

Sets: Medical Testing Example

Video companion

1 Example using set theory

VBS: “very bad syndrome”

X = set of people in a clinical trial

$S = \{x \in X : x \text{ has VBS}\}$

$H = \{x \in X : x \text{ does not have VBS}\}$

$$\begin{aligned} X &= S \cup H && \text{(you either have VBS or you don't)} \\ S \cap H &= \emptyset && \text{(no one both has and doesn't have it)} \end{aligned}$$

Point of medical testing to figure out whether a person is in S or in H

2 Test

$P = \{x \in X : x \text{ tests positive for VBS}\}$

$N = \{x \in X : x \text{ tests negative for VBS}\}$

$$\begin{aligned} P \cup N &= X && \text{(you either test positive or negative)} \\ P \cap N &= \emptyset && \text{(no one tests both positive and negative)} \end{aligned}$$

In a perfect world, S would equal P —the sick people would always test positive, and H would equal N —the healthy people would always test negative.

...but this is not always the case.

$S \cap P$	$H \cap N$	$S \cap N$	$H \cap P$
true positive	true negative	false negative	false positive

3 Cardinality

$\frac{|S|}{|X|}$ = proportion of people in the study who do genuinely have VBS

$\frac{|H|}{|X|}$ = proportion of people in the study without VBS

$$\frac{|S|}{|X|} + \frac{|H|}{|X|} = 1$$

$\frac{|S \cap P|}{|S|}$ true positive rate would like to be close to 1

$\frac{|H \cap P|}{|H|}$ false positive rate would like to be as small as possible

$\frac{|S \cap N|}{|S|}$ false negative rate would like to be as small as possible

$\frac{|H \cap N|}{|H|}$ true negative rate would like to be close to 1