Content

⇒ Submatrix (2D) sum queries

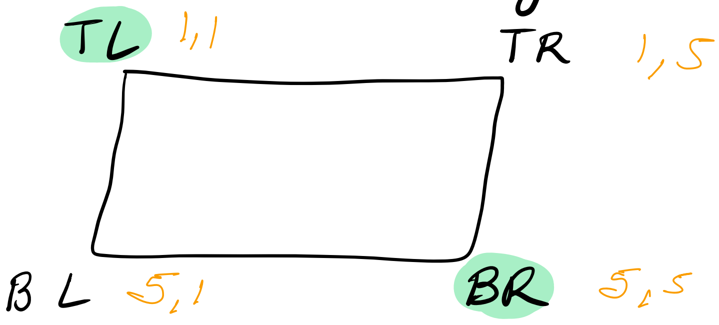
⇒ Sum of all submatrices

⇒ Max submatrix sum (sorted)

⇒ Search in sorted matrix

Q1 Given int mat[N][M], for each query q.
CRED find sum of given submatrix.
↳ Continuous part of matrix.

● How is submatrix given in query?



Eg -

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

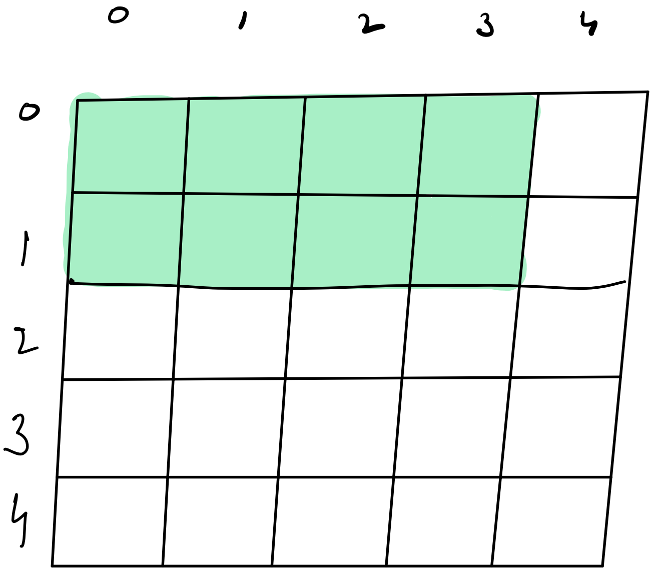
q → 2,1 , 4,2 (TL, BR)

Brute: Iterate on the whole submatrix

$O(n*m)$

q queries ⇒ $O(qnm)$

- **Idea:** Prefix sum



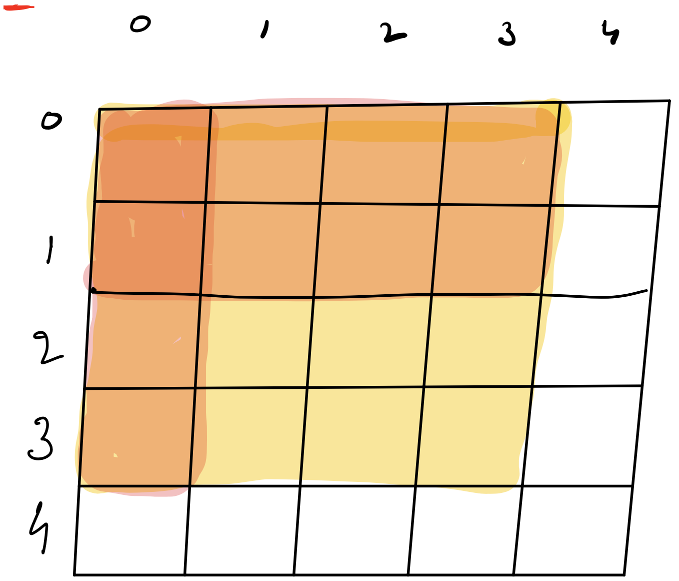
- ⇒ pf[1][3]

pf ⇒ sum of submatrix

TL ⇒ 0,0

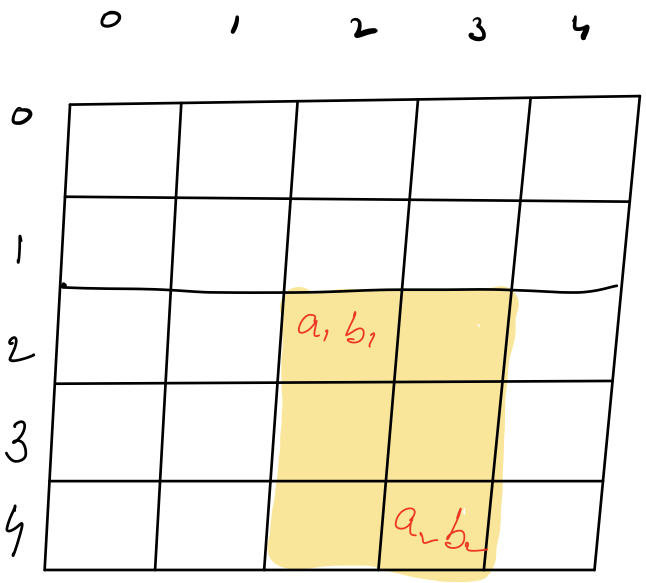
BR ⇒ i,j

- Assume pf is calculated. Answer Query.



2,1 to 3,3

= pf[3][3]
− pf[3][0]
− pf[1][3]
+ pf[1][0]



2,2 to 4,3

pf[4,3]
− pf[4,1]
− pf[1,3]
+ pf[1,1]

Generalize TL BR

a_1, b_1 a_2, b_2

$pf[a_2, b_2]$ $- pf[a_2][b_1 - 1]$
$- pf[a_1 - 1][b_2]$
$+ pf[a_1 - 1][b_1 - 1]$

Edge   pf[e] - pf[s-1]

```
int sum(int a1, int b1, a2, b2){
   sum = pf[a2][b2]
  if (b1-1 ≥ 0)
      sum -= pf[a2][b1-1]
  if (a1-1 ≥ 0)
      sum -= pf[a1-1][b2]
  if (a1-1 ≥ 0 && b1-1 ≥ 0)
      sum += pf[a1-1][b1-1]
}
```

Total TC: $O(n*m + q)$

- We can now answer queries using pf.
- How to build pf ?

VVV simple

⇒

Step 1) Apply row-wise prefix sum

Step 2) Apply column-wise prefix sum

	0	1	2
0	a_0	b_0	C_0
1	a_1	b_1	C_1
2	a_2	b_2	C_2

a_0	a_0+ b_0	a_0+ b_0+C_0
a_0+a_1	a_0+b_0 $+a_1+b_1$	$a_0+b_0+C_0$ $a_1+b_1+C_1$
a_0+ a_1+a_2	a_0+b_0 a_1+b_1 a_2+b_2	all

↓ row wise prefix su

a_0	a_0+ b_0	a_0+b_0 $+C_0$
a_1	a_1+b_1	a_1+b_1 $+C_1$
a_2	a_2+ b_2	a_2+b_2 $+C_2$

column wise pref sum →

a_0	a_0+ b_0	a_0+b_0 $+C_0$
a_0+ a_1	a_0+b_0 $+a_1+b_1$	a_0+b_0 $+C_0+a_1$ b_1+C_1
a_0+ a_1+a_2	a_0+b_0 a_1+b_1 a_2+b	all

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

row →

1	3	6
4	9	15
7	15	24

↓ col

1	3	6
5	12	21
12	27	45

TC: $O(n*m)$

break

back at 8:25

Q2 Given arr[N][M], Calc sum of all submatrix sums

Ex $\begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix}$

$[3] = 3$ $\quad [3, 1] = 4$

$[1] = 1$ $\quad [-1, -2] = -3$

$[-1] = -1$ $\quad \begin{bmatrix} 3 \\ -1 \end{bmatrix} = 2$

$[-2] = -2$ $\quad \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -1$

$\begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix} = 1$

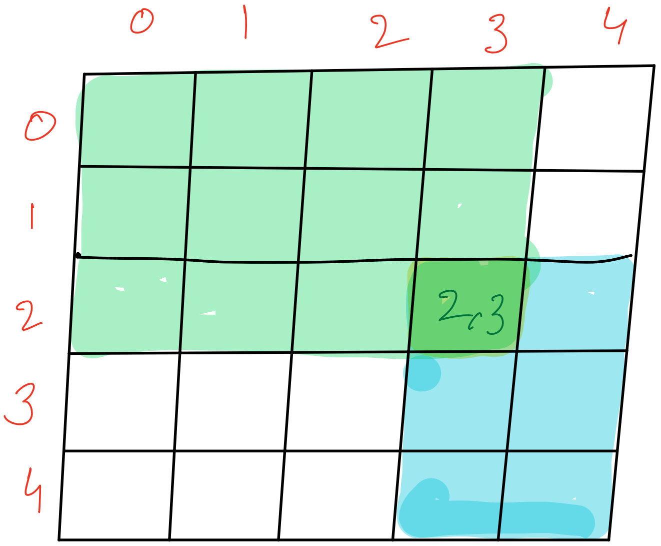
tot = 4

- Idea from intermediate for 1-D array

  Sum of all subarray sums ⇒

  Contribution Technique

Thus we will use the same technique here. Calculate number of submatrices which contain cell (i, j)



$TL = 3 \times 4$

$(2+1) \times (3+1)$

$BR = 3 \times 2$

$(5-2) \times (5-3)$

$3 \times 4 \times 3 \times 2 = 72$

$TL = (i+1)(j+1)$

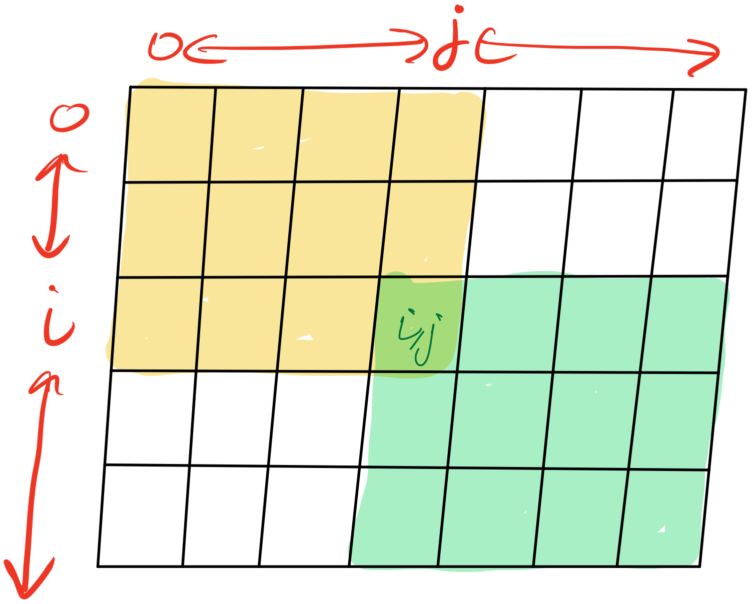
$BR = (n-i)(m-j)$

total no of submatrices

$= TL \times BR$

$(i+1)(j+1)(n-i)(m-j)$

For every TL & BR, we get unique submatrice



Total number =

$(i+1)(j+1)(n-i)(m-j)$

Code

```
for( i=0 ; i< n ; i++){
    for( j=0 ; j<m ; j++) {
        sum += a[i][j] *
                (i+1)(j+1)(n-i)(m-j)
    }
}
```

TC: O(nm)   SC: O(1)

Amazon

Q3 Given arr[N][M] find max sub sum. Array is row-wise & col-wise ~~sorted~~ ascending

Eg

	0	1	2	3
0	-20	-16	-4	8
1	-10	-8	12	14
2	-1	6	21	30
3	5	7	28	42

	0	1	2
0	-20	-16	-4
1	-10	-8	12
2	-1	6	21

Idea 1: Try all submatrices

Total no of submatrices

$$\Rightarrow \frac{n(n+1)}{2} \times \frac{m(m+1)}{2}$$

Obs: Always BR = $n-1, m-1$

TL = no idea

Create pf sum matrix

TL	BR
Try all points as top left	n-1, m-1

Code

1) Create pf array

```
ans = Integer.Min_Value
for(i=0;i<n;i++){
  for(j=0;j<m;j++){
      TL = i,j
      BR = n-1, m-1
     //Get sum of submatrix
      ans = max(sum, ans)
  }
}
```

TC: O(nm)   SC: O(1)

Q4 Given row-wise and col-wise sorted matrix, find k.

	0	1	2	3	4	5
0	-10	-5	-2	2	4	7
1	-7	-4	-1	3	6	9
2	-2	3	5	7	11	14
3	3	6	8	11	14	17
4	7	11	12	15	19	20
5	10	14	18	20	24	29

k =
12

Brute:

Iterate whole matrix

TC: O(n*m)

Idea: Start at Top right corner

```
If  arr[i][j] < k
        i++
    arr[i][j] > k
        j--
```

Code

```
bool search(int a[][], int k){
{   i=0            j=m-1
    while( i<n  && j>=0){
        if (a[i][j] == k)
            return true
        else if ( a[i][j] > k)
            j--
        else        // a[i][j] < k
            i++
    }
    return false
```

y

TC: $O(n+m)$ SC: $O(1)$

$m > n$

$O(m+m) = O(2m)$

$= O(m)$

{done}