

Contents

GCD(HCF)	3
LCM.....	3
1D Array.....	3
Initializing an Array	3
Input Elements in an Array	3
Output Elements in an Array.....	3
1D_ArrayList	4
Initializing 1D_Arraylist.....	4
Basic Operations	4
1. add :	4
2. get :	4
3. size :	4
4. set :	4
2D Array.....	5
Initializing an Array	5
Scanning Elements in an Array.....	5
Printing Elements in an Array	5
Equations & Expressions	5
2D ArrayList	6
Initializing 2D ArrayList	6
Basic Operations	6
1. get :	6
2. add :	6
3. set :	7
4. size :	7
String	8
Initializing and Scanning a String.....	8
Basic Operations	8
1. length() :	8
2. charAt() :	8
Time Complexity	9
Range of numbers.....	9
1. Include last number :	9
2. Exclude last number :	9

3. Arithmetic Progression(AP) :	9
4. Geometric Progression(GP) :	9
Log Basics	10
Big O notation	11
Space Complexity	12
Arrays	13
Prefix Sum	13
Carry Forward	13
Sub Arrays	14
2D Matrices	14
1. Diagonal Elements of matrices	14
2. Non-Diagonal Elements(secondary diagonal) of matrices	14
3. All diagonal Elements of matrices	15
4. Transpose Matrix	15
5. Rotating the matrices to 90°	16
6. Print boundary elements in clockwise	16
7. Print every boundary elements in clockwise	16
Sliding Window	17
Bit Manipulation	18
Binary to Decimal	18
Decimal to Binary	19
Binary Addition	20
Bitwise Operators	20
1. & (AND)	20
2. (OR)	20
3. ~ (NOT)	21
4. ^ (XOR)	21
5. << (Left Shift)	21
6. >> (Right Shift)	23
Storing and Retrieving -ve number	24
Storing -ve number	24
Retrieving -ve number	24
Max and Min numbers to be stored	24
Bitwise Properties	25
1. Commutative	25
2. Associative	25

GCD(HCF)

- Largest integer that can exactly divide both no without any remainder.
 - $(n1 \% i == 0) \ \&\& \ (n2 \% i == 0)$
HCF = i;

LCM

- Smallest integer that is perfectly divisible by both numbers.
 - $n1 * n2 = \text{GCD} * \text{LCM}$
 - $\text{LCM} = (n1 * n2) / \text{GCD}(\text{HCF})$

1D Array

Initializing an Array

```
dataType variable[] = new int[size]
```

or

```
dataType[] variable = new int[size]
```

Input Elements in an Array

```
for(int i=0; i<variable.length ;i++){  
    variable[i] = scn.nextType();  
}
```

Note: In scn.nextType Type-> datatype

Output Elements in an Array

```
for(int i=0; i<variable.length ;i++){  
    variable[i] = scn.nextType();  
}
```

Note: In scn.nextType Type-> datatype

1D ArrayList

Initializing 1D ArrayList

```
ArrayList<Integer> al = new ArrayList<Integer>();
```



Integer, Long, Double, String

Basic Operations

1. add :

- add() function used to add an element in the arraylist.

```
ArrayList<Integer> al = new ArrayList<Integer>();  
al.add(element)
```

Example :

```
al.add(1);  
al.add(-7);
```

Output : [1, -7]

2. get :

- get() function is used to get an element from particular index.
- If index is greater than size of the arraylist we will be getting an error called **index out of bound**.

```
ArrayList<Integer> al = new ArrayList<Integer>();  
al.get(index)
```

Example :

```
al.get(0); → 1  
al.get(1); → -7
```

3. size :

- size() function is used to get the size of the array list.

```
ArrayList<Integer> al = new ArrayList<Integer>();  
al.size(); → 2
```

4. set :

- set() is used to replace the current existing value of an index.
- set() can be used on existing indexes. If there is no index in the arraylist then it gives an error.

```
ArrayList<Integer> al = new ArrayList<Integer>();  
al.set(index, value)
```

Example:

```
al.get(0, 5);  
al.add(1, 7);
```

Output: [5,7]

2D Array

Initializing an Array

dataType variable[][] = new int[row][column]

or

dataType[][] variable = new int[row][column]

Scanning Elements in an Array

```
for(int row=0;row<N;row++){
    for (int col=0; col<M; col++){
        arr1[row][col] = scn.nextType();
    }
}
```

Note: In scn.nextType Type-> datatype

Printing Elements in an Array

```
for(int row=0;row<N;row++){
    for (int col=0; col<M; col++){
        System.out.print(result[row][col] + " ");
    }
    System.out.println();
}
```

Equations & Expressions

- `a.length;` *//getting the row length of array*
- `a[0].length;` *//getting the col length of array*
- dimensions of array -> $N \times M$ (row*column)
- Total no of elements -> $N \times M$
- Last element location of 2D array -> $N-1, M-1$

2D ArrayList

- ArrayList of arraylist

Initializing 2D ArrayList

```
ArrayList<ArrayList<Integer>> variable = new ArrayList<ArrayList<Integer>>();
```



Integer, Long, Double, String

- Example:

```
ArrayList < ArrayList <Integer>> list = new ArrayList < ArrayList <Integer>>();
```

```
ArrayList<Integer> li = new ArrayList<Integer>();
```

```
li.add(1);
```

```
li.add(2);
```

```
li.add(3)
```

```
li -> [1,2,3]
```

```
list.add(li)
```

```
list -> { [1,2,3] }
```

```
ArrayList<Integer> li2 = new ArrayList<Integer>();
```

```
li2.add(4);
```

```
li2.add(5);
```

```
li2.add(6)
```

```
li2 -> [4,5,6]
```

```
list.add(li2)
```

```
list -> { [1,2,3], [4,5,6] }
```

Basic Operations

1. get:

```
list.get(1) -> [4,5,6]
```

```
list.get(1).get(1) -> 5
```

2. add:

```
list.get(1).add(7)
```

```
list -> { [1,2,3], [4,5,6,7] }
```

3. set :

```
ArrayList <Integer> li3 = new ArrayList <Integer>();
```

```
li3.add(8);
```

```
li3.add(9);
```

```
li3.add(10)
```

```
li3 -> [8,9,10]
```

```
list -> { [1,2,3], [4,5,6,7] }
```

```
list.set(0,li3);
```

```
list -> { [8,9,10], [4,5,6,7] }
```

```
list.get(1).set(1,10);
```

```
list -> { [8,9,10], [4,10,6,7] }
```

4. size :

- Count of arraylists in list -> list.size() -> 2
- Length of 1st index arraylist -> list.get(0).size() ->3

String

- String is nothing but sequence of characters which is present between "".
- Characters :
 - [a-z]
 - [A-Z]
 - Special Characters
 - [0-9]

Initializing and Scanning a String

Scanner scn = new Scanner(System.in);

- **String str = scn.next(); //**
 - Java **next()** method can read the input before the space is found. It cannot read two words separated by space. It retains the cursor in the same line after reading the input.
- **String str = scn.nextLine(); //**
 - The **nextLine()** method of Scanner class is used to take a string from the user. The **nextLine()** method reads the text until the end of the line. After reading the line, it throws the cursor to the next line.

Basic Operations

1. length() :

- Used to get the length of the string
String name = "Mohnish"
name.length() → 7

2. charAt() :

- Used to get character at particular index of the string.
String name = "Mohnish"
name.charAt(3) → n

Time Complexity

- Sum of n natural numbers.
 $1 + 2 + 3 + \dots + n = (n * (n+1))/2$

Range of numbers

1. Include last number :

- [a,b] -> represents that b must be include in the range of numbers from a to b.
Example: [3,10]
Output: 3 4 5 6 7 8 9 10
- To get no of number in the particular range of 2 numbers we can use the following formula **b-a+1**.
Example: [3,10]
Output: $10-3+1 = 8$ // total there are 8 numbers from 3 to 10.

2. Exclude last number :

- [a,b) -> represents that b must be exclude in the range of numbers from a to b.
Example: [3,10)
Output: 3 4 5 6 7 8 9
- To get no of number in the particular range of 2 numbers we can use the following formula **b-a**.
Example: [3,10)
Output: $10-3 = 7$ // total there are 7 numbers from 3 to 10.

3. Arithmetic Progression(AP) :

3 6 9 12 15 18 21 24 n

- In the above example the difference between all numbers is 3 which is constant different and the terms are consecutive.
- Sum of n numbers in the AP can be calculated using the following formula
 $n/2[2a + (n-1)d]$
n = no of elements in the sequence.
a = starting number in the sequence.
d = difference between numbers.

4. Geometric Progression(GP) :

2 4 8 16 32 64 n

- In the above example the difference between all numbers is not constant but the ratio is common and the terms are consecutive terms.
- Sum of n natural numbers in GP is calculated using the following formula
 $a(r^n - 1) / r - 1$
a = starting number of the sequence
r = ratio between the numbers

Log Basics

1. $\log_b^a = ?$

- To get answer we need to find **to what power b need to be raised to get a.**

- Example

$$\log_2^{64} = ? \rightarrow a = 64, b = 2$$

To get the answer we need to find the power of 2(b) so that we get 64(a)

$$2^? = 64 \rightarrow 2^6 = 64$$

$$\rightarrow \log_2^{64} = 6$$

Since we need to raise the power 6 of 2 to get 64 hence 6 is the value of \log_2^{64} .

2. $\log_a^b = x$, where $b = a^x$

3. if $N = a^k \Rightarrow \log_a N = k$

Explanation:

$$N = 2^k$$

$$\log_2 N = \log_2^b, \text{ where } b = 2^k \rightarrow \text{Applied } \log_2 \text{ on both side of above equation}$$

$$\log_2 N = k \rightarrow \text{If we compare above equation's RHS to } 2^{\text{nd}} \text{ property we will be getting K}$$

$$N = 3^k$$

$$\log_3 N = \log_3^b, \text{ where } b = 3^k \rightarrow \text{Applied } \log_2 \text{ on both side of above equation}$$

$$\log_3 N = k \rightarrow \text{If we compare above equation's RHS to } 2^{\text{nd}} \text{ property we will be getting K}$$

Examples:

1. `int s=0;`

```
for(int i=0; i<=100; i++){
```

```
    s = s+i;
```

```
}
```

- The above loop iterates 101 times since in the condition both 0 and 100 are involved
- So we can assume as $[0,100] \Rightarrow 100-0+1 \rightarrow 101$ ($[a,b] \Rightarrow b-a+1$).

2. `s = 0`

```
for(int i=35; i<=87; i=i+1){
```

```
    s=s+i;
```

```
}
```

Ans $\Rightarrow [35,87] \Rightarrow 87-35+1 \Rightarrow 53$ **$[(35\ 36\ 37\ 38\ \dots\ 87) \rightarrow \text{total 53 terms}]$**

3. `s = 0`

```
for(int i=1; i<=N; i=i+1){
```

```
    s=s+i;
```

```
}
```

Ans $\Rightarrow [1,N] \Rightarrow N-1+1 \Rightarrow N$ **$[(1\ 2\ 3\ 4\ \dots\ N) \rightarrow \text{total N terms}]$**

4. $s = 0$
 for(int i=1; i<=N; i=i+1){
 $s=s+i$;
 }

for(int i=1; i<=M; i=i+1){
 $s=s+i$;
 }

Ans $\rightarrow N+M$

Explanations:

- The first loop runs N times
- The second loop runs M times
- So if we add both loops its runs **N+M** times

5. $s = 0$
 for(int i=1; i*i<=N; i=i+1){
 $s=s+i$;
 }

Ans $\rightarrow \sqrt{N}$

Explanation:

- $i*i \leq N \rightarrow i \leq \sqrt{N}$
- $[1, \sqrt{N}] \Rightarrow \sqrt{N} - 1 + 1 \rightarrow \sqrt{N}$

6. $s = 0$
 for(int i=1; i*i<=N; i=i+1){
 for(int j = i; j<=N; j++){
 $s=s+1$;
 }
 }

Ans $\rightarrow N^2$

Explanations:

- The first loop runs N times
- The second loop runs N times
- Since second loop is inside first loop we need to multiply both loop hence its N^2

Big O notation

- Calculate no of iterations.
- Ignore lower order terms.
- Ignore constant coefficient.

Example to ignore lower order terms and constant coefficient

1. $100\log N \rightarrow O(\log N)$ [here 100 is constant coefficient].
2. $N/100 \rightarrow O(N)$ [here 1/100 is the constant coefficient].
3. $4N^2 + 5N + 6 \rightarrow O(N^2)$ [here N^2 is the higher order term so remaining lower order and constant terms are ignored].

Space Complexity

- Total space taken by the algorithm which is completely based on the variable space.

- **Example:**

1. `fun(int N){` → N = 4 bits space
 `int x = N;` → x = 4 bits space
 `int y = x*x;` → y = 4 bits space
 `long z = x + y;` → z = 8 bits space
 `}`

Total = 4+4+4+8 → 20bits (constant bits)

So O(1).

2. `fun(int N){` → N = 4 bits space
 `int x = N;` → x = 4 bits space
 `int y = x*x;` → y = 4 bits space
 `long z = x + y;` → z = 8 bits space
 `int[] arr = new int[N]` → arr = 4N bits space
 `}`

Total = 4+4+4+8+4N → 20Nbits (20 is constant so ignored but has a higher order terms which is N)

So O(N).

3. `fun(int N){` → N = 4 bits space
 `int x = N;` → x = 4 bits space
 `int y = x*x;` → y = 4 bits space
 `long z = x + y;` → z = 8 bits space
 `int[] arr = new int[N]` → arr = 4N bits space
 `int[] arr1 = new int[N][N]` → arr1 = 4N² bits space
 `}`

Total = 4+4+4+8+4N+4N² → 20Nbits (20 is constant so ignored but has a higher order terms which is N²)

So O(N²).

Arrays

- Time complexity of single element in the array is $O(1)$.
- Time complexity of access (scan, print) of the array is $O(N)$.

Prefix Sum

- Prefix sum is the technique where you precompute & store the cumulative sum of the sequence of elements that allows fast sum calculation of any continuous range.
- Let's say we have a sequence of elements A as mentioned below-
 $A = \{a_0, a_1, a_2, a_3, a_4, a_5\}$

- So Prefix Sum P will be calculated as
 $P = \{p_0, p_1, p_2, p_3, p_4, p_5\}$

where

$$\begin{aligned}p_0 &= a_0 \\p_1 &= a_1 + a_0 \\p_2 &= a_0 + a_1 + a_2 \\p_3 &= a_0 + a_1 + a_2 + a_3 \\p_4 &= a_0 + a_1 + a_2 + a_3 + a_4 \\p_5 &= a_0 + a_1 + a_2 + a_3 + a_4 + a_5\end{aligned}$$

- Q) Say we need to sum get sum of all elements from indices
 $[2 \text{ to } 5] \Rightarrow [a_2 + a_3 + a_4 + a_5]$ or $[p_5 - p_1]$
 $[1 \text{ to } 4] \Rightarrow [a_1 + a_2 + a_3 + a_4]$ or $[p_4 - p_1]$
 $[0 \text{ to } 4] \Rightarrow [a_0 + a_1 + a_2 + a_3 + a_4]$ or $[p_4]$

Carry Forward

- Carry forward is a process where we will tracing the array from $[n-1 \text{ to } 0]$ and perform required operations or conditions or both.

Sub Arrays

- We know that an array is a contiguous memory block. Similarly, a sub-array is any contiguous part of that array, which may consist of any number of elements with at least one element in it.
- Let us write all the subarrays for the array: [3, 5, 1, 2, 7, 4]
- The sub-arrays for this array are:

Index of Element	Subarrays Possible	Count
0 (element 3)	{3}, {3, 5}, {3, 5, 1}, {3, 5, 1, 2}, {3, 5, 1, 2, 7}, {3, 5, 1, 2, 7, 4}	6
1 (element 5)	{5}, {5, 1}, {5, 1, 2}, {5, 1, 2, 7}, {5, 1, 2, 7, 4}	5
2 (element 1)	{1}, {1, 2}, {1, 2, 7}, {1, 2, 7, 4}	4
3 (element 2)	{2}, {2, 7}, {2, 7, 4}	3
4 (element 7)	{7}, {7, 4}	2
5 (element 4)	{4}	1

- Total number of subarrays possible in an array of length $N = (N*(N+1))/2$

2D Matrices

1. Diagonal Elements of matrices

0	1	2	3	4
1				
2				
3				
4				

- $i = 0 \rightarrow N-1 \rightarrow a[i][i]$

2. Non-Diagonal Elements(secondary diagonal) of matrices

0	1	2	3	4
1				
2				
3				
4				

- $i = 0 \rightarrow N-1$
- $j = N-1 \rightarrow 0$
- $a[i][j]$

3. All diagonal Elements of matrices

0	1	2	3	4
1				
2				
3				
4				

- The above matrices is divided into 2 parts
- Part 1:**

0	1	2	3	4
1				
2				
3				
4				

- Column $\rightarrow 0 - M-1$
- $i \rightarrow 0 - N-1$
- $j \rightarrow c - 0$ (c indicate every iteration of column).

o $a[i][j]$

Part 2:

0	1	2	3	4
1				
2				
3				
4				

- Row $\rightarrow 0 - N-1$
- $i \rightarrow r - N-1$ (r indicates every iteration of row).
- $j \rightarrow M-1 - 0$

4. Transpose Matrix

- Converting rows of a matrix into columns and columns of a matrix into row is called transpose of a matrix.

0	1	2	3	4
1				
2				
3				
4				

- If we see the above matrices it is divided into 3 parts
- Part 1 : Upper triangle
- Part 2 : Diagonals
- Part 3 : Lower triangle
- We have to swap upper triangle value to lower triangle.
- Since when we convert row to column and column to row the diagonals will be in its absolute location so we have to swap upper and lower triangle.
- $i \rightarrow 0 - N-1$
 - $j \rightarrow i+1 - N$

▪ Swap $a[i][j]$ & $a[j][i]$

0	1	2	3	4
1				
2				
3				
4				

- After performing transpose process the matrix will as above (the blue and orange part has been swapped).

5. Rotating the matrices to 90°

- We have to perform 2 steps to rotate matrices to 90°
- First step : we have to perform transpose matrix process.
- Second step : we have to reverse the elements for each and every columns.

6. Print boundary elements in clockwise

0	1	2	3
11			4
10			5
9	8	7	6

- We have to use 4 loops and default value $i = 0, j = 0$
- Loop 1 → Top wall [$k \rightarrow 1 - N - 1, j++$]
- Loop 2 → Right wall [$k \rightarrow 1 - N - 1, i++$]
- Loop 3 → Bottom wall [$k \rightarrow 1 - N - 1, j--$]
- Loop 4 → Left wall [$k \rightarrow 1 - N - 1, i--$]

7. Print every boundary elements in clockwise

0	1	2	3	4
15	16	17	18	5
14	23	24	19	6
13	22	21	20	7
12	11	10	9	8

- There are 2 steps
- $i = 0, j = 0$
- Step 1: while loop [$\text{sizeOfArray} - N$]
 - In step1 there are 4 steps
 - Loop 1 → Top wall [$k \rightarrow 1 - N - 1, j++$] print $a[i][j]$
 - Loop 2 → Right wall [$k \rightarrow 1 - N - 1, i++$] print $a[i][j]$
 - Loop 3 → Bottom wall [$k \rightarrow 1 - N - 1, j--$] print $a[i][j]$
 - Loop 4 → Left wall [$k \rightarrow 1 - N - 1, i--$] print $a[i][j]$
 - $N = N - 2, i++, j++$
- Step 2: if condition ($N == 1$) print $a[i][j]$

Sliding Window

- **Carry forward technique + fixed size of subarray = Sliding Window**
- Given N element, return max subarray sum of length k ?

Example:

```

      0 1 2 3 4 5 6 7 8 9
ar[10] = -3 4 -2 5 3 -2 8 2 -1 4
k = 5

```

$0 - 4 = 7$	$= \text{sum}$
$1 - 5 = 7 - (-3) + (-2)$	$= \text{sum} - a[0] + a[5] = 8$
$2 - 6 = 8 - 4 + 5$	$= \text{sum} - a[1] + a[6] = 9$
$\cdot \quad \cdot \quad \cdot$	$\cdot \quad \cdot \quad \cdot$
$\cdot \quad \cdot \quad \cdot$	$\cdot \quad \cdot \quad \cdot$
$\cdot \quad \cdot \quad \cdot$	$\cdot \quad \cdot \quad \cdot$
$\cdot \quad \cdot \quad \cdot$	$\cdot \quad \cdot \quad \cdot$
$a[s] - a[e]$	$= \text{sum} - a[s] + a[e]$

Bit Manipulation

- Decimal number system :- { 0 – 9 }
- 1 2 3 → 3 place is units(0) place, 2 place is tens(1) place, 3 place is hundreds(2) place.....
 $3 * 10^0 + 2 * 10^1 + 1 * 10^2$ → here 10 represents there are total 10 numbers
- $\sum \text{Digits} + \text{Base}^{\text{position}}$
Here,
Digits are 1 2 3
Base is 10
Position is digits places
- So base for decimal number system is 10
- Binary number system :- { 0, 1 }
- Since in binary number there are 2 numbers the base is 2.

Binary to Decimal

- $\sum \text{Digits} + \text{Base}^{\text{position}}$ (Base = 2)
- Example

1. $110 \rightarrow 0 * 2^0 + 1 * 2^1 + 1 * 2^2 = 0 + 2 + 4 = 6$

$$110 = 6$$

2. $101 \rightarrow 1 * 2^0 + 0 * 2^1 + 1 * 2^2 = 1 + 0 + 4 = 5$

$$101 = 5$$

3. $111001 \rightarrow ?$

$$1 * 2^0 + 0 * 2^1 + 0 * 2^2 + 1 * 2^3 + 1 * 2^4 + 1 * 2^5$$

$$1 + 0 + 0 + 8 + 16 + 32$$

$$57$$

$$(111001)_2 = (57)_{10}$$

Decimal to Binary

- **Number % 2 (Until the quotient is 0)**
- Example

1. 57

57%2

Base	Number	Remainder
2	57	1
2	28	0
2	14	0
2	7	1
2	3	1
2	1	1
2	0	

Combine remainder from bottom to top 1 1 1 0 0 1

$$(57)_{10} = (111001)_2$$

2. 25

25%2

Base	Number	Remainder
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
2	0	

Combine remainder from bottom to top 1 1 1 0 0 1

$$(25)_{10} = (111001)_2$$

- Repeatedly divide with 2
Until number becomes 0
Read remainder from bottom to top

Binary Addition

- Rules
 - $0 + 0 = 0$
 - $1 + 0 = 1$
 - $0 + 1 = 1$
 - $1 + 1 = 10$ (Here 1 will be carried forward)
- Example

1. $0111001 + 0100101$

$$\begin{array}{r} 1 \\ 0111001 \\ + 0100101 \\ \hline 1011110 \end{array} \qquad \begin{array}{r} 57 \\ + 37 \\ \hline 94 \end{array}$$

Bitwise Operators

- The following operations can be performed

1. & (AND)

A	B	A&B
0	0	0
0	1	0
1	0	0
1	1	1

- If any of input is 0, result is 0.
- If N is a number then
 - $N \& 1 \rightarrow 1$ [N is odd] (set)
 - $N \& 1 \rightarrow 0$ [N is even] (unset)
- $N \& 0 \rightarrow 0$

2. | (OR)

A	B	A B
0	0	0
0	1	1
1	0	1
1	1	1

- If any of input is 1, result is 1.
- If N is a number then
 - $N | 1 \rightarrow N$ [N is odd]
 - $N | 1 \rightarrow N + 1$ [N is even]
- $N | 0 \rightarrow N$

3. ~ (NOT)

A	~A
0	1
1	0

- The result will be opposite to input.

4. ^ (XOR)

A	B	A^B
0	0	0
0	1	1
1	0	1
1	1	0

- If both input is same, result is 0.
- If both input is different, result is 1.
- If N is a number then
 - $N \wedge N \rightarrow 0$
 - $N \wedge 0 \rightarrow N$
 - $N \& 0 \rightarrow 0$

5. << (Left Shift)

- The left shift operator moves all bits by a given number of bits to the left.
- **Example**

$a = 00111001$ $57 \rightarrow 57 \times 2^0$
 $a \ll 1 = 01110010$ (1 bit has been moved to left) $114 \rightarrow 57 \times 2^1$
 $a \ll 2 = 11100100$ (2 bit has been moved to left) $228 \rightarrow 57 \times 2^2$
 $a \ll 3 = 11001000$ (3 bit has been moved to left) $456 \rightarrow 57 \times 2^3$

$a \ll n = a \times 2^n$ (If there is no overflow [only 0 moved to left])

- $N \mid (1 \ll i)$ — $\begin{cases} N, \text{ if } i^{\text{th}} \text{ bit is 1 (Set)} \\ > N, \text{ if } i^{\text{th}} \text{ bit is 0 (UnSet)} \end{cases}$

➤ **Example**

$N = 57 \rightarrow 111001$

Case 1

$i = 3$
 $N \mid (1 \ll i)$
 $111001 \mid 1 \ll 3$

111001
 001000

 $111001 \rightarrow 57 = N$

- If we see the above example the i^{th} bit is set(1) so the operations gives N value as the result.

Case 2

$$\begin{array}{l}
 i = 2 \\
 N \mid (1 \ll i) \\
 111001 \mid 1 \ll 2 \\
 \\
 \begin{array}{r}
 111\textcolor{red}{0}01 \\
 000100 \\
 \hline
 111101 \rightarrow 61 = >N
 \end{array}
 \end{array}$$

- If we see the above example the i^{th} bit is unset(0) so the operations gives $>N$ value as the result.

$$\circ \quad N \& (1 \ll i) \begin{cases} 1 \ll i, & \text{if } i^{\text{th}} \text{ bit is } 1 \\ 0, & \text{if } i^{\text{th}} \text{ bit is } 0 \end{cases}$$

➤ Example

$$N = 57 \rightarrow 111001$$

Case 1

$$\begin{array}{l}
 i = 3 \\
 N \& (1 \ll i) \\
 111001 \& 1 \ll 3 \\
 \\
 \begin{array}{r}
 11\textcolor{red}{1}001 \\
 001000 \\
 \hline
 001000 \rightarrow 1 \ll 3 = 1 \ll i
 \end{array}
 \end{array}$$

- If we see the above example the i^{th} bit is set(1) so the operations gives $1 \ll i$ value as the result.

Case 2

$$\begin{array}{l}
 i = 2 \\
 N \& (1 \ll i) \\
 111001 \& 1 \ll 2 \\
 \\
 \begin{array}{r}
 111\textcolor{red}{0}01 \\
 000100 \\
 \hline
 000000 \rightarrow 0
 \end{array}
 \end{array}$$

- If we see the above example the i^{th} bit is unset(0) so the operations gives 0 value as the result.

- $N \wedge (1 \ll i)$
 - $< N$, if i^{th} bit is 1
 - $> N$, if i^{th} bit is 0

➤ **Example**

$N = 57 \rightarrow 1\ 1\ 1\ 0\ 0\ 1$

Case 1

$i = 3$
 $N \wedge (1 \ll i)$
 $1\ 1\ 1\ 0\ 0\ 1 \wedge 1 \ll 3$

$1\ 1\ 1\ 0\ 0\ 1$
 $0\ 0\ 1\ 0\ 0\ 0$

 $1\ 1\ 0\ 1\ 1\ 1 \rightarrow <N$

- If we see the above example the i^{th} bit is set(1) so the operations gives $<N$ value as the result.

Case 2

$i = 2$
 $N \& (1 \ll i)$
 $1\ 1\ 1\ 0\ 0\ 1 \& 1 \ll 2$

$1\ 1\ 1\ 0\ 0\ 1$
 $0\ 0\ 0\ 1\ 0\ 0$

 $1\ 1\ 1\ 1\ 0\ 1 \rightarrow >N$

- If we see the above example the i^{th} bit is unset(0) so the operations gives $>N$ value as the result.

6. >> (Right Shift)

- The right shift operator moves all bits by a given number of bits to the right.
- **Example**

$a = 57 = 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1$ $57 \rightarrow 57/2^0$
 $a \gg 1 = 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0$ (1 bit has been moved to right) $28 \rightarrow 57/2^1$
 $a \gg 2 = 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0$ (2 bit has been moved to right) $14 \rightarrow 57/2^2$
 $a \gg 3 = 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1$ (3 bit has been moved to right) $7 \rightarrow 57/2^3$

$a \gg n = a/2^n$

Storing and Retrieving -ve number

Storing -ve number

- Step 1 : remove -ve sign of number
- Step 2 : convert to 1's complement(toggle all the bits)
- Step 3 : convert to 2's complement(add 1 to number at step2)

Retrieving -ve number

- Step 1: consider the number at step 3 in the storing process
- Step 2: consider the highest position bit as MSB
- Step 3: considered MSB bit is -ve number
- Step 4: add all the digital number of 1's with MSB

Example

N = -57

Storing

Step 1: 57 (1 1 1 0 0 1)

Step 2: 1 1 1 0 0 1 \rightarrow 0 0 0 1 1 0

Step 3: 0 0 0 1 1 0 + 1 = 0 0 0 1 1 1

Retrieving

Step 1: 0 0 0 1 1 1

Step 2: 0 0 0 1 1 1 (highlighted bit is the MSB)

Step 3: 5th position is MSB $-2^5 = -32$

Step 4:

0	0	0	1	1	1
-2^5	2^4	2^3	2^2	2^1	2^0
-64	0	0	4	2	1

$-64 + 4 + 2 + 1 = -57$

Max and Min numbers to be stored

Range	Bits	Min	Max
	8 bits	-2^7	$2^7 - 1$
int	32 bits	-2^{31}	$2^{32} - 1$
long	64 bits	-2^{63}	$2^{63} - 1$

Bitwise Properties

1. Commutative

$$a \wedge b = b \wedge a$$

$$a \& b = b \& a$$

$$a \mid b = b \mid a$$

2. Associative

$$(a \& b) \& c = a \& (b \& c)$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$(a \mid b) \mid c = a \mid (b \mid c)$$