Reconfigurable Heterogeneous Quorum Systems

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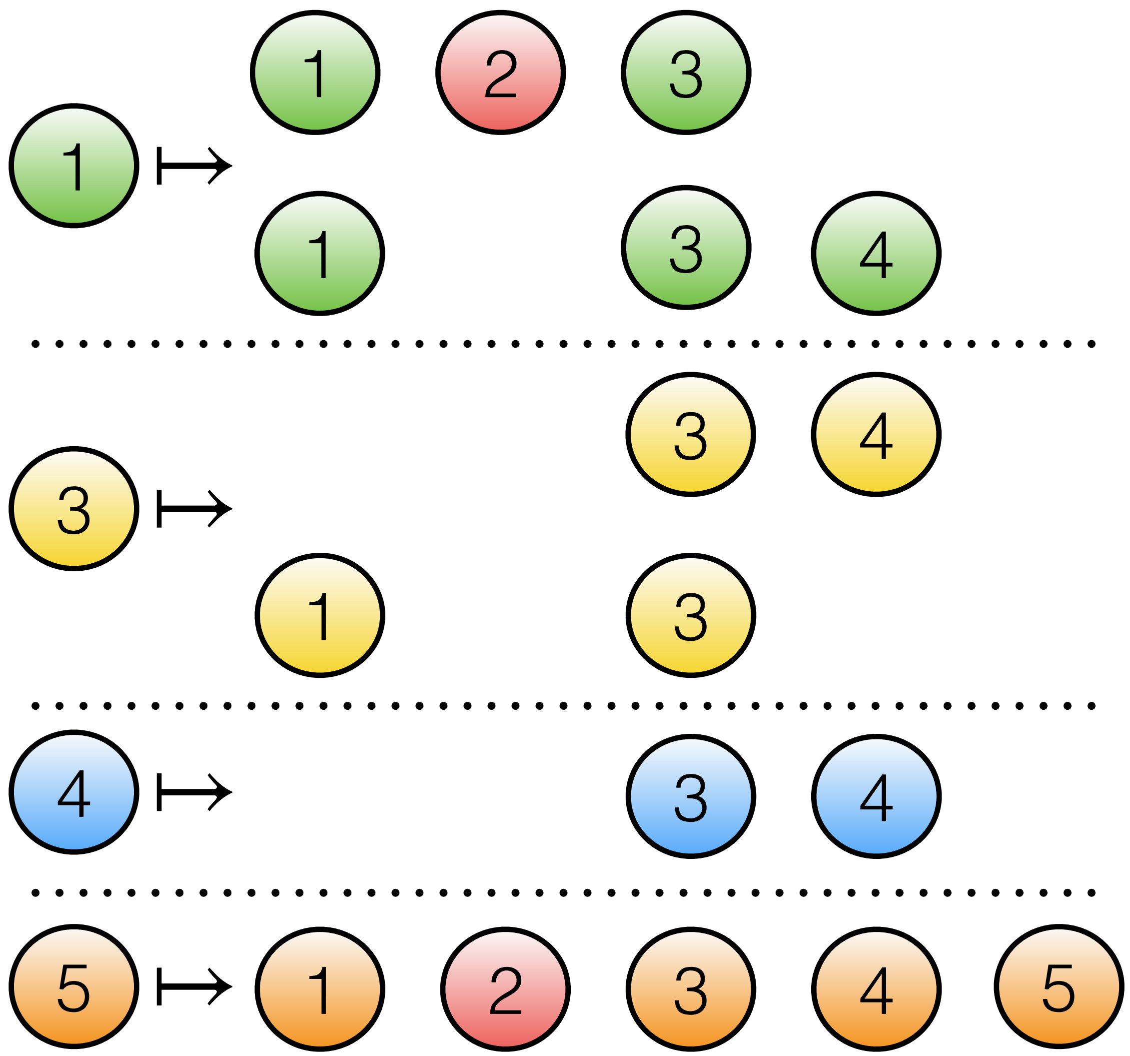
Contributions

- A graph characterization of heterogeneous quorum systems, and its application to optimize reconfiguration and a sink discovery protocol
- Trade-offs between reconfiguration guarantees
- Reconfiguration protocols for joining and **leaving of a process**, and adding and removing of a quorum, and their proofs of correctness

Heterogeneous Quorum Systems (HQS)

$$\mathcal{P} = \mathcal{W} \cup \mathcal{B}, \quad \mathcal{W} = \{1,3,4,5\}, \quad \mathcal{B} = \{2\}$$

$$\begin{aligned}\mathcal{Q} = \{1 &\mapsto \{\{1,2,3\},\{1,4\}\},\\ 3 &\mapsto \{\{3,4\},\{1,3\}\}\\ 4 &\mapsto \{\{3,4\}\}\\ 5 &\mapsto \{\{1,2,3,5\}\}\}\end{aligned}$$



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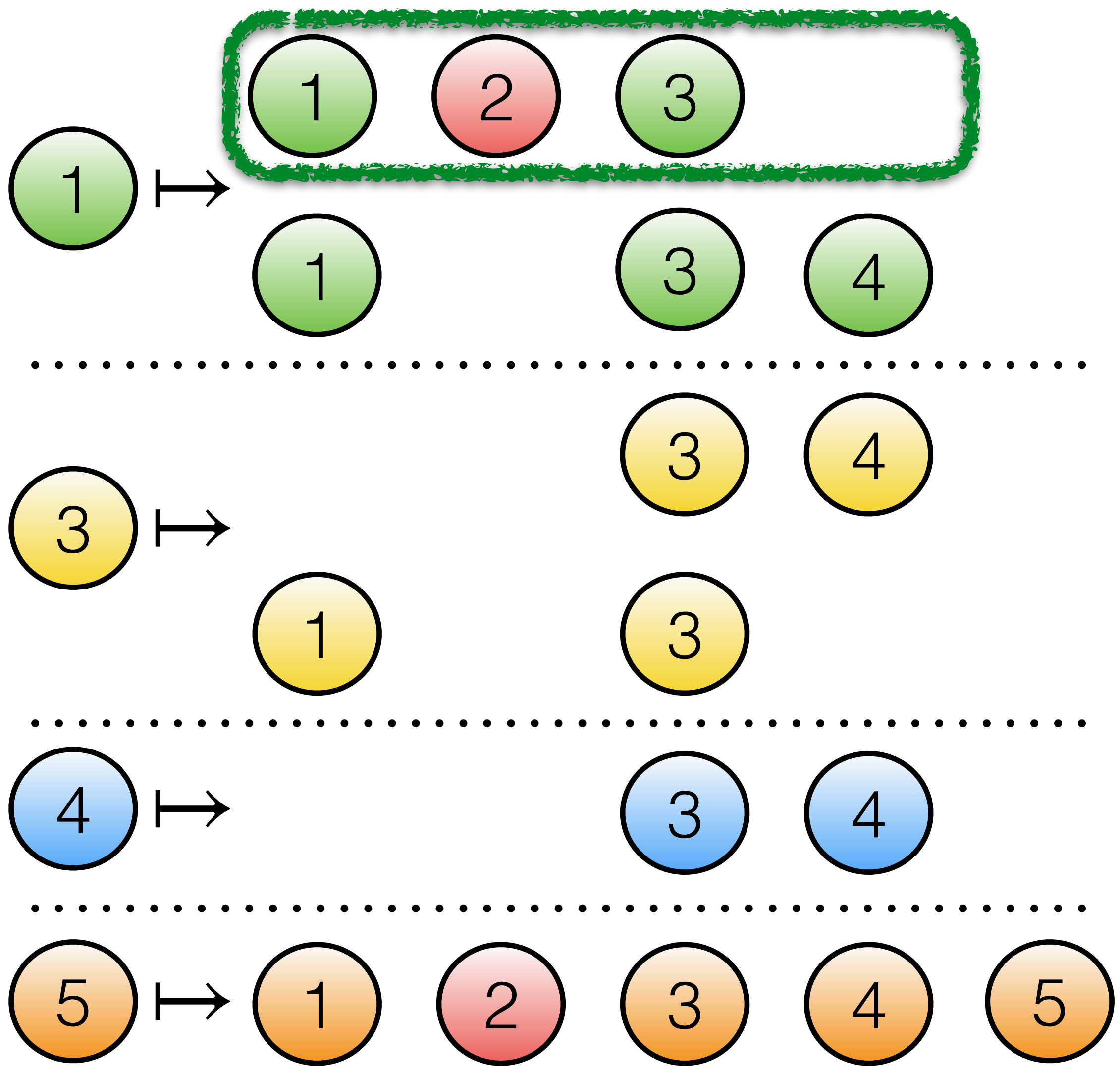

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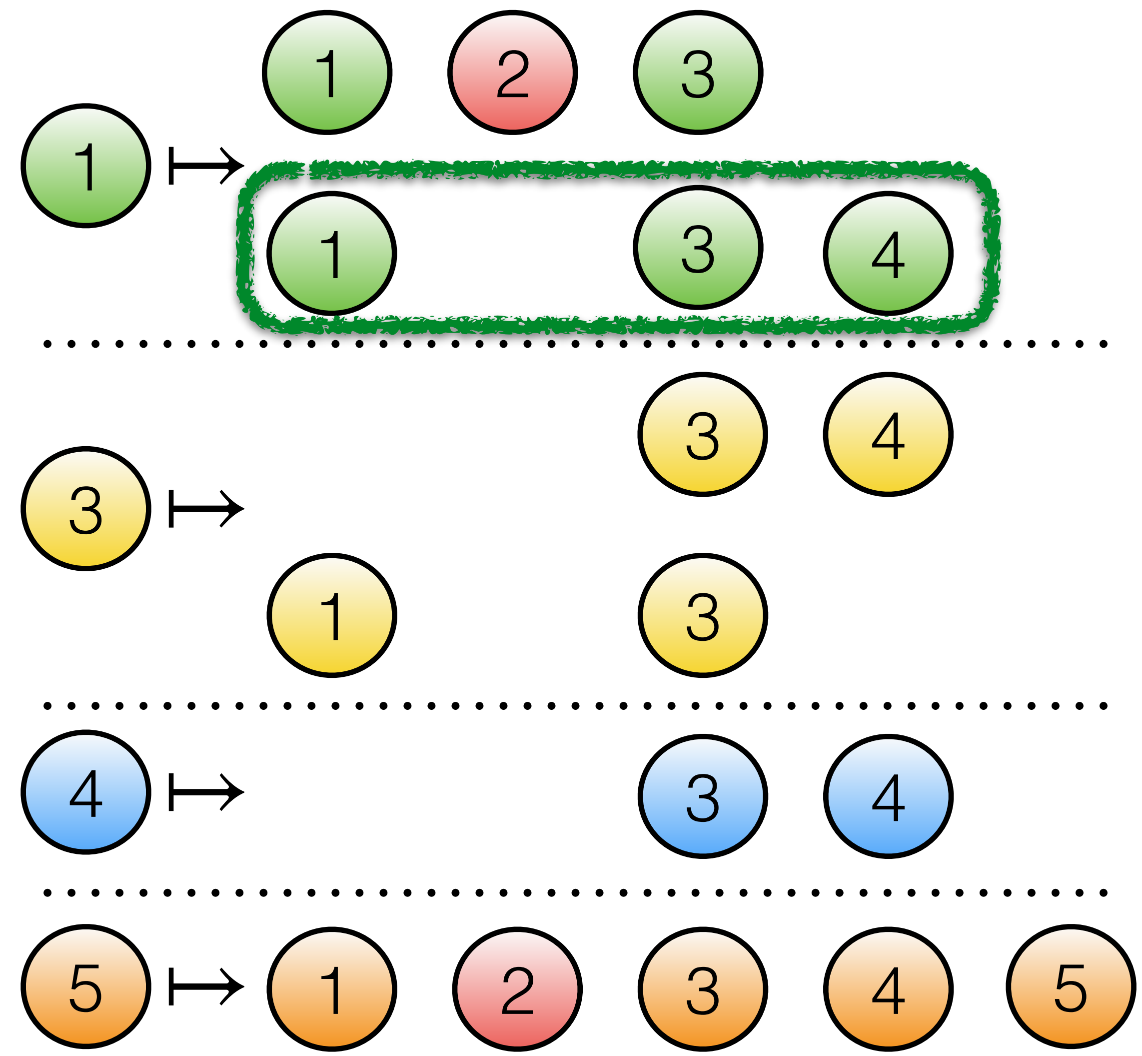
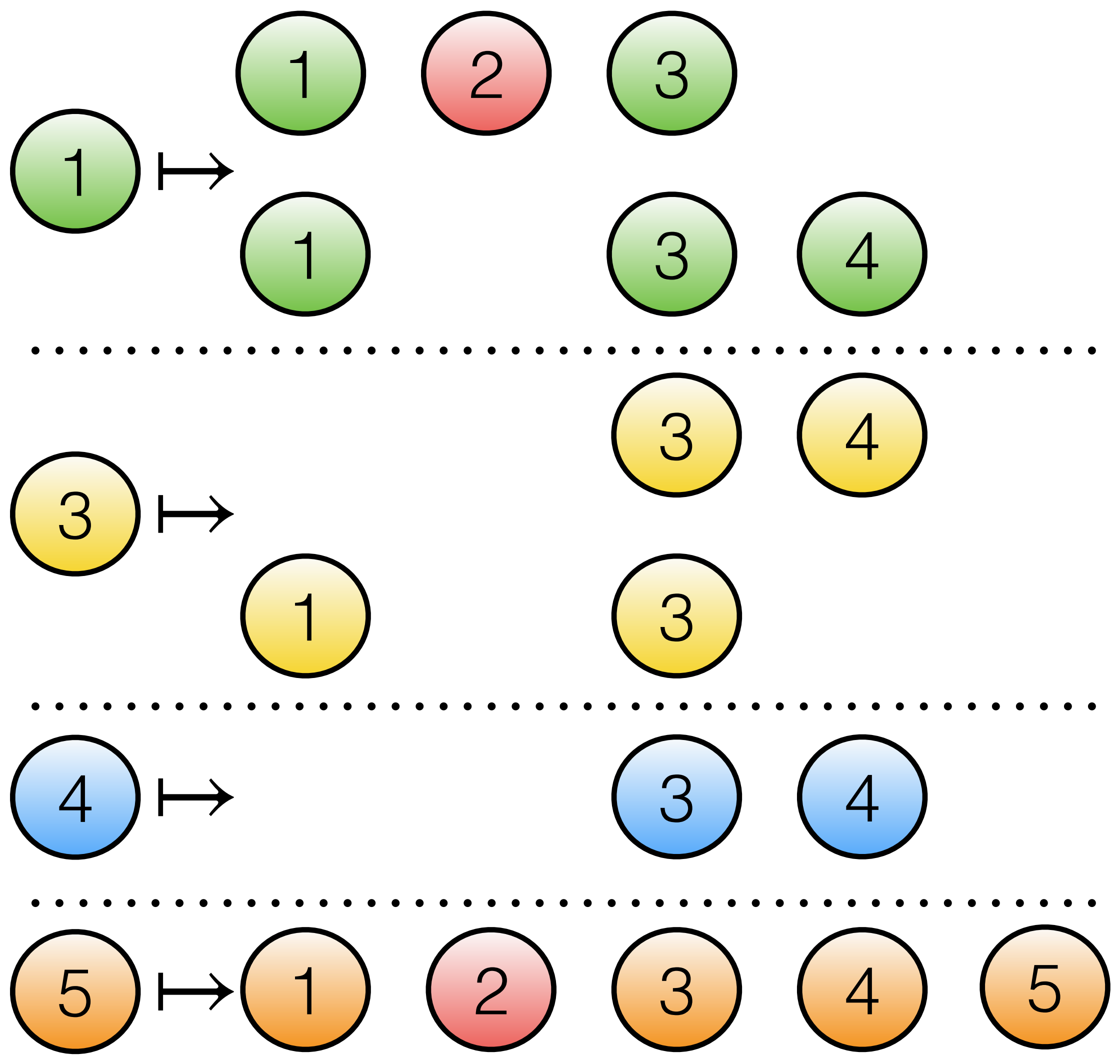


Quorum Intersection at $\mathcal{O}$



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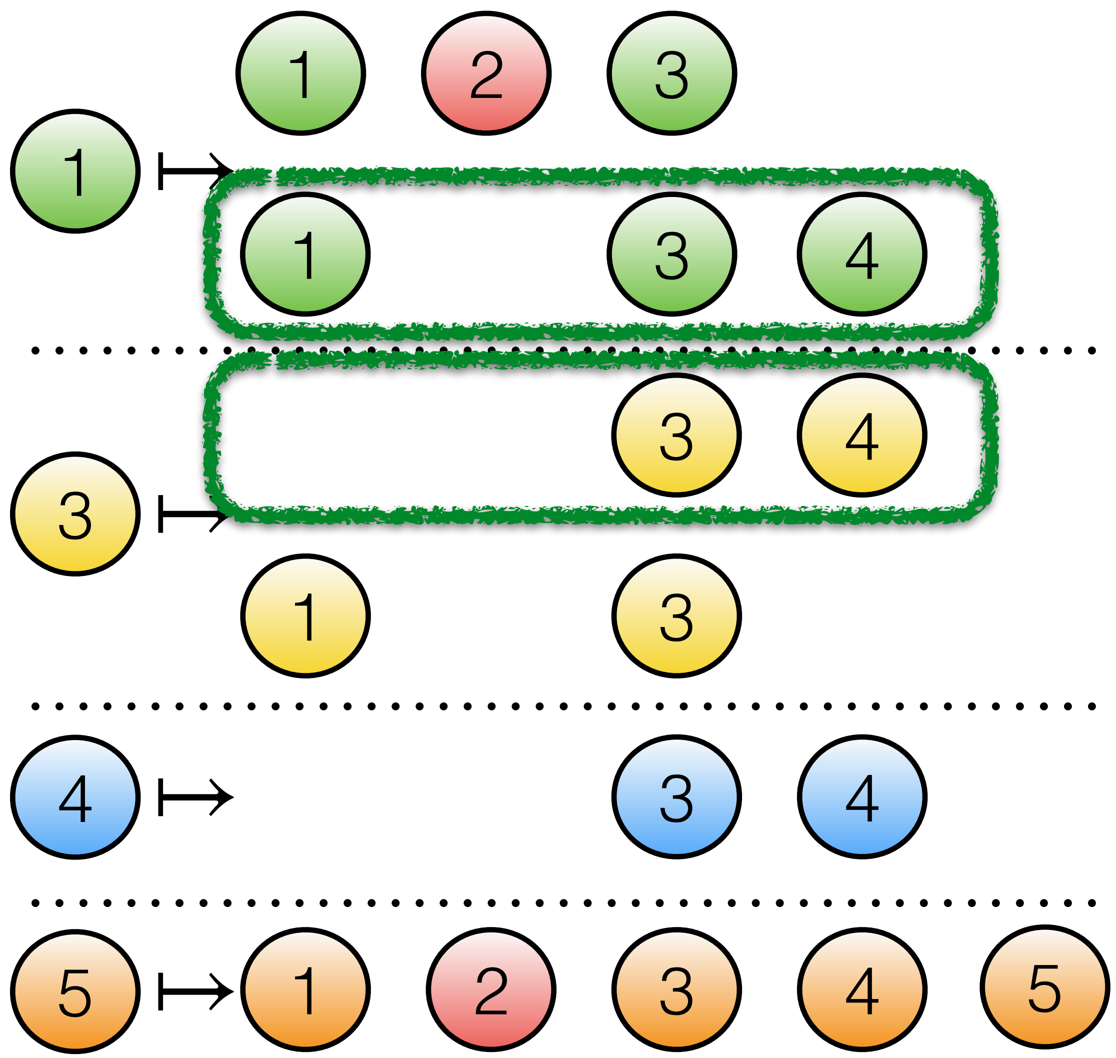

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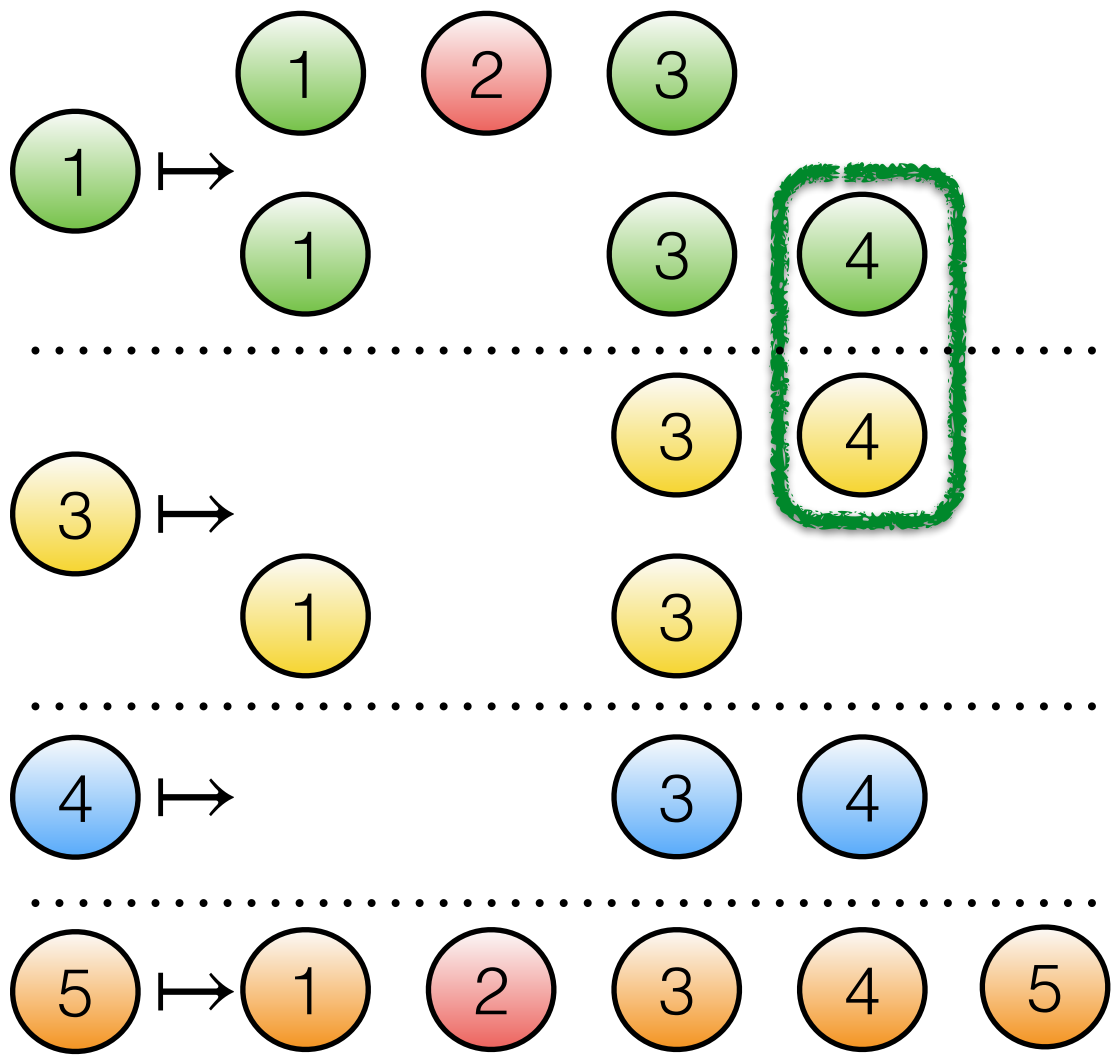

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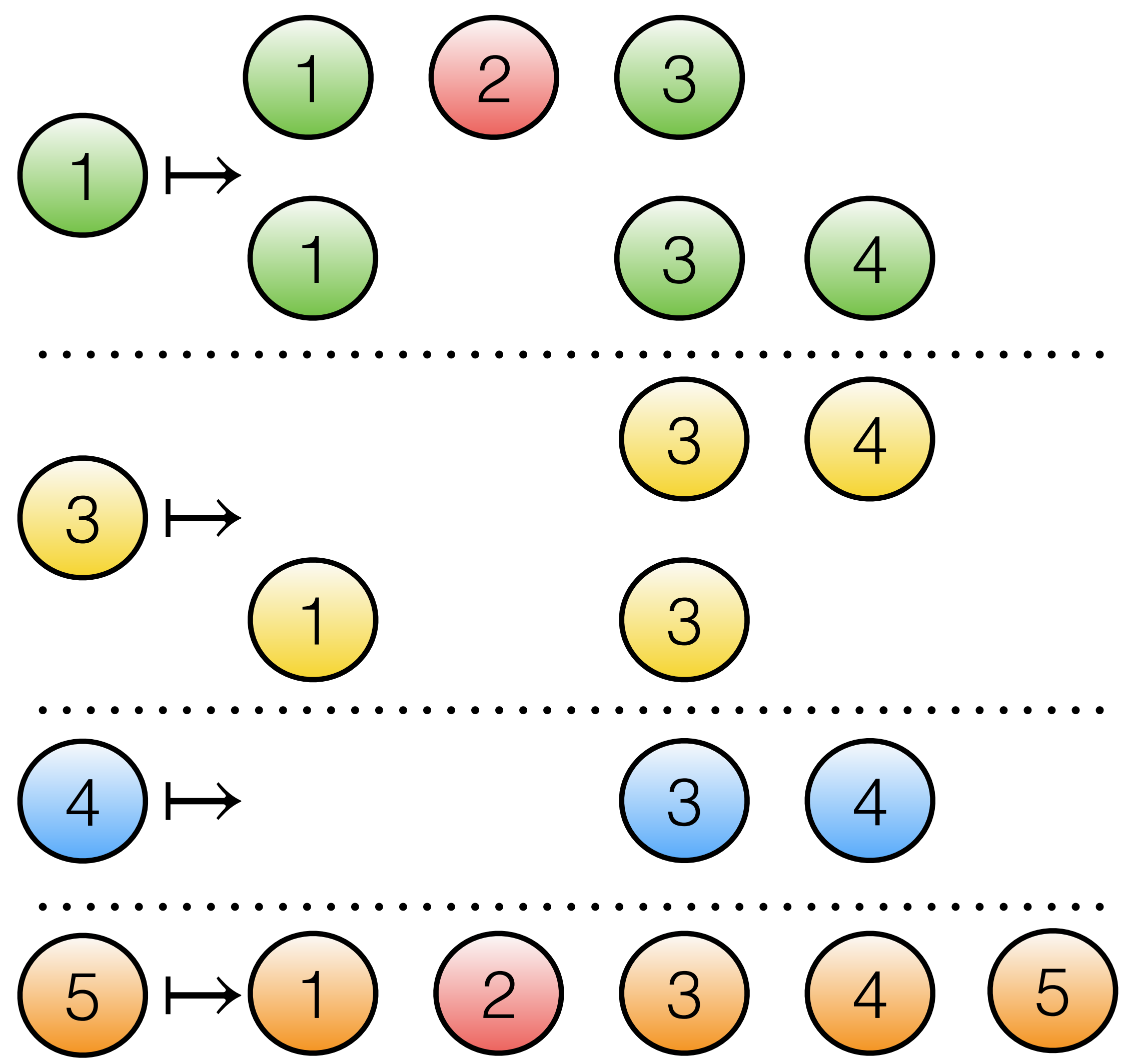
Availability inside $\mathcal{O}$



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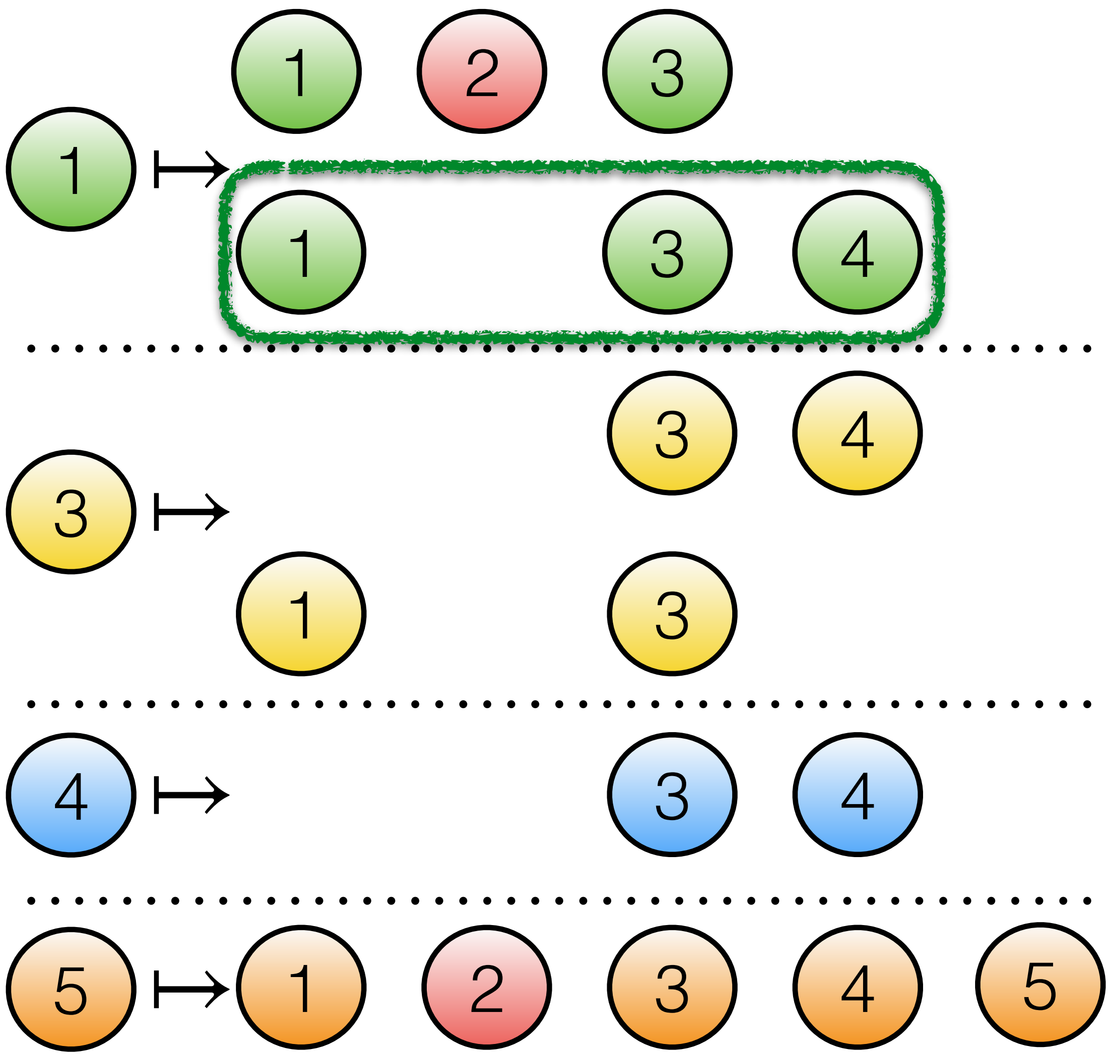

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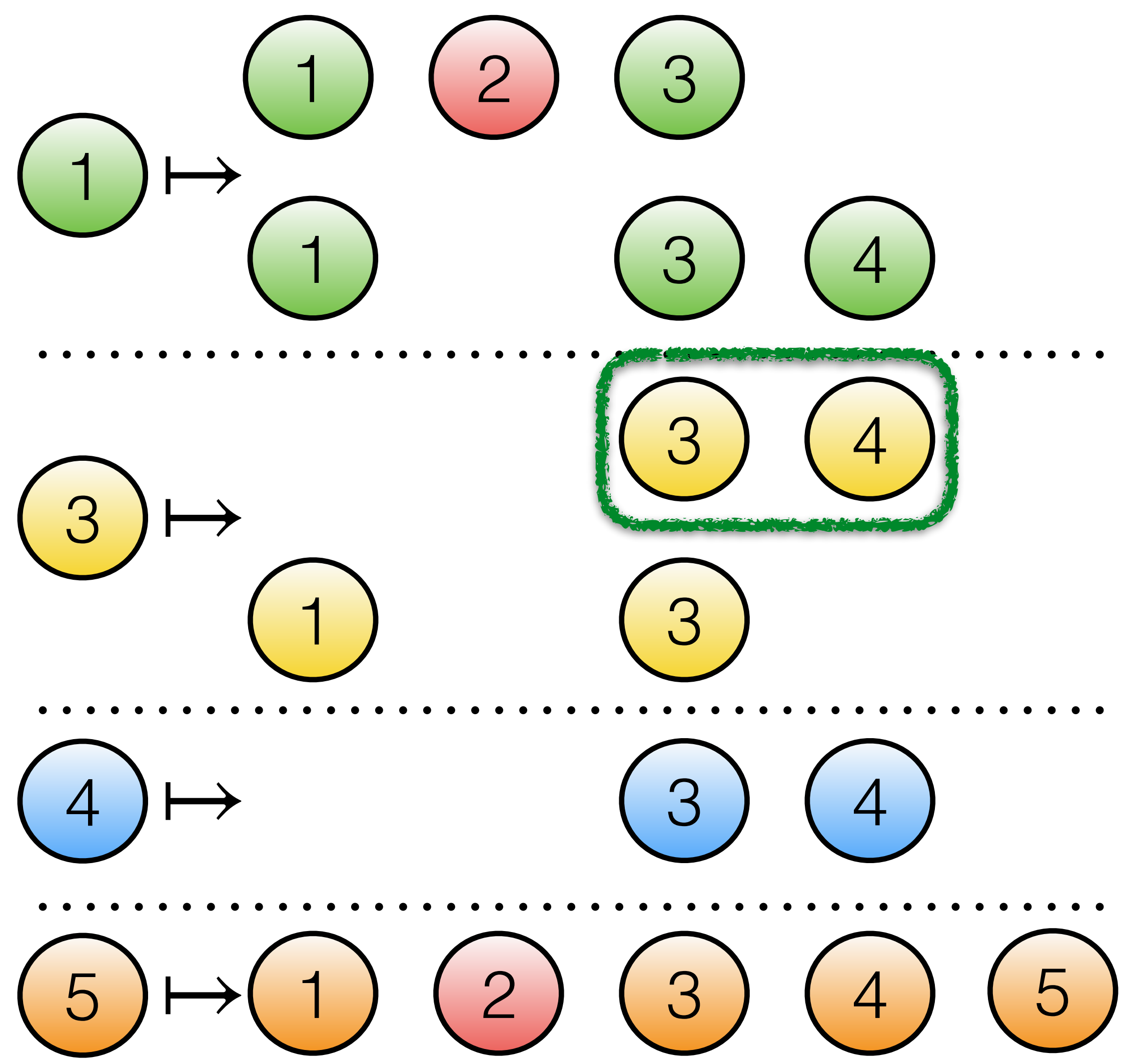

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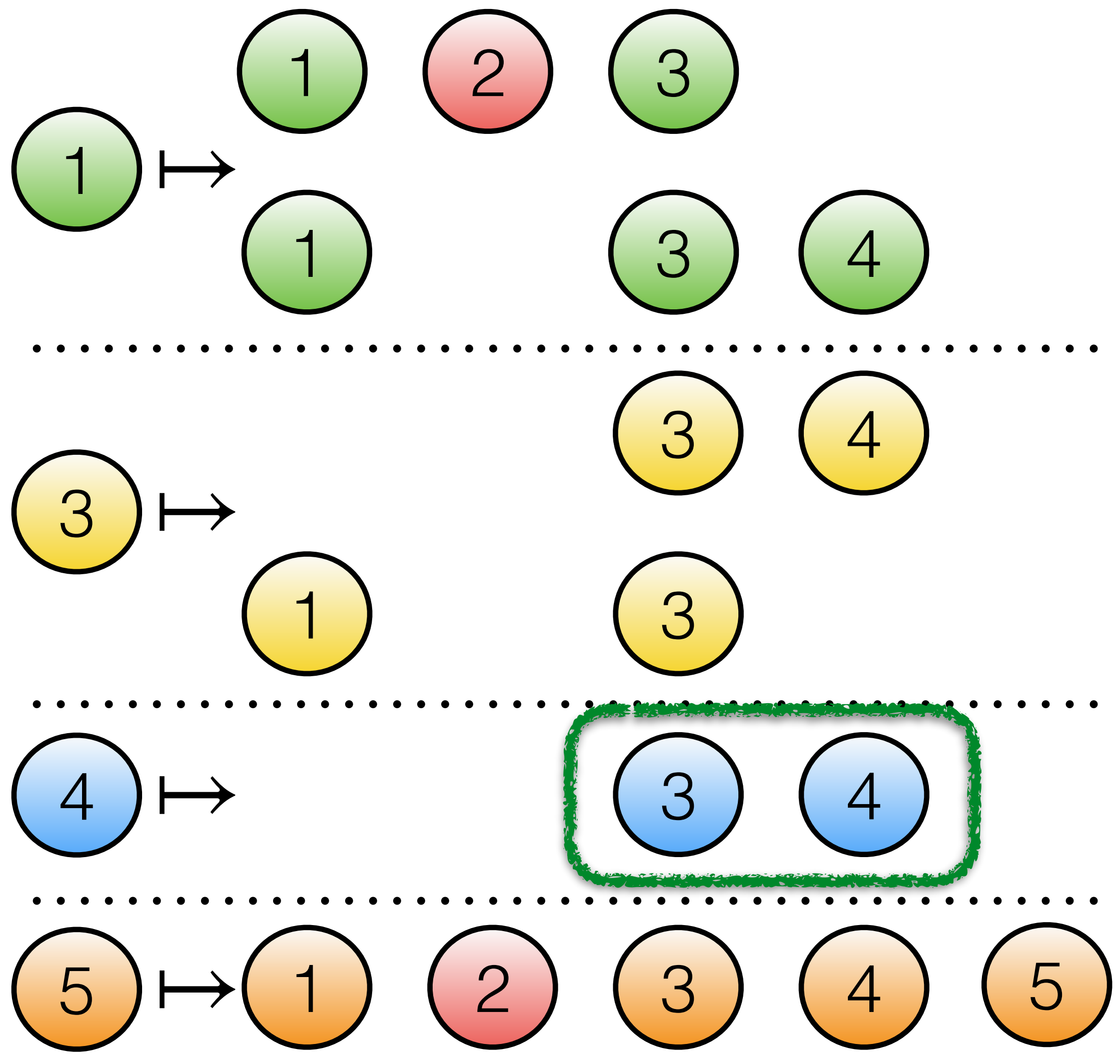

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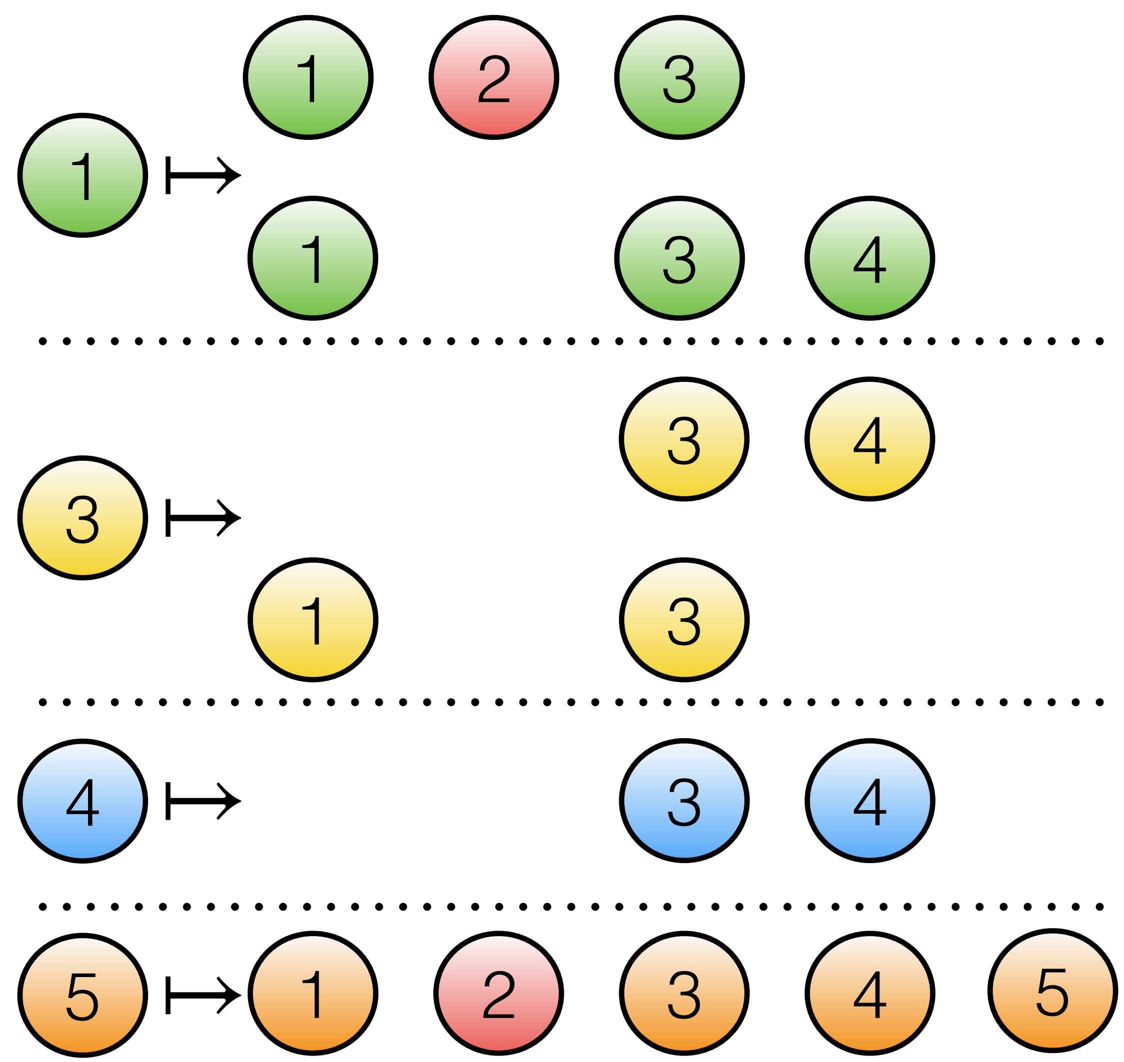

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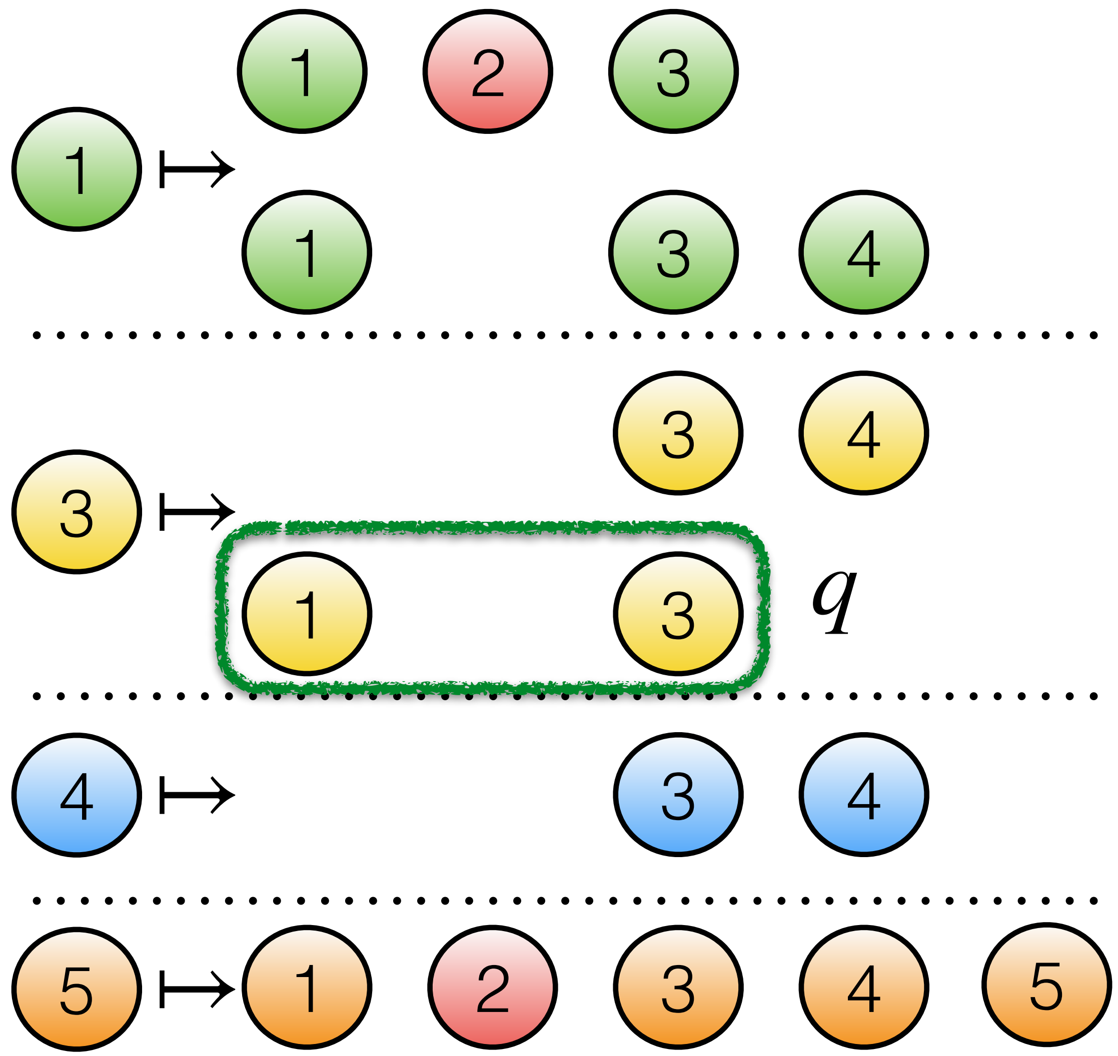

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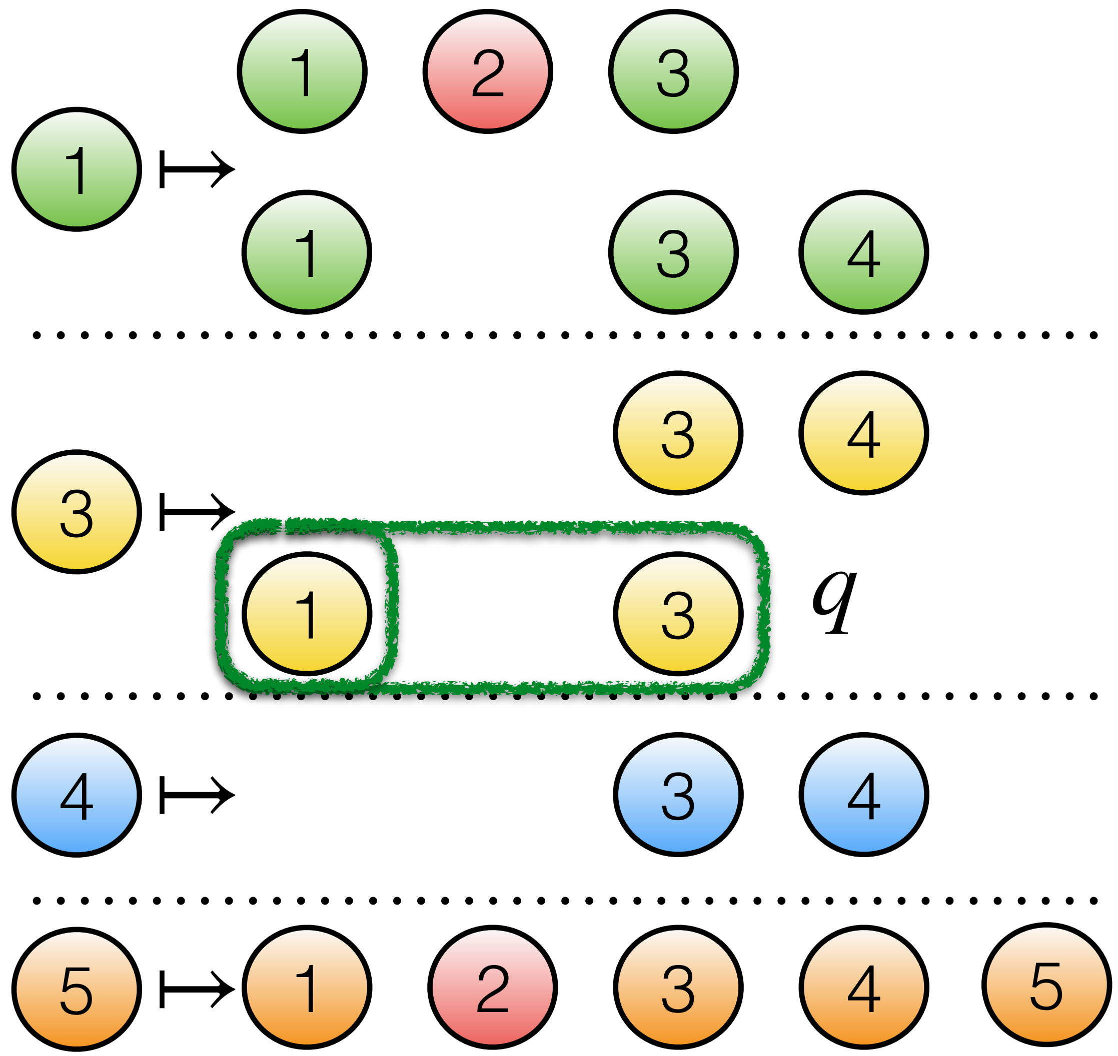

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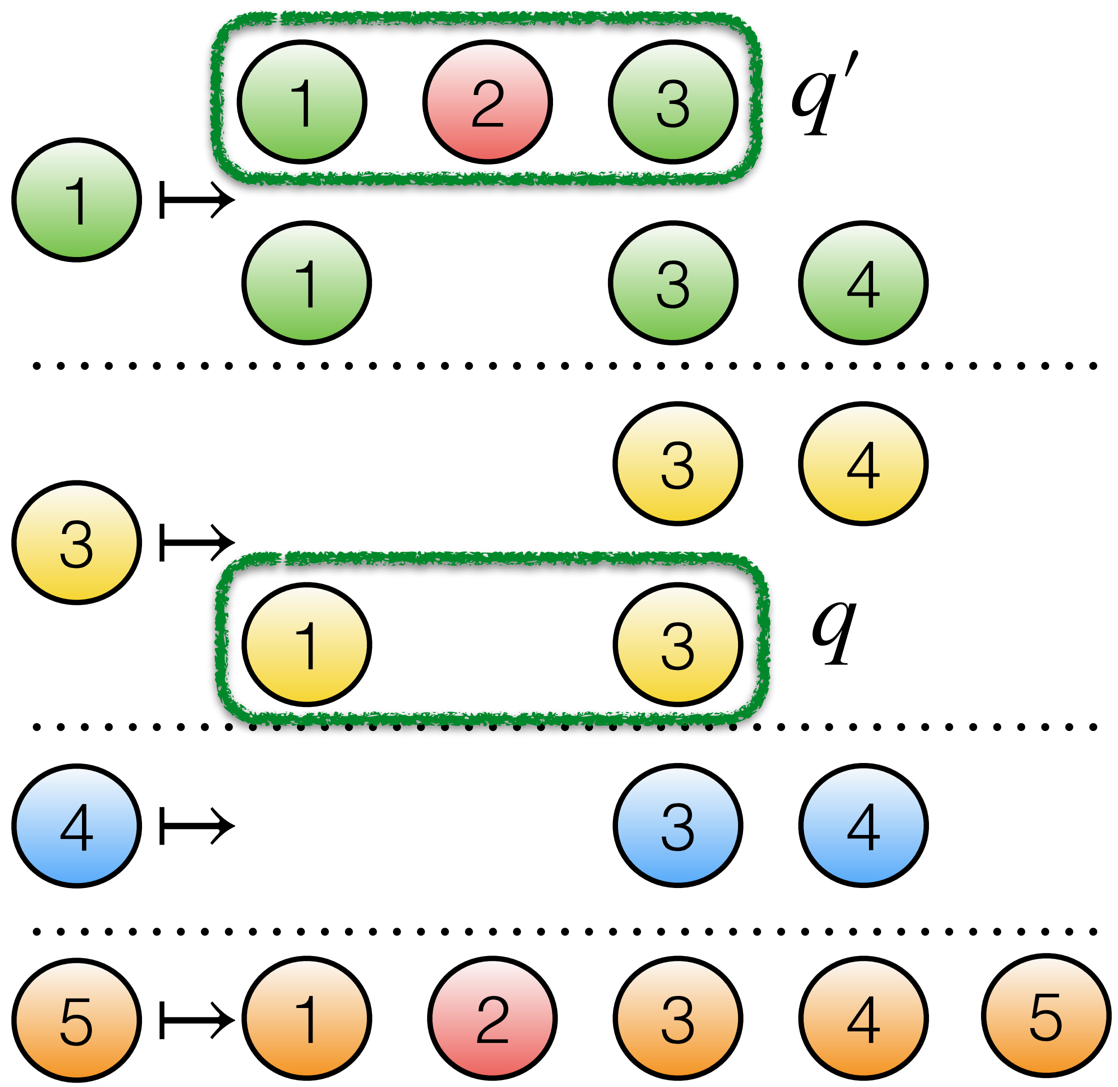

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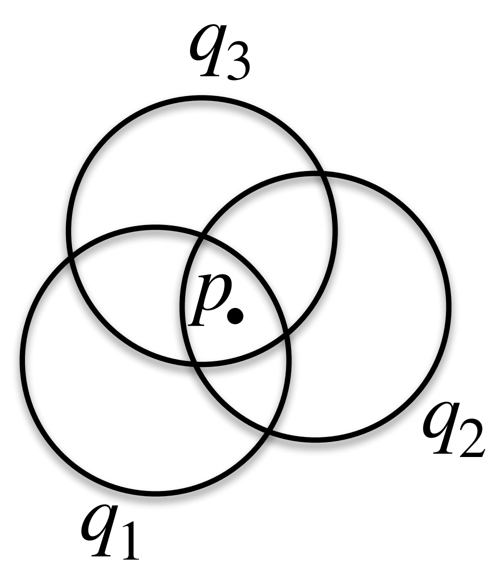
There is a set of well-behaved processes $\mathcal{O}$ such that the quorum system has

- quorum intersection at $\mathcal{O}$,
- quorum availability inside $\mathcal{O}$, and
- quorum inclusion for $\mathcal{O}$.

Blocking Set

p

Blocking Set



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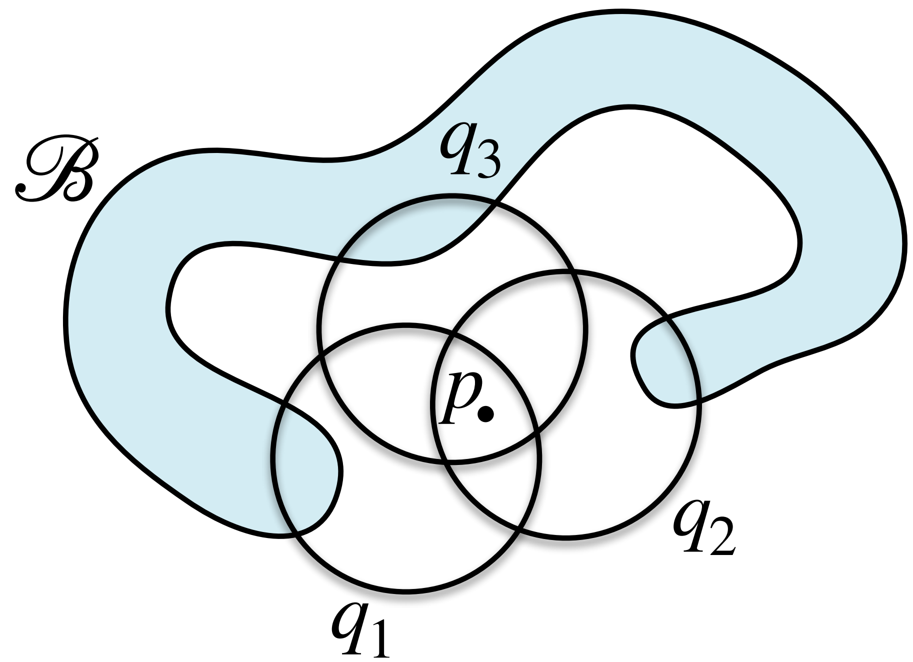
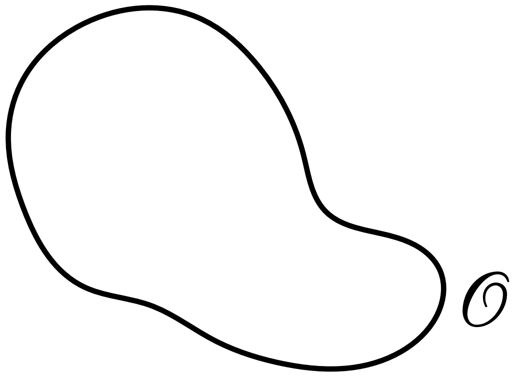


Lemma for blocking set

If

- Quorum system $\mathscr{Q}$ is available inside $\mathscr{O}$



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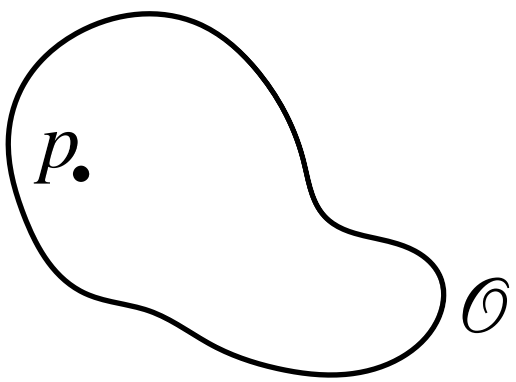

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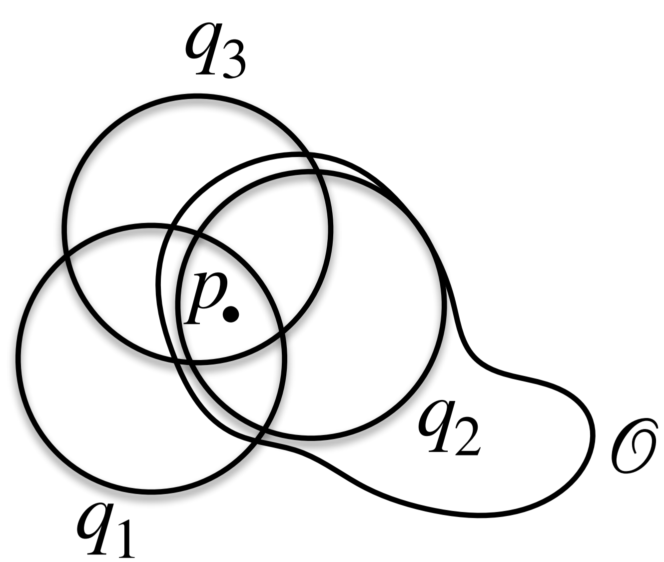

Lemma for blocking set

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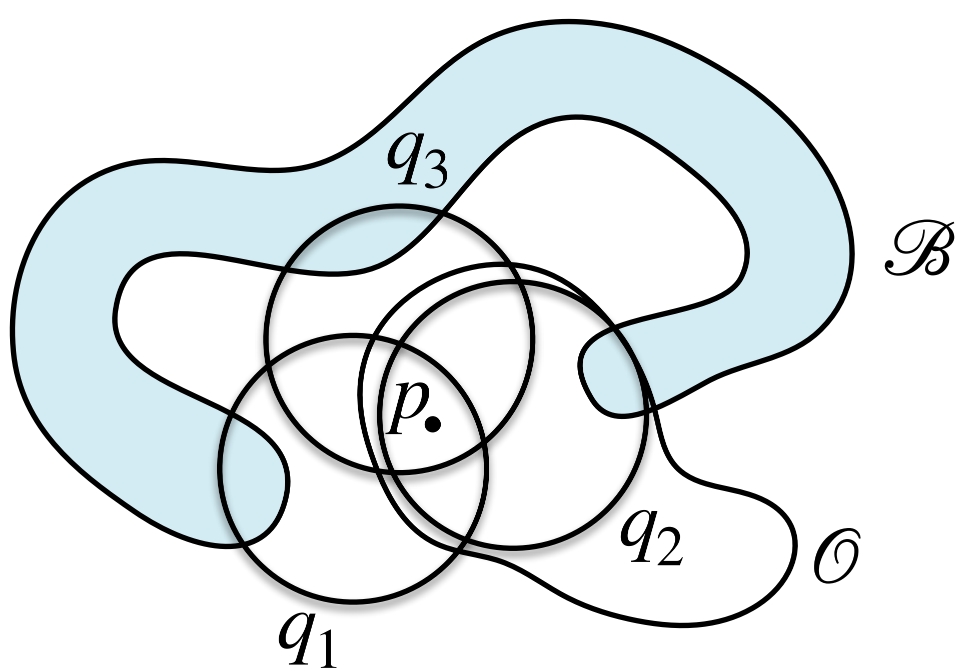

Lemma for blocking set

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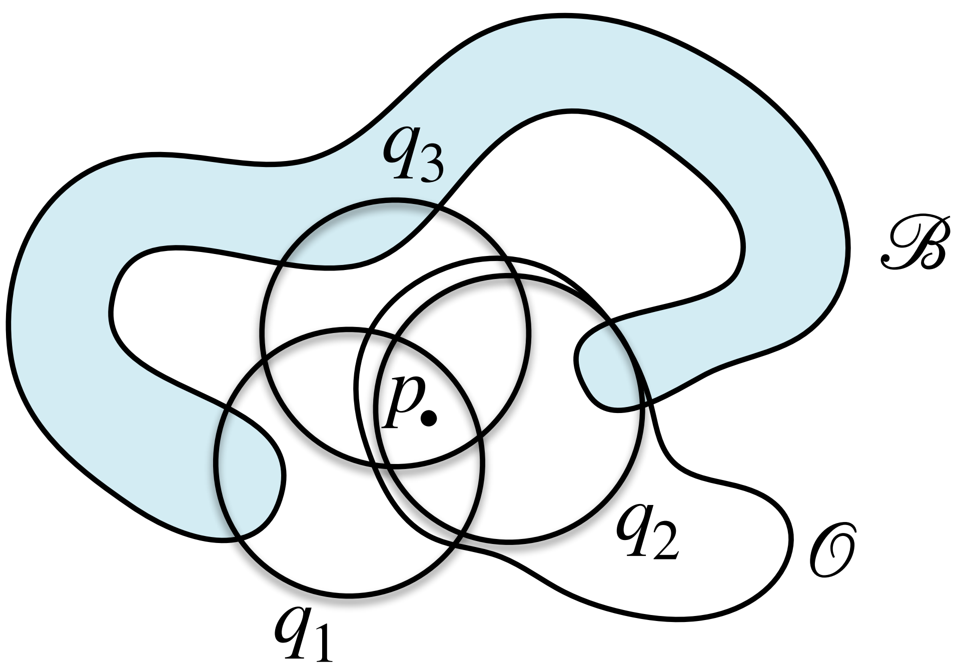
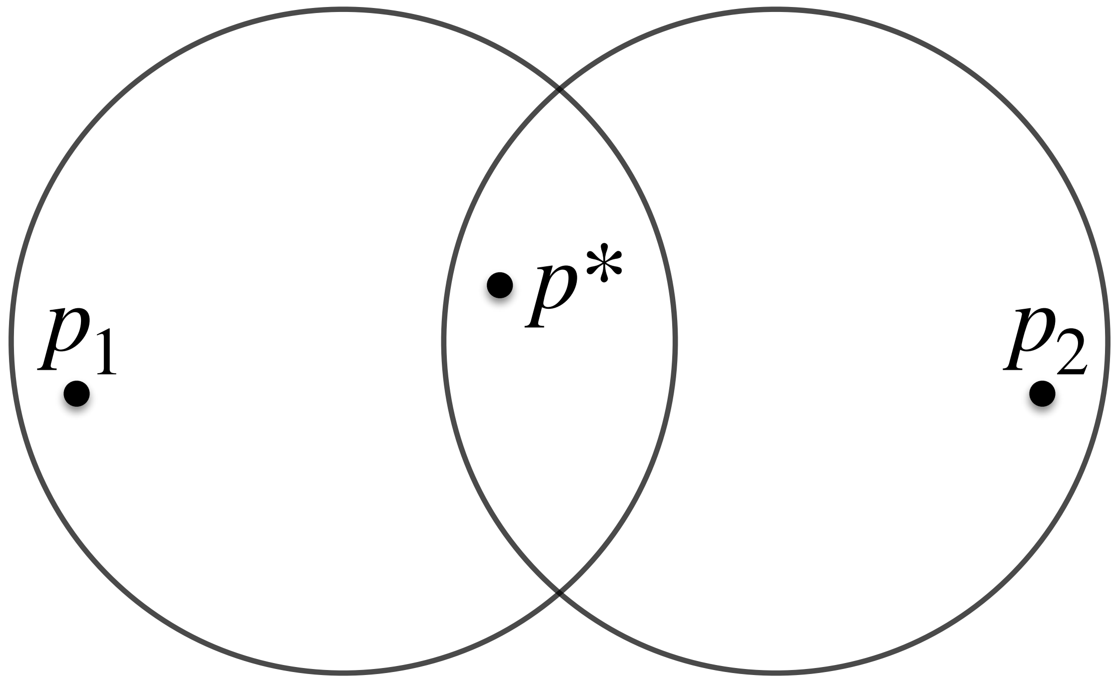
Then

- $\mathscr{B}$ intersects with $\mathscr{O}$



Leave Protocol



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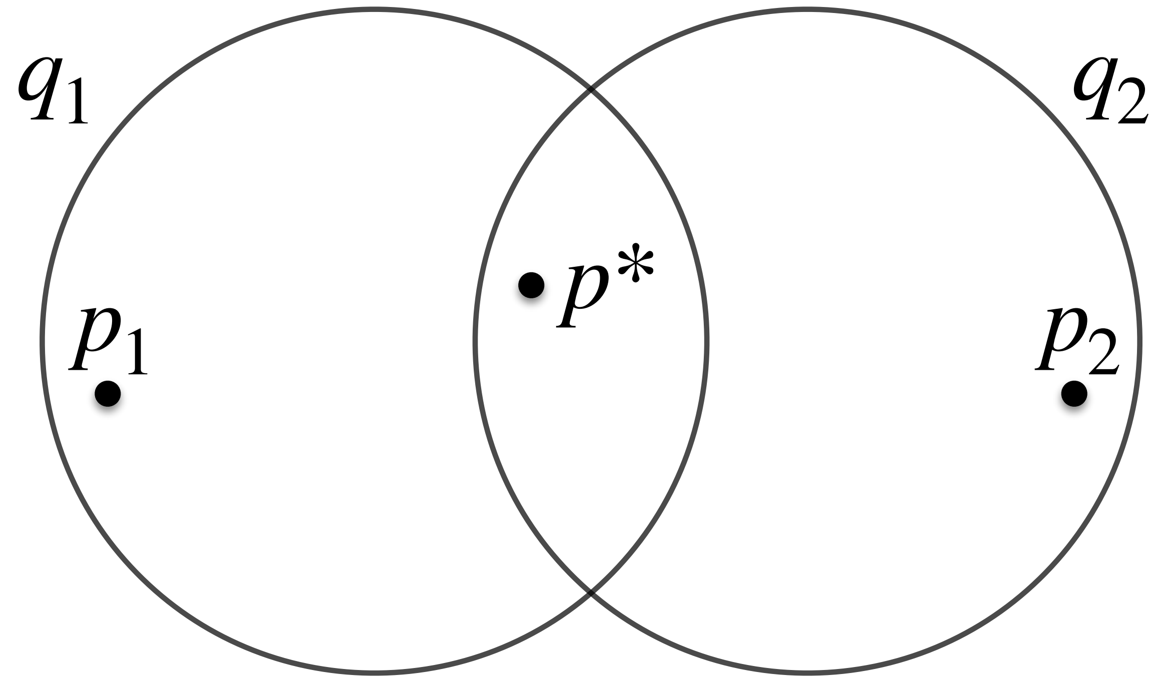

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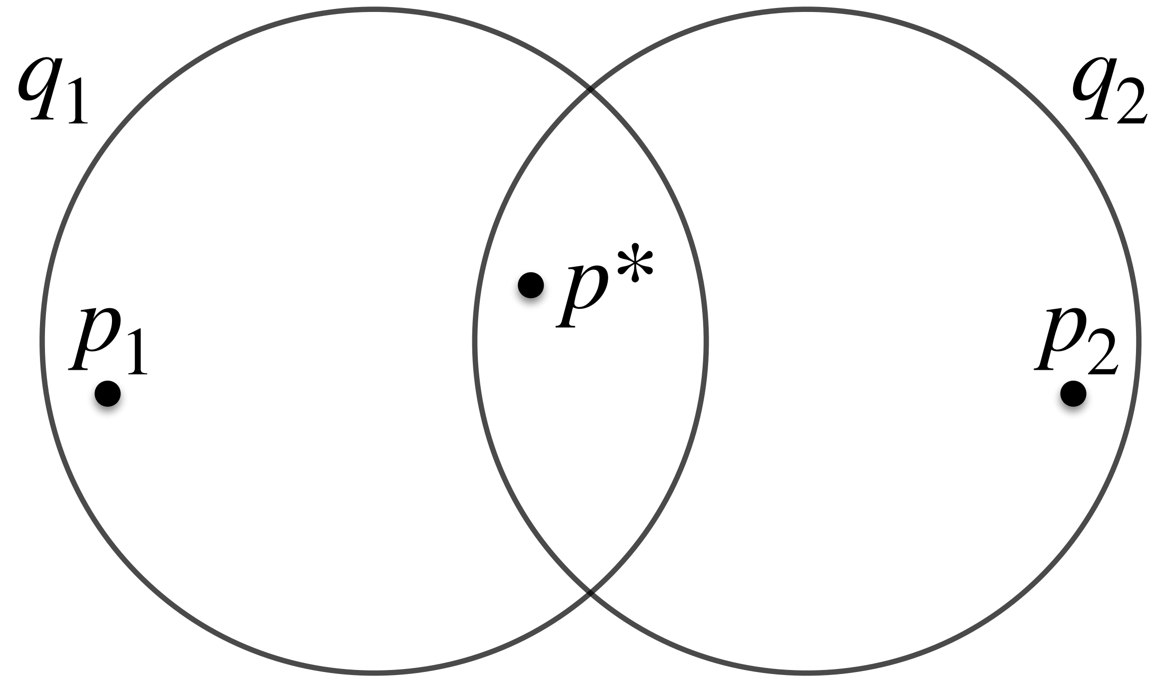

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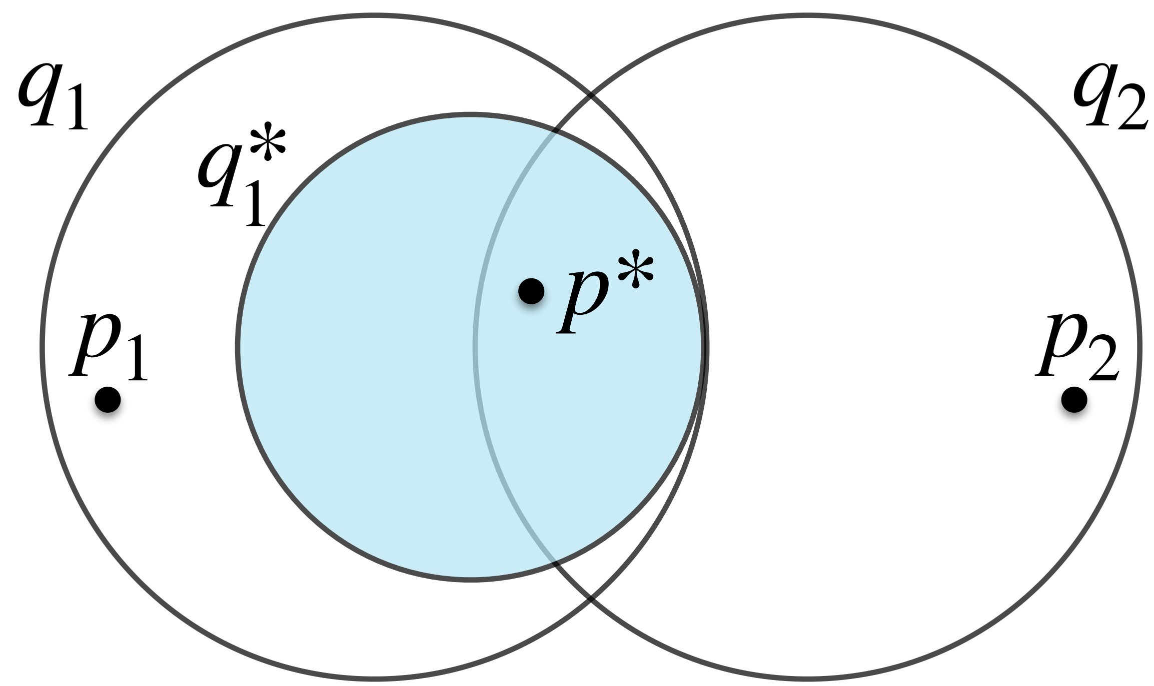

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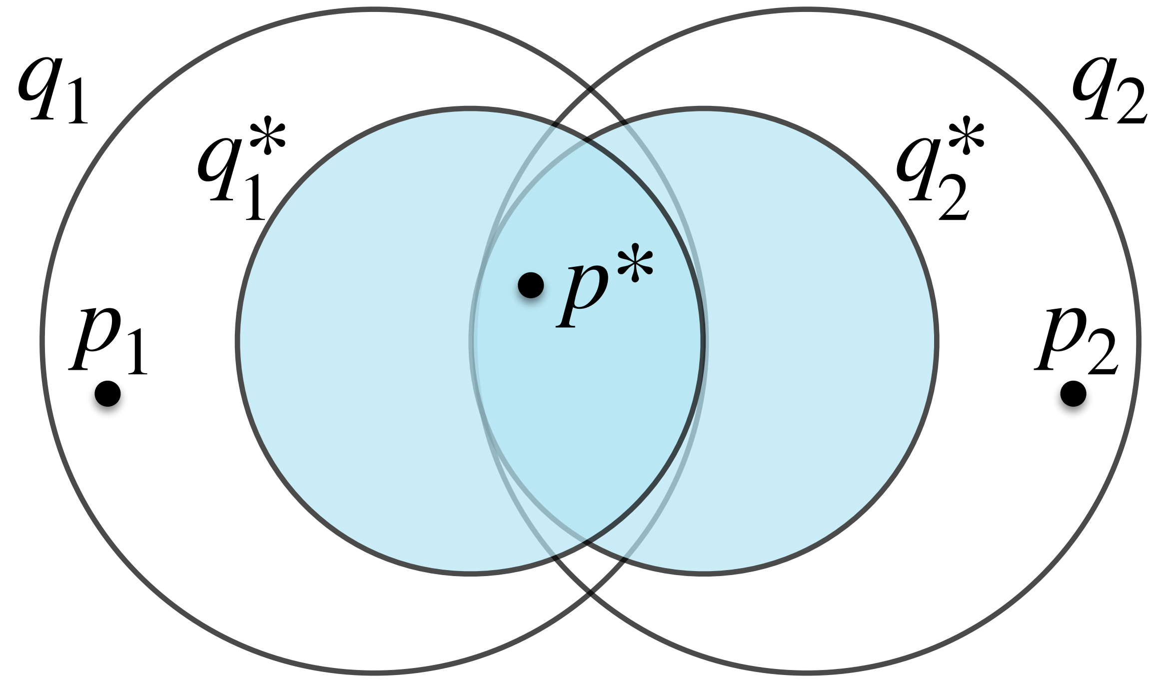

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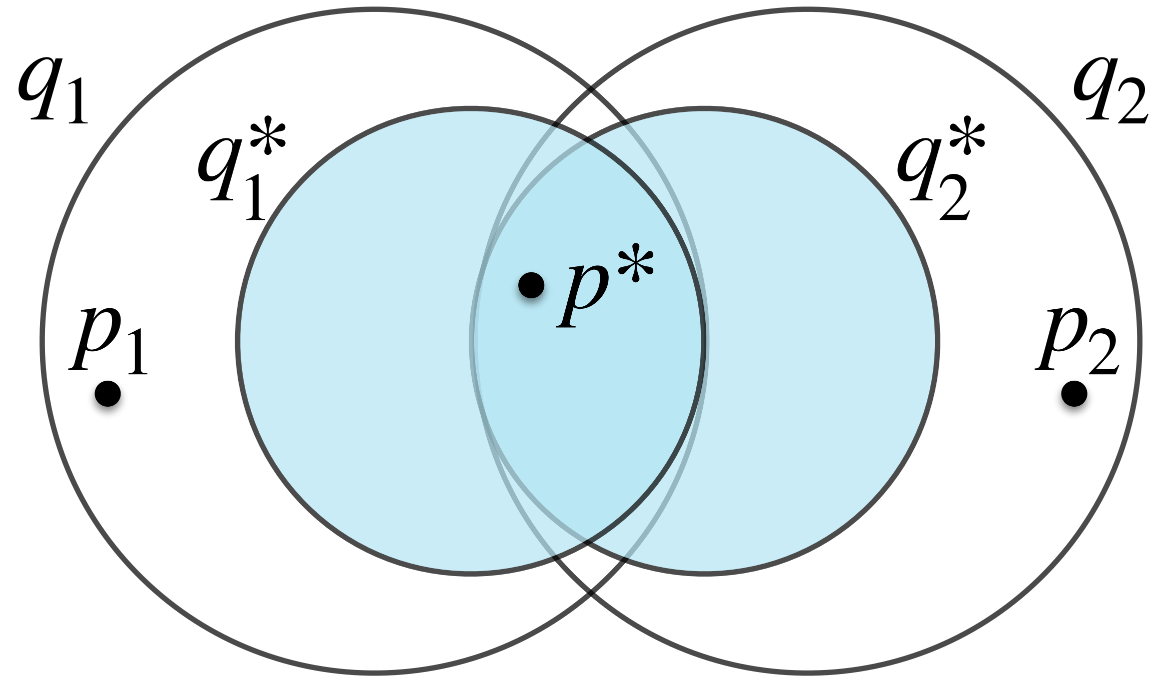

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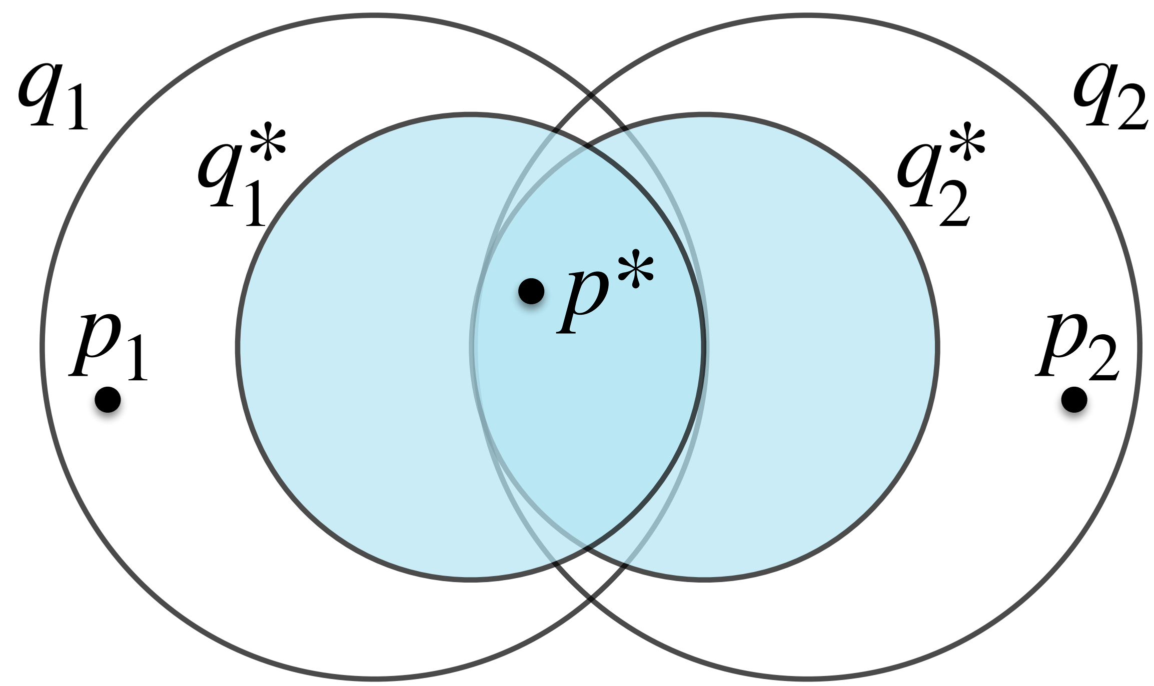

$(q_1^* \cap q_2^*) \setminus \{p^*\}$ is p^*-blocking

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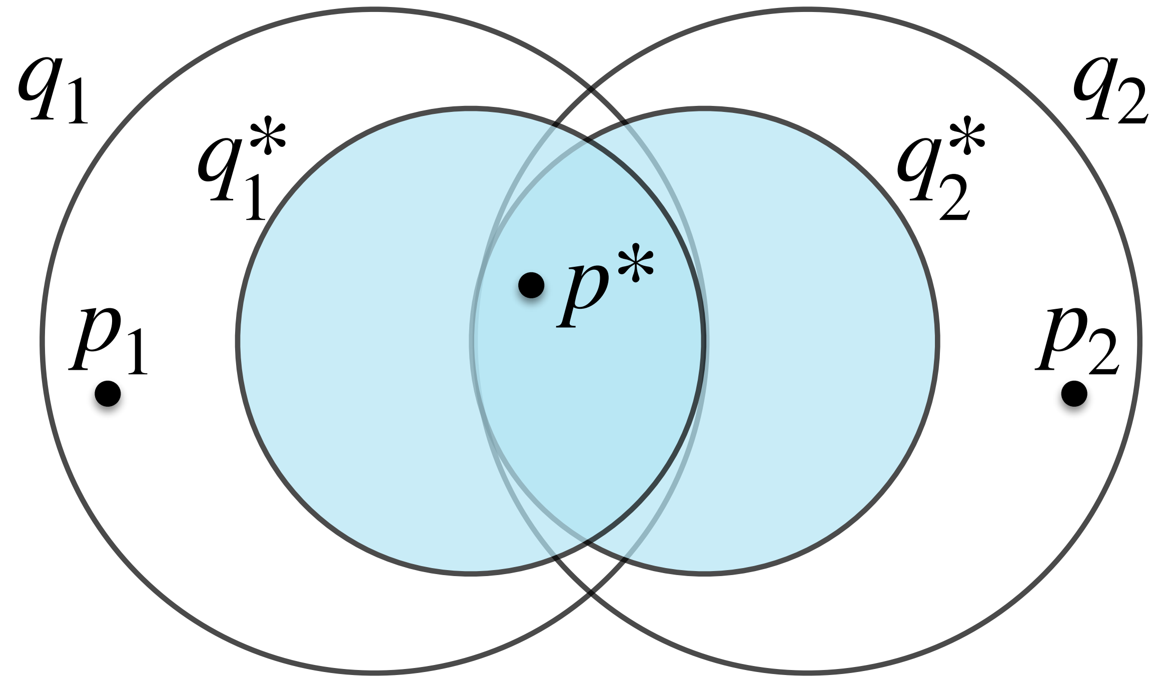

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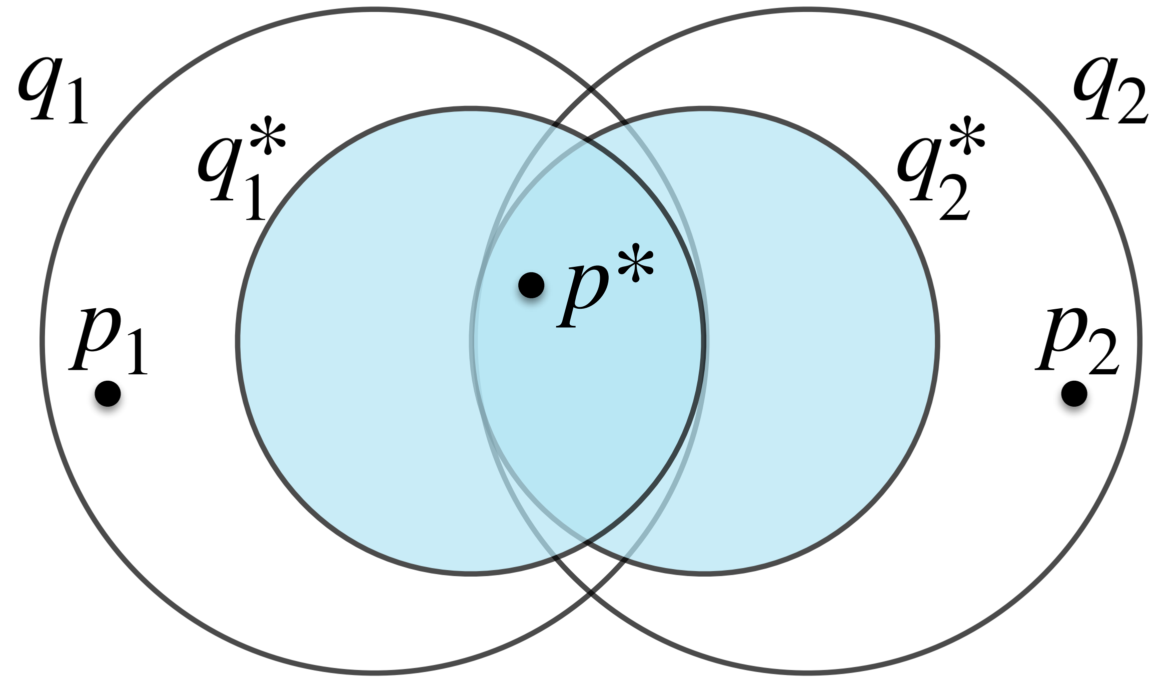

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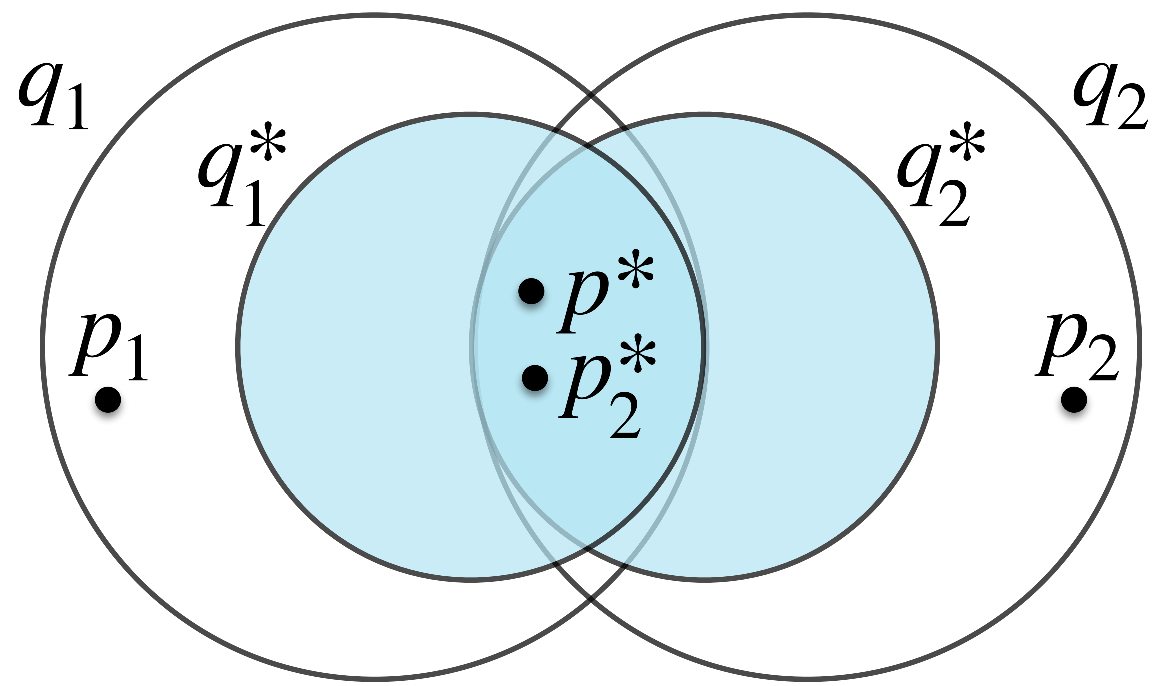

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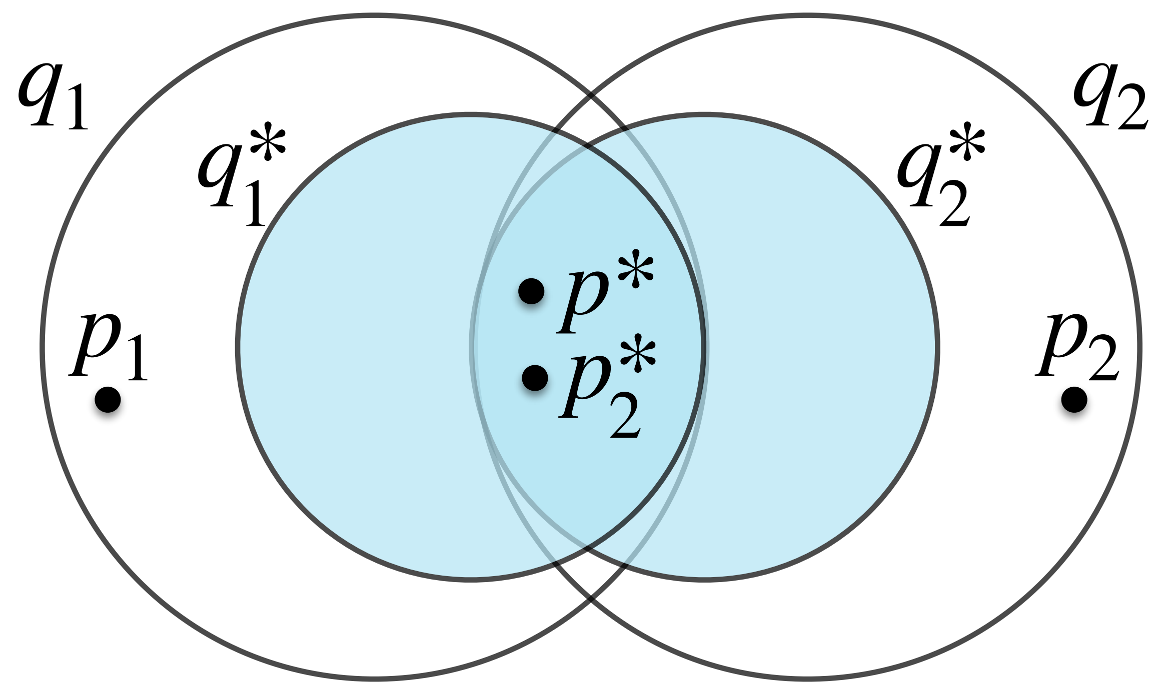

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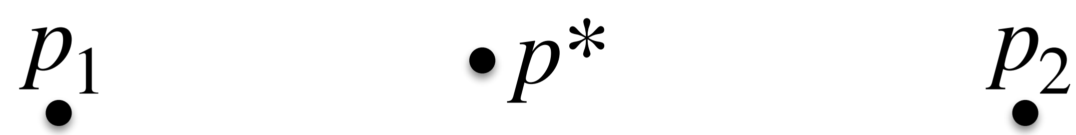

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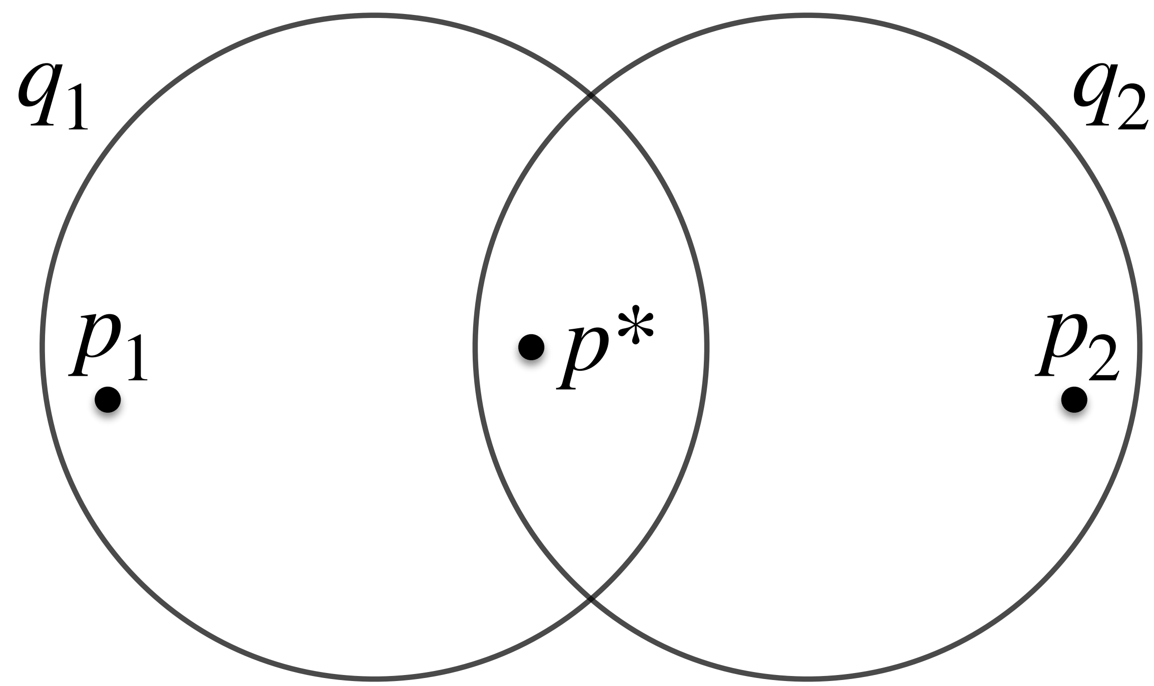

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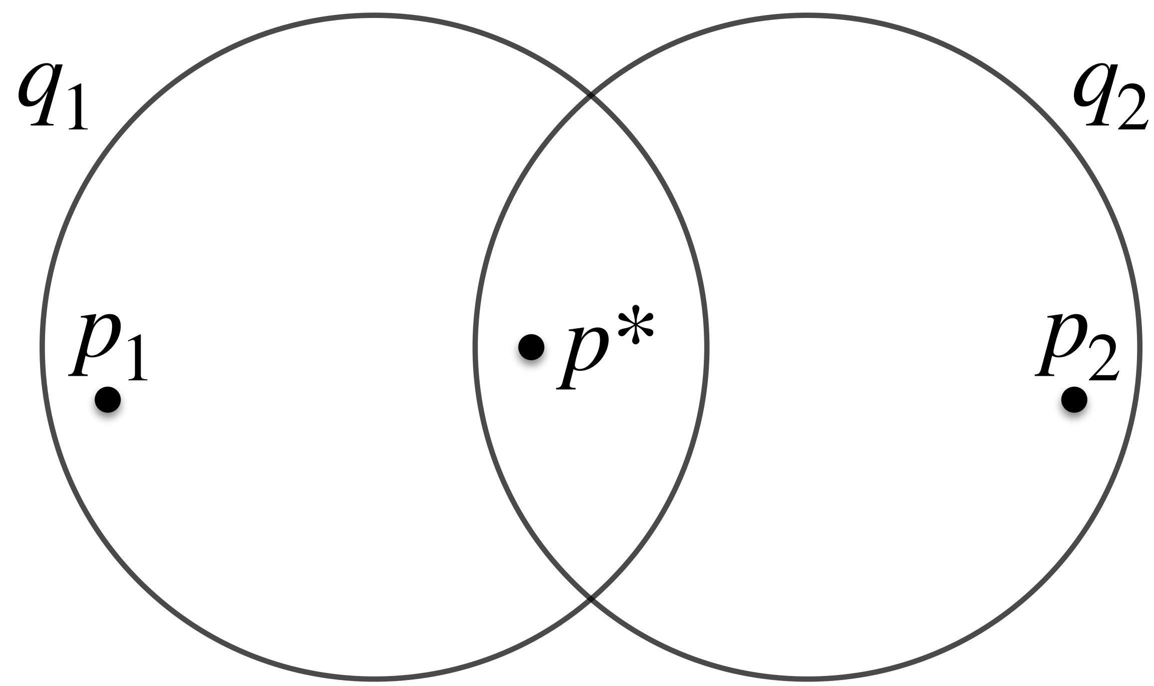

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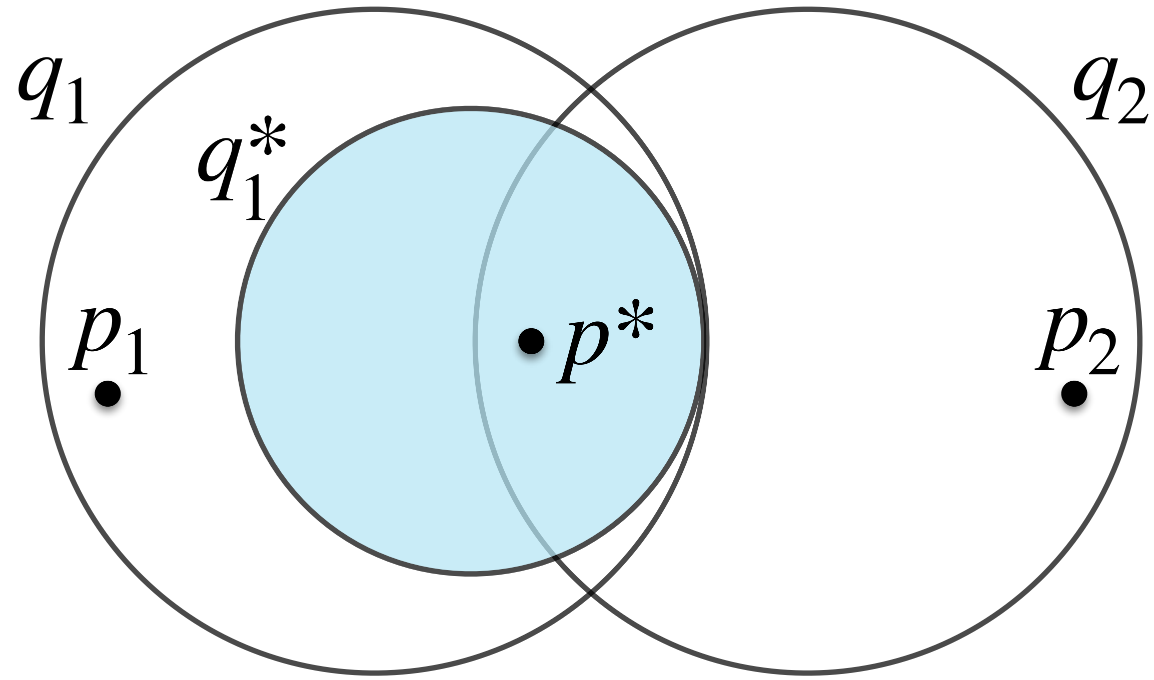

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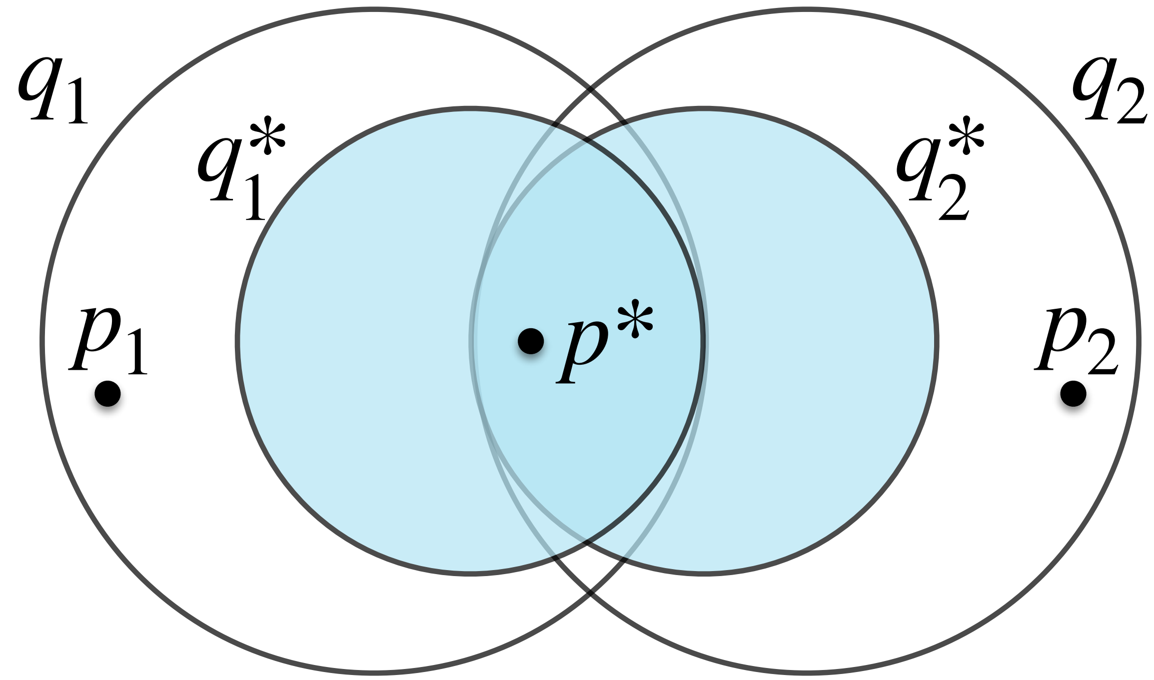

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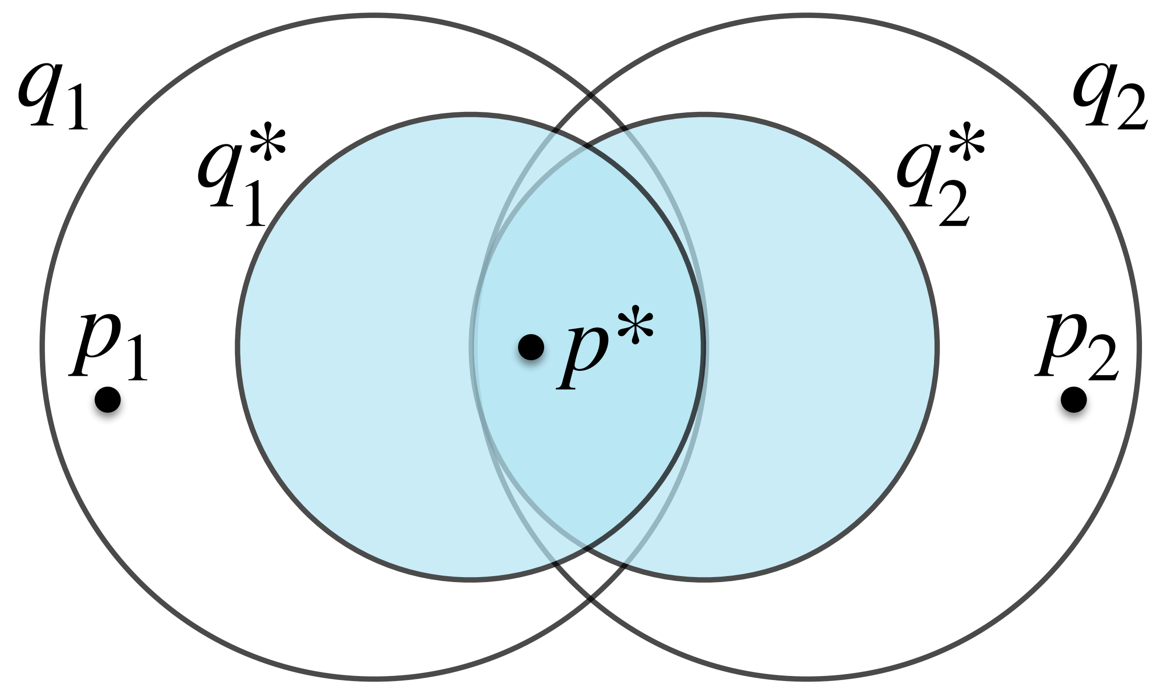

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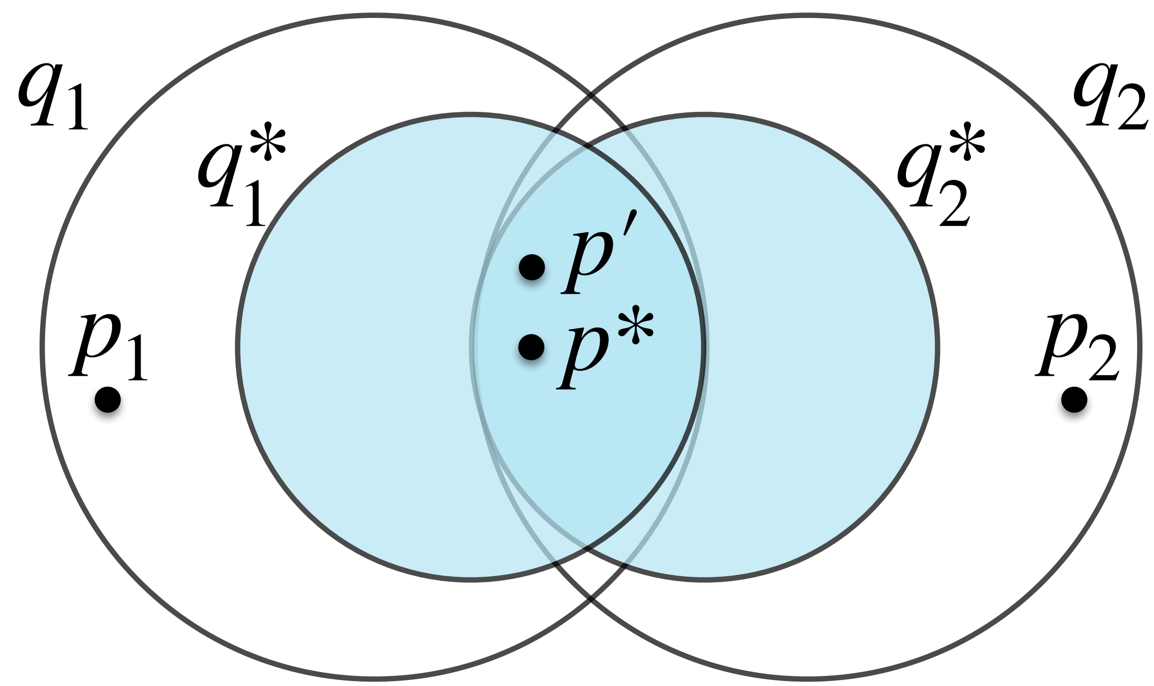

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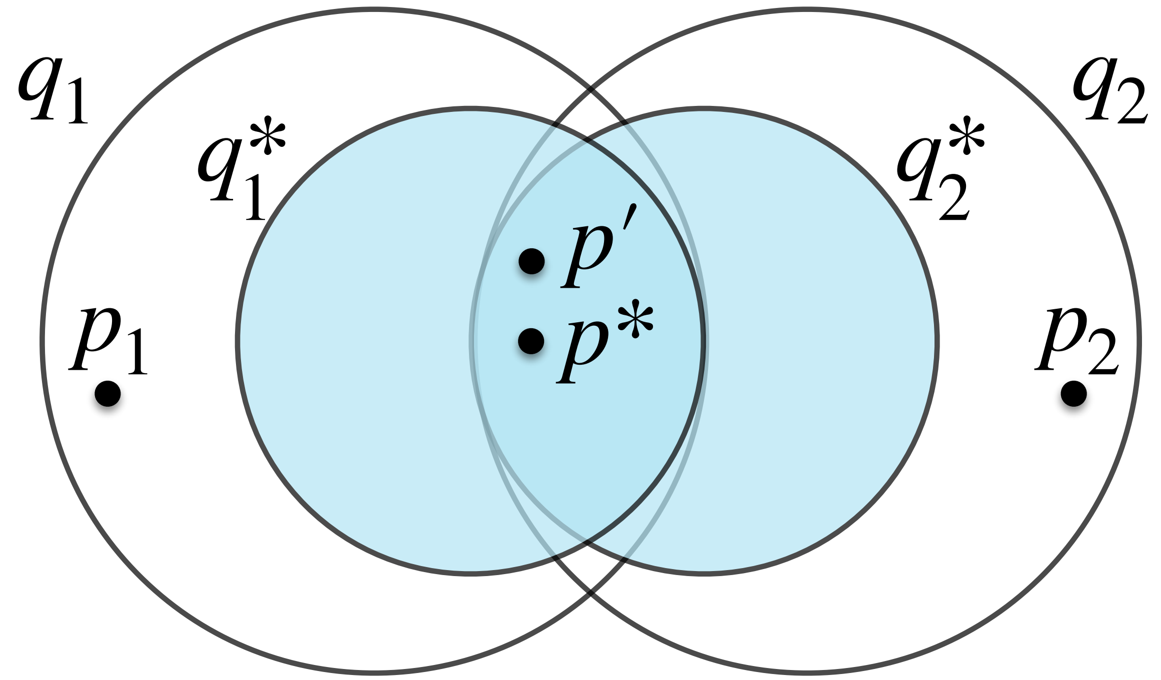
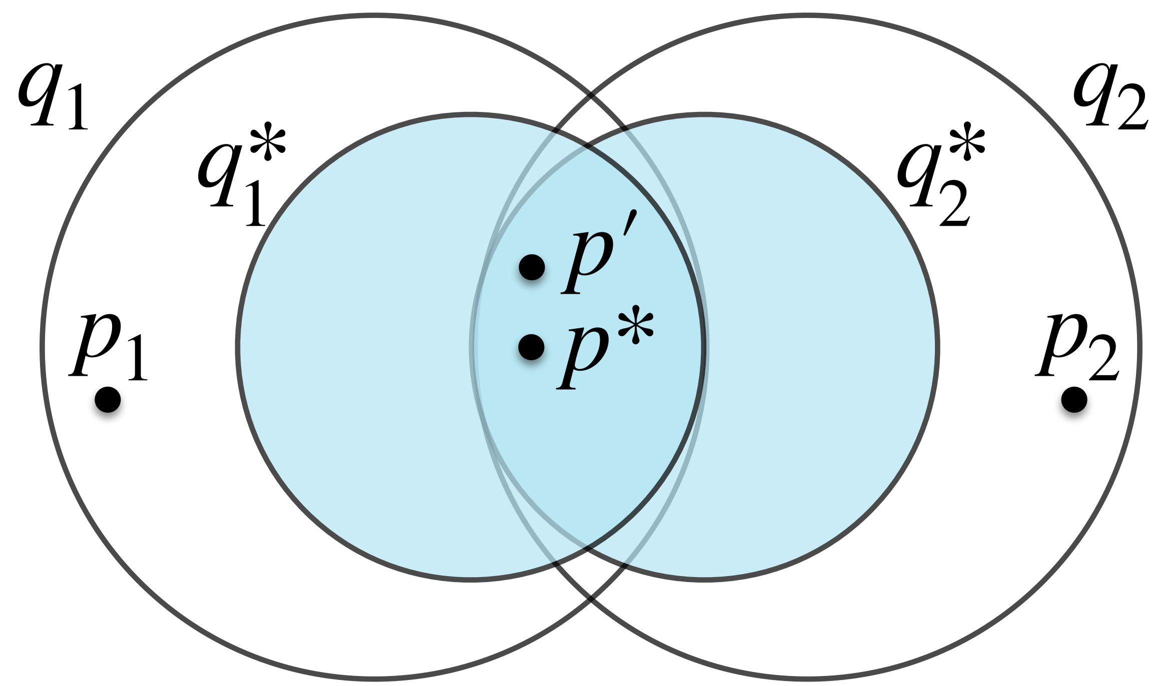


$p' \in tomb$

Leave Protocol



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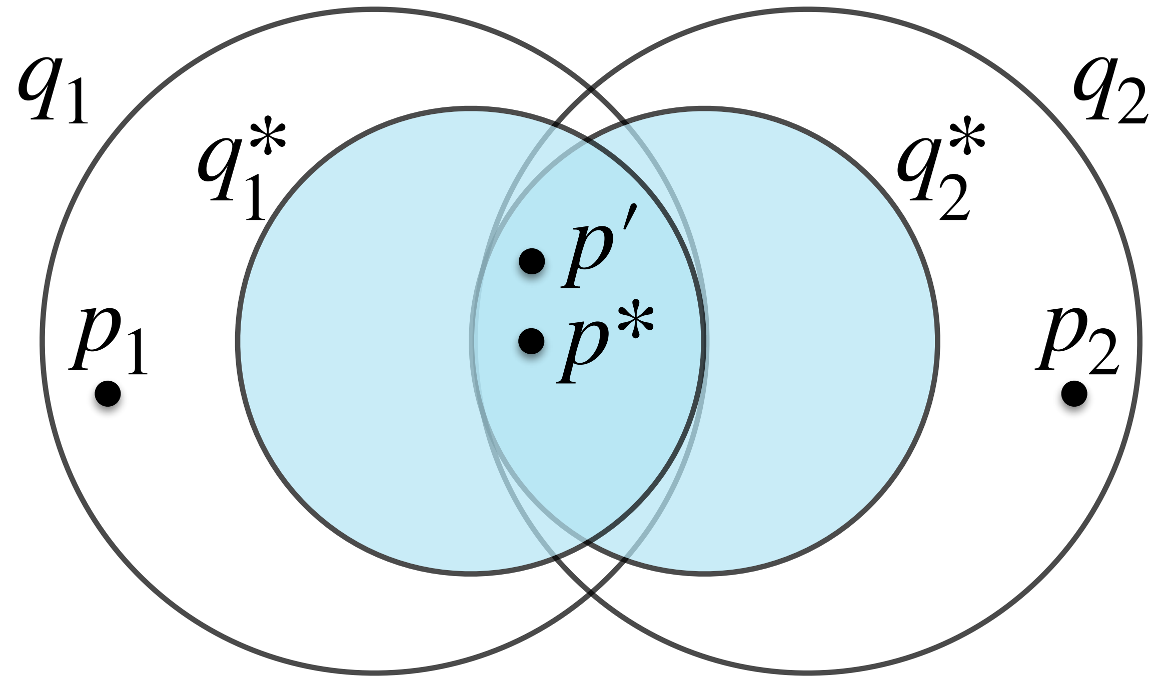

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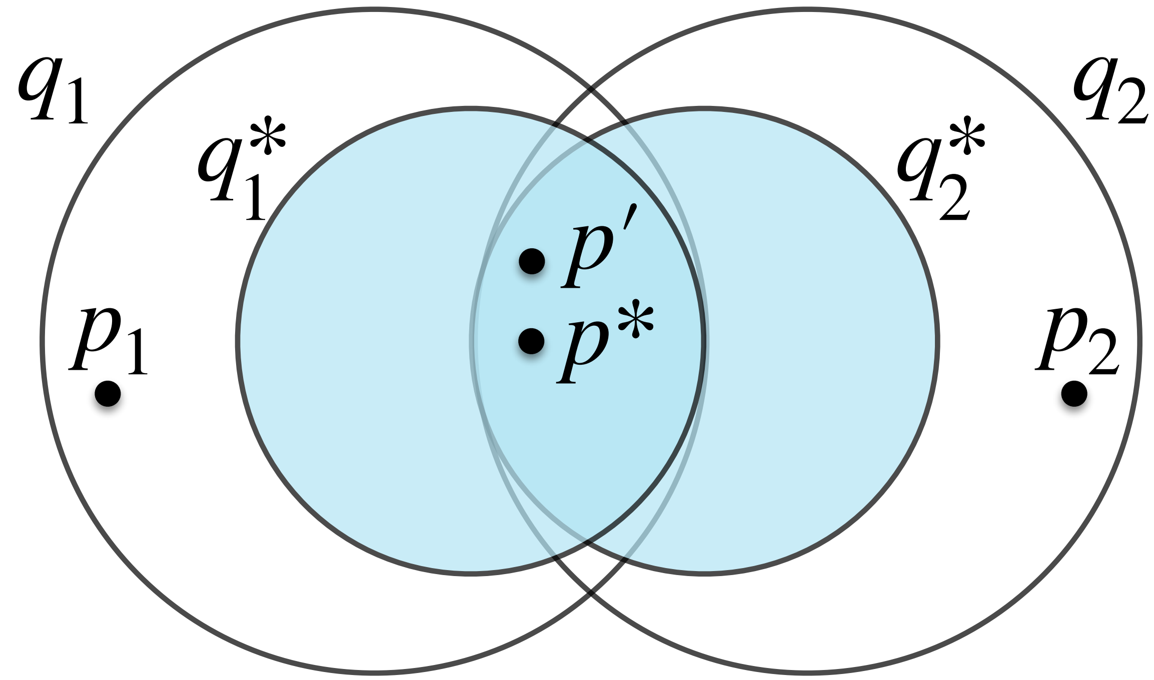
Leave Protocol



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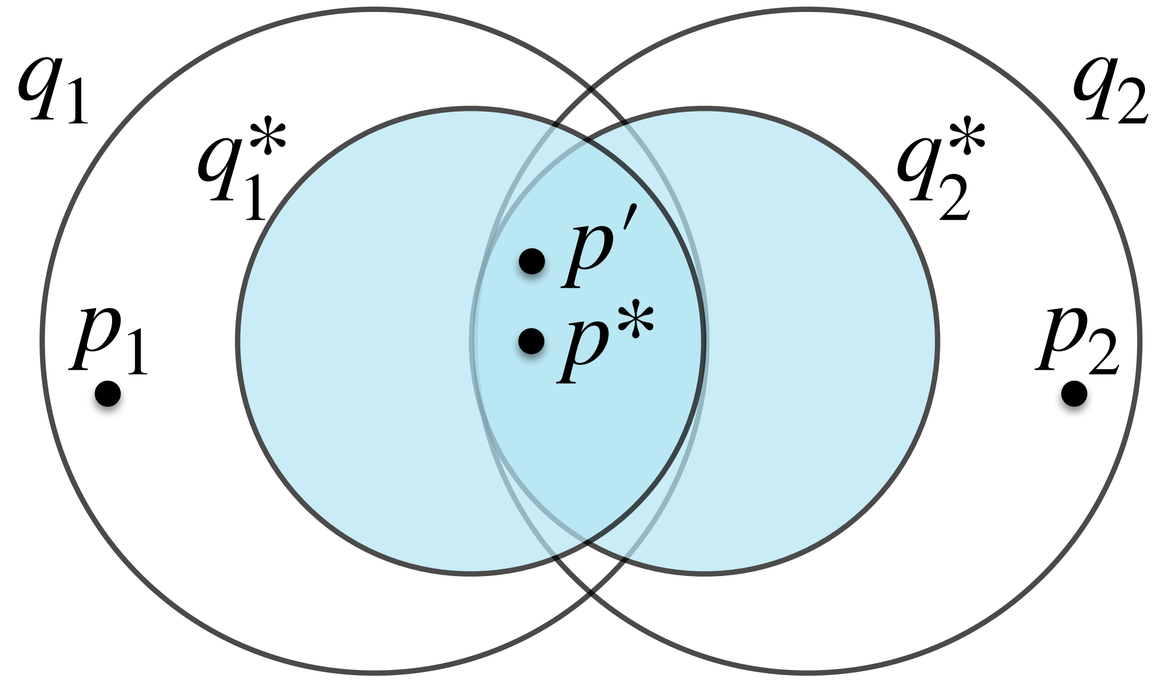

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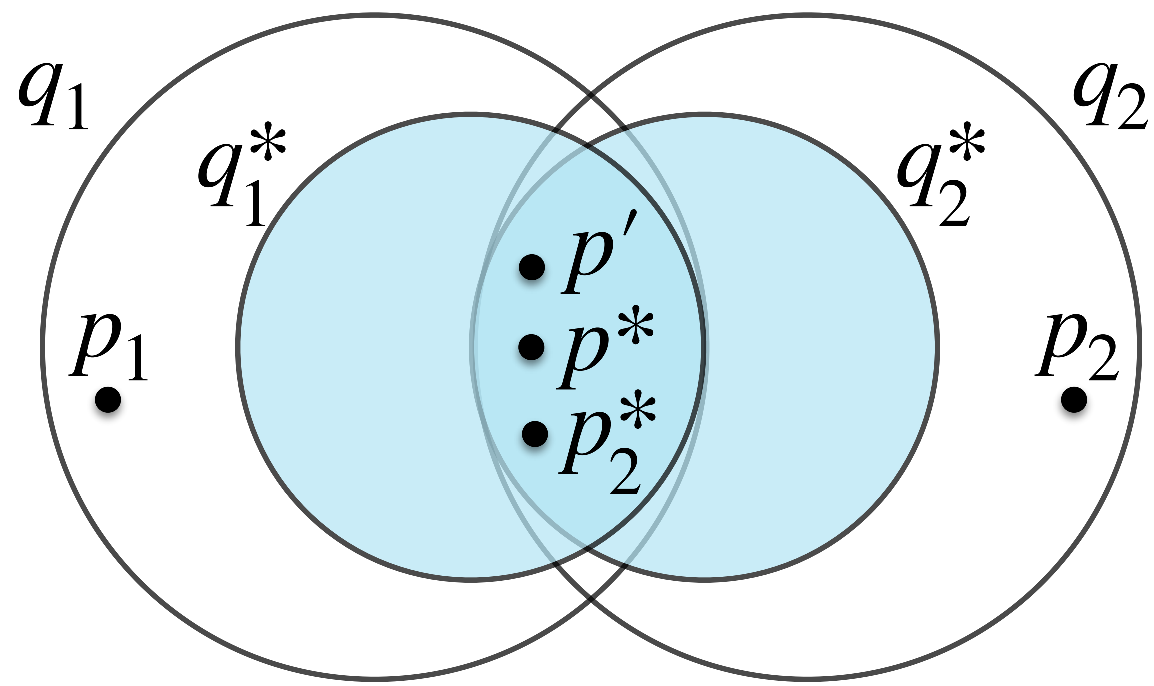

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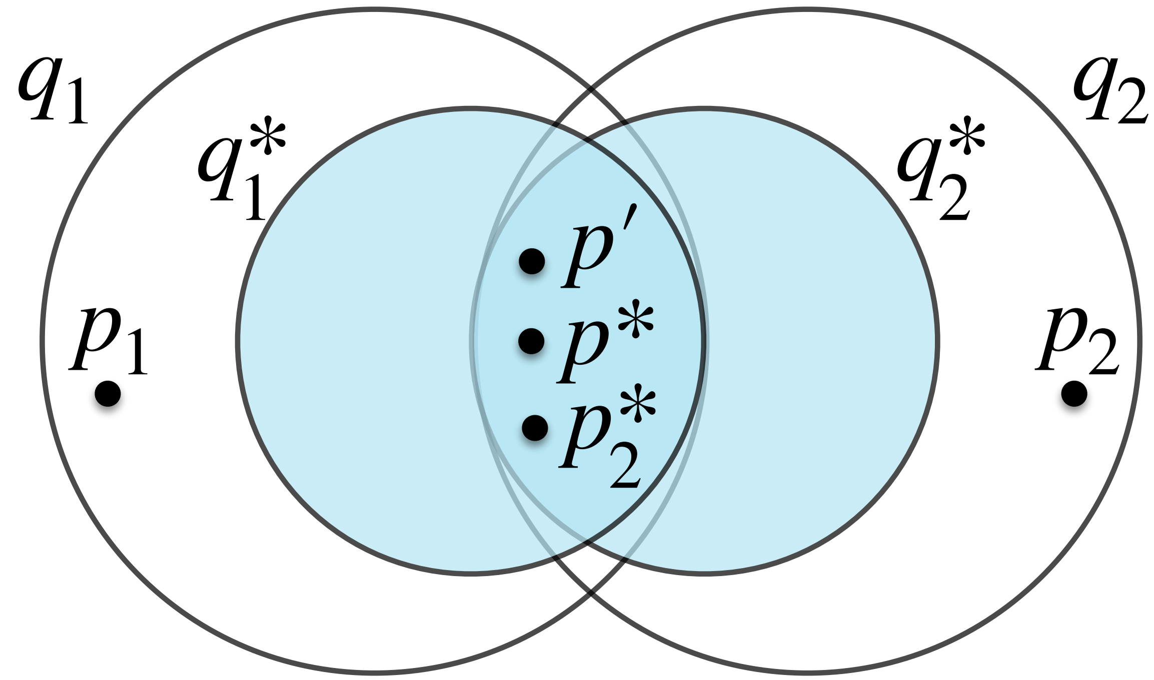

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Reconfigurable Heterogeneous Quorum Systems

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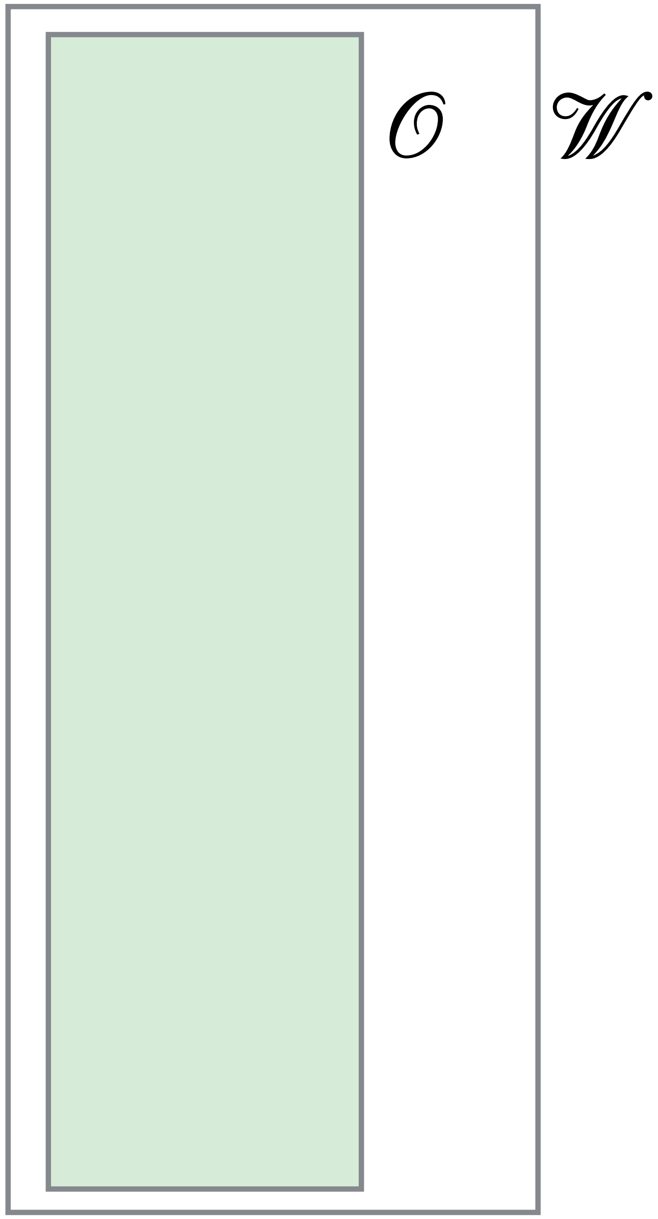
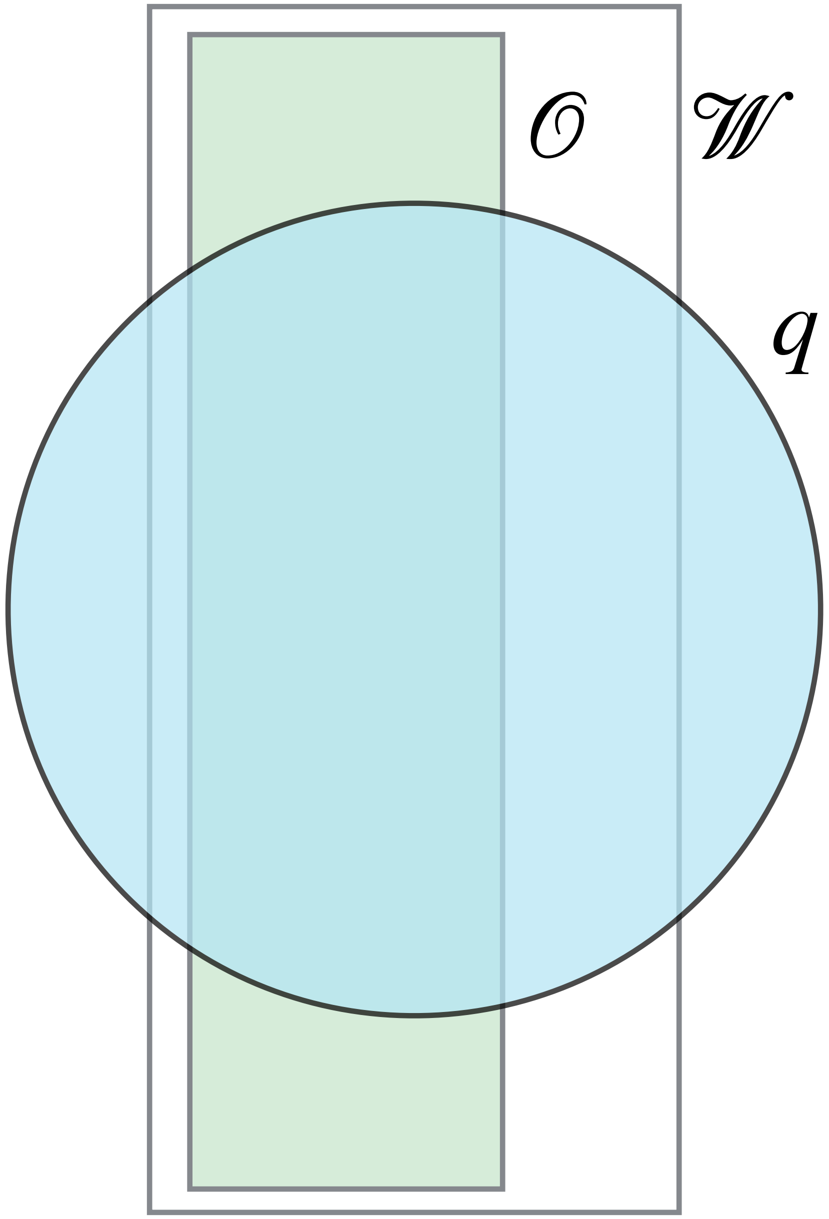
Reconfigurability, the last missing property

	Proof-of-work	Proof-of-stake	Byzantine Replication	HQS	Reconfigurable HQS
Heterogenous trust	✓	✓	✗	✓	✓
Reconfigurability (Openness)	✓	✓	✗	✗	✓
Energy efficiency	✗	✓	✓	✓	✓
Consistency	✓	✓	✓	✓	✓
Finality	✗	✗	✓	✓	✓
Equity	✓	✗	✓	✓	✓


$\mathscr{O}$
$\mathscr{W}$

Quorum Inclusion for $\mathscr{O}$



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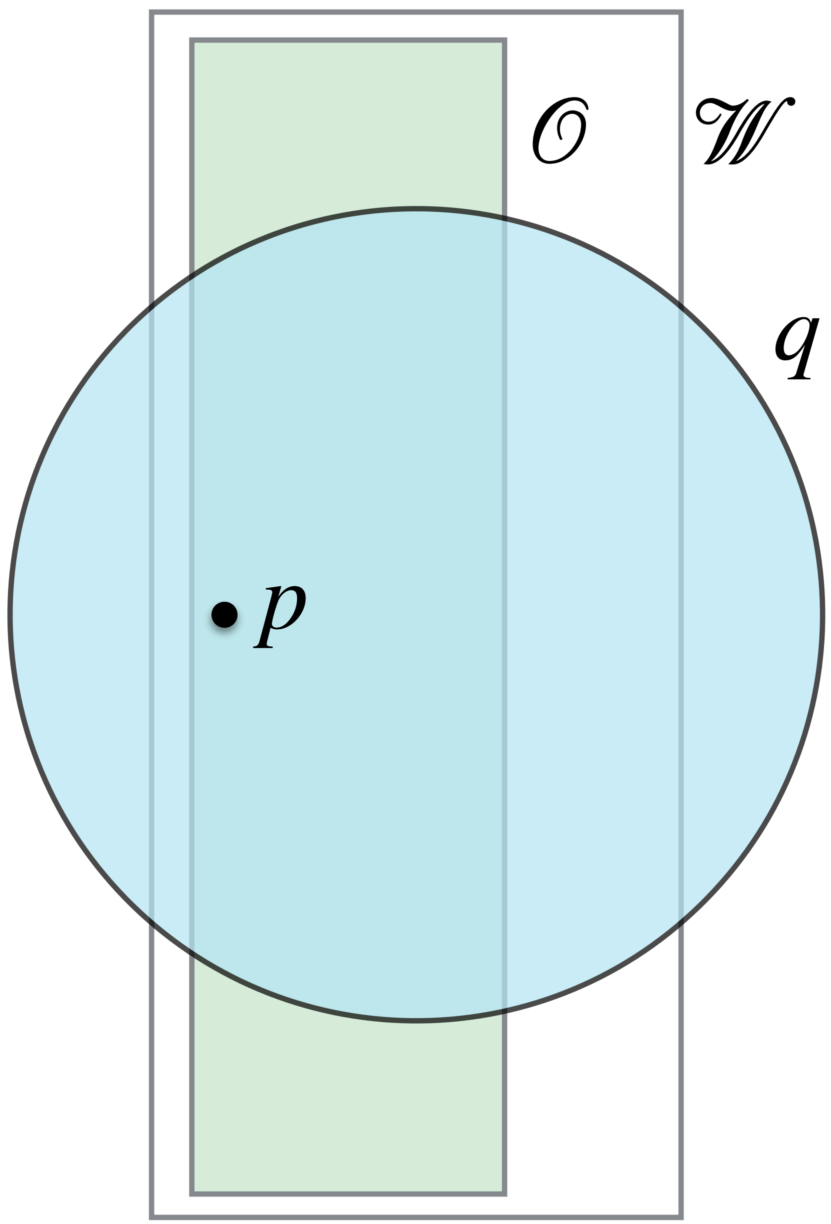

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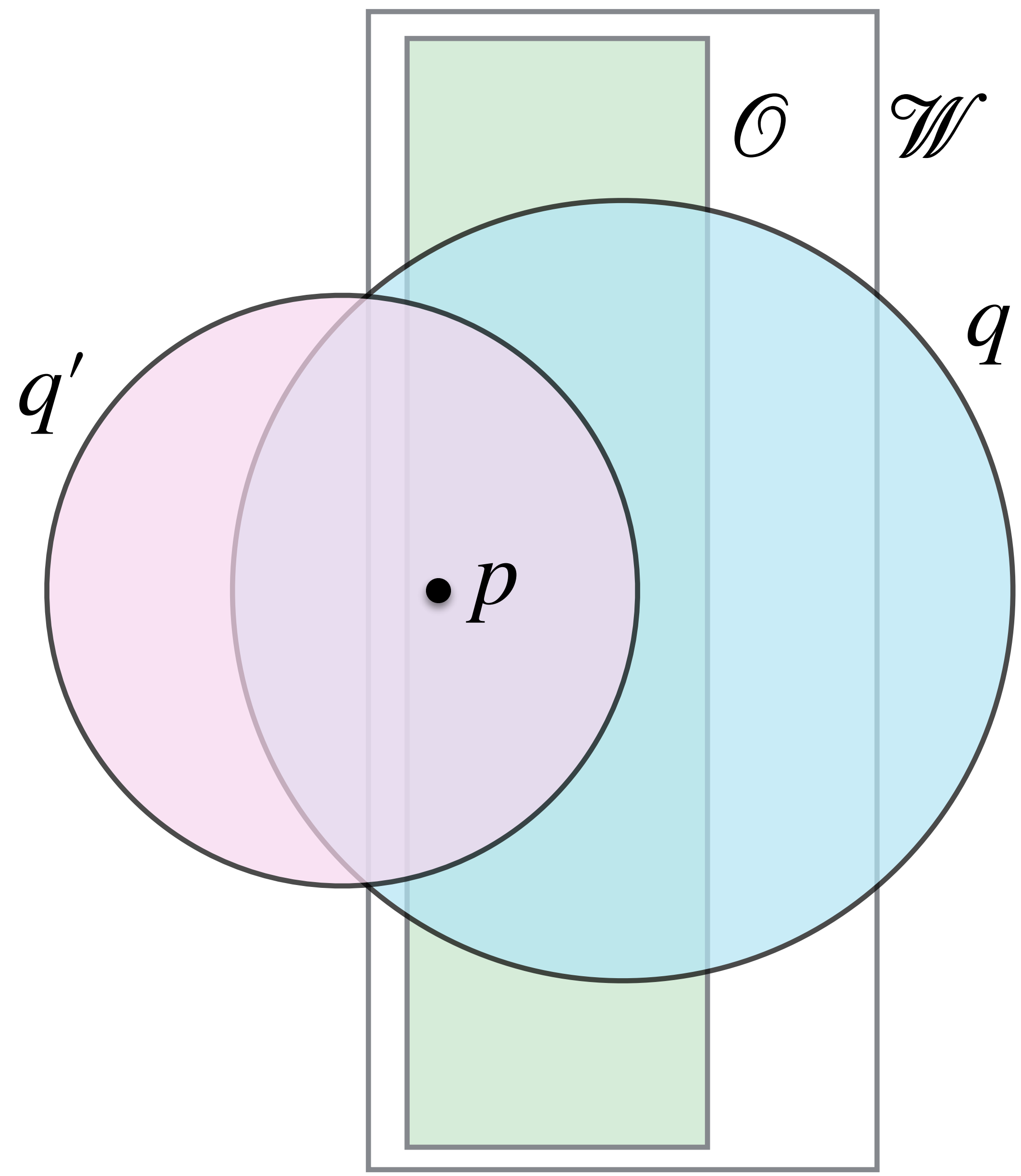

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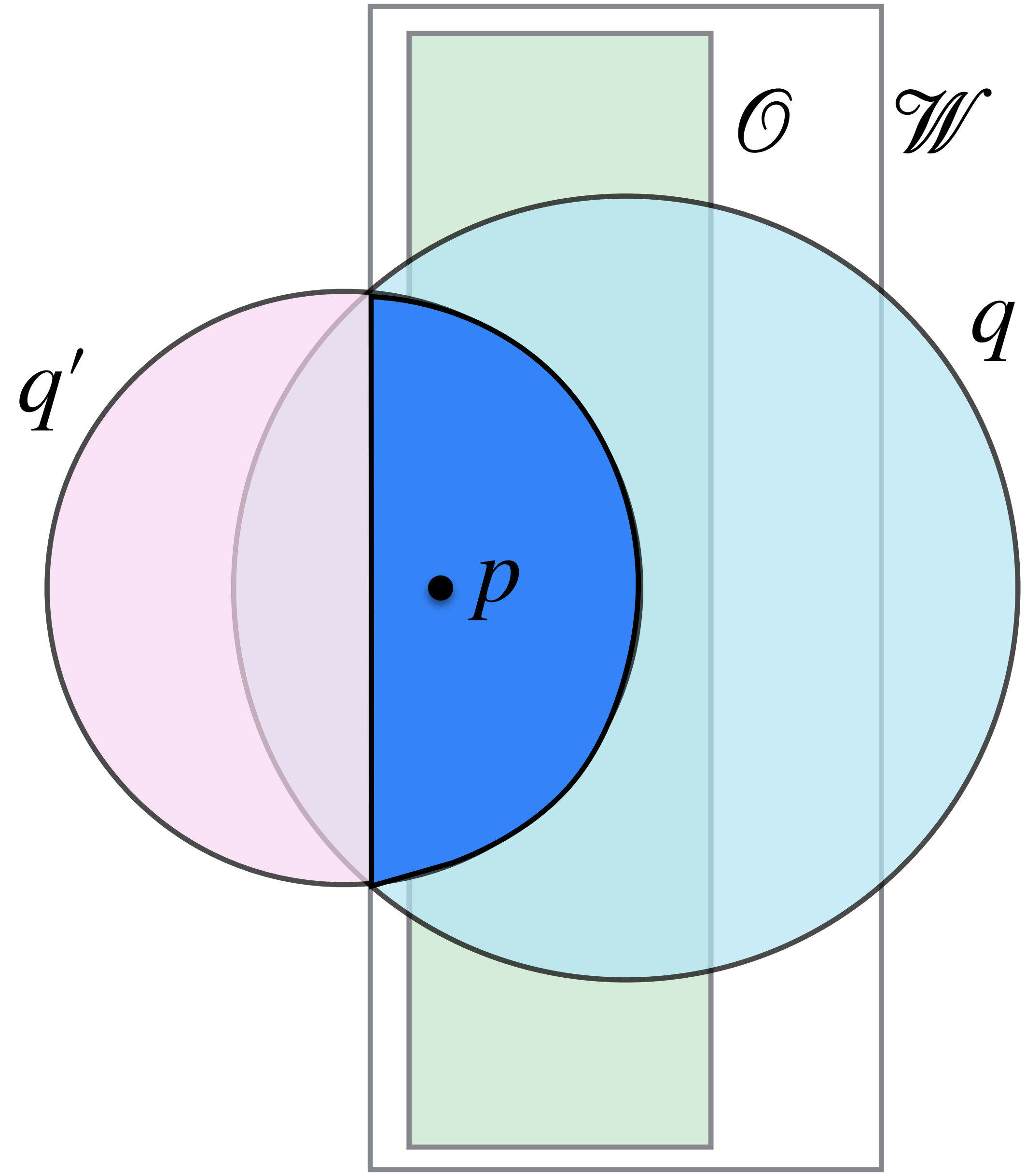
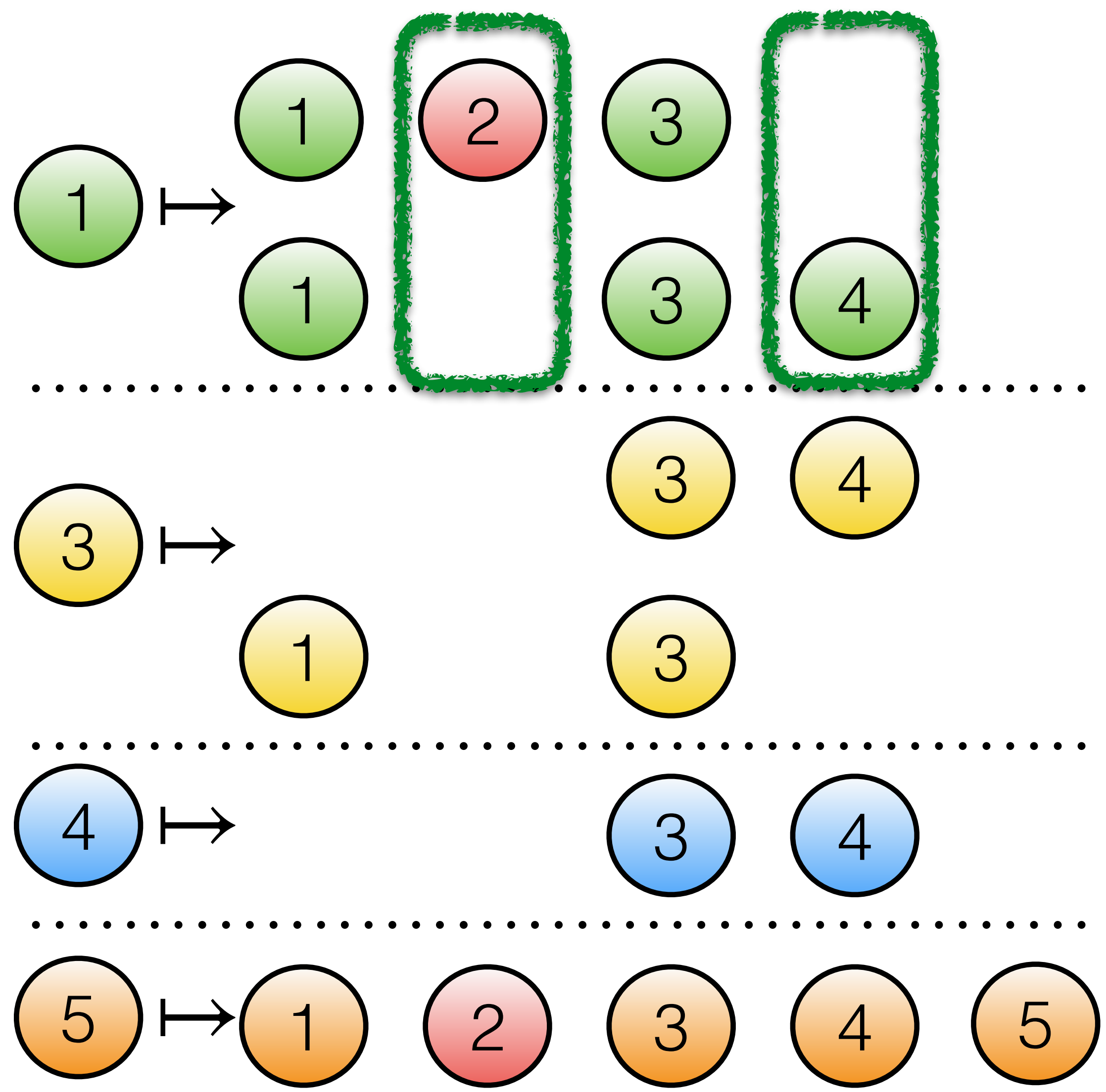


Blocking set

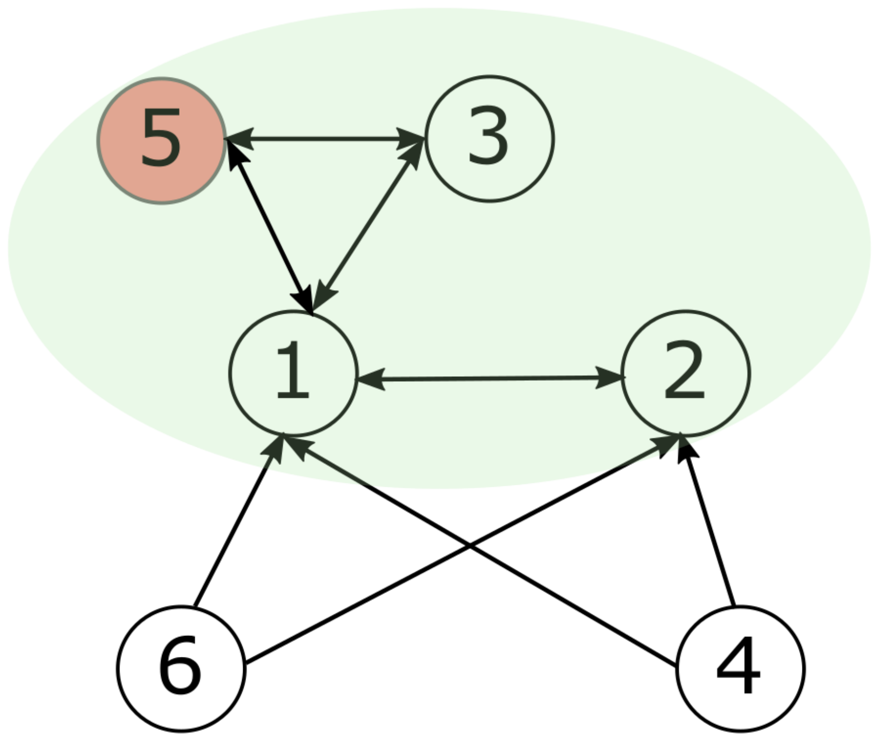


$$\mathscr{B} = \{2, 4\}$$

Quorum Graphs and the Sink Component

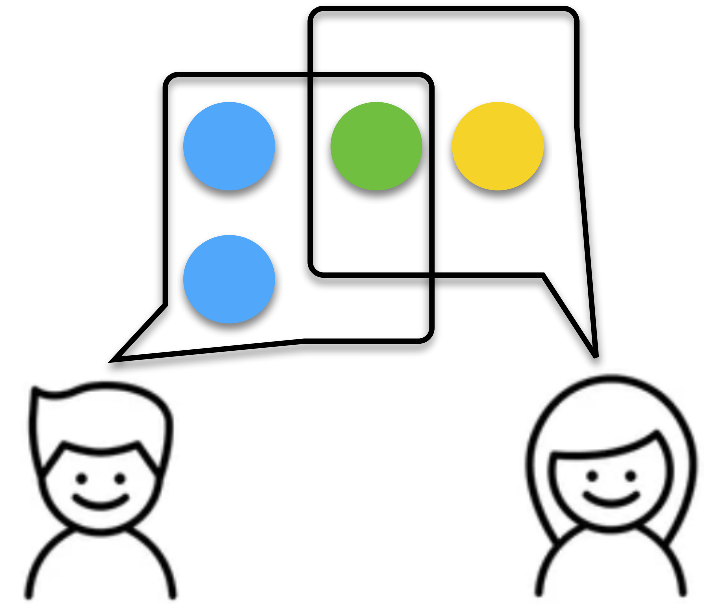
Theorem: In any quorum system with consistency and quorum sharing,
there is one sink component.
all well-behaved processes of the minimal quorums are in the sink component.

$$\mathcal{P} = \{1, 2, 3, 4, 5, 6\}, \mathcal{B} = \{5\},$$
$$\mathcal{Q}(1) = \{\{1, 2\}, \{1, 3, 5\}\},$$
$$\mathcal{Q}(2) = \{\{1, 2\}\},$$
$$\mathcal{Q}(3) = \{\{1, 3, 5\}\},$$
$$\mathcal{Q}(4) = \{\{1, 2, 4\}\},$$
$$\mathcal{Q}(5) = \{\{1, 3, 5\}\},$$
$$\mathcal{Q}(6) = \{\{1, 2, 6\}\}$$
$$MQ(\mathcal{Q}) = \{\{1, 2\}, \{1, 3, 5\}\}$$

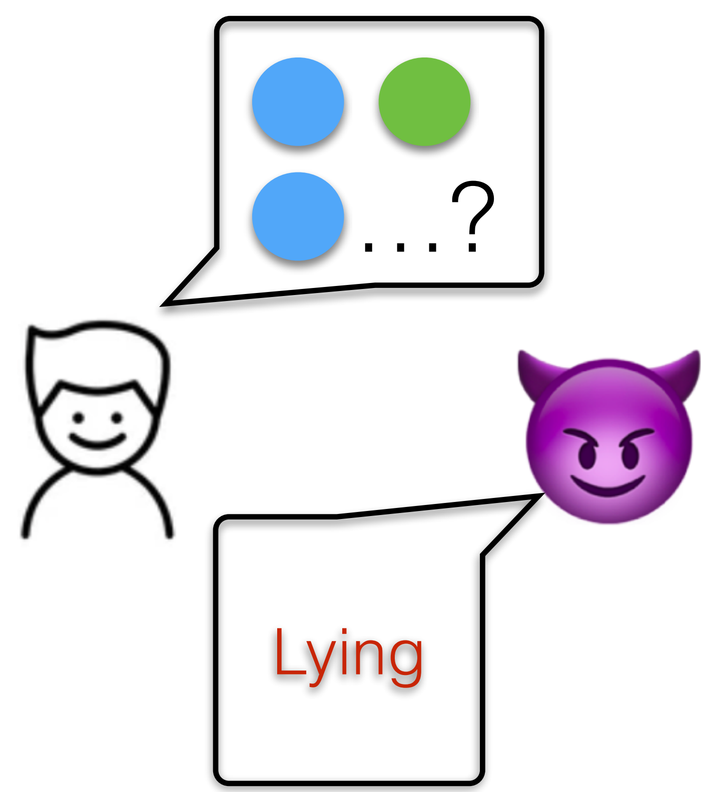


Motivation for Heterogeneous Quorum Systems

- **Uniform Trust**:
  All processes trust the same sets of quorums.
  No personal preference.



- **Public Trust**:
  All processes know the quorums that others trust.
  Not feasible in an open network.
  Byzantine nodes can lie about quorums.



Reconfiguration Trade-offs

Theorem: There is no Leave or Remove reconfiguration protocol that is policy-preserving, availability-preserving and terminating.

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Theorem: There is no Add reconfiguration protocol that is consistency-preserving, policy-preserving, and terminating.