Neurosymbolic AI:
Integrating Learning and Reasoning

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November 12, 2025

UC Santa Cruz Seminar

Outline

1. Introduction
2. Basic Concepts
3. integrating learning and reasoning
4. Neurosymbolic AI approaches and applications
5. Perspectives of the field

Outline

Recommended Background:

Top objective literature review:
A Mathematical Framework and a Suite of Learning Techniques for Neural-Symbolic Systems: C. Dickens, C. Pryor, C. Gao, A. Albalak, E. Augustine, W. Wang, S. Wright, L,. Getoor. Arxiv 2025.

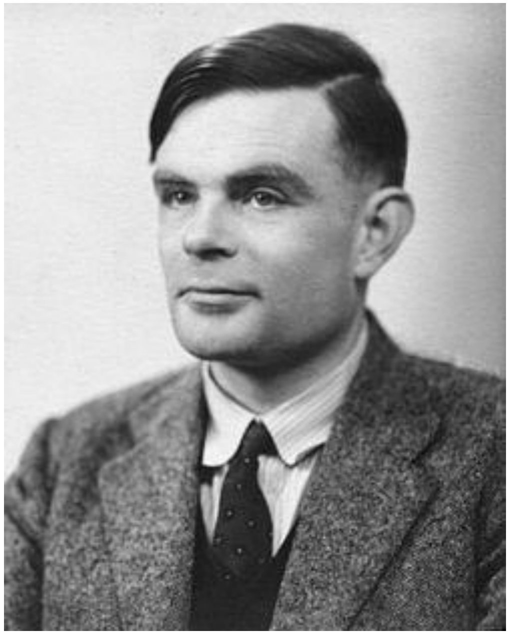
Perspectives of different approaches:
Neurosymbolic AI: the 3rd Wave. A. Garcez and L.C. Lamb. Arti. Int. Rev. (2023).

A BIG QUESTION/CHALLENGE

“Can Digital Machines Think?”

The whole thinking process is still rather mysterious to us, but I believe that the ***attempt to make a thinking machine will help us greatly in finding out how we think ourselves****.*

Alan Turing, 15 May 1951, “Can Digital Machines Think” BBC.



Source: Wikipedia

Remember Descartes:
Cogito, ergo sum.

Early Neurosymbolic AI

McCulloch & Pitts, 1943. A Logical Calculus of the Ideas Immanent in Nervous Activity.

S. C. Kleene, 1951. Representation of Events in Nerve Nets and Finite Automata

Summary: To what kinds of events can a McCulloch—Pitts nerve net respond by firing a certain neuron? More generally, to what kinds of events can any finite automaton respond by assuming one of certain states? This memorandum is devoted to an elementary exposition of the problems and of results obtained on it during investigations in August 1951.

REPRESENTATION OF EVENTS

IN NERVE NETS AND FINITE AUTOMATA

S. C. Kleene

AI + Machine Learning Today

Machine learning: the power and promise of computers that learn by example
Issued: April 2017 DES4702
ISBN: 978-1-78252-259-1

Source: ROYAL SOCIETY report: April 2017

These can benefit from Neurosymbolic ai

A new wave of research

Machine learning is a vibrant field of research, with a range of exciting areas for further development across different methods and applications. There is a collection of specific research questions where progress would directly address potential public concerns around machine learning, or constraints on its wider use:

- **Interpretability:** Can we create powerful machine learning systems where the reasons for particular decisions or recommendations can be understood or interrogated?
- **Verification and validation:** Can we create more advanced, and accurate, methods of verifying machine learning systems?
- **Privacy:** What are the technical solutions that can maintain the privacy of datasets, while allowing them to be used in new ways?
- **Fairness and dealing with real-world data:** How can real-world data be curated into usable forms, addressing 'real-world' messiness, and systemic – or social – bias?
- **Causality:** How can machine learning methods discover cause-effect relationships?
- **Human-machine interaction:** How do we design machine learning systems so they can work with humans safely and effectively?
- **Security:** How do we ensure machine learning systems are not vulnerable to cyber-attack?

Support for research in these areas is needed to help ensure continued public confidence in the deployment of machine learning systems.

Computer Science, the Web and AI changed the world

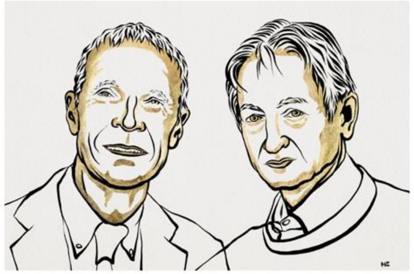
T. Berners-Lee, web, 1989

http://news.bbc.co.uk/2/hi/science/nature/1329623.stm

Kasparov v. Deep Blue, IBM, 1997

https://www.theatlantic.com/technology/archive/2012/05/picture-of-the-day-deep-blue-defeats-kasparov/257055/

Nobel Prize in Physics 2024



John J. Hopfield and Geoffrey Hinton.

Ill. Niklas Elmehed © Nobel Prize Outreach

Quick facts

- Awarded to John J. Hopfield and Geoffrey Hinton
- Prize motivation: “for foundational discoveries and inventions that enable machine learning with artificial neural networks”
- The prize is awarded by the Royal Swedish Academy of Sciences

Read more about the prize

The Nobel Prize in Chemistry 2024

David Baker

“for computational protein design”

David Baker. Ill. Niklas Elmehed © Nobel Prize Outreach

Demis Hassabis

“for protein structure prediction”

Demis Hassabis. Ill. Niklas Elmehed © Nobel Prize Outreach

John Jumper

“for protein structure prediction”

John Jumper. Ill. Niklas Elmehed © Nobel Prize Outreach

AI researchers win two Nobel Prizes, 2024

A scientific relevant area

“We need some language for describing the alternative algorithms that a network of neurons may be implementing”. (CACM 2011)

“The aim is to identify a way of looking at and manipulating commonsense knowledge that […] can support […] the two most fundamental aspects of intelligent cognitive behavior:
the ability to learn from experience and the ability to reason from what has been learned.
We are therefore seeking a semantics of knowledge that can computationally support the basic phenomena of intelligent behavior” (J ACM 2003)

Leslie Valiant, ACM Turing Award Winner

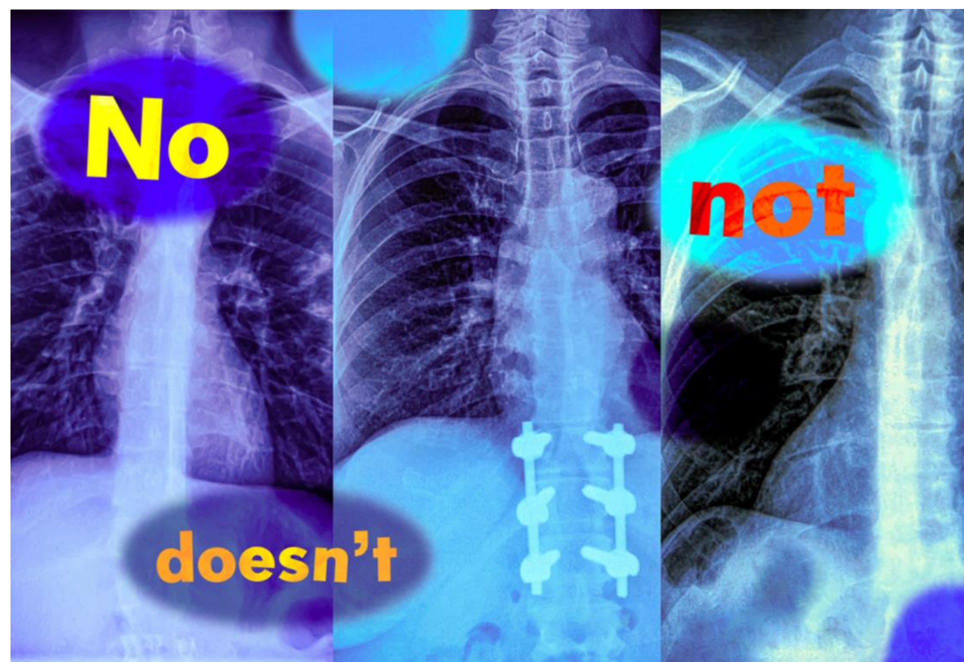
MIT News
ON CAMPUS AND AROUND THE WORLD

Study shows vision-language models can’t handle queries with negation words

Words like “no” and “not” can cause this popular class of AI models to fail unexpectedly in high-stakes settings, such as medical diagnosis.

Adam Zewe | MIT News
May 14, 2025

One reason for this failure is a shortcut the researchers call affirmation bias — VLMs ignore negation words and focus on objects in the images instead.



Transparency and Explainability

Neurosymbolic AI is an interesting research direction ***for creating more transparent and explainable AI models […]***
Unlike less interpretable deep learning models, symbolic reasoning offers ***clearer insights into how models work*** *and allows for direct modifications of the model's knowledge through expert feedback.*

Source: AI Index Report, 2024.

Can we develop ***self-explaining architectures*** *that can help* ***anticipate failures*** *instead of providing justifications post hoc?*
When explanations are wrong, there is no way to correct them. This is a necessity for ***safety-critical systems*** *[..]*

L. Gilpin, Accountability Layers: Explaining Complex System Failures by Parts, AAAI 2023.

2005-2010: deep learning

In the early 2000s top ML/AI Conferences:
AI "Winter": fewer papers made of use of **Artificial Neural Networks**

(Neural-Symbolic Learning: Lamb, Borges & d'Avila Garcez, AAAI 2007; IJCAI-07 Bader, Hitzler, Hölldobler, Witzel)

2006:
before the 'deep learning revolution':

A fast learning algorithm for deep belief nets.
GE Hinton, S Osindero, YW Teh.
Neural computation 18 (7), 1527-1554, 2006.

We began to see remarkable results in AI and innovation across both research and business.

LETTER **Communicated by Yann Le Cun**

A Fast Learning Algorithm for Deep Belief Nets

Geoffrey E. Hinton
hinton@cs.toronto.edu
Simon Osindero
osindero@cs.toronto.edu
Department of Computer Science, University of Toronto, Toronto, Canada M5S 3G4

Yee-Whye Teh
tehyw@comp.nus.edu.sg
Department of Computer Science, National University of Singapore, Singapore 117543

But there are well known challenges…

Large Language Models (LLMs):

- powerful at learning statistical patterns in language, but

- can make **logical errors** or contradict themselves (hallucinations).

- Expensive to train, maintain, update, and deploy.

- Integrating **logic,** as constraints, post-processing, or embedded logical / symbolic reasoning helps to improve **consistency, semantics, and explainability**.

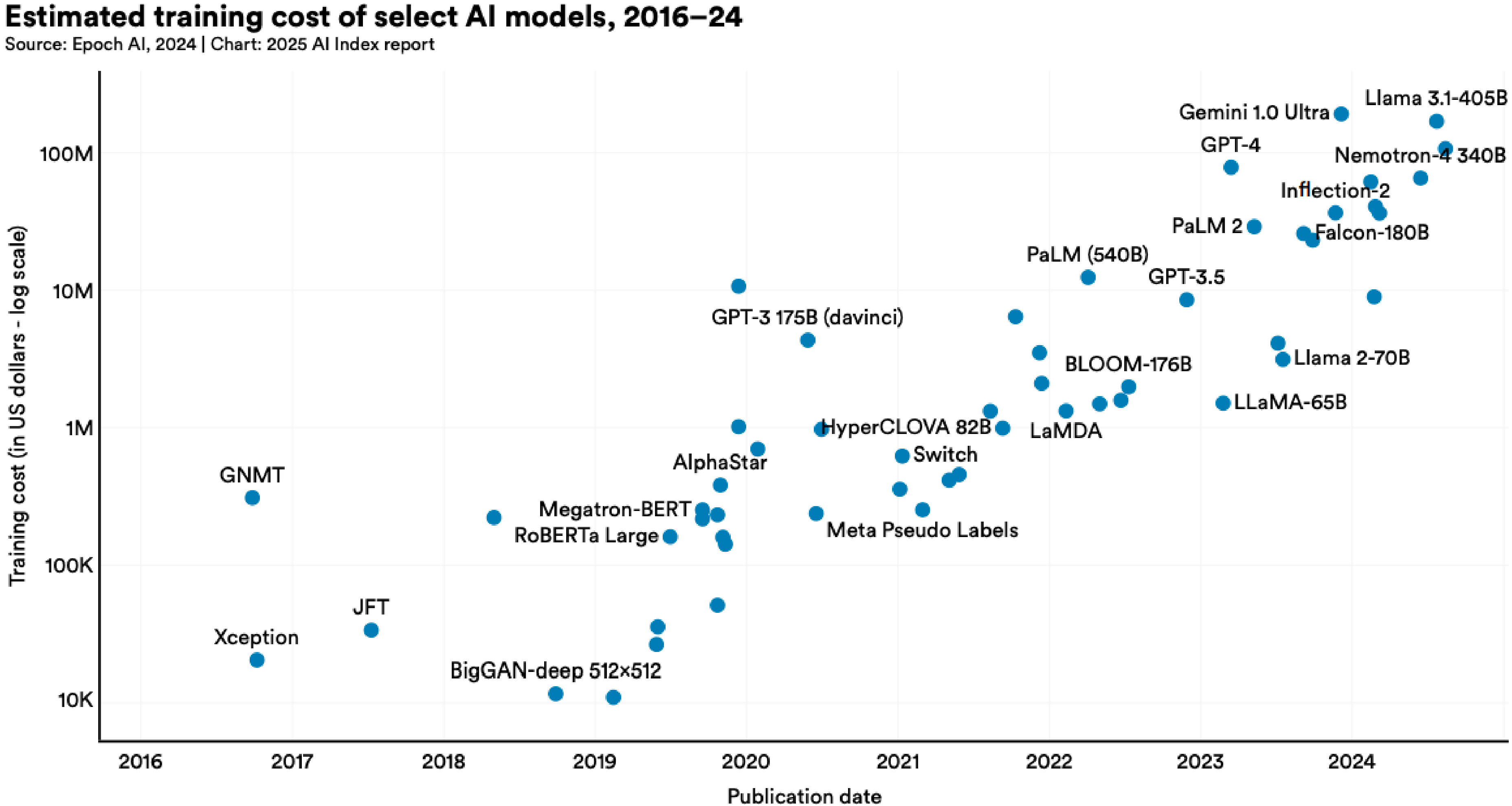


Figure 1.3.25

Figure 1.3.24

AI Index Report, Stanford HAI 2025

https://hai.stanford.edu/assets/files/hai_ai_index_report_2025.pdf

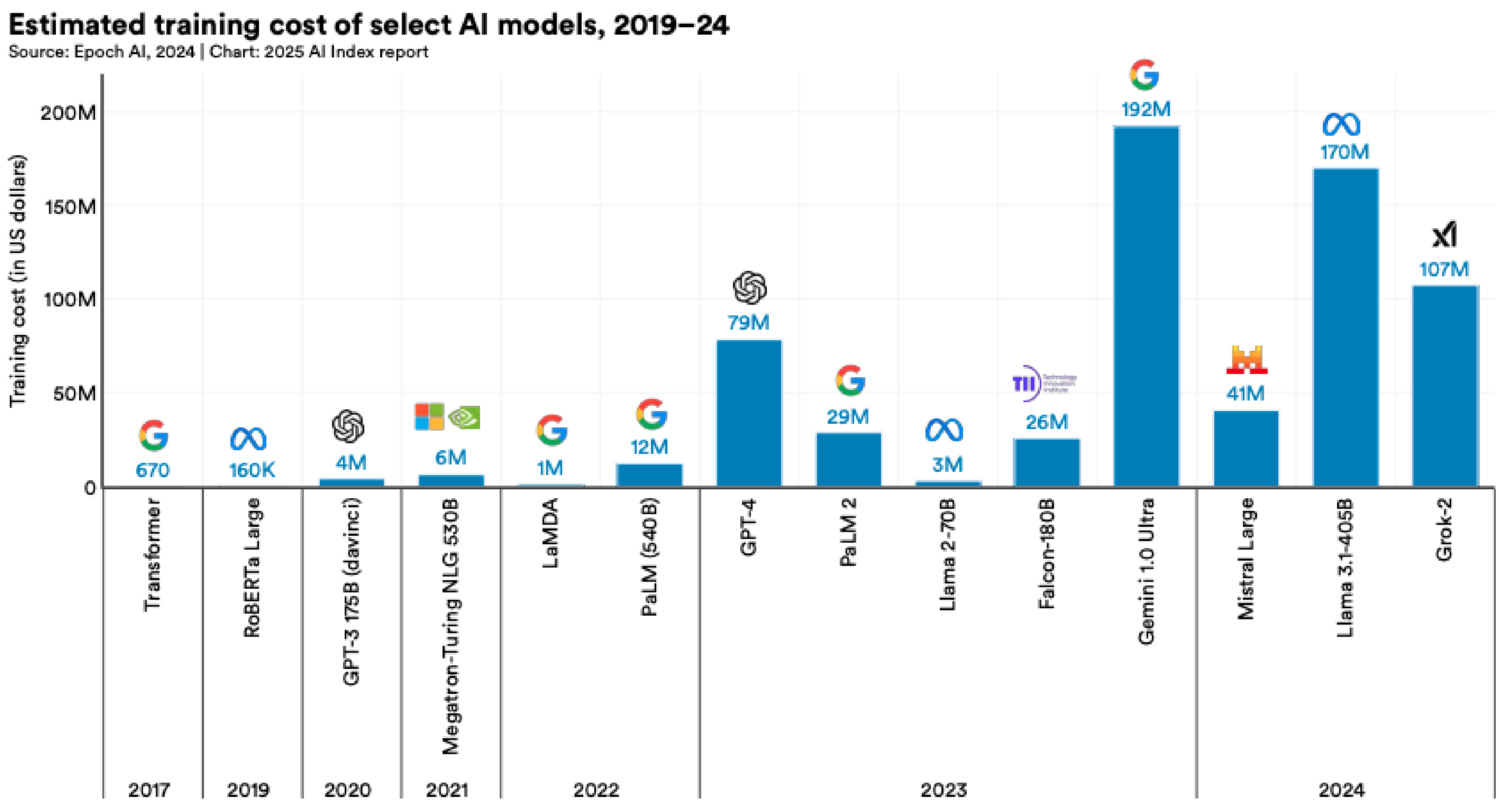
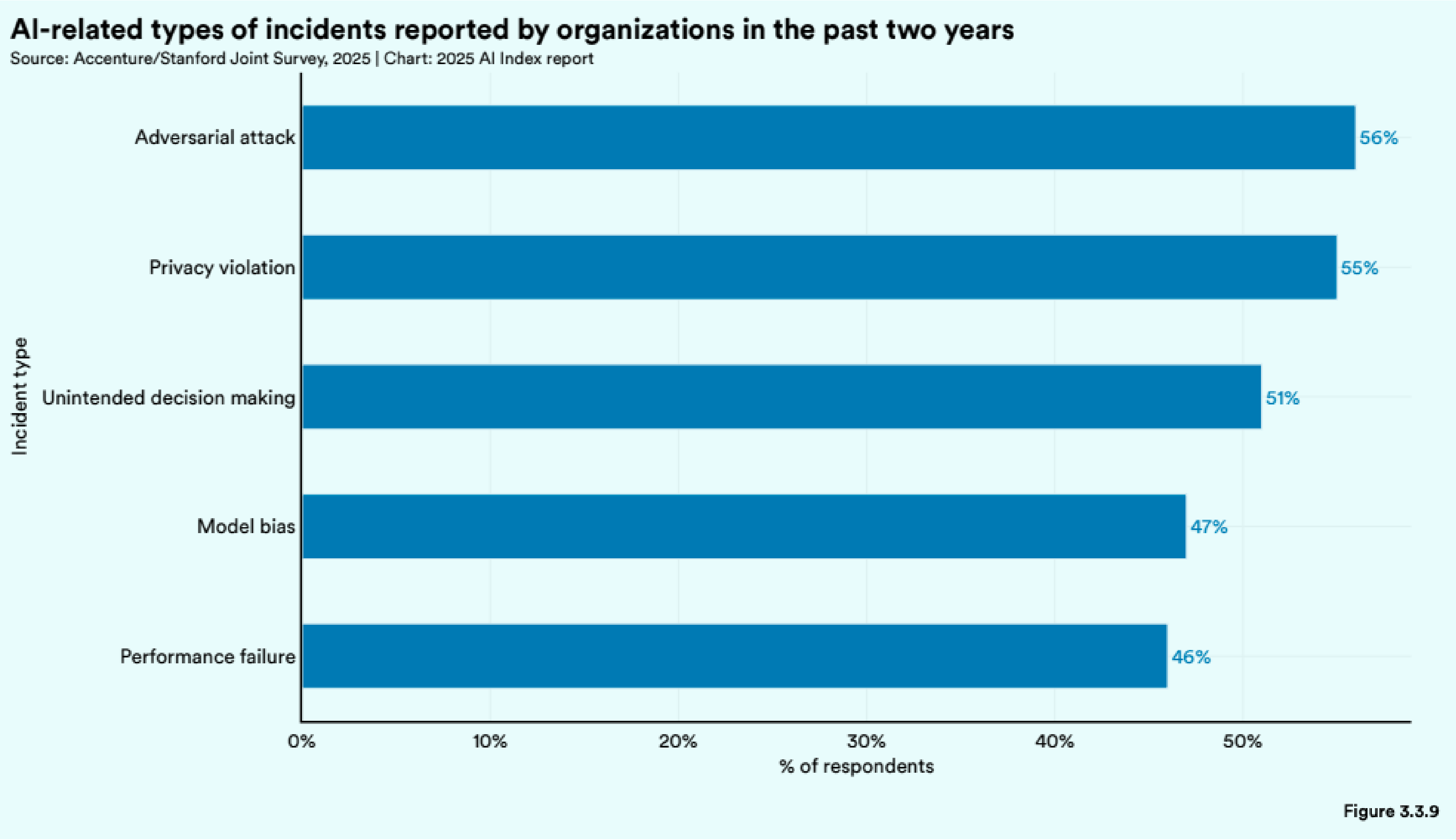


Figure 1.3.24

AI Index Report, Stanford HAI 2025

https://hai.stanford.edu/assets/files/hai_ai_index_report_2025.pdf

Security issues have also been identified:



AI Index Report, Stanford HAI 2025

https://hai.stanford.edu/assets/files/hai_ai_index_report_2025.pdf

"When no one knew ***why*** **things worked**, even radical technological shifts **failed to generate the applications that made them useful in production processes.**

Scientific Background to the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2025.
The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel.

"When no one knew ***why* things worked**, even radical technological shifts **failed to generate the applications that made them useful in production processes.** In **such a world**, economic resources were not allocated towards improving technologies, as potential investors were just as likely to **waste valuable resources** on fruitless inventions that would never work, such as perpetual motion machines or artificially created gold (Mokyr, 2002, p. 31–32)."

Nobel Prize announcement, 2025.

And quoted by Professor Joshua Gans, October 13, 2025.

Scientific Background to the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2025.

The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel.

2. integrating learning and reasoning

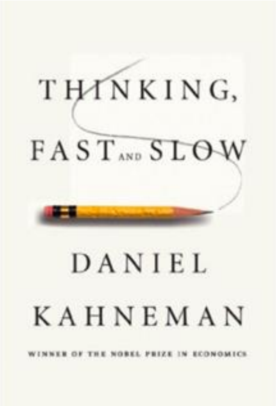
How to build safe, reliable technologies?

Dual Process Theory

Kahneman: system 1 & system 2

system 1 - intuitive, fast parallel system - is capable of understanding language – Deep Learning system 1

System 2, deliberative, reflexive, most probably performing symbol manipulation as argued by Gary Marcus.



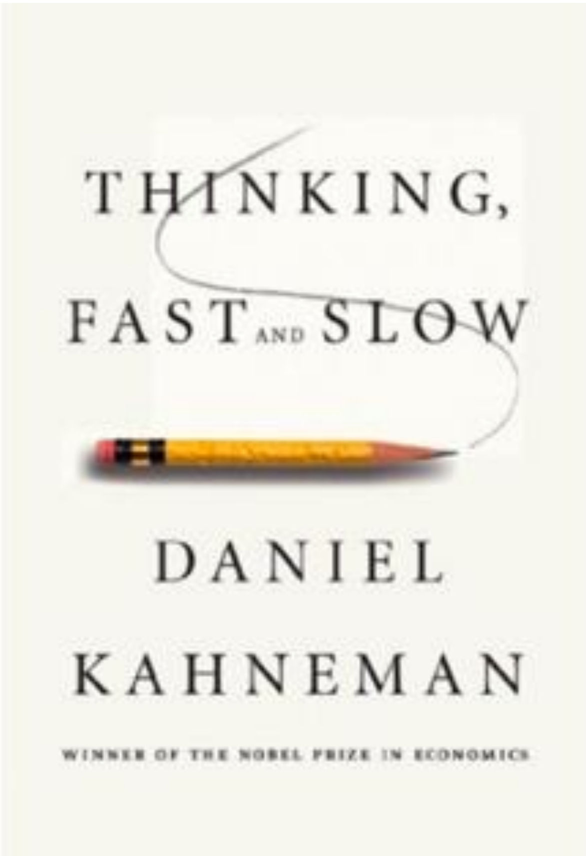
Kahneman:

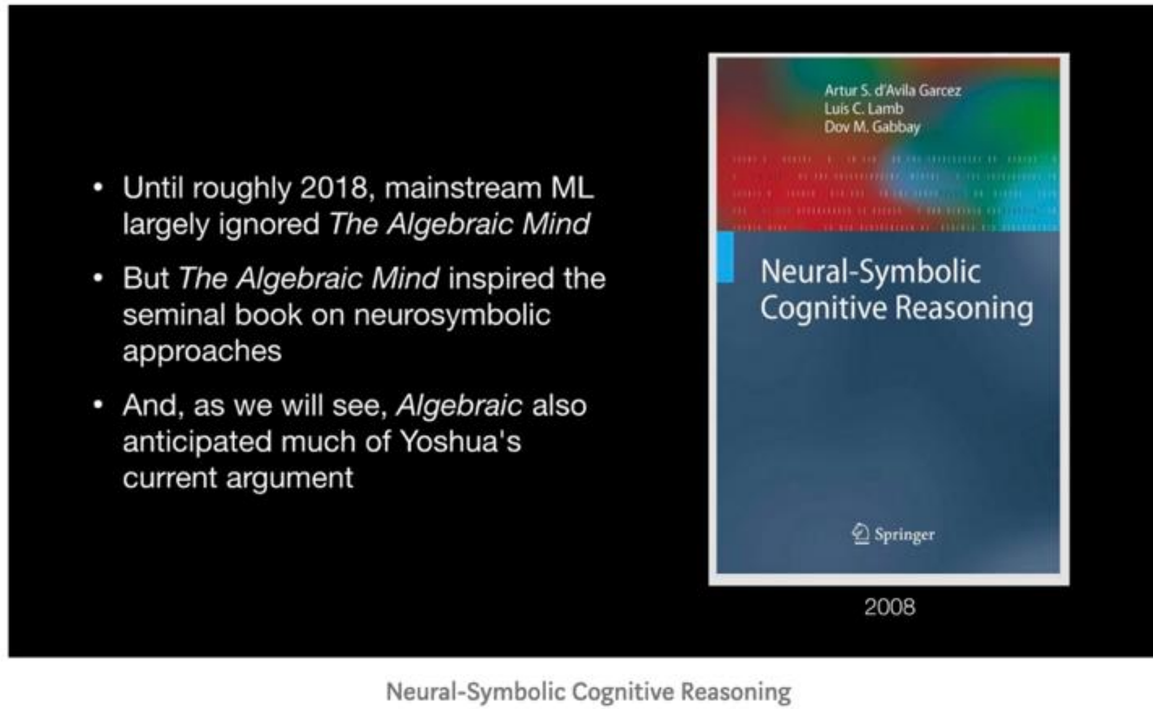
"...so far as I'm concerned, System 1 certainly knows language... System 2... does involve certain manipulation of symbols." AAAI2020

Neurosymbolic AI

- Fireside conversation: Hinton, LeCun, Bengio; Francesca Rossi and Daniel Kahneman on thinking fast and slow in AI and relationship with neurons and symbol manipulation**, 2020.**
- Henry Kautz's 'The Third AI Summer' – R.S. Engelmore Memorial Award Lecture: taxonomy for neurosymbolic computing
- AI DEBATE 2: Moving AI Forward.
- AAAI2021 Panel on Neurosymbolic AI.
- IBM Seminars 2022,2023.
- 20 Years of the NeSy conference
- AlphaGo, AWS Guardrails.

The First great ai debate: integrating learning & reasoning





Gary Marcus in the Great AI Debate, Montreal
23 december 2019.

Henry Kautz

Neurosymbolic Taxonomy

In "The Third AI Summer"

AAAI Robert S. Engelmore Memorial Lecture

Thirty-fourth AAAI Conference on Artificial Intelligence, New York, Feb. 10, 2020

Symbolists and Connectionists

http://diva.library.cmu.edu/Newell/biography.html

Alan Newell and Herbert Simon: “symbolists”?



Y. Bengio, G. Hinton, Y. LeCun

Photos:
http://amturing.acm.org/

- **2010**: Leslie **Valiant:** PAC **learning**; computational complexity; parallel/distributed computing;

- **2011:** Judea **Pearl:** Bayesian/**causal reasoning and learning in AI.**

Neurosymbolic AI

Neurosymbolic AI:
Learning from experience and reasoning about what has been learned from an uncertain environment in a computationally efficient way.

They use abduction, deduction and induction.

Motivation: there is a need for systems that:

-Learn from changes in the environment (think of temporal logics).

-Reason about commonsense knowledge (space/time/uncertainty…).

-Integrate reasoning (computation) and learning: we combine the logical nature of reasoning and the statistical nature of learning, see Valiant.

-But… Reasoning can be hard.

Why nonclassical logics?

Nonclassical logics have been shown adequate in expressing several reasoning features, allowing for the representation of temporal, epistemic and probabilistic abstractions in computer science and AI.
(Fagin et al. 1995; Halpern 2005)

Examples of nonclassical logics:
temporal logics; fuzzy logics; probabilistic logics; modal logics; description logics; intuitionistic logics.

Classic (and most valuable) Literature:

Ronald Fagin, Joseph Y. Halpern, Yoram Moses, Moshe Y. Vardi:
Reasoning About Knowledge. MIT Press 1995.

Joseph Y. Halpern: **Reasoning about uncertainty.** MIT Press 2005

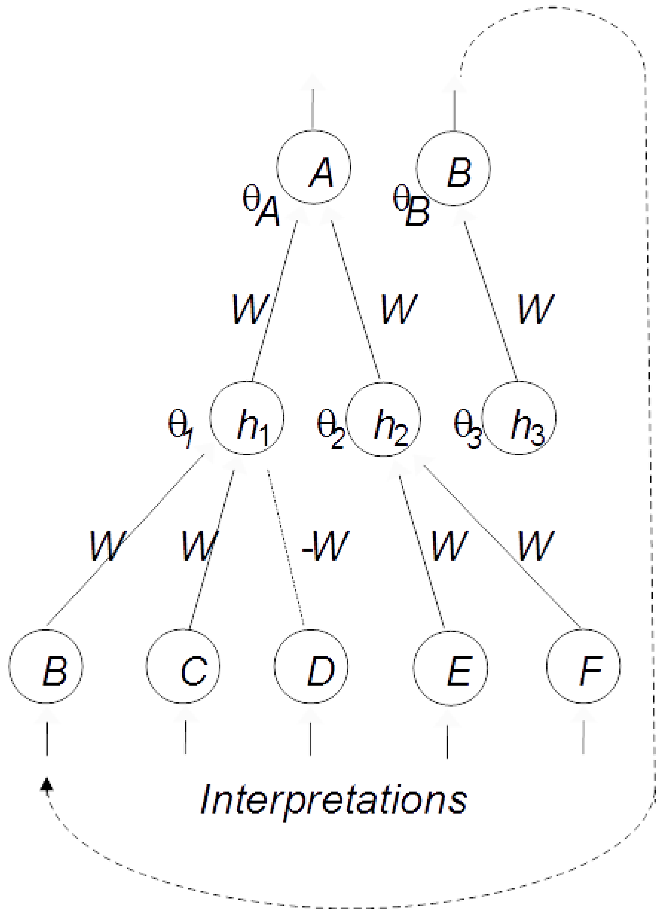
Representing Rules

Rules and their corresponding neural model

r_1: $A \leftarrow B, C, \sim D;$

r_2 : $A \leftarrow E, F;$

r_3 : $B \leftarrow$



G. Towell and J. Shavlik. Knowledge-based artificial neural networks. Artif. Intel.,70:119–165, 1994.

THEOREM: For any logic program P there exists a neural network N such that N computes P. *Garcez, Zaverucha, Appl. Intell. J, 1999.*

Learning Computational Logics

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(Fagin et al. 1995; Halpern 2005)

Learning Computational Logics

temporal, epistemic, and probabilistic logic/abstractions
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Joseph Y. Halpern: **Reasoning about uncertainty.** MIT Press 2005

Conectionist Modal Logics

Modal logic goes beyond propositional reasoning: Vardi, 1997.

A proposition is necessary (*box*) in a possible world (state of affairs) if it is true in all worlds which are possible in **relation** to that world.

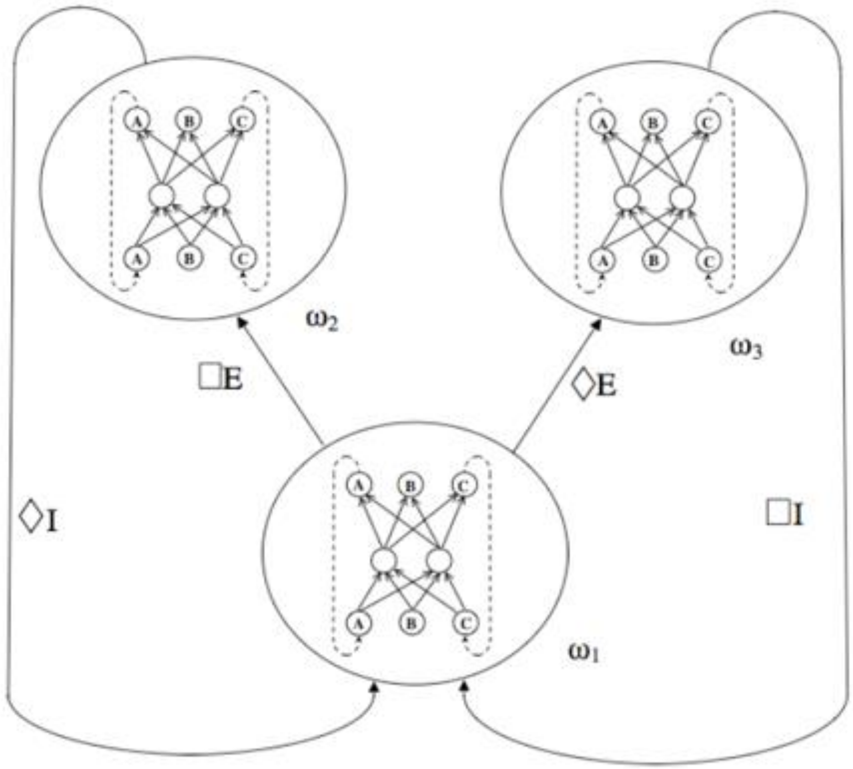
A proposition is possible (*diamond*) in a possible world (state of affairs) if it is true in at least one world which is possible in relation to that same world (reference state).

Modalities also used for reasoning about uncertainty (following J. Halpern).

Relational learning/reasoning is notoriously hard.

Learning to reason in neural networks

- Insight: *assume that neurons are possible worlds.*
- Propositional Modal Logic = decidable fragment of FOL with two variables.
- Full solution of Muddy Children puzzle and other testbeds.
- Ensembles of NNs that are seen as possible world.



Garcez, Lamb, Gabbay. Connectionist Modal Logic. *Theoretical Computer Science*, 371: 34-53, 2007.

Garcez, Lamb. Connectionist Model for Epistemic and Temporal Reasoning. *Neural Computation,* 18:1711-1738, July 2006

Connectionist Modal Logics: Reasoning Rules

Table 1
Rules for modality operators

$$\dfrac{\begin{array}{c}[R(\omega, g_\varphi(\omega))] \\ \vdots \\ g_\varphi(\omega) : \varphi\end{array}}{\omega : \Box\varphi}\ \Box I$$	$$\dfrac{\omega_1 : \Box\varphi,\ R(\omega_1, \omega_2)}{\omega_2 : \varphi}\ \Box E$$
$$\dfrac{\omega : \Diamond\varphi}{f_\varphi(\omega) : \varphi,\ R(\omega, f_\varphi(\omega))}\ \Diamond E$$	$$\dfrac{\omega_2 : \varphi,\ R(\omega_1, \omega_2)}{\omega_1 : \Diamond\varphi}\ \Diamond I$$

Let $\mathcal{P} = \{\omega_1 : r \rightarrow \Box q;\ \omega_1 : \Diamond s \rightarrow r;\ \omega_2 : s;\ \omega_3 : q \rightarrow \Diamond p;\ \mathcal{R}(\omega_1,\omega_2),\ \mathcal{R}(\omega_1,\omega_3)\}$.

Connectionist Modal Logics: Rules and Reasoning

Let $\mathcal{P} = \{\omega_1 : r \rightarrow \Box q;\ \omega_1 : \Diamond s \rightarrow r;\ \omega_2 : s;\ \omega_3 : q \rightarrow \Diamond p;\ \mathcal{R}(\omega_1,\omega_2),\ \mathcal{R}(\omega_1,\omega_3)\}$.

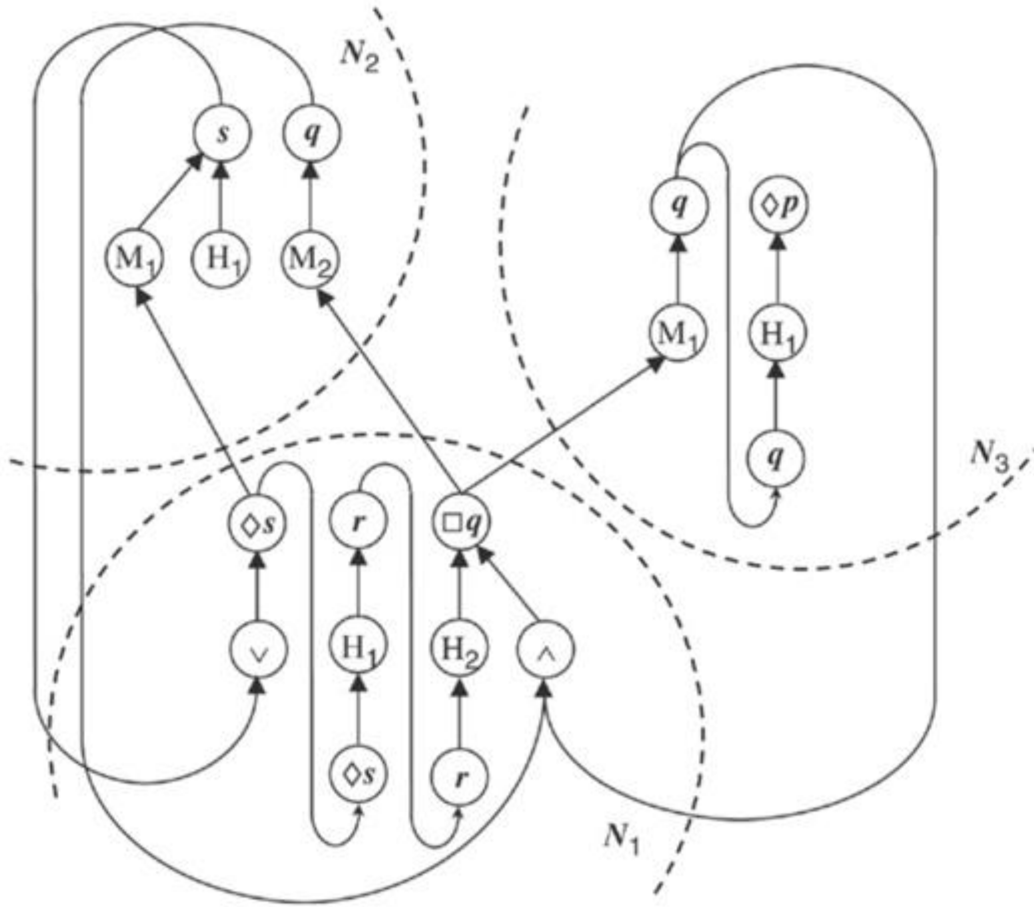


Fig. 5. The ensemble of networks $\{\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3\}$ that represents $\mathcal{P}$.

Connectionist Modal Logics – translation or embedding

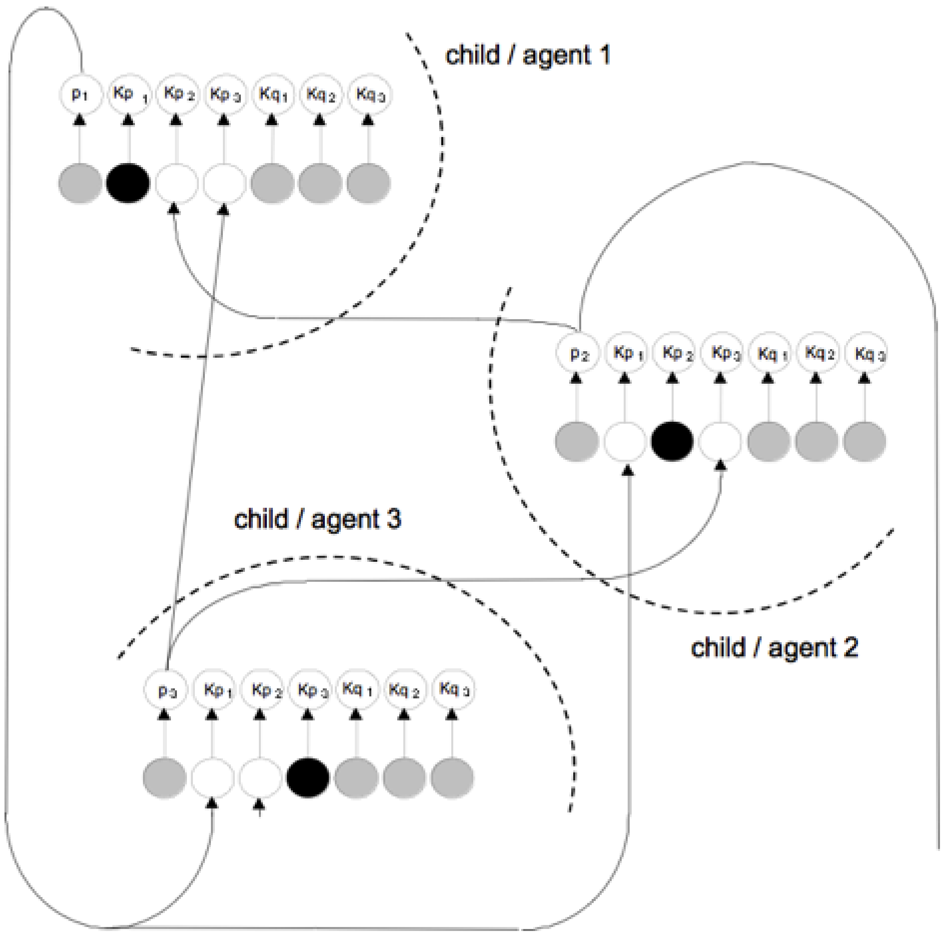
Algorithm 2: Translation of ●-based programs

```
●-based_Translation(P)
    Define (max_P(k, μ) − 1) / (max_P(k, μ) + 1) ≤ A_min < 1;
    Define W ≥ (ln(1 + A_min) − ln(1 − A_min)) / (max_P(k, μ)(A_min − 1) + A_min + 1) · 2/β;
    foreach C_l ∈ Clauses(P) do
        AddHiddenNeuron(N, h_l);
        foreach α ∈ body(C_l) do
            if in_α ∉ Neurons(N) then
                AddInputNeuron(N, in_α);
                ActivationFunction(in_α) ← g(x);
            AddLink(N, in_α, h_l, W);
        end
        foreach ~ α ∈ body(C_l) do
            if in_α ∉ Neurons(N) then
                AddInputNeuron(N, in_α);
                ActivationFunction(in_α) ← g(x);
            AddLink(N, in_α, h_l, −W);
        end
        α ← head(C_l);
        if out_α ∉ Neurons(N) then
            AddOutputNeuron(N, out_α);
        AddLink(N, h_l, out_α, W);
        Threshold(h_l) ← ((1 + A_min)(k_l − 1) / 2) W;
        Threshold(out_α) ← ((1 + A_min)(1 − μ_l) / 2) W;
        ActivationFunction(h_l) ← h(x);
        ActivationFunction(out_α) ← h(x);
    end
    foreach α ∈ atoms(P) do
        if (in_α ∈ neurons(N)) ∧ (out_α ∈ neurons(N)) then
            AddLink(N, out_α, in_α, 1)

    end
    foreach in_α ∈ neurons(N) do
        if (α = ●^n β) then
            if ∃i < n(out_{●^i β} ∈ neurons(N)) then
                j ← maximum(i);
                AddDelayedLink(N, n − j, out_{●^j β}, in_α);
            else
                AddInputDelay(N, n, in_α)


    end
    return N;
end
```

Lamb, Borges, Garcez, AAAI 2007

Connectionist Modal & Temporal Logics

Neural network ensembles correspond to possible worlds/states; modularity for learning; accessibility relations, disjunctive information.



THEOREM 2: For any modal/temporal logic program *P* there exists an ensemble of neural networks *N* such that *N* computes *P*. *Garcez, Lamb, 2006, Neural Computation, vol 18(7).*

Connectionist Modal/Temporal Logics

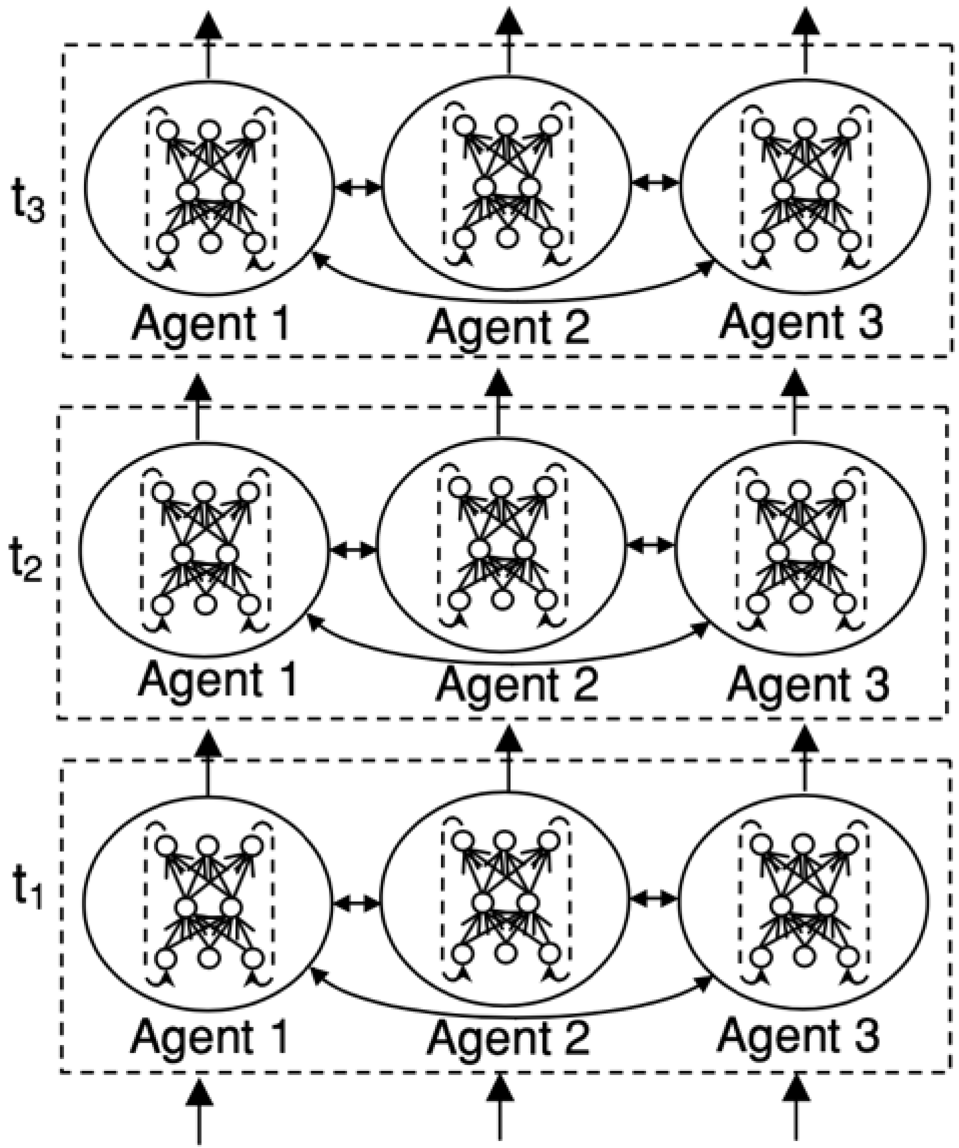


Figure 4: Evolving knowledge through time.

Adding a temporal dimension is crucial in numerous applications

3. Neurosymbolic AI Approaches and Applications

Software engineering, safe systems

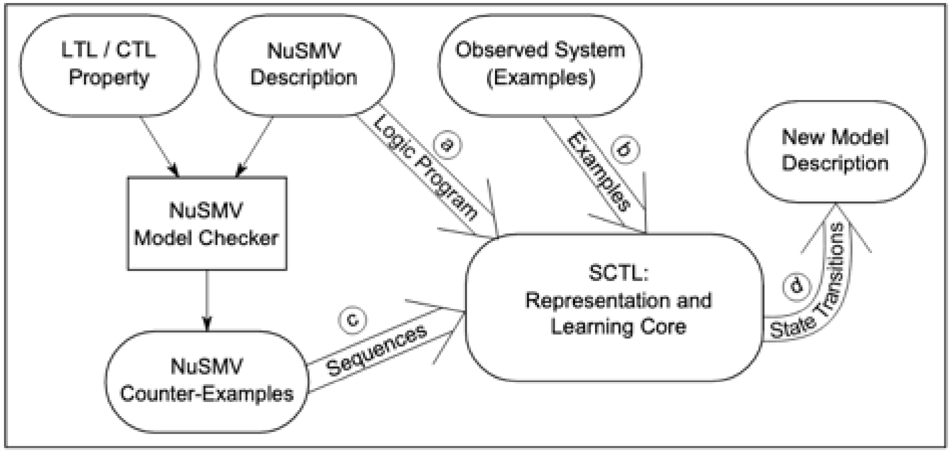


Figure 1: General diagram of the proposed framework

An initial description of a model can be expressed in NuSMV or as a temporal logic program.

This model can then be subject to verification by NuSMV.
If the model does not satisfy the property, a set of counter-examples is returned.

Counter-examples are used as input of the adaptation engine in order to obtain a new, improved model.

This new model can be subject to the same process until the properties are satisfied.

Source: R. Borges, A. Garcez, LC Lamb, IEEE Transactions on Neur. Net. 2011, ICSE 2011, ASE2011 and more.

See also:
Labor Division with Movable Walls: Composing Executable Specifications with Machine Learning and Search.
Harel et al. AAAI 2019

APPLICATIONS IN LARGE-SCALE SYSTEMS

THE WALL STREET JOURNAL.

World Business U.S. Politics Economy Tech Markets & Finance Opinion Arts Lifestyle

STEVEN ROSENBUSH

Meet Neurosymbolic AI, Amazon's Method for Enhancing Neural Networks

A hybrid approach to AI is powering Amazon's Rufus shopping assistant and cutting-edge warehouse robots

By Steven Rosenbush Follow

Aug. 12, 2025 11:00 am ET

"There's this notion of neurosymbolic AI, that's the sort of moniker under which you might call automated reasoning," […]

"The rise of interest in neurosymbolic AI caused people, while they were using the tool, to realize how important this work was."

Byron Cook, Distinguished Scientist and Vice President at AWS
VentureBeat, Aug. 2025

AWS News Blog

Minimize AI hallucinations and deliver up to 99% verification accuracy with Automated Reasoning checks: Now available

by Danilo Poccia | on 06 AUG 2025 | in Amazon Bedrock, Amazon Bedrock Guardrails, Announcements, Artificial Intelligence, Generative AI, Launch, News, Responsible AI | Permalink | Comments | Share

NSAI helps mitigate hallucinations using sound techniques to validate / correct, and logically explain the information generated by the AI models.

NSAI guardrails use automated reasoning (Lean) checks to validate AI outputs, ensuring factual accuracy and detecting hallucinations in language models.

The system enables customizable validation, policy feedback, and scalable rule-based governance over AI behavior (e.g., via Amazon Bedrock Guardrails)

Identifies correct model responses with up to 99% accuracy “the first and only generative AI safeguard to do so.”

https://aws.amazon.com/bedrock/guardrails/?trk=e61dee65-4ce8-4738-84db-75305c9cd4fe&sc_channel=el

"At our inference time or after inference, you can ***move the statements coming out of a language model****,* ***put them into logic and then prove or disprove the correctness*** *of them. Automated reasoning is the symbolic manipulation of, like you move symbols around and you deduce things that are true about the semantics that those formally represent. You can do all kinds of things* ***in combination with machine learning****."*

Byron Cook, AWS Vice President.

"ChatGPT and other generative AI tools […] they can deliver even extraordinarily wrong answers with irrational confidence."

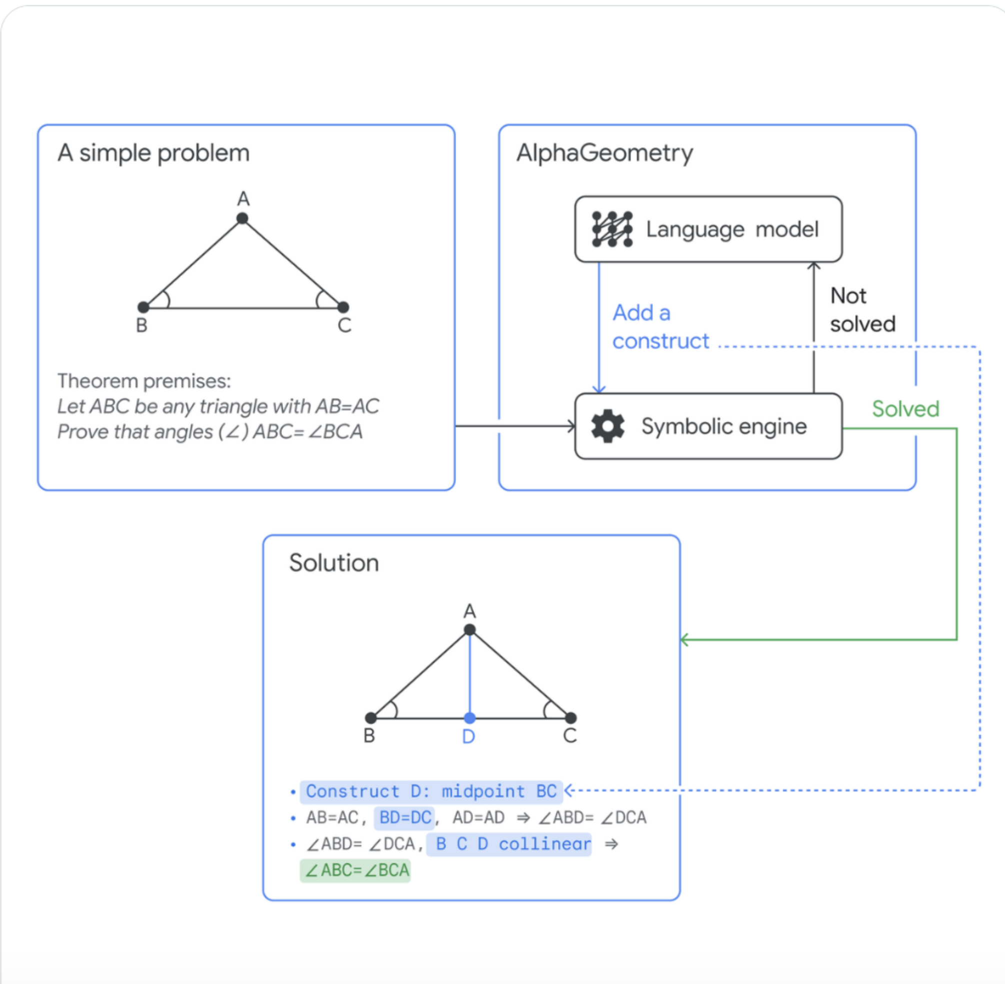
WSJ, 2025.

Google DeepMind
@GoogleDeepMind

AlphaGeometry is a system made up of 2 parts:

- A neural language model, which can predict useful geometry constructions to solve problems
- A symbolic deduction engine, which uses logical rules to deduce conclusions

Both work together to find proofs for complex geometry theorems.



AlphaGeometry is a **neuro-symbolic** system made up of a neural language model and a symbolic deduction engine, which work together to find proofs for complex geometry theorems.

Akin to the idea of "thinking, fast and slow", one system provides fast, "intuitive" ideas, and the other, more deliberate, rational decision-making.

"AlphaGeometry's output is impressive because it's both verifiable and clean."

Solving olympiad geometry without human demonstrations

Trieu H. Trinh, Yuhuai Wu, Quoc V. Le, He He & Thang Luong

Nature **625**, 476–482 (2024) | Cite this article

“AlphaGeometry’s ability to synthesise millions of theorems and proofs, using a neural language model trained on large-scale synthetic data to guide a symbolic deduction engine, makes it a groundbreaking **example of how Neuro-Symbolic AI can achieve advanced problem-solving capabilities**, bridging gaps across multiple domains of AI research.” (google/DeepMind)

The symbolic deduction engine

The core functionality of the engine is deducing new true statements given the theorem premises. Deduction can be performed by means of geometric rules such as ‘If X then Y’, in which X and Y are sets of geometric statements such as ‘A, B, C are collinear’. We use the

How do neuro(al) and symbolic systems interact?

How do neuro(al) and symbolic systems interact?
A Taxonomy for Neurosymbolic Systems

How do neuro(al) and symbolic systems interact?

Type 1.symbolic Neuro symbolic

Type 2.Symbolic[Neuro]

Type 3.Neuro;Symbolic

Type 4.Neuro:Symbolic → Neuro

Type 5.$\text{Neuro}_{\text{Symbolic}}$

Type 6.Neuro[Symbolic]

Kautz taxonomy, AAAI 2020.

How do neuro(al) and symbolic systems interact?

1. symbolic Neuro symbolic

Standard deep learning; a stretch, but included to note that the input and output of an ANN can be made of symbols. e.g. language translation, QA.

2. Symbolic[Neuro]

Hybrids, like AlphaGo systems where the core neural network is coupled with a symbolic problem solver such as Monte Carlo tree search. E.g. AlphaGo; AlphaZero

Kautz taxonomy, AAAI 2020.

How do neuro(al) and symbolic systems interact?

Type 3. Neuro;Symbolic

hybrid system where a NN focus one task (e.g. object detection) & interacts via input/output with a symbolic system working in a complementary task (e.g.Q&A).

E.g. neuro-symbolic concept learner (MIT)

Type 4. Neuro:Symbolic → Neuro

Here, symbolic knowledge is compiled into the training set of a neural network.
Rule A → B becomes an input-output training pair (A, B)
e.g. Deep Learning for Symbolic Mathematics (Lample & Charton 2020)

Kautz taxonomy, AAAI 2020.

How do neuro(al) and symbolic systems interact?

Type 5. Neuro$_{\text{Symbolic}}$

neural-symbolic systems where a symbolic logic rule is mapped onto a distributed representation (an embedding) and acts as a soft-constraint (a regularizer) on the network's loss function.

Symbolic rules are used as *templates* for structures within the neural network

Logic Tensor Networks - Badredine et. al., Artificial Intelligence Journal 2022

Kautz taxonomy, AAAI 2020.

How do neuro(al) and symbolic systems interact?

Type 6. Neuro[Symbolic]

True symbolic reasoning inside a neural engine.
Fully-integrated system.

Our work in achieved this (CML, etc), but in a localist rather than a distributed architecture.

Kautz adds that such a system should be capable of combinatorial reasoning.

“Enable super-human and super-neuro combinatorial reasoning.”
Towards thinking, fast and slow?

Building Neurosymbolic Tools For Research

Neurosymbolic Tools Research:

Logic Tensor Networks:

- Combines many-valued First Order logic as a representation language for deep learning.
- Maps logical formulas to real-valued tensors, supporting fuzzy truth values.
- Formulas are grounded in real-valued tensors, enabling each logical statement to have a truth value in [0,1].
- Tensorization of logical objects, predicates, functions, formulas are represented as tensors.
- Learns logical grounding using tensor computations.
- Logic and learning operate jointly: logic statements are constraints/queries on a tensorized representation and uses differentiable optimization in frameworks like TensorFlow.
- supports fuzzy reasoning and smooth integration with neural network architectures.
- Reasoning is “soft” and resource-intensive as the number of encoded formulas grows.

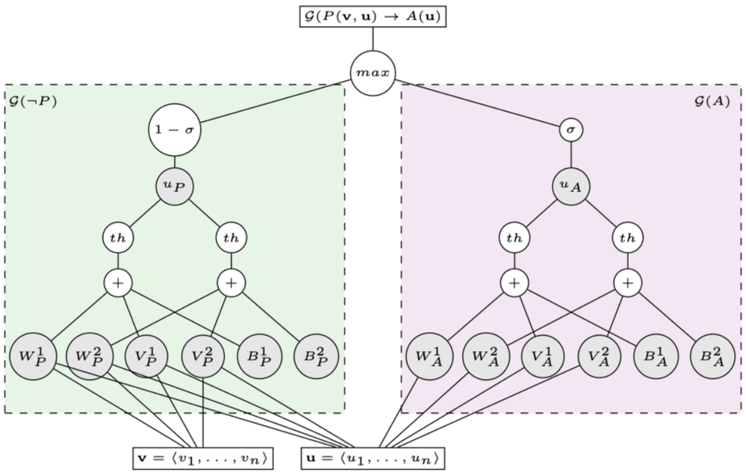


Fig. 1. Tensor net for $P(x,y) \to A(y)$, with $\mathcal{G}(x) = \mathbf{v}$ and $\mathcal{G}(y) = \mathbf{u}$ and $k = 2$.

Badreddine et al, AIJ 2022

Neurosymbolic Tools Research:

Logical Neural Networks (Fagin et al 2022-)

- Every neuron has a meaning as a component of a formula in a weighted real-valued logic.
- Neurons represent literals or logical gates, and bounds on truth values are maintained using interval arithmetic.
- Applicable to many real-world problems that require explainable models.
- Learns logical reasoning within the networks.
- Ensures consistency when possible; supports incomplete and partially specified logical information (via bounds); enables interpretability and debugging of logical inference steps.
- Can be challenging in scaling to expressive first-order logic with many variables.

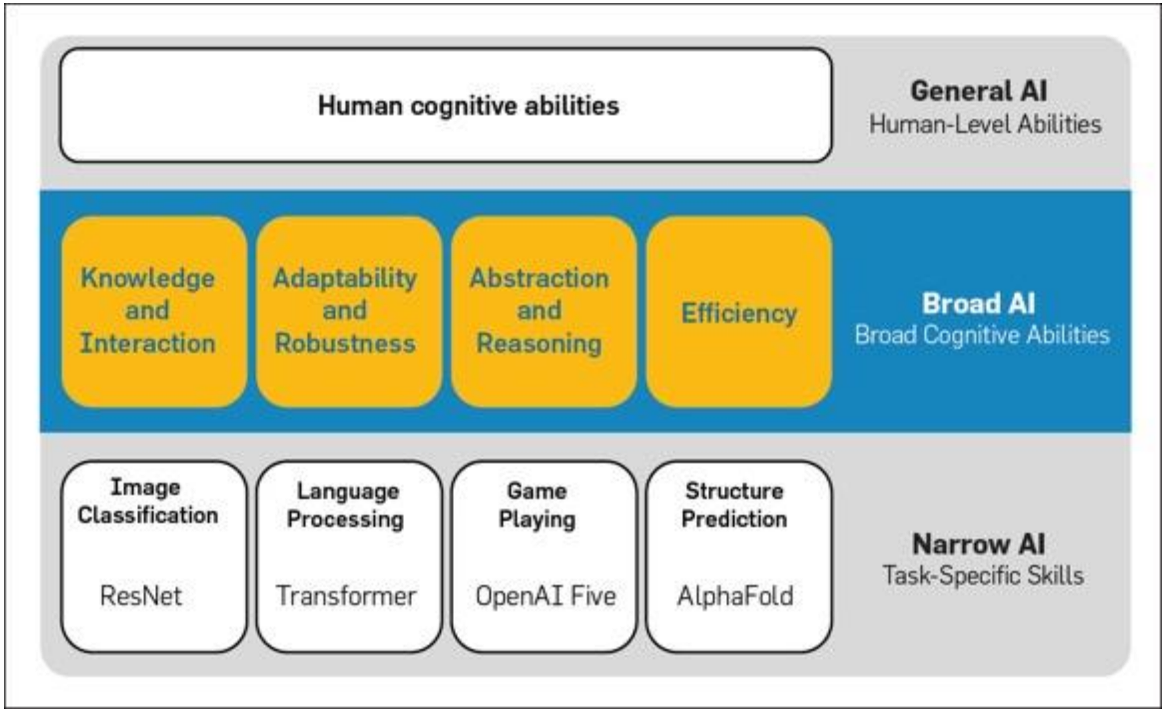
Neurosymbolic Tools Research:

DeepProbLog: (Manhaeve et al, 2021-)

- Integrates probabilistic logic programming language with NNs allowing “neural predicates” within the logic program.
- The neural predicate represents probabilistic facts whose probabilities are parameterized by neural networks.
- Neural predicate weights learned by backprop; logic program structure hand-defined.
- The logic program facts, rules, and queries are augmented with neural components (e.g., CNNs for vision), and probabilistic inference (with neural prediction) runs end-to-end.
- Fit for program induction, probabilistic reasoning, and applications where explicit probabilistic modeling is crucial (noisy perception and symbolic reasoning/representation).
- Scalability can be bottlenecked by the required logic program inference when the symbolic part is complex; model design may be less flexible outside the logic programming paradigm.

Conclusions

Building broader, safe AI Technologies



S. Hochreiter: Towards a Broad AI, CACM, April 2022.

“A broad AI […] performs any cognitive task by virtue of its sensory perception, previous experience, and learned skills.”

“The most promising approach to a broad AI is a neuro-symbolic AI”

“GNNs are the predominant models of neural-symbolic computing.[6]”

6. Lamb, L.C., Garcez, A., Gori, M., Prates, M., Avelar, P. and Vardi, M. Graph Neural Networks Meet Neural-Symbolic Computing: A Survey and Perspective.
IJCAI (2020)

Neurosymbolic AI Open Challenges

(1) First-order logic and higher-order knowledge extraction from very large networks that is provably **sound and efficient**, explains the entire model and local network interactions and accounts for different levels of abstraction.

(2) Goal-directed commonsense and efficient combinatorial reasoning about what has been learned by a complex deep network trained on large amounts of multimodal data.

e.g. **Learning to Solve NP-Complete Problems:** A Graph Neural Network for Decision TSP. Prates, Avelar, Lemos, Lamb, Vardi. **AAAI-2019.**

(3) Human-network communication as part of a multiagent system that promotes communication/argumentation protocols between the user and an agent that can ask questions and check her understanding.

Neurosymbolic AI Open Challenges

(4) Axiomatizing Neurosymbolic AI:
Building a consensus around a shared set of foundational principles for neural-symbolic AI. […] a unifying theoretical foundation to:

- reduce the overhead of describing new systems,
- facilitate collaboration across subfields
- foster clearer communication [within the] area.
- Avoids continued fragmentation and redundancy.

C. Pryor, L. Getoor: Neural-Symbolic Architectural Axioms of Integration: A Manifesto. NeSy 2025; G+L+G: Neural-symbolic cognitive Reasoning, 2009.

(5) Neurosymbolic AI Education: designing tobust tools and a semester long course, and more.
I provided a first version at Dagstuhl 25452, Nov. 2025.

Building broader, safe AI Technologies

As we have seen, Neurosymbolic AI:

Contributes to integrate reasoning and learning, an AI scientific challenge that can support richer technologies.

Neural networks learn from data, and logic reasons over structure.

Neurosymbolic supports building **trustworthy AI:** reasoning can be verified, learning can be justified, and semantics is provided.

A challenge is scaling neurosymbolic techniques into **large models and complex systems.**

Industry relevance is crucial: e.g. Amazon, google, Microsoft applications.

Finally, Neurosymbolic AI is not just technical, it's human driven.

It draws from philosophy, cognitive science, and linguistics to contribute to better technologies or systems that "reason, learn, and communicate" in a clear way.

References

R.Riegel, A. Gray, F. Luus, N.Khan, N. Makondo, I. Akhalwaya, H.Qian, R.Fagin, F. Barahona, U.Sharma, S.Ikbal, H.Karanam, S.Neelam, A.Likhyani, S.Srivastava: **Logical Neural Networks**. CoRR 2006.13155 (2020) & PNAS (2023).

R. Manhaeve, S. Dumančić, A. Kimmig, T. Demeester, L. De Raedt . DeepProbLog: Neural Probabilistic Logic Programming. https://arxiv.org/abs/1805.10872. NeuRIPS, 2018.

S. Badreddine, A. Garcez, L, Serafini, M. Spranger: Logic Tensor Networks. AI Journal (2022)

Survey Papers:

A Mathematical Framework and a Suite of Learning Techniques for Neural-Symbolic Systems: C. Dickens, C. Pryor, C. Gao, A. Albalak, E. Augustine, W. Wang, S. Wright, L,. Getoor. 2025.

Neurosymbolic AI: the 3rd Wave. A. Garcez and L.C. Lamb. Arti. Int. Rev. (2023).

S. Bader and P. Hitzler: **Dimensions of Neural-symbolic Integration** – A Structured Survey, College Pub. 2005.

T.R. Besold, A. d'Avila Garcez, S. Bader, H. Bowman, P. Domingos, P. Hitzler, K-U. Kühnberger, LC. Lamb, P. Lima, L. de Penning, G. Pinkas, H. Poon, G. Zaverucha:
Neural-Symbolic Learning and Reasoning: A Survey and Interpretation. Neuro-Symbolic Artificial Intelligence, 2022.

References

Books:

Artur S. d'Avila Garcez, Luis C. Lamb and Dov M. Gabbay. Neural-Symbolic Cognitive Reasoning. Cognitive Technologies. Springer, 2009.

Barbara Hammer, Pascal Hitzler (Eds), Perspectives of Neural-Symbolic Integration. Studies in Computational Intelligence, Vol. 77. Springer, 2007.

Pascal Hitzler, Md. Kamruzzaman Sarker (Eds):
Neuro-Symbolic Artificial Intelligence: The State of the Art. IOS Press 2022.

Pascal Hitzler, Md. Kamruzzaman Sarker, Aaron Eberhart (Eds):
Compendium of Neurosymbolic Artificial Intelligence. IOS Press 2023.

Paulo Shakarian, Chitta Baral, Gerardo I. Simari, Bowen Xi, Lahari Pokala:
Neuro Symbolic Reasoning and Learning. Springer, 2023.

Evolution of Neurosymbolic AI- Recent Pointers

R.Riegel, A. Gray, F. Luus, N.Khan, N. Makondo, I. Akhalwaya, H.Qian, R.Fagin, F. Barahona, U.Sharma, S.Ikbal, H.Karanam, S.Neelam, A.Likhyani, S.Srivastava: **Logical Neural Networks**. CoRR 2006.13155 (2020)

Gabriele Ciravegna, Pietro Barbiero, Francesco Giannini, Marco Gori, Pietro Lió, Marco Maggini, Stefano Melacci: **Logic Explained Networks**. AI Journal (2022)

S. Badreddine, A. Garcez, L, Serafini, M. Spranger: **Logic Tensor Networks**. AI Journal 2022

V. Belle: **Symbolic Logic meets Machine Learning:** A Brief Survey in Infinite Domains. CoRR abs/2006.08480 (2020)

Md. Kamruzzaman Sarker, Lu Zhou, Aaron Eberhart, Pascal Hitzler:
Neuro-Symbolic Artificial Intelligence: Current Trends. 2022

A. R. Tavares, P. Avelar, J. Flach, M. Nicolau, Luís C. Lamb, Moshe Y. Vardi: **Understanding Boolean Function Learnability on Deep Neural Networks**. NeSy 2025.

Artur Garcez, Luís C. Lamb: **Neurosymbolic AI: The 3rd Wave**. AI Review, 2023

Pascal Hitzler, Md. Kamruzzaman Sarker (Editors). **Neuro-Symbolic Artificial Intelligence.** IOS, 2022.

References

Tavares,A., P. Avelar, J. Flach, M. Nicolau, Luís C. Lamb, Moshe Y. Vardi: **Understanding Boolean Function Learnability on Deep Neural Networks**. NeSy Conference (2025).

Gabriele Ciravegna, Pietro Barbiero, Francesco Giannini, Marco Gori, Pietro Lió, Marco Maggini, Stefano Melacci: **Logic Explained Networks**. AI Journal (2023)

A.D. Garcez, M. Gori, L. C. Lamb, L. Serafini, M. Spranger, S.Tran: Neural-symbolic Computing: An Effective Methodology for Principled Integration of Machine Learning and Reasoning. FLAP 6(4): 611-632 (2019)

Luís C. Lamb, Artur S. d'Avila Garcez, Marco Gori, Marcelo O. R. Prates, Pedro H. C. Avelar, Moshe Y. Vardi: Graph Neural Networks Meet Neural-Symbolic Computing: A Survey and Perspective. IJCAI 2020: 4877-4884

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Artificial Intelligence Review, 2023:
https://link.springer.com/article/10.1007/S10462-023-10448-W

https://arxiv.org/pdf/2012.05876.pdf