# Assignment 4: Exploring Image Morphological Operators, Image Compression & Enhancement

## Homeworks Guidelines and Policies

- ***What you must hand in.*** It is expected that the students submit an assignment report (HW4_[student_id].pdf) as well as required source codes (.m or .py) into an archive file (HW4_[student_id].zip). Please combine all your reports just into a single .pdf file.
- ***Pay attention to problem types.*** Some problems are required to be solved *by hand* (shown by the 📝 icon), and some need to be implemented (shown by 🐍 the icon). Please do not use implementation tools when it is asked to solve the problem by hand, otherwise you will be penalized and lose some points.
- ***Don't bother typing!*** You are free to solve by-hand problems on a paper and include their pictures in your report. Here, cleanness and readability are of high importance. Images should also have appropriate quality.
- ***Reports are critical.*** Your work will be evaluated mostly by the quality of your report. Do not forget to explain your answers clearly, and provide enough discussions when needed.
- ***Appearance matters!*** In each homework, 5 points (out of a possible 100) belong to compactness, expressiveness, and neatness of your report and codes.
- ***MATLAB is also allowable.*** By default, we assume you implement your codes in Python. If you are using MATLAB, you have to use the equivalent functions when it is asked to use specific Python functions.
- ***Be neat and tidy!*** Your codes must be separated for each question, and for each part. For example, you have to create a separate .m file for part b. of question 3, which must be named 'p3b.m'.
- ***Use bonus points to improve your score.*** Problems with bonus points are marked by the ⭐ icon. These problems usually include uncovered related topics, or those that are only mentioned briefly in the class.
- ***Moodle access is essential.*** Make sure you have access to Moodle, because that is where all assignments as well as course announcements are posted. Homework submissions are also made through Moodle.

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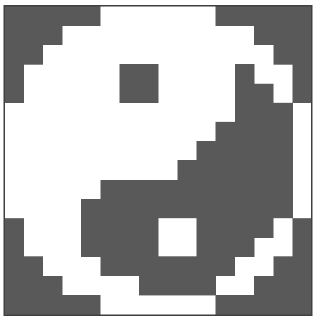
- ***Assignment Deadline.*** Please submit your work **before the end of July 3rd**.
- ***Delay policy.*** During the semester, students are given only 10 free late days which they can use them in their own ways. Afterwards, there will be a 15% penalty for every late day, and no more than four late days will be accepted.
- ***Collaboration policy.*** We encourage students to work together, share their findings, and utilize all the resources available. However you are not allowed to share codes/answers or use works from the past semesters. Violators will receive a zero for that particular problem.
- ***Any questions?*** If there is any question, please do not hesitate to contact us through the Telegram group chat or following email addresses: m.ebadpour@aut.ac.ir , Peymanhashemi@aut.ac.ir , and atiyeh.moghadam@aut.ac.ir .

## 1. Mastering Image Morphological Operators Calculations (17 Pts.)

**Keywords**: *Image Morphology, Image Logical Operations, Structuring Element, Image Dilation, Image Erosion, Image Opening, Image Closing*

The first task includes several problems concerning **Morphological Operations** for binary images, which you are required to solve by hand.



Given below is a $16\times16$ binary image used in each part of this problem. First, we apply four different unknown morphological operations among **Erosion**, **Dilation**, **Opening** and **Closing** on the input image. Determine the type of morphological operation used alongside the given **Structuring Elements** to give the results in the following figure.

a.

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |

b.

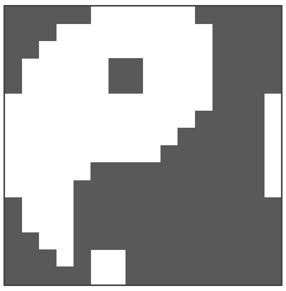
| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

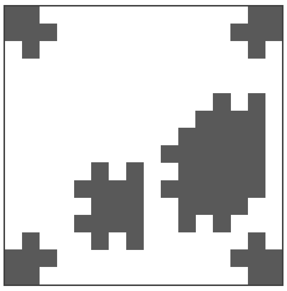
c.

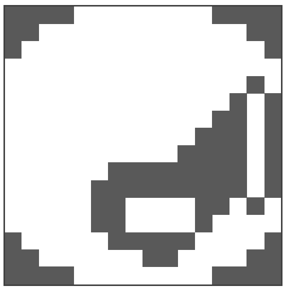
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 0 | 0 |

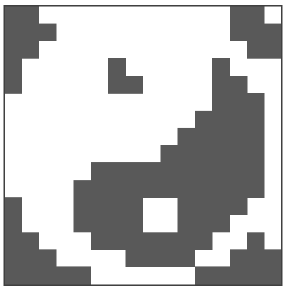
d.

| | | |
|---|---|---|
| 0 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |


**(a)**

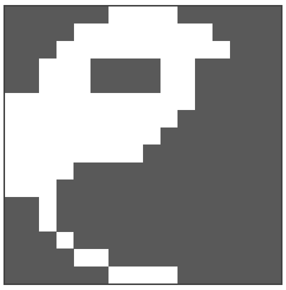

**(b)**

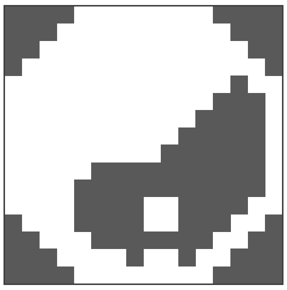

**(c)**

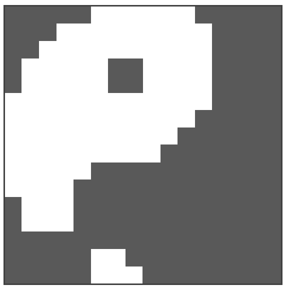

**(d)**

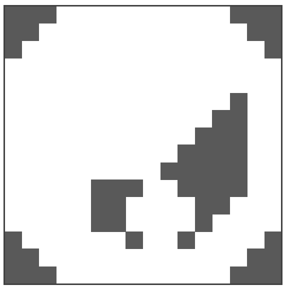
Consider another set of results obtained by applying certain morphological operations on the input image. Given the type of the operations used, find the $3\times3$ structuring elements for each part. Note that the origin of these structuring elements is always in the centre.

e. Erosion
f. Closing
g. Opening
h. Dilation


**(e)**


**(f)**


**(g)**


**(h)**

In the final part, the goal is to investigate more complicated morphological algorithms. Perform the following tasks on the input image.

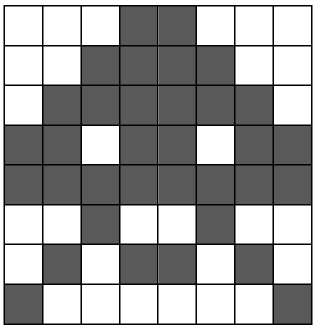
i. Determine the positions of the corners.
j. Extract the boundaries.
k. Fill the holes.
l. Extract the connected components.
m. Obtain the skeleton of the shape.

## 2. Fundamentals of Image Compression (16 Pts.)

**Keywords**: *Image Compression, Huffman Coding, Zig-Zag Ordering, Run-Length Coding, Entropy Coding, Peak Signal-to-Noise Ratio*

The goal of this problem is to evaluate your fundamental knowledges of **Image Compression** techniques.

First, consider the following binary image.



a. Find the runlength representation. Note that you must assume a line always starts with "white", and specify the white runlength and then the black runlength alternatively. Put a "EOL" symbol at the end of each line.
b. Find the corresponding zig-zag ordered vector to this image.
c. We want to use a single Huffman codebook to code the white and black runlengths. Write down the probability distribution of the symbols to be coded.
d. Next, generate the Huffman code for all possible runlength symbols. You must consider "EOL" symbols as well.
e. Calculate the average number of bits per coded symbol. How does it compare with the entropy of the symbol?
f. What is the average bit rate (bits/pixel) for this method? What about the compression ratio compared to using 1 bit for each pixel to indicate whether the pixel is black or white?

Now consider the following table as the initial dictionary for LZW algorithm.

| Index | Entry |
|---|---|
| 1 | a |
| 2 | - |
| 3 | r |
| 4 | t |

g. Output of the LFW encoder on a sequence is below. Decode the sequence.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 4 | 6 | 8 | 4 | 2 | 1 | 2 | 5 | 10 | 6 | 11 | 13 | 6 |

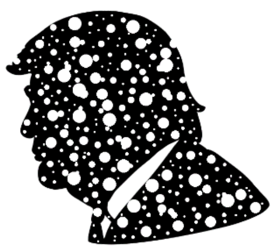
h. Encode the decoded sequence using the same initial dictionary. Does your answer match the sequence given above? Explain.

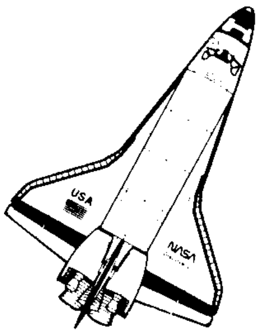
## 3. Warming Up with Binary Images (19 Pts.)

**Keywords**: *Image Morphology, Image Logical Operations, Structuring Element, Image Dilation, Image Erosion, Image Opening, Image Closing*

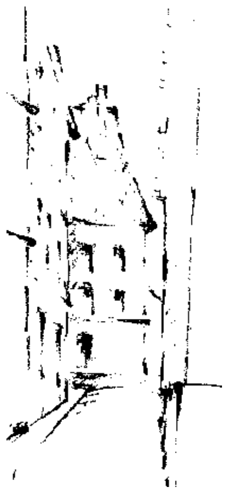
**Image Morphological Operations** can be more clearly explained in binary images. The effect of each morphological operation on an image and the role of structuring element and its properties like position of the origin and size are more perceivable when these operations are applied to 1-bit images. For this reason, let’s start with some binary images first. Note that you are not allowed to use built-in functions to perform morphological operations, i.e. you have to implement them yourself.

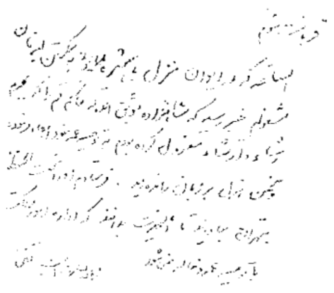
a. Consider the image in Figure 1, part a. Using an appropriate procedure consisting of proper morphological operation(s) to fill the holes inside the figure. The final results must be as clear as possible, similar to the image in part b.
b. Morphological operations can also be used to extract relevant features from an image. Try to use these operations in order to remove certain features (like diagonal lines) from the image given in Figure 1, part b.
c. Assume the binary image in Figure 1-c. Use thickening alongside other morphological operations to process this image. Reduce all lines to a single pixel width and obtain their maximum length.
d. Figure 1-d displays a letter which is said to be wrote by Amirkabir to Naser al-Din Shah Qajar. As can be seen, the letters are separated and sentences can hardly be read. Try to fill the gaps and obtain a clear readable result.


(a)



(b)


(c)


[illegible]

(d)

*Figure 1 Input images provided for each part (a) Binary image of Donal Trump with white holes of different sizes (b) A schematic of a NASA shuttle (c) A painting consisting of some houses in an alley (d) Handwriting attributed to Amirkabir*

## 4. Beyond Bi-Level Images: Broadening Image Morphology Applications (16 Pts.)

**Keywords**: *Image Morphology, Structuring Element*

As you are very well aware, the scope of **Image Morphological Operations** is not limited to binary images. One can apply these operations on grayscale images and obtain fine results based on the intended application.

Here you are going to practice some of the related problems. Note that you are not necessarily supposed to obtain perfect results, as some of the tasks are a bit difficult to deal with. You must try your best nonetheless.

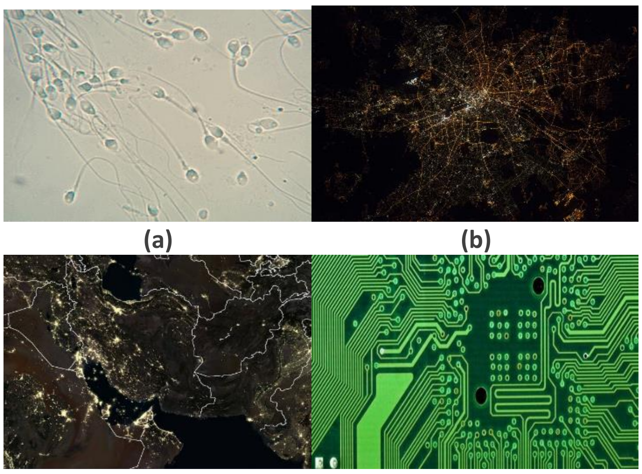


*Figure 2 Input images (a) Microscopic image of human body cells (b) Satellite image of Berlin at night (c) Satellite image of Iran (d) An unknown printed circuit board*

a. Given below are two microscopic images of human body cells which can be used in various medical applications. Your task is to implement an algorithm to count the number of cells inside each image.

b. Now comes the third task, in which the goal is to create a street map from a satellite image taken from the city of Berlin at night. You must use appropriate morphological operations to detect the main roads and remove the remaining areas of the image. Highlight the streets properly.

c. This part is somehow similar to the previous one, as it utilizes another satellite image taken at night. This time the goal is to find and display the most crowded cities in this map in the Figure 2-c. Display the map with the whereabouts of the 10 most crowded (or brightest) cities clearly highlighted with colored points.

d. Finally, you are an image of a printed circuit board. Use morphological operations to remove the lines within the board, while keeping the circles. Then try to remove the circles while keeping the lines. Note that the output of the colored input must also be colored.

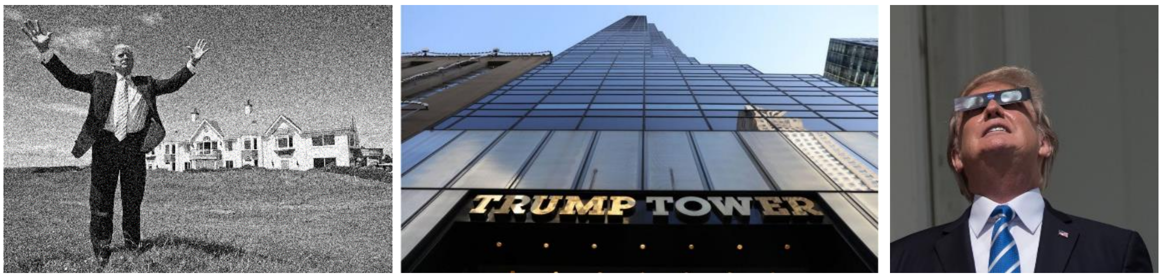
## 5. Applying Image Morphological Operations for Filtering Images (16 Pts.)

**Keywords**: *Grayscale Image Morphology, Image Morphological Filtering, Morphological Noise Reduction, Morphological Edge Detection*

A more complicated application of **Image Morphology** in grayscale images is **Image Filtering**, where the goal is to apply different types of filtering, e.g. smoothing filter, using a combination of **Morphological Operations**.

In this problem, your task is to examine some of these operations, and compare their results.

a. Load the image "trump_dance.jpg" and apply morphological smoothing using dilation and erosion. How morphological smoothing can be applied on RGB images.?
b. Repeat part a. with opening and closing. Compare the results obtained from part a. and b., and comment on their performance in **Noise Reduction**.
c. Load the image "trump_tower.jpg" and apply morphological 2nd derivatives using dilation and erosion.
d. Repeat part c. with opening and closing.
e. Repeat part c. with dilation, erosion, opening and closing. Compare the results obtained from part c., d., and e., and comment on their performance in **Edge Detection**.
f. Load the image "trump_eclipse.jpg" and apply morphological gradient using dilation and erosion.
g. Repeat part f. with opening and closing.
h. Repeat part f. with dilation, erosion, opening and closing. Compare the results obtained from part f., g., and h., and comment on their performance in **Edge Detection**.



**(a)** **(b)** **(c)**

*Figure 3 Input images of the problem (a) Part a and b (b) Part c to e (c) Part f to h*

## 6. Restoration and Enhancement of Historical Photos (26 Pts.)

**Keywords**: *Image Enhancement, Image Restoration*

Due to incapability of hardware and lack of technology, historical photos hugely suffer from different types of degradations. **Image Restoration** and **Image Enhancement** techniques are powerful tools to improve these photos, making them more appropriate for publishing or researching purposes.

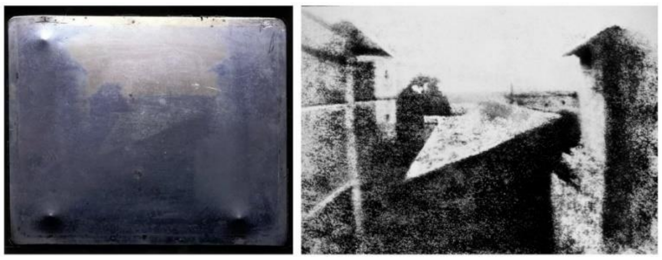


*Figure 5 The oldest surviving camera photograph, known as "View from the Window at Le Gras", taken in 1826 or 1827 by Nicéphore Niépce [link]. Left: original plate. Right: enhanced version*

In this problem, you are provided with some of the oldest surviving photos of the history, and your task is to increase their quality using all you have learnt from image restoration and image enhancement techniques.

a. Please load the image "le_gras.jpg". Considering the degradation you observe, use any image enhancement (like histogram equalization, sharpening filters, etc.) or image restoration (like median filter, Wiener filter, etc.) techniques you wish to improve the quality of the image. You are free to apply more than one method as well. Display all intermediate versions of your work, and include them in the report.

b. Repeat part a. for the image "still_life_in_studio.jpg".

c. Repeat part a. for the image "boulevard_du_temple.jpg".

d. Repeat part a. for the image "robert_cornelius_selfie.jpg".

e. Repeat part a. for the image "william_jennings_lightning.jpg".

f. Repeat part a. for the image "john_quincy_adams.jpg“.

g. Repeat part a. for the image "berkowski_solar_eclipse.jpg“.

h. Repeat part a. for the image "louis_daguerre.jpg“.

i. Repeat part a. for the image "self_portrait_as_a_drowned_man.jpg“.

j. Repeat part a. for the image "tartan_ribbon.jpg“.
**Note**: Please keep images format as is.

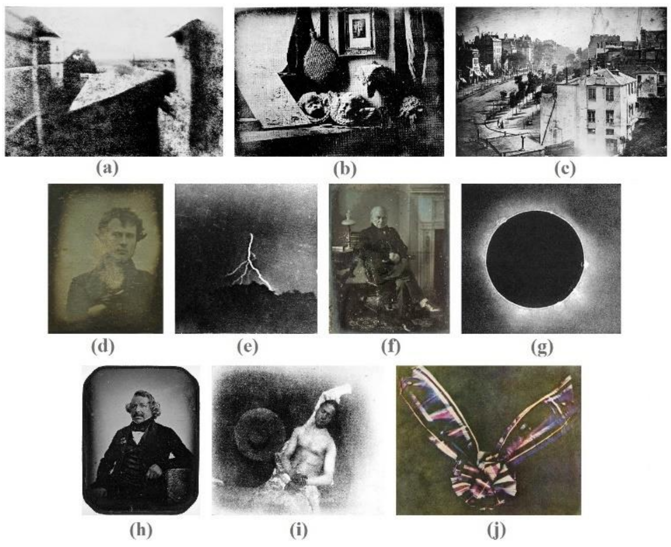


*Figure 4 Some of the oldest known surviving photographs in the history, given as the input images of this problem (a) Oldest known photograph in the history (1826) (b) Some plaster casts in Louis Daguerre's room (1837) (c) First ever picture of a living person (on the bottom-left of the image) (1838) (d) Oldest selfie (1839) (e) First photo of a lightning (1887) (f) First known photo of a US President, John Quincy Adams (1843) (g) First photo of a solar eclipse (1851) (h) Louis Daguerre, one of the fathers of photography (1830) (i) First hoax photograph, known as "Self Portrait as a Drowned Man" (1840) (j) First color photograph, taken of a tartan ribbon (1861)*

## 7. Getting More Familiar with Some of the Applications of Image Interpolation (15 Pts.)

**Keywords**: *Image Interpolation, Image Warping, Image Inpainting, Image Watermarking, Image Morphing*

In addition to resolution enhancement, **Image Interpolation** can also be used in various image processing applications. In this problem, you will get hands-on experience in some of them.

**a.** **Image Warping** is the process of manipulating a digital image so that different parts in the image have been significantly distorted. It may be used in many purposes; from correcting image distortion to creating funny images.
The goal of this part is to apply warping on the input image, "trump_shout.jpg". You are free to choose any warping pattern you like.

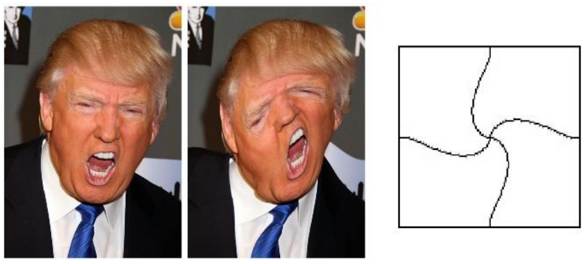


*Figure 6 Input image alongside a warping example obtained by Photoshop using the given pattern*

b. **Image Inpainting** is the usage of some image processing techniques to reconstruct lost or corrupted parts on an image. Load the image "queen_elizabeth_wedding.jpg", and try to replace the corrupted parts using neighborhood information.
**Note**: There are several fold effects in the input image, but you just have to remove the most prominent ones which can be seen as two horizontal (in the middle) and vertical (in the left) lines in the picture.

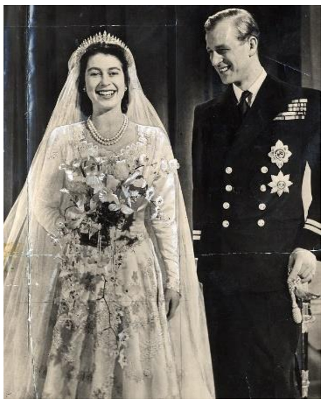


*Figure 7 Photograph of Queen Elizabeth II in her wedding, with fold effects appear as two horizontal and vertical lines*

c. **Image Watermarking** is the process of embedding a type of marker in an image, typically to identify the owner or creator of the image. Your goal in this problem is to remove watermark (appears as image caption) from the image "harry_meghan.jpg".
**Note:** You just have to remove the caption inside the image. Your result might look unnatural.

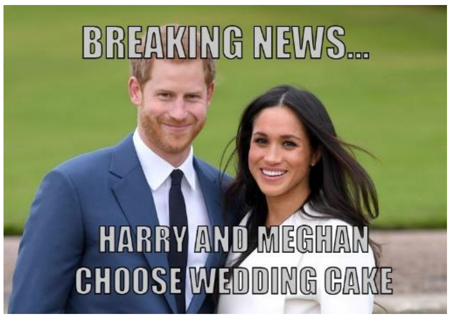


*Figure 8 An internet meme containing a note, which is desired to be removed*

## 8. Comparison of Different Methods for Resolution Enhancement (20 Pts.)

**Keywords**: *Image Interpolation, Image Super-Resolution, Nearest-Neighbor Interpolation, Bilinear Interpolation, Nearest Neighbor Value Interpolation, Non-uniform Interpolation*

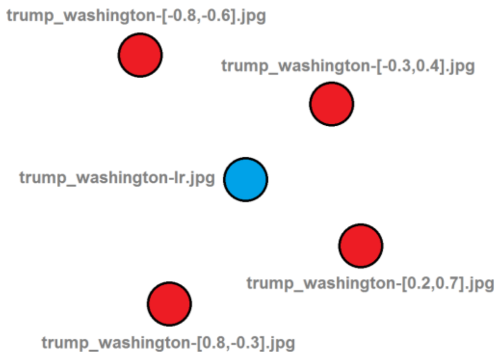
**Image Interpolation** is an image processing method by which the number of pixels comprising an image is increased. **Nearest-Neighbor Interpolation** is the simplest way of image interpolation, where the values of an unknown pixel is determined by the value of its nearest pixel, while **Bilinear Interpolation** works by interpolating pixel intensity value using more pixel neighbors, based on their distances to the unknown pixel.

All image interpolation methods use only local pixel information, therefore they cannot add new details to the image. **Image Super-Resolution**, on the other hand, is a different class of image processing techniques which not only enhances image resolution, but also tries to add new information to the image.

The purpose of this problem is to practice enhancing image resolution using these methods. You are going to work with image "trump_washington-lr.jpg", as the input image.

*Figure 9 Low-resolution input of part a. to part c.*

a. Rescale the input image by the factor of 2 using nearest-neighbor interpolation method.
b. Rescale the input image by the factor of 2 using bilinear interpolation method.
c. Rescale the input image by the factor of 2 using nearest neighbor value interpolation. Note that in this method, the missing pixel value is estimated by the nearest value rather than the distance. In other words, the nearest neighbor value interpolation considers the four pixels directly surrounding the empty location, and selects the one whose value is almost equal to the value obtained by bilinear interpolation method.
   **Useful Links**: [1], [2]
d. You are provided with four low-resolution images (instead of one), each with a subpixel shift with respect to each other. Considering the given shift information and images names in Figure 10, rescale the input images by the factor of 2 using non-uniform interpolation.
   **Note**: Although this is considered as a super-resolution technique, the procedure is very similar to the prior interpolation methods.


trump_washington-[-0.8,-0.6].jpg
trump_washington-[-0.3,0.4].jpg
trump_washington-lr.jpg
trump_washington-[0.2,0.7].jpg
trump_washington-[0.8,-0.3].jpg
Figure 10 Pixel positions of the given low-resolution images for part d. (shown in red), with respect to the main low-resolution image pixel (shown in blue)

e. Calculate and compare PSNR of part a. to part c. and d.. results. You can use the given groundtruth image "trump_washington-gt.jpg". Also comment on visual appearance of the results.

*Good Luck!*
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