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Uncertainty quantification in game theory

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ABSTRACT

This work examines the applicability of Uncertainty Quantification (UQ) in Game Theory. We consider the classical games “matching pennies” and “Hawk an Dove” in situations involving uncertainty. The first game examined is “matching pennies”: a first situation concerns the game where the probabilities of choice between Heads and Tails are unknown and must be determined from observations. A second situation concerns fluctuations in the implementation of the Nash equilibrium. Instability is evidentiated and a strategy based on statistical estimation is introduced. A third situation considers random payoffs having an unknown distribution: observations are used to generate an UQ representation of the real distribution of the payoffs, without any supplementary assumption on the nature of the distribution. Finally, we analyze the effects of uncertainties on the associated replicator dynamics: UQ is applied to generate mean trajectories and mean orbits - in this step, we need to manipulate statistics of curves, which are objects defined by functions, belonging to infinitely dimensional vector spaces. The second game is “Hawk and Dove”. We examine the situation where the reward and the cost of an injury are both uncertain and only a small sample of values is available. The methods of UQ are applied to determine the mean evolution of the system and confidence intervals for the evolution of the fractions of Hawks and Doves. The UQ methods involved are described and simple examples are given to facilitate understanding and application to other situations.

1. Introduction

Uncertainty Quantification (UQ) is a field of knowledge which develops methods for the analysis of numerical data, namely when uncertainty or variability is involved. The general aim of UQ is to characterize the observed variability in a quantity X by using a random variable U. UQ searches for representations of the observed variability that can be useful for numerical calculations involving X. Indeed, if some information may be obtained on (X, U), then it can be used to analyze the behavior of X and to obtain a model representing its variability, namely by using U as explanation of the observed randomness of X. Typical information used by UQ are samples (see, for instance, [1–6]), but UQ may exploit also other types on information, such as numerical problems involving both the variables (see, for instance, [7–14]). In practice, it often happens that U or its distribution is unknown : variability of X is observed, but the explanation is unknown - UQ proposes also methods for this kind of situation (see, for instance, [15–19]). UQ techniques may deal also with infinitely dimensional objects, such as curves and surfaces. For instance, UQ proposes methods for the determination of means, medians and confidence intervals of families of curves and surfaces, such as Pareto fronts, trajectories and orbits [20–22]. UQ methods are general enough to cover a wide range of application areas (to cite a few among a large number, [23–40]), but their use in Game Theory (GT) is still incipient. We present in this work an example of application of these methods to a simple game and the associated replicator dynamics. The methods of UQ are used to determine, for instance, unknown distributions of probability, mean trajectories, mean orbits and also to estimate probabilities of some events connected to the game or the replicator dynamics. The focus of this work is UQ and not GT, even if some elements of GT are recalled, by sake of clarity.

2. GT and uncertainty

GT encounters uncertainty right from the start, as the result of the game may depends on a random variable. For example, when betting on the outcome of a coin toss - Head (H) or tail (T), the result is random. In addition, a player may decide to choose heads with a given probability p and tails with its complement $1 - p$: his strategy mixes the pure strategies H and T and a supplementary randomness is introduced in the game. As a consequence, the gains (or losses) of the players are also random variables. Supplementary uncertainty arises if the payoffs associated to each possible result are affected by variability or randomness - for exam-

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