad \cdots\cdots(1)$$

The values of y in (1) are negative or zero, $i.e, y \ngtr 0$.

Now $y^2 = 1 - 2x \qquad ; y \ngtr 0$ [Squaring both sides]

$$1 - 2x = y^2 \qquad ; y \ngtr 0$$

$$2x = 1 - y^2 \qquad ; y \ngtr 0$$

$$x = \frac{1 - y^2}{2} \qquad ; y \ngtr 0$$

Here, x is defined for $y \leq 0$.

Therefore, the range of the given function is

$$R_f = \{y : y \leq 0\}$$

$$= (-\infty, 0] \qquad \textbf{(Ans).}$$

H.W:

Find the domain and range of the following functions

1. $y=\sqrt{2x-1}$ Ans: $D_f=\left[\frac{1}{2},\propto\right)$ and $R_f=[0,\propto)$
2. $y=\sqrt{1-5x}$ Ans: $D_f=\left(-\propto,\frac{1}{5}\right]$ and $R_f=[0,\propto)$
3. $y=\sqrt{2x-1}+5$ Ans: $D_f=\left[\frac{1}{2},\propto\right)$ and $R_f=[5,\propto)$
4. $y=\sqrt{x+6}-3$ Ans: $D_f=[-6,\propto)$ and $R_f=[-3,\propto)$
5. $y=5-\sqrt{8-2x}$ Ans: $D_f=(-\propto,4]$ and $R_f=[5,-\propto)$
6. $y=-\sqrt{x-1}$ Ans: $D_f=[1,\propto)$ and $R_f=(-\propto,0]$
7. $y=-\sqrt{1-4x}$ Ans: $D_f=\left(-\propto,\frac{1}{4}\right]$ and $R_f=(-\propto,0]$

Problem 07: Find the domain and range of the function $y=\sqrt{x^2-4x+3}$.

Solution: Given function is,

$$y=\sqrt{x^2-4x+3}$$

Here, y gives real values iff,

$$x^2-4x+3\geq 0$$

$$or,\ x^2-3x-x+3\geq 0$$

$$or,\ x(x-3)-1(x-3)\geq 0$$

$$or,\ (x-3)(x-1)\geq 0$$

This inequality is satisfied if

$$x\leq 1\ or\ x\geq 3$$

Therefore, the domain of the given function is,

$$D_f=\{x:x\leq 1\}\cup\{x:x\geq 3\}$$

$$=(-\infty,1]\cup[3,\infty)$$

$$=R-(1,3)$$

Again, we have,

$$y=\sqrt{x^2-4x+3}\ \cdots\cdots(1)$$

The values of y in (1) are positive or zero *i.e*, , $y\not< 0$.

Now, $y^2=x^2-4x+3\qquad ;y\not< 0\qquad$ [Squaring both sides]

$$x^2-4x+3-y^2=0\qquad ;y\not< 0$$

$$x^2-4x+\left(3-y^2\right)=0 \qquad ; y \nless 0$$

In the above equation the values of x will be real if and only if it's $Discriminant \geq 0$.

$$i.e,\ (-4)^2-4\times1.\left(3-y^2\right)\geq 0 \qquad ; y \nless 0 \qquad [b^2-4ac\geq 0]$$

$$or,\ 16-4\left(3-y^2\right)\geq 0 \qquad ; y \nless 0$$

$$or,\ 16-12+4y^2\geq 0 \qquad ; y \nless 0$$

$$or,\ 4+4y^2\geq 0 \qquad ; y \nless 0$$

$$or,\ 1+y^2\geq 0 \qquad ; y \nless 0$$

Here, x is defined for $y\geq 0$.

So the range of the given function is

$$R_f=\{y : y\geq 0\}$$

$$=[0,\infty) \quad \textbf{(Ans).}$$

Problem 08: Find the domain and range of the function $y=\sqrt{x^2+1}$.

Solution: Given function is,

$$y=\sqrt{x^2+1}$$

Here, y gives real values iff,

$$x^2+1\geq 0$$

This inequality is satisfied for all real values of x.

Therefore the domain of the given function is,

$$D_f=R.$$

Again, we have,

$$y=\sqrt{x^2+1} \quad \ldots \ldots (1)$$

The values of y in (1) are positive and lowest value is 1, *i.e*, $y \nless 1$.

$$\text{Now} \quad y^2=x^2+1 \qquad ; y \nless 1 \qquad \text{[Squaring both sides]}$$

$$\Rightarrow x^2+1-y^2=0 \qquad ; y \nless 1$$

$$\Rightarrow x^2+0.x+\left(1-y^2\right)=0 \quad ; y \nless 1$$

In the above equation the values of x will be real if and only if its $Discriminant \geq 0$.

$$\text{i.e,} \quad 0^2-4.1.\left(1-y^2\right)\geq 0 \qquad ; y \nless 1 \qquad [b^2-4ac\geq 0]$$

$$or,\ -4\left(1-y^2\right)\geq 0 \qquad ; y \nless 1$$

$$or,\ 4y^2-4\geq 0 \qquad ; y \nless 1$$

$$or,\ y^2-1\geq 0 \qquad ; y \nless 1$$

Here, x is defined for all $y\geq 1$.

$$R_f=\{\mathrm{y} : \mathrm{y}\geq 1\}$$

$$=[1,\infty) \quad \textbf{(Ans).}$$

Problem 09: Find the domain and range of the function $y=\sqrt{4-x^2}$.

Solution: Given function is,

$$y=\sqrt{4-x^2}$$

Here, y gives real values iff,

$$4-x^2 \geq 0$$

$$or,\ (2+x)(2-x) \geq 0$$

This inequality is satisfied if,

$$-2 \leq x \leq 2$$

Therefore, the domain of the given function is,

$$D_f = \{x : -2 \leq \ x \leq 2\}$$

$$= [-2,\ 2]$$

Again, we have,

$$y=\sqrt{4-x^2} \quad \dots \dots (1)$$

The values of y in (1) are positive and lowest value is zero, *i.e,* $y \nless 0$.

$$\text{Now} \quad y^2 = 4-x^2 \qquad ;\ y \nless 0 \qquad \text{[Squaring both sides]}$$

$$\Rightarrow x^2+y^2-4=0 \qquad ; y \nless 0$$

$$\Rightarrow x^2+0.x+\left(y^2-4\right)=0 \qquad ; y \nless 0$$

In the above equation the values of x will be real if and only if it's *Discriminant* ≥ 0 .

$$\text{i.e,} \quad 0^2-4.1.\left(y^2-4\right) \geq 0 \qquad ; y \nless 0 \qquad [\ b^2-4ac \geq 0\ ]$$

$$or,\ -4y^2+16 \geq 0 \qquad ; y \nless 0$$

$$or,\ y^2-4 \leq 0 \qquad ; y \nless 0 \qquad \text{[Dividing by -4]}$$

Here, x is defined for all $0 \leq y \leq 2$.

Therefore the range of the given function is,

$$R_f = \{\text{y} : 0 \leq \text{y} \leq 2\}$$

$$= [0, 2] \qquad \text{(Ans.)}$$

H.W:

Find the domain and range of the following functions

1. $y=\sqrt{x^2-3}$ Ans: $D_f = R-\left(-\sqrt{3}, \sqrt{3}\right)$ and $R_f = [0,\ \infty)$
2. $y=\sqrt{x^2-25}$ Ans: $D_f = R-(-5,5)$ and $R_f = [0,\ \infty)$
3. $y=\sqrt{x^2+3x}$ Ans: $D_f = R-(-3,0)$ and $R_f = [0,\ \infty)$
4. $y=\sqrt{x^2-2x}$ Ans: $D_f = R-(0,2)$ and $R_f = [0,\ \infty)$

5. $y=\sqrt{x^2+3}$ Ans: $D_f=R$ and $R_f=[3,\propto)$
6. $y=\sqrt{x^2+25}$ Ans: $D_f=R$ and $R_f=[25,\propto)$
7. $y=\sqrt{16-x^2}$ Ans: $D_f=[-4,4]$ and $R_f=[0,4]$
8. $y=\sqrt{x^2-2x+2}$ Ans: $D_f=R$ and $R_f=[0,\infty)$

Problem 10: Find the domain and range of the function $y=\dfrac{1}{\sqrt{2x+3}}$.

Solution: Given function is,

$$y=\frac{1}{\sqrt{2x+3}}$$

Here, y gives real values iff,

$$2x+3>0$$
$$or,\ 2x>-3$$
$$or,\ x>-\frac{3}{2}$$

Therefore the domain of the given function is $D_f=\{x:x>-\frac{3}{2}\}$.

$$D_f=\left(-\frac{3}{2},\infty\right)$$

Again, we have,

$$y=\frac{1}{\sqrt{2x+3}} \quad \cdots\cdots(1)$$

The values of y in (1) are positive and lowest value is near to 0, *i.e*, $y>0$.

$$\text{Now, } y^2=\frac{1}{2x+3} \qquad ;\ y>0$$
$$or,\ 2x+3=\frac{1}{y^2} \qquad ;\ y>0$$
$$or,\ 2x=\frac{1}{y^2}-3 \qquad ;\ y>0$$
$$or,\ x=\frac{1}{2}\left(\frac{1}{y^2}-3\right) \qquad ;\ y>0$$

Here, x is defined for all $y>0$.

Therefore the range of the given function is

$$R_f=\{y:y>0\}$$
$$=(0,\infty) \qquad \textbf{(Ans)}$$

Problem 11: Find the domain and range of the function $f(x)=\sqrt{\dfrac{2x+3}{x-5}}$.

Solution: Given function is,

$$y=\sqrt{\frac{2x+3}{x-5}}$$

Here, y gives real values iff,

$$\frac{2x+3}{x-5}\geq 0$$

This inequality is satisfied if $x\leq -\dfrac{3}{2}$ or $x>5$.

Therefore the domain of the given function is,

$$D_f=\{x:x\leq -\frac{3}{2}\}\cup\{x:x>5\}.$$
$$=(-\infty,-\frac{3}{2}]\cup(5,\infty).$$

Again, we have,

$$y=\sqrt{\frac{2x+3}{x-5}} \quad \cdots\cdots\cdots (1)$$

The values of y in (1) are positive or zero, *i.e*, $y \nless 0$.

$$\text{Now,}\quad y^2=\frac{2x+3}{x-5} \qquad ; y\nless 0 \qquad \text{[Squaring both-sides]}$$
$$or,\ xy^2-5y^2=2x+3 \quad ; y\nless 0$$
$$or,\ xy^2-2x=5y^2+3 \quad ; y\nless 0$$
$$or,\ x\left(y^2-2\right)=5y^2+3 \quad ; y\nless 0$$
$$or,\ x=\frac{5y^2+3}{y^2-2} \qquad ; y\nless 0$$
$$or,\ x=\frac{5y^2+3}{y^2-\left(\sqrt{2}\right)^2} \qquad ; y\nless 0$$
$$or,\ x=\frac{5y^2+3}{\left(y-\sqrt{2}\right)\left(y+\sqrt{2}\right)} \qquad ; y\nless 0$$

Here, x is defined for all $y\geq 0$ except $y=\sqrt{2}$.

So, the range of the given function is

$$R_f=\left\{y:y\geq 0\ ;\ y\neq\sqrt{2}\right\}$$
$$=\left[0,\infty\right)-\sqrt{2} \qquad \textbf{(Ans.)}$$

<u>Problem 12:</u> Find the domain and range of $y = e^x$.

Solution: Given function is,

$$y = e^x$$

Here, *y* gives real values for all real values of *x*.
So, the domain of the given function is,

$$D_f = R$$

Again, we have,

$$y = e^x$$
$$or,\ \ln y = x$$
$$or,\ x = \ln y$$

Here, *x* gives real values iff $y > 0$.
So, the range of the given function is,

$$R_f = \{y : y > 0\}$$

$$= (0, \infty) \quad \textbf{(Ans).}$$

<u>Problem 13:</u> Find the domain and range of $y = \ln\left(\dfrac{1+x}{1-x}\right)$.

Solution: Given function is,

$$y = \ln\left(\frac{1+x}{1-x}\right)$$

Here, *y* gives real values iff

$$\frac{1+x}{1-x} > 0$$

This inequality is satisfied if $-1 < x < 1$.
So, the domain of the given function is,

$$D_f = \{x : -1 < x < 1\}$$
$$= (-1, 1)$$

Again, we have,

$$y = \ln\left(\frac{1+x}{1-x}\right)$$
$$or,\ \frac{1+x}{1-x} = e^y$$
$$or, 1 + x = e^y - xe^y$$
$$or, xe^y + x = e^y - 1$$
$$or, x\left(e^y + 1\right) = e^y - 1$$

$$or, x = \frac{e^y - 1}{e^y + 1}$$

Here, x gives real values for all real values of y.
So, the range of the given function is,

$$R_f = \{y : -\infty < y < \infty\}$$

$$= (-\infty, \infty)$$

$$= R$$ **(Ans).**

Problem 14: Find the domain and range of $y = \sin x$.

Solution: Given function is,

$$y = \sin x$$

Here, y gives real values for all real values of x.
So, the domain of the given function is,

$$D_f = \{x : -\infty < x < \infty\}$$
$$= (-\infty, \infty)$$
$$= R$$

Again, we have,

$$y = \sin x$$
$$or,\ x = \sin^{-1} y$$

Here, x gives real values for $-1 \le y \le 1$.
So, the range of the given function is,

$$R_f = \{y : -1 < y < 1\}$$

$$= [-1, 1]$$ **(Ans).**

Problem 15: Find the domain and range of $y = \tan x$.

Solution: Given function is,

$$y = \tan x$$

Here, y gives real values for all real values of x except $x = (2n+1)\frac{\pi}{2}$; $where, n = 0, \pm 1, \pm 2, \cdots \cdots \cdots$

So, the domain of the given function is,

$$D_f = R - \left\{\cdots \cdots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots \cdots\right\}$$

Again, we have,

$$y = \tan x$$
$$or,\ x = \tan^{-1} y$$

Here, x gives real values for all real values of y.

So, the range of the given function is,

$$R_f = \{y : -\propto < y < \propto\}$$

$$= (-\propto, \propto)$$

$$= R \qquad \textbf{(Ans).}$$

Problem 16: Find the domain and range of $y = \cot x$.

Solution: Given function is,

$$y = \cot x$$

Here, *y* gives real values for all real values of x except $x = n\pi \;\; ; where, n = 0, \pm 1, \pm 2, \cdots\cdots\cdots$

So, the domain of the given function is,

$$D_f = R - \{\cdots\cdots -2\pi, -\pi, 0, \pi, 2\pi, \cdots\cdots\}$$

Again, we have,

$$y = \cot x$$

$$or,\ x = \cot^{-1} y$$

Here, *x* gives real values for all real values of y.

So, the range of the given function is,

$$R_f = \{y : -\propto < y < \propto\}$$

$$= (-\propto, \propto)$$

$$= R \qquad \textbf{(Ans).}$$

Problem 17: Find the domain and range of $y = \sec x$.

Solution: Given function is,

$$y = \sec x$$

Here, *y* gives real values for all real values of x except $x = (2n+1)\frac{\pi}{2} \;\; ; where, n = 0, \pm 1, \pm 2, \cdots\cdots\cdots$

So, the domain of the given function is,

$$D_f = R - \left\{\cdots\cdots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots\cdots\right\}$$

Again, we have,

$$y = \sec x$$

$$or,\ x = \sec^{-1} y$$

Here, *x* gives real values for all real values of y except $-1 < y < 1$.

So, the range of the given function is,

$$R_f = (-\propto, -1] \cup [1, \propto)$$

$$= R - (-1, 1) \qquad \textbf{(Ans).}$$

Problem 18: Find the domain and range of $y = \cos ec x$.

Solution: Given function is,

$$y = \cos ec x$$

Here, y gives real values for all real values of x except $x = n\pi$; $where, n = 0, \pm 1, \pm 2, \cdots\cdots\cdots$

So, the domain of the given function is,

$$D_f = R - \{\cdots\cdots -2\pi, -\pi, 0, \pi, 2\pi, \cdots\cdots\}$$

Again, we have,

$$y = \cos ec x$$

$$or,\ x = \cos ec^{-1} y$$

Here, x gives real values for all real values of y except $-1 < y < 1$.

So, the range of the given function is,

$$R_f = (-\infty, -1] \cup [1, \infty)$$

$$= R - (-1, 1) \qquad \textbf{(Ans).}$$

H.W:

Find the domain and range of the following functions:

1. $y = e^{(x-2)}$ $\qquad$ $Ans: D_f = R \ \ and \ \ R_f = (0, \infty)$
2. $y = \ln(x-2)$ $\qquad$ $Ans: D_f = (2, \infty) \ \ and \ \ R_f = R$
3. $y = \cos x$ $\qquad$ $Ans: D_f = R \ \ and \ \ R_f = [-1, 1]$

Graph of Functions

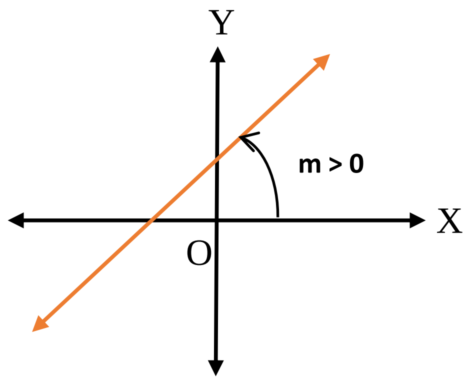
If $f: A \to B$ denotes a function, then the graph of the function $f(x)$ is the set of all ordered pairs $(x, f(x))$ for all values of x in the domain A.

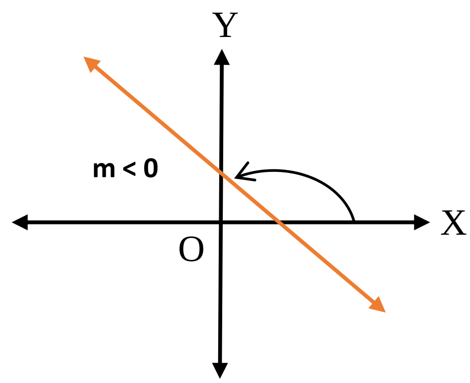
$$\therefore \text{Graph of } f(x) = \{(x,y): x \epsilon A, y = f(x) \epsilon B\}$$

Therefore, Graph is the geometrical/Pictorial representation of a function or visualization of a function.

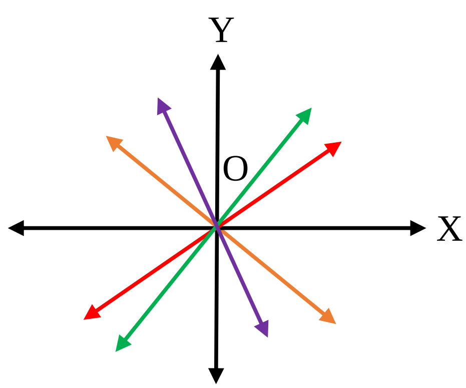
Graph of some elementary functions:

❖ ***Graph of*** $\boldsymbol{y = mx + c}$

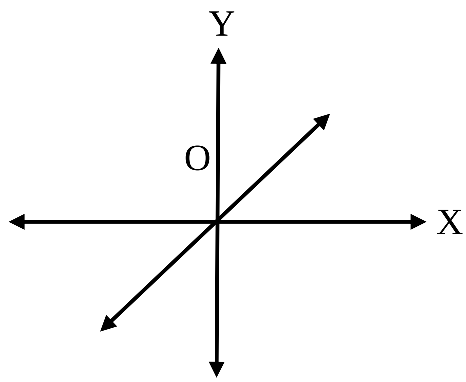




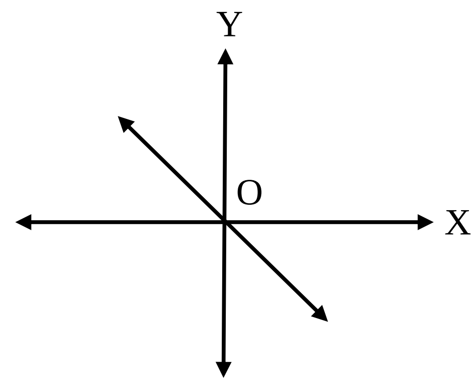
❖ ***Graph of*** $\boldsymbol{y = mx}$



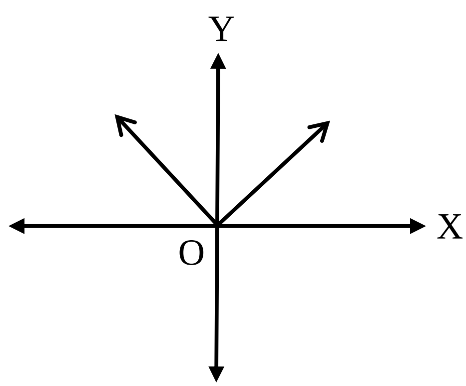
❖ ***Graph of*** $\boldsymbol{y = x}$



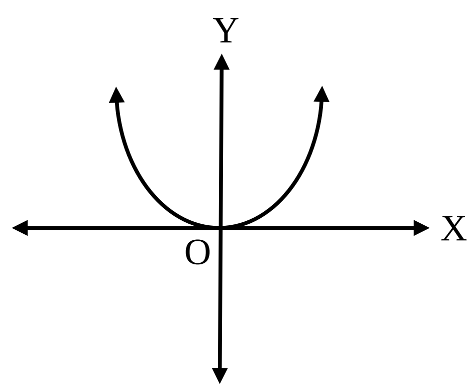
◆ ***Graph of*** $\boldsymbol{y = -x}$



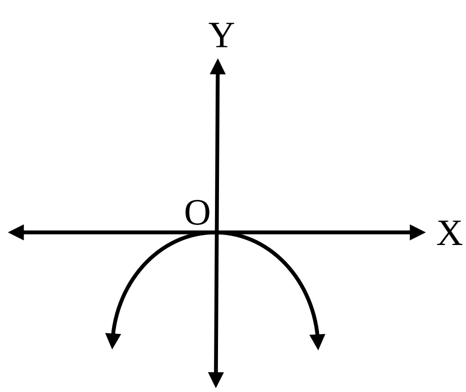
❖ ***Graph of*** $\boldsymbol{y = |x|}$



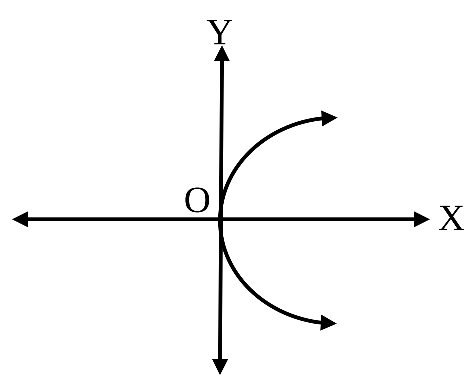
◆ ***Graph of*** $\boldsymbol{y = x^2}$



❖ ***Graph of*** $\boldsymbol{y = -x^2}$

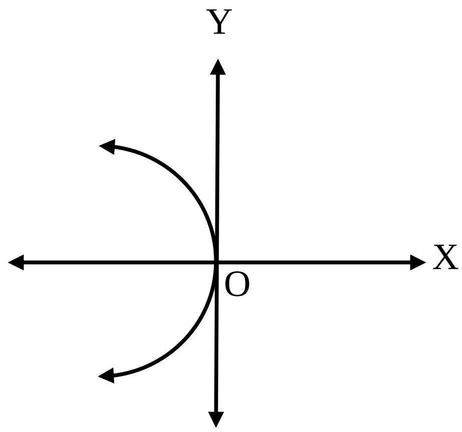


◆ ***Graph of*** $\boldsymbol{x = y^2}$

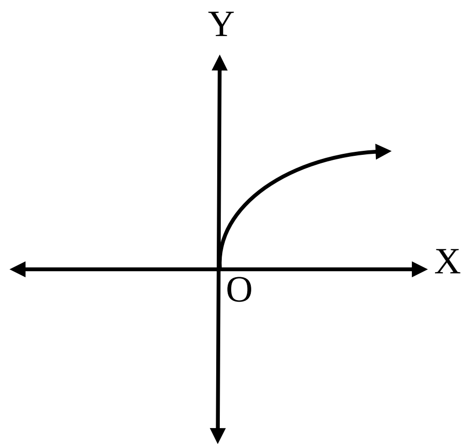


Note: *when power of the variable increases then graph will be wider.*

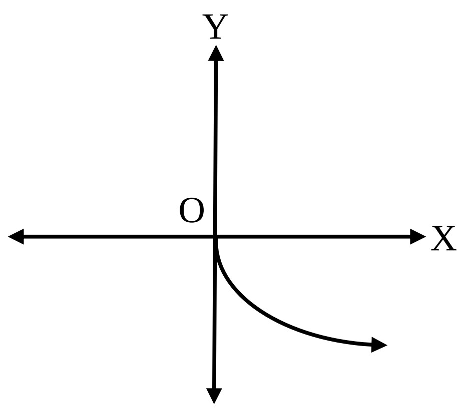
❖ ***Graph of*** $x = -y^2$



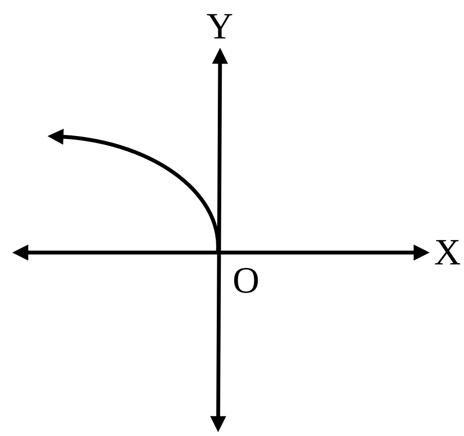
■ ***Graph of*** $y = \sqrt{x}$



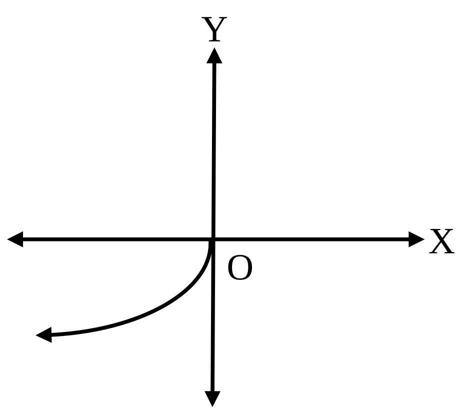
❖ ***Graph of*** $y = -\sqrt{x}$



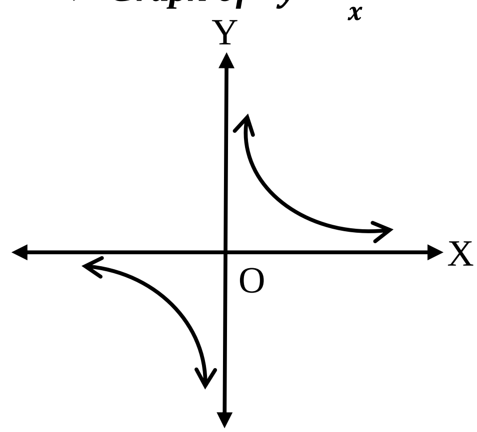
■ ***Graph of*** $y = \sqrt{-x}$



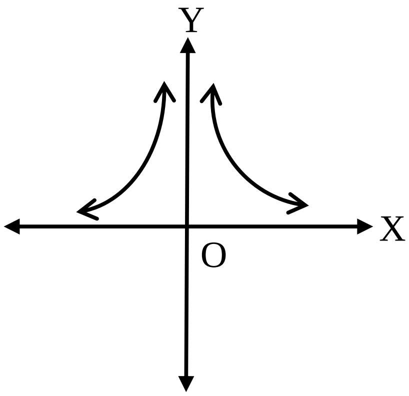
❖ ***Graph of*** $y = -\sqrt{-x}$



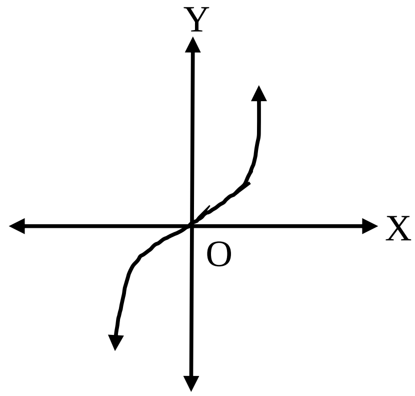
■ ***Graph of*** $y = \frac{1}{x}$



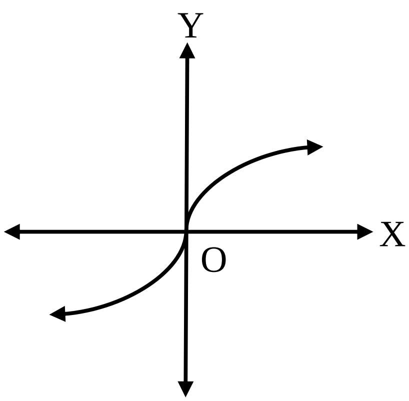
❖ ***Graph of*** $y = \frac{1}{x^2}$



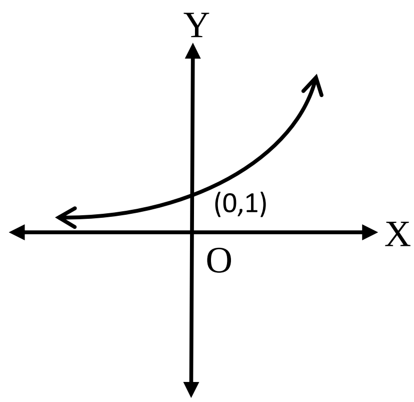
- ***Graph of*** $y = x^3$



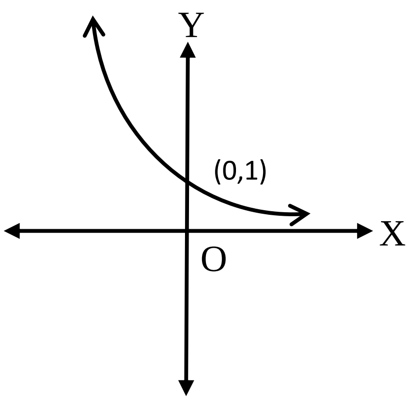
❖ ***Graph of*** $y = \sqrt[3]{x}$



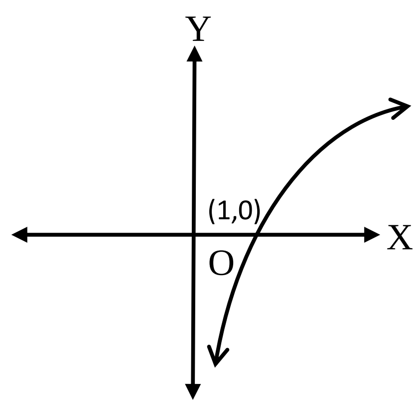
- ***Graph of*** $y = e^x$



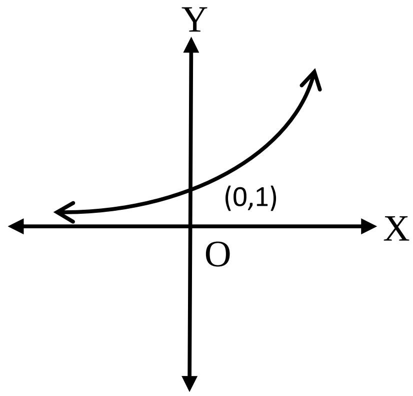
❖ ***Graph of*** $y = e^{-x}$



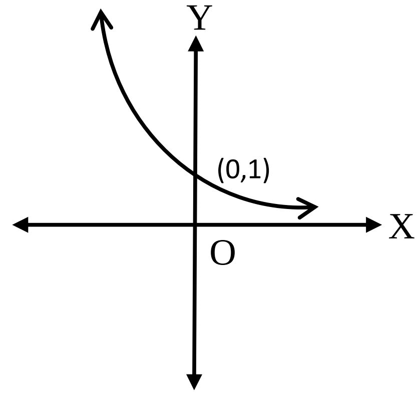
- ***Graph of*** $y = ln|x|$



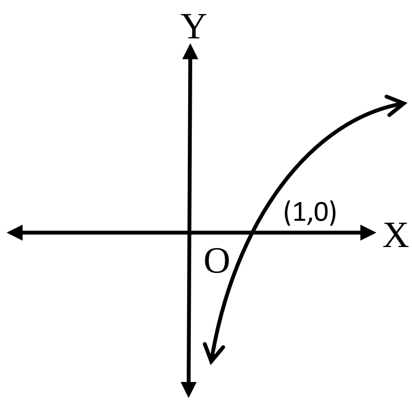
❖ ***Graph of*** $y = a^x$, $a > 1$



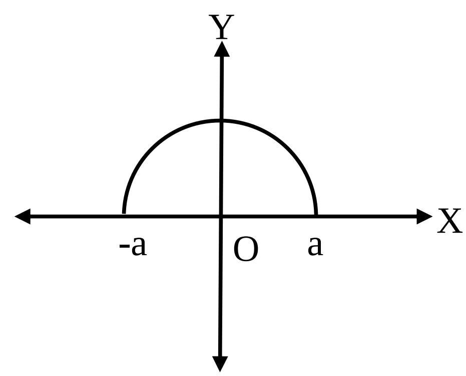
● ***Graph of*** $y = a^{-x}$



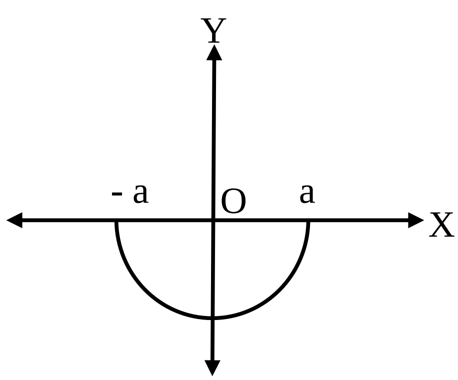
❖ ***Graph of*** $y = log_a|x|,\ \ a > 1$



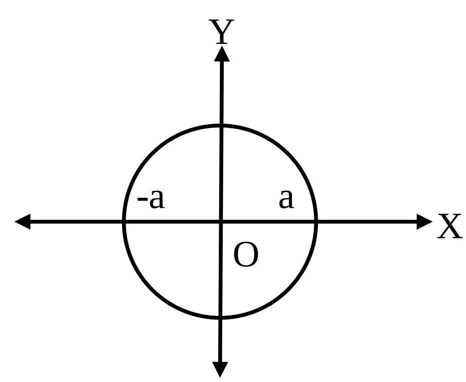
● ***Graph of*** $y = \sqrt{a^2 - x^2}$



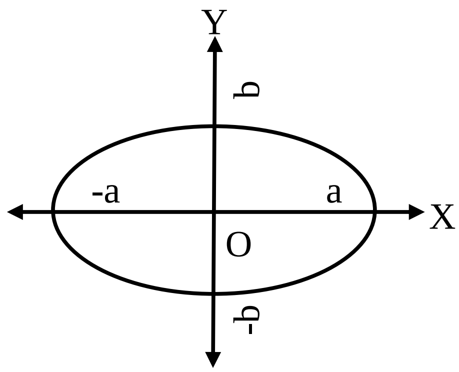
❖ ***Graph of*** $y = -\sqrt{a^2 - x^2}$



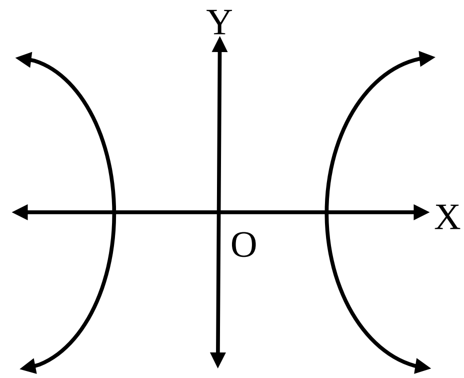
● ***Graph of*** $x^2 + y^2 = a^2$



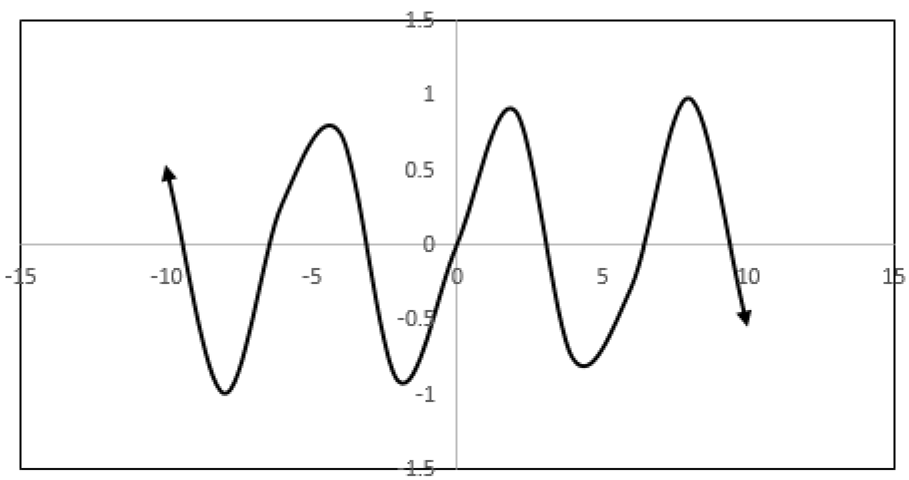
❖ ***Graph of*** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



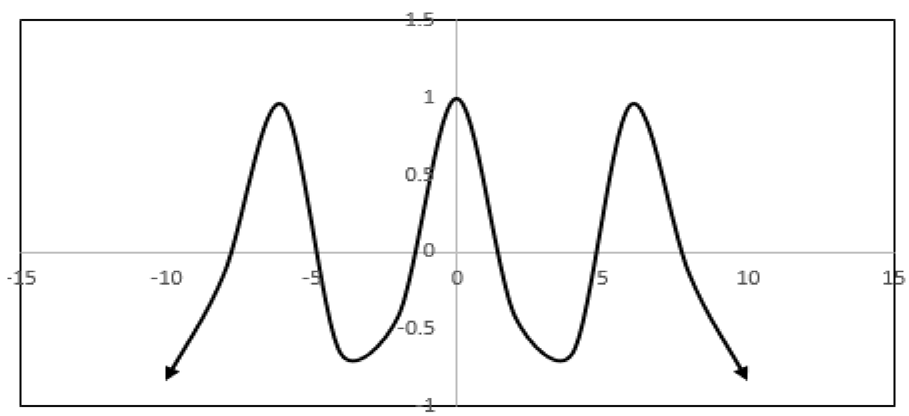
Graph of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$



❖ ***Graph of*** $y = sinx$



❖ ***Graph of*** $y = cosx$



Transformation of Function

Transformation of a function is any kind of change in the function such as move or resize the graphs of functions. There are two types of transformation of the functions such as,

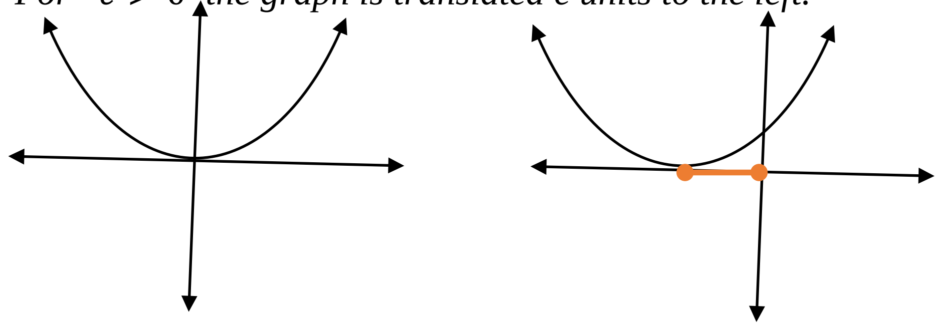
1. ***Translation/Shifting:*** Any kind of shifting of the graph of a function is called translation of the function that means changing the location of the graph without changing its size and shape is called translation.
2. ***Scaling:*** Scaling of a graph of a function is a transformation in which the size and shape of the graph is changed.

➢ *Translation*

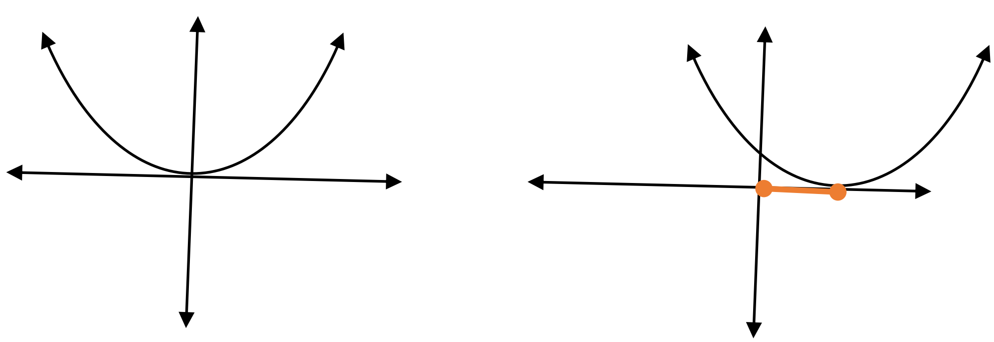
Horizontal translation:

Function: $g(x) = f(x + c)$

For $c > 0$ *the graph is translated c units to the left.*

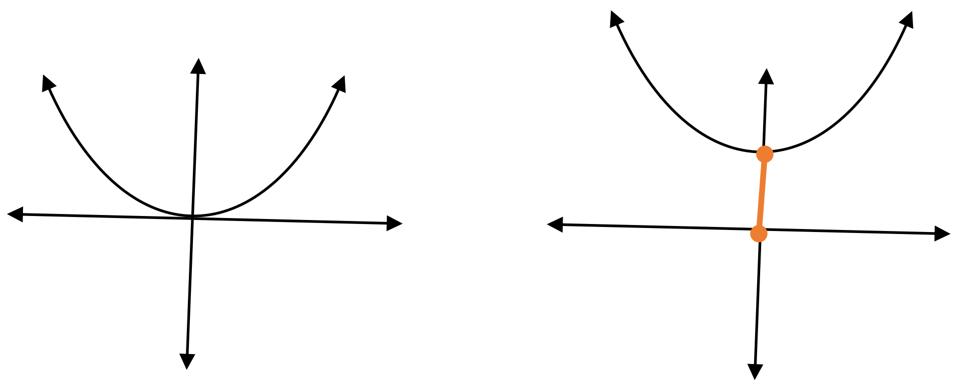


For $c < 0$ *the graph is translated c units to the right.*

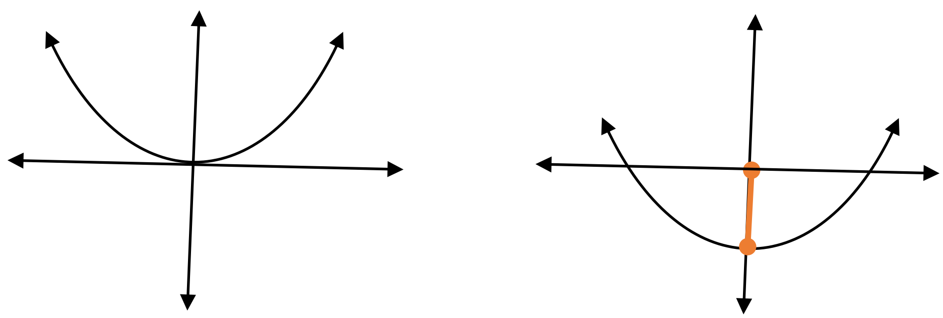


Vertical Translation:

Function: $g(x) = f(x) + c$
For $c > 0$ *the graph is translated c units upward.*

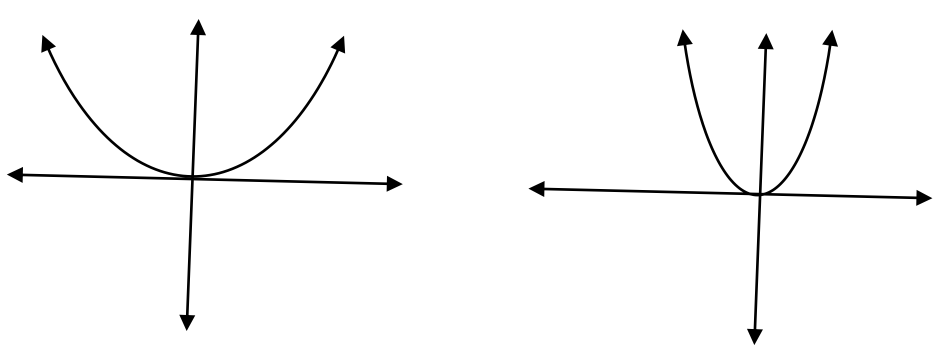


For $c < 0$ *the graph is translated c units downward.*



➢ ***Scaling***

Function: $g(x) = cf(x)$
For $|c| > 1$ $(integer)$ *the graph is compressed.*



For $|c| < 1$ $(integer)$ *the graph is stretched.*

<u>Problem- 01:</u> Sketch the graph of the function $y = x^2 + 6x + 10$.

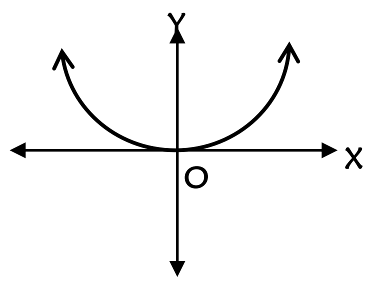
Solution: The equation of the given function is,

$$y = x^2 + 6x + 10$$

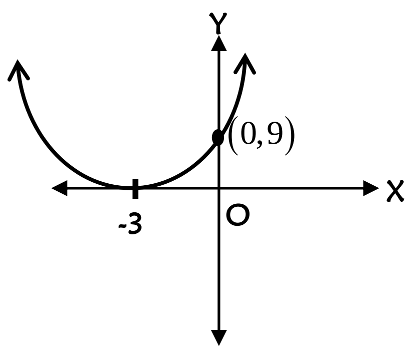
Completing the given equation in a square form it becomes as

$$\begin{aligned} y &= x^2 + 6x + 10 \\ &= x^2 + 2.x.3 + 3^2 - 3^2 + 10 \\ &= (x+3)^2 + 1 \end{aligned}$$

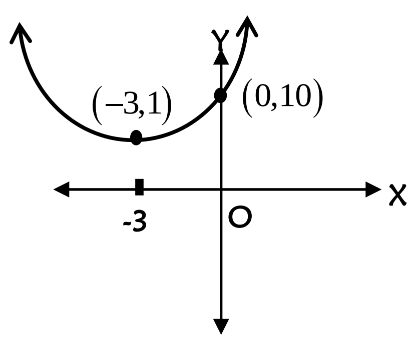
The graph of the standard function $y = x^2$ is as follows



Translating or shifting the above graph 3 units to the left, we get the graph of the function $y = (x+3)^2$.



Translating or shifting the above graph 1 units upward, we get the graph of the function $y = (x+3)^2 + 1$.



(Desired Graph)

H.W:

Sketch the graph of the following functions

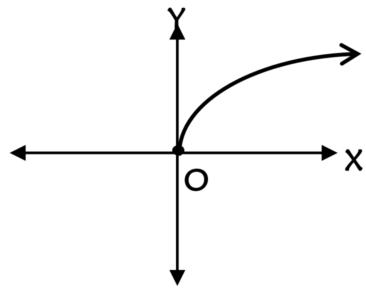
1. $y = x^2 + 4x + 10$
2. $y = 2x^2 + 5x + 10$
3. $y = x^2 - 4x + 5$
4. $y = 2x^2 - 5$
5. $y = 2x^2 + 5$
6. $y = -2(x+1)^2 - 3$

Problem -02: Sketch the graph of the function $y = \sqrt{x-2} + 5$.

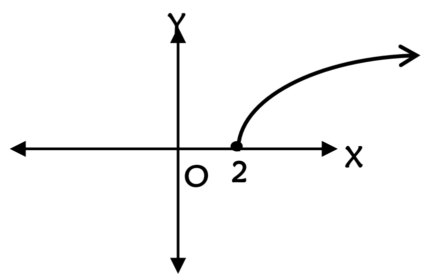
Solution: The equation of the given function is,

$$y = \sqrt{x-2} + 5$$

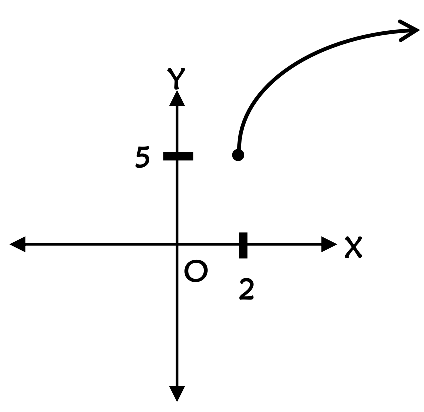
The graph of the standard positive square root function $y = \sqrt{x}$ is as follows



Translating or shifting the above graph 2 units to the right, we get the graph of the function $y = \sqrt{x-2}$.



Translating or shifting the above graph 5 units upward, we get the graph of the function $y = \sqrt{x-2} + 5$.



(Desired Graph)

H.W:

Sketch the graph of the following functions

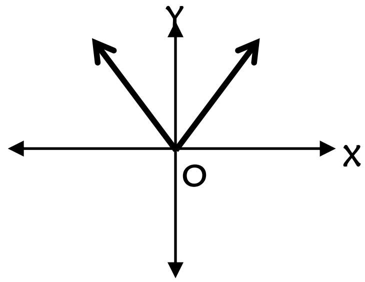
1. $y=\sqrt{x+2}$
2. $y=\sqrt{2x+5}$
3. $y=2-\sqrt{x+5}$
4. $y=1-\sqrt[3]{x+2}$
5. $y=\sqrt{5-x^2}+6$
6. $y=\dfrac{1}{(x-3)^5}$
7. $y=\sqrt{x^2-4x+4}$
8. $y=2-\dfrac{1}{x+1}$

<u>Problem -03:</u> Sketch the graph of the function $y=2-|x+2|$.

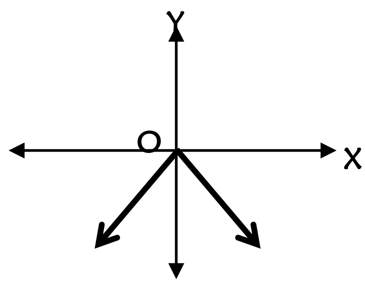
Solution: The equation of the given function is,

$$y=2-|x+2|$$

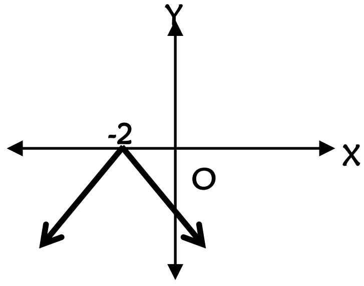
The graph of the standard absolute value function $y=|x|$ is as follows



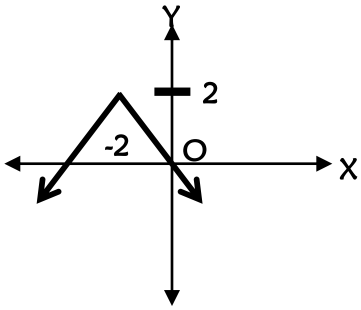
Therefore the graph of the standard absolute value function $y=-|x|$ is as follows



Translating or shifting the above graph 2 units to the left, we get the graph of the function $y=-|x+2|$.



Translating or shifting the above graph 2 units upward, we get the graph of the function $y = -|x+2| + 2$ or $y = 2 - |x+2|$.



(Desired Graph)

H.W: Sketch the graph of the following functions:

1. $y = |x+2| - 2$
2. $y = |x-2| + 3$
3. $y = 1 - |x-3|$

Piecewise function: A **piecewise-defined function** (also called a **piecewise function** or a **hybrid function**) is a function which is defined by multiple sub-functions, each sub-function applying to a certain interval of the main function's domain (a sub-domain).

For example: The following function is the piecewise function

$$y = f(x) = \begin{cases} f_1(\text{x}) \quad , & x < a \\ f_2(\text{x}) \quad , & a \le x < b \\ f_3(\text{x}) \quad , & x \ge b \end{cases}$$

Note that:

1. The function $f_1(x)$ is defined on the interval $(-\infty, a)$.
2. The function $f_2(x)$ is defined on the interval $[a, b)$.
3. The function $f_3(x)$ is defined on the interval $[\text{b}, \infty)$.

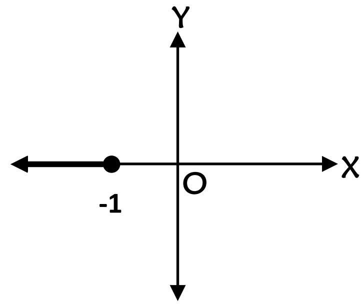
Mathematical Problem

__**Problem -01:**__ Sketch the graph of the function $f(x) = \begin{cases} 0 & ; x \le -1 \\ \sqrt{1-x^2} & ; -1 < x < 1 \\ x & ; x \ge 1 \end{cases}$. Also find domain and range of the function.

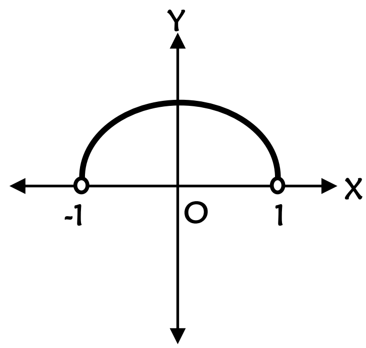
Solution: Given function is

$$y = f(x) = \begin{cases} 0 & ; x \le -1 \\ \sqrt{1-x^2} & ; -1 < x < 1 \quad [say] \\ x & ; x \ge 1 \end{cases}$$

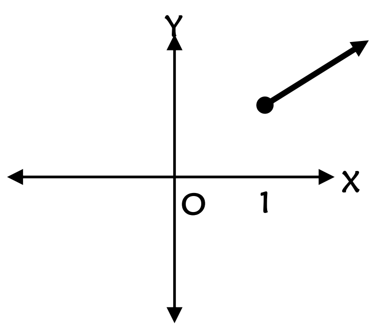
In the interval $x \le -1$ or $(-\infty, -1]$, the graph of the function $y = 0$ is,



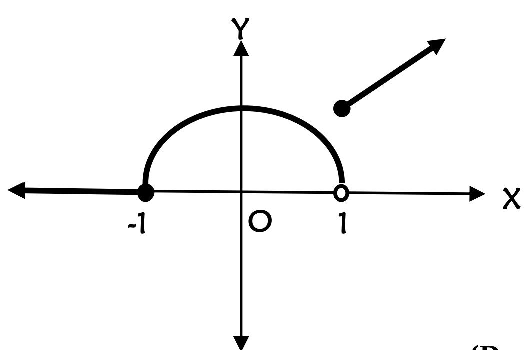
In the interval $-1 < x < +1$ or $(-1,1)$, the graph of the function $y = \sqrt{1-x^2}$ which is an upper semi-circle of radius 1 units and Centre at origin is,



In the interval $x \ge 1$ or $[1, \infty)$, the graph of the function $y = x$ is,



Therefore, the graph of the given function is as follows:



(Desired Graph)

Again, the domain is,

$$D_f = (-\infty, -1] \cup (-1, 1) \cup [1, \infty)$$
$$= (-\infty, \infty)$$
$$= R$$

And the range is,

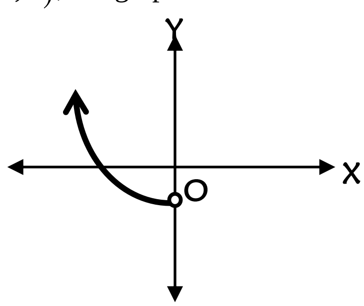
$$R_f = \{0\} \cup (0, 1] \cup [1, \infty)$$
$$= [0, \infty) \qquad \textbf{(Ans.)}$$

<u>Problem -02:</u> Sketch the graph of the function $f(x) = \begin{cases} x^2 - 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$. Also find domain and range of the function.

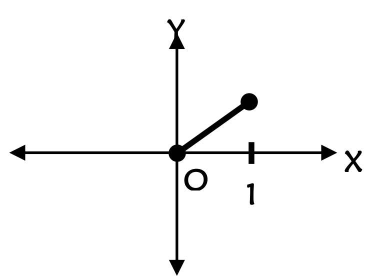
Solution: Given function is,

$$y = f(x) = \begin{cases} x^2 - 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases} \qquad [say]$$

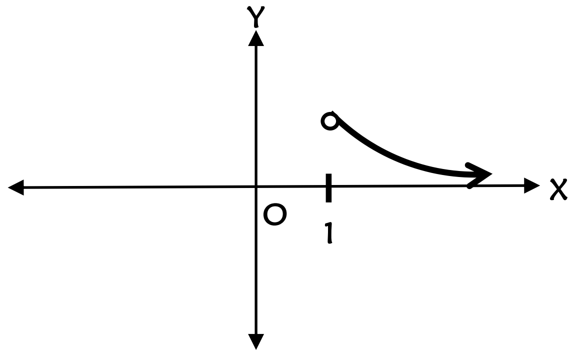
In the interval $x < 0$ or $(-\infty, 0)$, the graph of the function $y = x^2 - 1$ is,



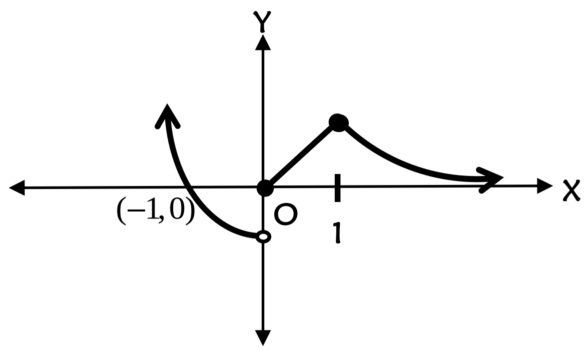
In the interval $0 \le x \le 1$ or $[0, 1]$, the graph of the function $y = x$ is,



In the interval $x \geq 1$ or $[1, \infty)$, the graph of the function $y = \dfrac{1}{x}$ is,



Finally, the graph of the given function is as follows,



(Desired Graph)

Again, the domain is, $D_f = (-\infty, 0) \cup [0, 1] \cup (1, \infty)$

$$= (-\infty, \infty)$$

$$= R$$

And the range is, $R_f = [-1, \infty) \cup [0, 1] \cup (0, 1)$

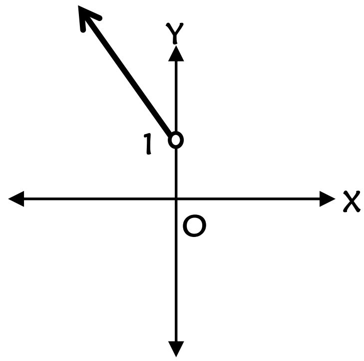
$$= (-1, \infty)$$ **(Ans.)**

Problem -03: Sketch the graph of the function $f(x) = \begin{cases} -2x+1 & ; x < 0 \\ 1 & ; 0 \leq x < 1 \\ 2x-1 & ; x \geq 1 \end{cases}$. Also find domain and range of the function.

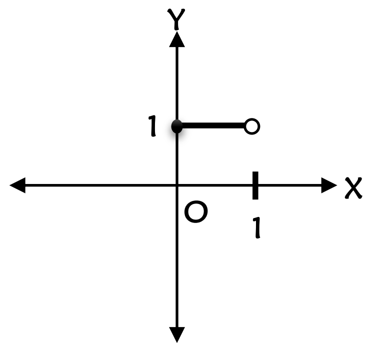
Solution: Given function is,

$$y = f(x) = \begin{cases} -2x+1 & ; x < 0 \\ 1 & ; 0 \leq x < 1 \\ 2x-1 & ; x \geq 1 \end{cases} \qquad [Say]$$

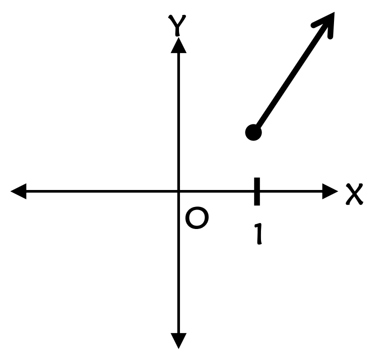
In the interval $x<0$ or $(-\infty,0)$, the graph of the function $y=-2x+1$ is,



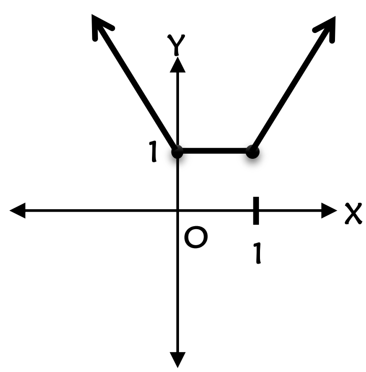
In the interval $0\le x<1$ or $[0,1)$, the graph of the function $y=1$ is,



In the interval $x\ge 1$ or $[1,\infty)$, the graph of the function $y=2x-1$ is,



Finally, the graph of the given function is as follows:



(Desired Graph)

Again, the domain is,

$$\begin{aligned} D_f &= (-\infty, 0) \cup [0,1) \cup [1, \infty) \\ &= (-\infty, \infty) \\ &= R \end{aligned}$$

And the range is,

$$\begin{aligned} R_f &= (1, \infty) \cup \{1\} \cup [1, \infty) \\ &= [1, \infty) \qquad \textbf{(Ans.)} \end{aligned}$$

H.W: Sketch the graph of the following piecewise functions:

1. $f(x) = \begin{cases} x^2 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$

2. $f(x) = \begin{cases} x^2 + 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ \dfrac{1}{x} & ; x > 1 \end{cases}$

3. $f(x) = \begin{cases} 1 - x & ; -1 \le x < 1 \\ 0 & ; 1 \le x \le 2 \\ x^2 - 4 & ; x > 2 \end{cases}$

4. $f(x) = \begin{cases} x^2 + 1 & ; x < 0 \\ x & ; 0 \le x \le 1 \\ 0 & ; x > 1 \end{cases}$

5. $f(x) = \begin{cases} 0 & ; 1 < x \\ 1 + x & ; -1 \le x < 0 \\ 1 - x & ; 0 \le x \le 1 \end{cases}$

6. $f(x) = \begin{cases} 2 - x & ; x \ge 1 \\ x & ; 0 < x \le 1 \\ -x & ; x \le 0 \end{cases}$

7. $f(x)=\begin{cases} 0 & ;|x|>1 \\ 1+x & ;-1\le x\le 0 \\ 1-x & ;0<x<1 \end{cases}$

8. $f(x)=\begin{cases} \dfrac{x}{|x|} & ;x\neq 0 \\ 0 & ;x=0 \end{cases}$

Modulus/absolute function: The modulus or absolute value of x is denoted by the symbol $|x|$ and is defined as follows,

$$|x|=\begin{cases} x & ;\ x\ge 0 \\ -x & ;x<0 \end{cases}$$

Geometrically the modulus or absolute value of a number represents the distance of that number from the origin. The absolute value of x is always positive or zero.

A function together with modulus or absolute value sign is called modulus function.

For example: The function $f(x)=5|x+3|+2|x-2|$ is an absolute value function or Modulus function.

Breaking point of a function: Breaking point of a function is a point at which the function changes.

For example: The function $f(x)=5|x+3|+2|x-2|$ has two breaking points are $x=-3$ & $x=2$.

Procedure of Graphing Absolute value function:

1. At first convert the modulus function into piecewise function according to its number of breaking points.
2. After that sketch the graph as piecewise function.

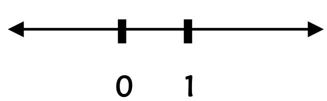
Mathematical Problem

<u>**Problem -01:**</u> Sketch the graph of the function $f(x)=|x|+|x-1|$.

Solution: Given absolute value function is,

$$y=f(x)=|x|+|x-1| \qquad [Say]$$

For breaking points $x=0$ and $x-1=0 \Rightarrow x=1$.



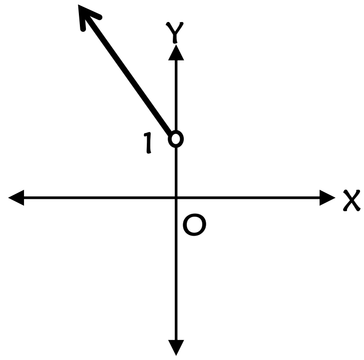
There are two breaking points in this mathematical problem such as $x=0$ & $x=1$ and these points divide real number line into three intervals. Therefore, we define this absolute value function section-ally by three parts.

Now, $y=|x|+|x-1|$

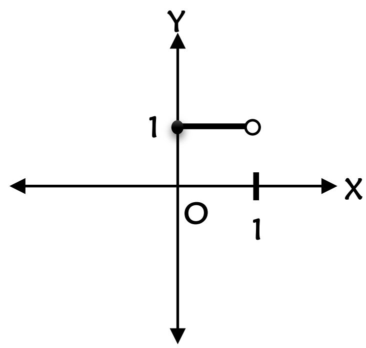
$$=\begin{cases} x+(x-1) & ;x\ge 1 \\ x+(-(x-1)) & ;0\le x<1 \\ (-x)+(-(x-1)) & ;x<0 \end{cases}$$

$$=\begin{cases} 2x+1 & ;x\ge 1 \\ 1 & ;0\le x<1 \\ -2x+1 & ;x<0 \end{cases}$$

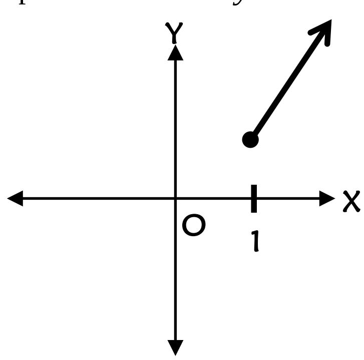
Graph: In the interval $x<0$ or $(-\infty,0)$, the graph of the function $y=-2x+1$ is,



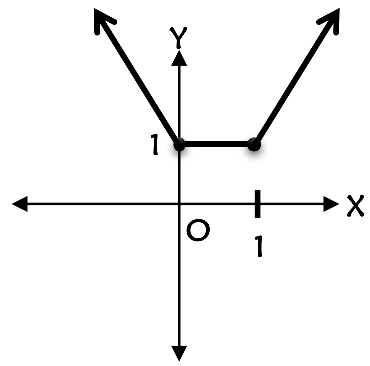
In the interval $0\le x<1$ or $[0,1)$, the graph of the function $y=1$ is,



In the interval $x\ge 1$ or $[1,\infty)$, the graph of the function $y=2x-1$ is,



Finally, the graph of the given function is as follows:



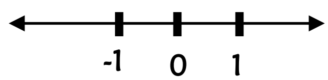
(Desired Graph)

<u>Problem -02:</u> Sketch the graph of the function $f(x)=|x-1|+|x|+|x+1|$.

Solution: Given absolute value function is,

$$y=f(x)=|x-1|+|x|+|x+1| \qquad [Say]$$

For breaking points $x-1=0 \Rightarrow x=1$ and $x=0$ an also $x+1=0 \ \Rightarrow x=-1$



There are three breaking points in this mathematical problem such as $x=-1$, $x=0$ & $x=1$ and those points divide real number line into four intervals. Therefore, we define this absolute value function section-ally by four parts.

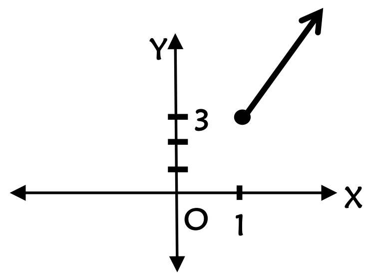
Now,

$$y=|x-1|+|x|+|x+1|$$

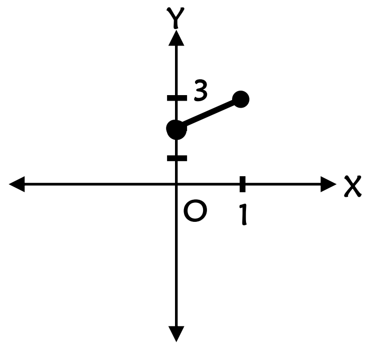
$$=\begin{cases} (x-1)+x+(x+1) & ;x\ge 1 \\ -(x-1)+x+(x+1) & ;0\le x<1 \\ -(x-1)+(-x)+(x+1) & ;-1\le x<0 \\ -(x-1)+(-x)+(-(x+1)) & ;x<-1 \end{cases}$$

$$=\begin{cases} 3x & ;x\ge 1 \\ x+2 & ;0\le x<1 \\ -x+2 & ;-1\le x<0 \\ -3x & ;x<-1 \end{cases}$$

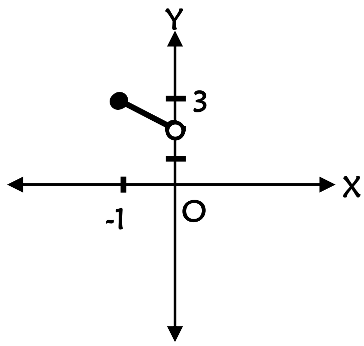
Graph: In the interval $x \geq 1$ or $[1, \infty)$, the graph of the function $y = 3x$ is,



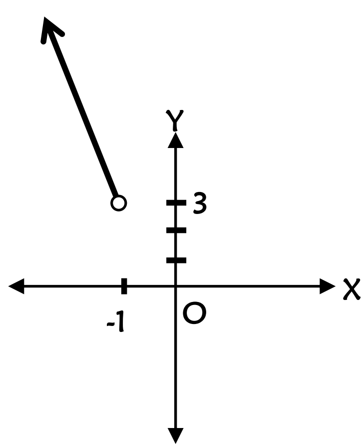
In the interval $0 \leq x < 1$ or $[0,1)$, the graph of the function $y = x + 2$ is,



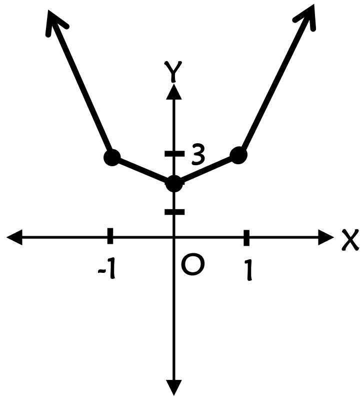
In the interval $-1 \leq x < 0$ or $[-1, 0)$, the graph of the function $y = -x + 2$ is,



In the interval $x < -1$ or $[-1, 0)$, the graph of the function $y = -3x$ is,



Finally, the graph of the given function is as follows:



(Desired Graph)

H.W:

Sketch the graph of the following absolute value functions:

1. $f(x) = |x| - x$
2. $f(x) = |x| + |x+1|$
3. $f(x) = |x+1| + |x-1|$
4. $f(x) = |x+1| + |x-2|$
5. $f(x) = |x+2| + |x-2|$
6. $f(x) = |x| + |x-1| + |x-2|$