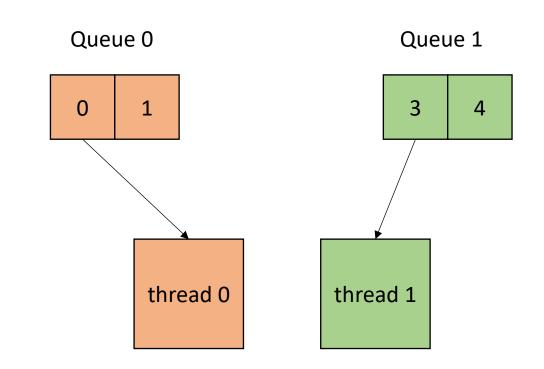
CSE113: Parallel Programming

• Topics:

- DOALL loops
- Static scheduling
- Workstealing



Announcements

Midterm graded

• HW 1 is graded

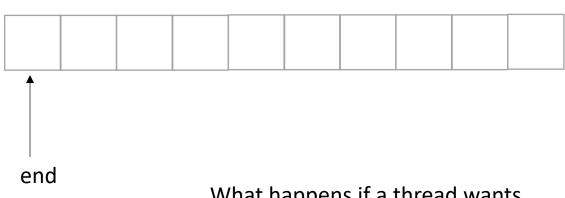
• HW 2 graded this week.

- HW 3 due this week
 - You should be able to do part 2 after today's lecture.

Input/output queues use atomic increments and decrements to protect against threads that are trying to concurrently enqueue and dequeue

○ True

False



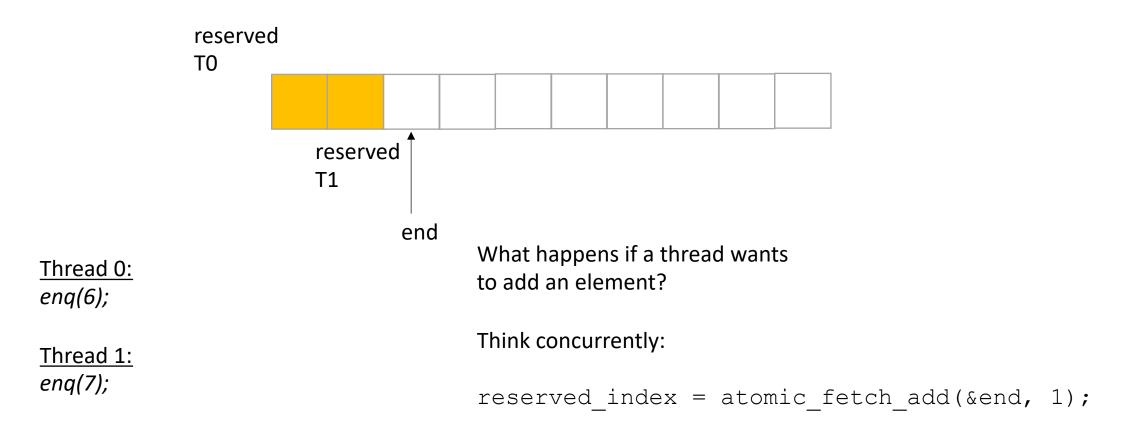
Thread 0: enq(6);

Thread 1: enq(7);

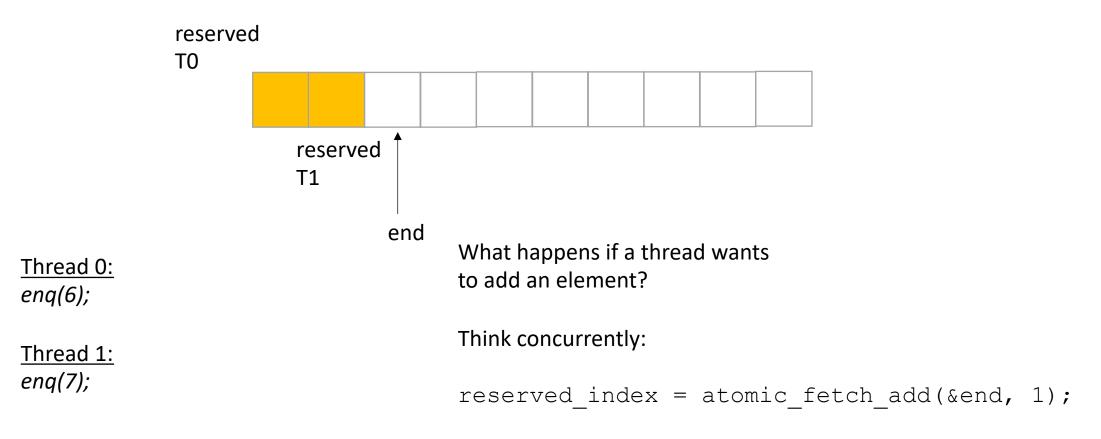
What happens if a thread wants to add an element?

Think concurrently:

```
reserved_index = atomic_fetch_add(&end, 1);
```



does it matter which order threads add their data?



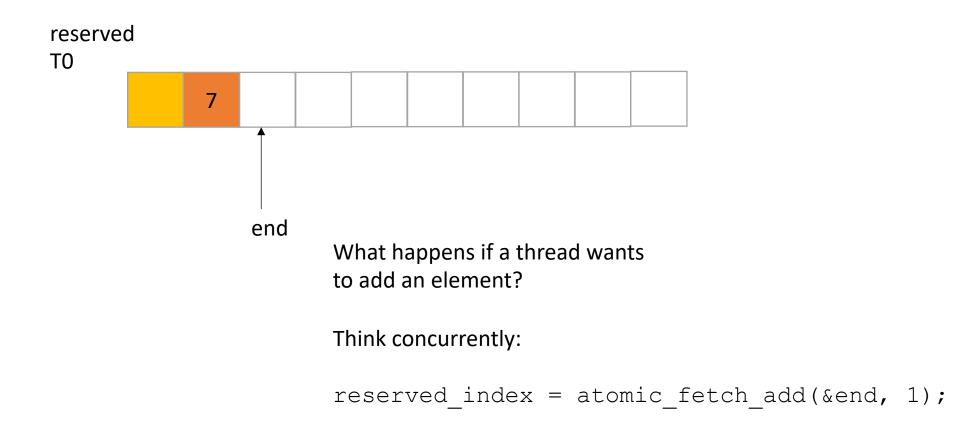
Thread 0:

Thread 1:

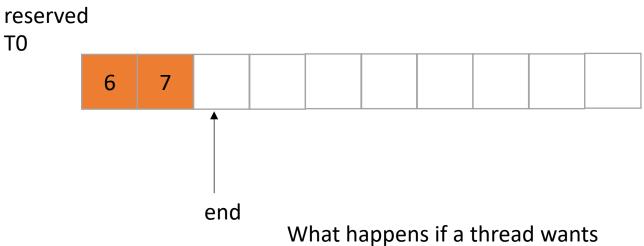
enq(7);

enq(6);

does it matter which order threads add their data?



does it matter which order threads add their data? No! Because there are no deqs!



Thread 0: enq(6);

Thread 1: enq(7);

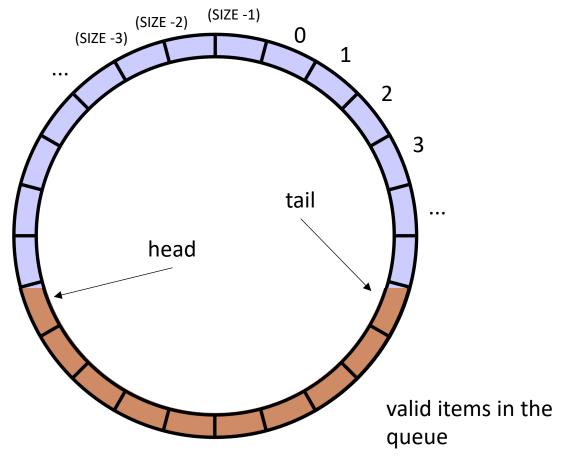
What happens if a thread wants to add an element?

Think concurrently:

```
reserved_index = atomic_fetch_add(&end, 1);
```

Write a few questions about the pros and cons of using a specialized concurrent queue (e.g. an IO queue) and a fully general concurrent queue.

```
class InputOutputQueue {
 private:
    atomic int front;
    atomic int end;
    int list[SIZE];
 public:
    InputOutputQueue() {
        front = end = 0;
    void enq(int x) {
        int reserved index = atomic fetch add(&end, 1);
        list[reserved index] = x;
    void deq() {
       int reserved index = atomic fetch add(&front, 1);
       return list[reserved index];
     int size() {
        return end.load() - front.load();
```



```
class ProdConsQueue {
 private:
    atomic int head;
    atomic_int tail;
    int buffer[SIZE];
 public:
   void enq(int x) {
      // wait for there to be room
      // store value at head
         increment head
    int deq() {
      // wait while queue is empty
      // get value at tail
         increment tail
```

The performance of an application using a producer-consumer queue depends most on:

- If the queue is implemented using mutex or not
- O The rate at which the consumer enqueues elements
- The rate at which the producer enqueues elements

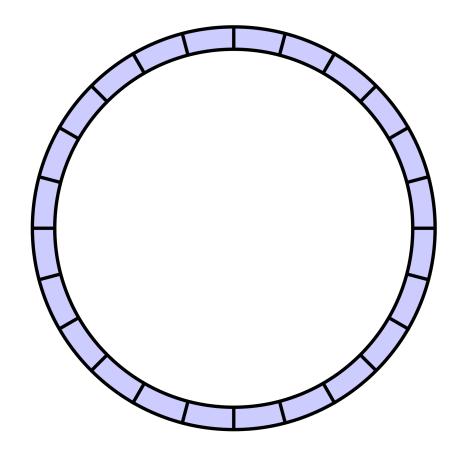
A circular buffer is:

- O A useful data representation for fixed-length queues
- O Part of the C++ standard library
- A special type of memory that is organized in circular patters

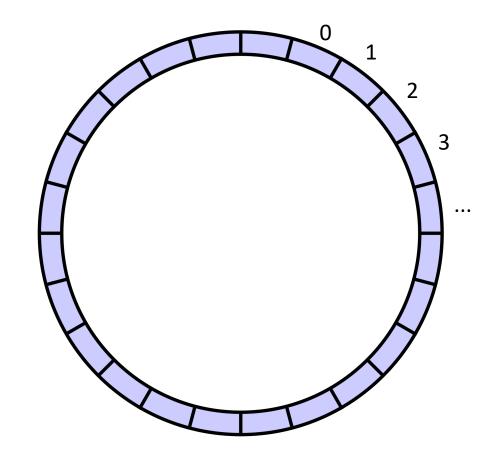
Start with a fixed size array



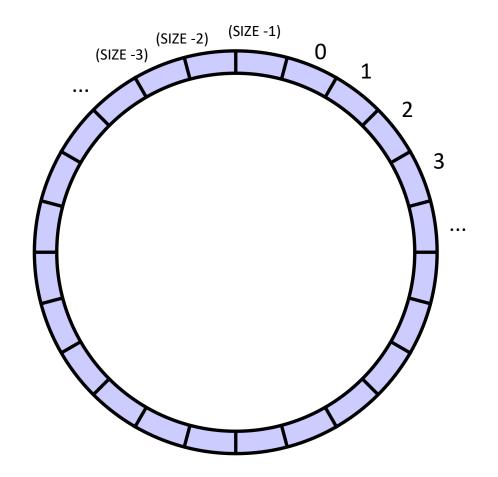
Start with a fixed size array



Start with a fixed size array



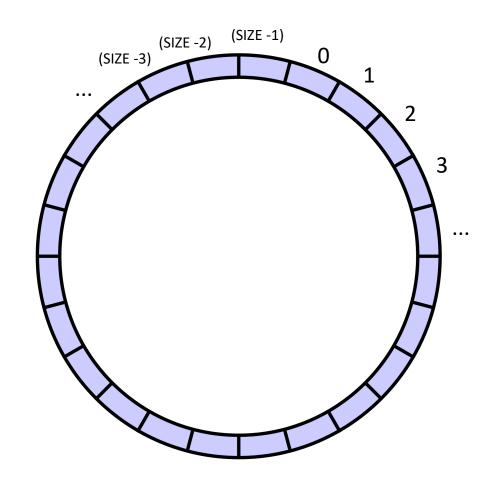
Start with a fixed size array



Start with a fixed size array

we will assume modular arithmetic:

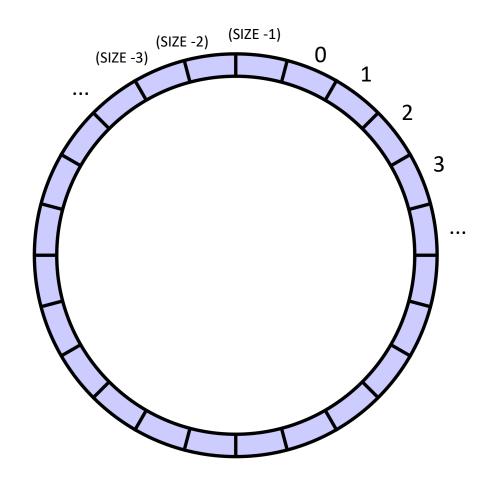
if
$$x = (SIZE - 1)$$
 then $x + 1 == 0$;



Start with a fixed size array

Two variables to keep track of where to deq and enq:

head and tail

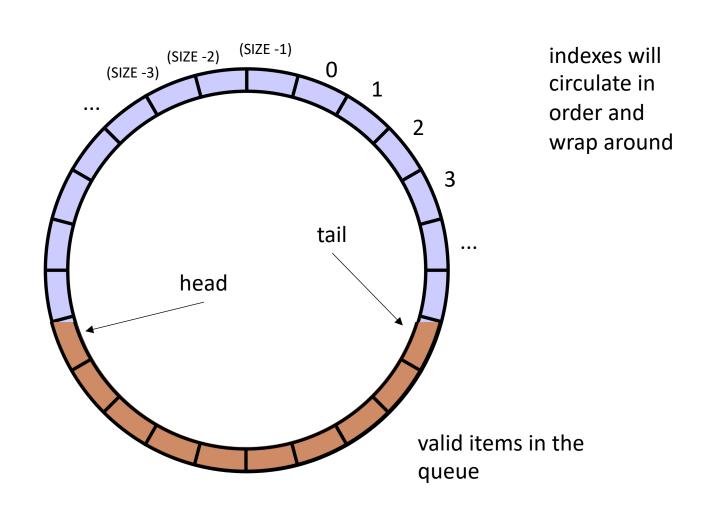


Start with a fixed size array

Two variables to keep track of where to deq and enq:

head and tail:

enq to the head, deq from the tail

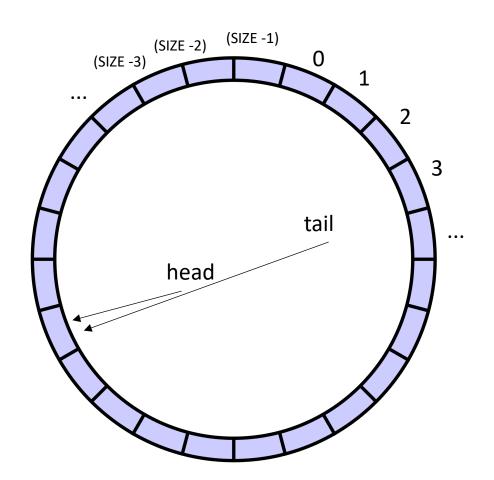


Start with a fixed size array

Two variables to keep track of where to deq and enq:

head and tail

Empty queue is when head == tail



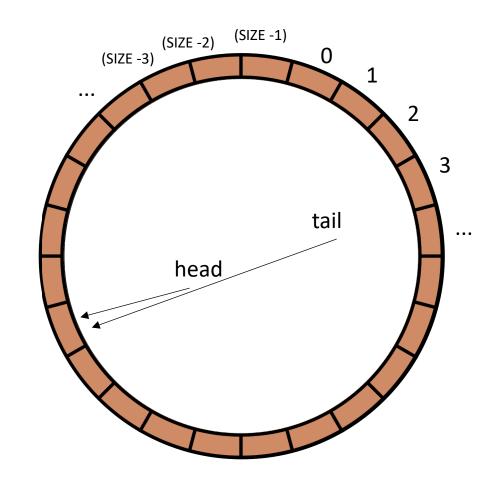
Start with a fixed size array

Two variables to keep track of where to deq and enq:

head and tail

Empty queue is when head == tail

Full queue is when head == tail?



indexes will circulate in order and wrap around

conceptually it is a circle

Start with a fixed size array

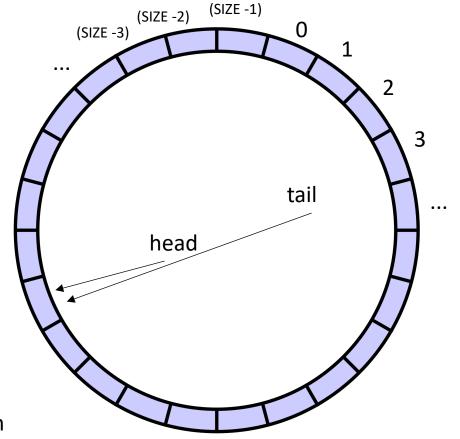
Two variables to keep track of where to deq and enq:

head and tail

Empty queue is when head == tail

Full queue is when head == tail?

but then how to tell full queue from empty?



indexes will circulate in order and wrap around

conceptually it is a circle

Start with a fixed size array

Two variables to keep track of where to deq and enq:

head and tail

Empty queue is when head == tail

Full queue is when head + 1 == tail

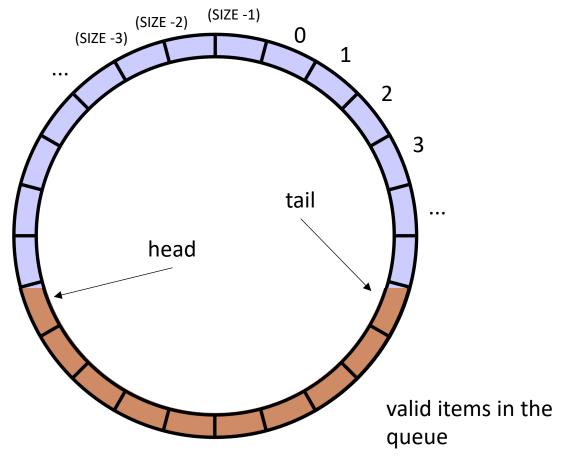
(SIZE -1) (SIZE -2) (SIZE -3) tail head

indexes will circulate in order and wrap around

wasting one location, but its okay...

conceptually it is a circle

review



```
class ProdConsQueue {
 private:
    atomic int head;
    atomic_int tail;
    int buffer[SIZE];
 public:
   void enq(int x) {
      // wait for there to be room
      // store value at head
         increment head
    int deq() {
      // wait while queue is empty
      // get value at tail
         increment tail
```

On to new stuff!

Work stealing

adds two arrays

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

adds elements with neighbors

```
for (int i = 0; i < SIZE; i++) {
  a[i] += a[i+1]
}</pre>
```

are they the same if you traverse them backwards?

adds two arrays

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

```
for (int i = SIZE-1; i >= 0; i--) {
  a[i] = b[i] + c[i];
}
```

adds elements with neighbors

```
for (int i = 0; i < SIZE; i++) {
   a[i] += a[i+1]
}</pre>
```

```
for (int i = SIZE-1; i >= 0; i--) {
  a[i] += a[i+1]
}
```

are they the same if you traverse them backwards?

adds two arrays

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

```
for (int i = SIZE-1; i >= 0; i--) {
  a[i] = b[i] + c[i];
}
```

adds elements with neighbors

```
for (int i = 0; i < SIZE; i++) {
  a[i] += a[i+1]
}</pre>
```

```
for (int i = SIZE-1; i >= 0; i--) {
  a[i] += a[i+1]
}
```

No!

adds two arrays

what about a random order?

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

```
for (pick i randomly) {
   a[i] = b[i] + c[i];
}
```

adds elements with neighbors

```
for (int i = 0; i < SIZE; i++) {
  a[i] += a[i+1]
}</pre>
```

```
for (pick i randomly) {
  a[i] += a[i+1]
}
```

adds two arrays

what about a random order?

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

```
for (pick i randomly) {
   a[i] = b[i] + c[i];
}
```

adds elements with neighbors

```
for (int i = 0; i < SIZE; i++) {
  a[i] += a[i+1]
}</pre>
```

```
for (pick i randomly) {
  a[i] += a[i+1]
}
```

No!

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

These are **DOALL** loops:

- Loop iterations are independent
- You can do them in ANY order and get the same results

```
for (int i = 0; i < SIZE; i++) {
  a[i] = b[i] + c[i];
}</pre>
```

These are **DOALL** loops:

- Loop iterations are independent
- You can do them in ANY order and get the same results
- Most importantly: you can do the iterations in parallel!
- Assign each thread a set of indices to compute

DOALL Loops

- Given a nest of For loops, can we make the outer-most loop parallel?
 - Safely
 - Efficiently

DOALL Loops

- We will consider a special type of for loop, common in scientific applications:
 - Operates on N dimensional arrays (only side-effects are array writes)
 - Array bases are disjoint and constant
 - Bounds, indexes are a function of loop variables, input variables and constants
 - Loops Increment by 1

```
for (int i = 0; i < dim1; i++) {
  for (int j = 0; j < dim3; j++) {
    for (int k = 0; k < dim2; k++) {
      a[i][j] += b[i][k] * c[k][j];
    }
}</pre>
```

DOALL Loops

- We will consider a special type of for loop, common in scientific applications:
 - Operates on N dimensional arrays (only side-effects are array writes)
 - Array bases are disjoint and constant
 - Bounds, indexes are a function of loop variables, input variables and constants
 - Loops Increment by 1 (optional). Transform to 1.

DOALL Loops

- Given a nest of *candidate* For loops, determine if we can we make the outer-most loop parallel?
 - Safely
 - efficiently
- Criteria: every iteration of the outer-most loop must be independent
 - The loop can execute in any order, and produce the same result

- How do we check this?
 - If the property doesn't hold then there exists 2 iterations, such that if they are re-ordered, it causes different outcomes for the loop.
 - Write-Write conflicts: two distinct iterations write different values to the same location
 - **Read-Write conflicts**: two distinct iterations where one iteration reads from the location written to by another iteration.

- Criteria: every iteration of the outer-most loop must be independent
- the loop must produce the same result for any order of the iterations

```
for (i = 0; i < size; i++) {
   a[index(i)] = loop(i);
}</pre>
```

- Criteria: every iteration of the outer-most loop must be independent
- the loop must produce the same result for any order of the iterations

```
for (i = 0; i < size; i++) {
   a[index(i)] = loop(i);
}</pre>
```

index calculation based on the loop variable

- Criteria: every iteration of the outer-most loop must be independent
- the loop must produce the same result for any order of the iterations

```
for (i = 0; i < size; i++) {
   a[index(i)] = loop(i);
}</pre>
```

index calculation based on the loop variable Computation to store in the memory location

- Criteria: every iteration of the outer-most loop must be independent
- the loop must produce the same result for any order of the iterations

```
for (i = 0; i < size; i++) {
   a[index(i)] = loop(i);
}</pre>
```

Write-write conflicts:

for two distinct iteration variables:

```
i_x != i_y
Check:
index(i_x) != index(i_y)
```

- Criteria: every iteration of the outer-most loop must be independent
- the loop must produce the same result for any order of the iterations

```
for (i = 0; i < size; i++) {
   a[index(i)] = loop(i);
}</pre>
```

Write-write conflicts:

for two distinct iteration variables:

```
i_x != i_y
Check:
index(i_x) != index(i_y)
```

Why?

```
Because if index(i_x) == index(i_y)
```

```
then:

a[index(i_x)] will equal

either loop(i_x) or loop(i_v)
```

depending on the order

• Criteria: every iteration of the outer-most loop must be independent

```
for (i = 0; i < size; i++) {
    a[write_index(i)] = a[read_index(i)] + loop(i);
}</pre>
```

Read-write conflicts:

for two distinct iteration variables:

```
i_x != i_y Check: write_index(i_x) != read_index(i_y)
```

• Criteria: every iteration of the outer-most loop must be independent

```
for (i = 0; i < size; i++) {
    a[write_index(i)] = a[read_index(i)] + loop(i);
}</pre>
```

Read-write conflicts:

for two distinct iteration variables:

```
i_x != i_y
Check:
write_index(i_x) != read_index(i_y)
```

Why?

if i_x iteration happens first, then iteration i_y reads an updated value.

if $\mathbf{i}_{\mathbf{y}}$ happens first, then it reads the original value

```
for (i = 0; i < 128; i++) {
   a[i] = a[i]*2;
}</pre>
```

```
for (i = 0; i < 128; i++) {
    a[i]= a[i]*2;
}

for (i = 0; i < 128; i++) {
    a[i]= a[0]*2;
}</pre>
```

```
for (i = 0; i < 128; i++) {
    a[i] = a[i] * 2;
}

for (i = 0; i < 128; i++) {
    a[i] = a[0] * 2;
}</pre>
for (i = 1; i < 128; i++) {
    a[i] = a[0] * 2;
}
```

```
for (i = 0; i < 128; i++) {
  a[i] = a[i] *2;
for (i = 0; i < 128; i++) {
   a[i] = a[0] *2;
for (i = 0; i < 128; i++) {
  a[i\%64] = a[i]*2;
```

```
for (i = 1; i < 128; i++) {
   a[i] = a[0]*2;
}</pre>
```

```
for (i = 0; i < 128; i++) {
   a[i] = a[i] *2;
for (i = 0; i < 128; i++) {
                                       for (i = 1; i < 128; i++) {
   a[i] = a[0] *2;
                                          a[i] = a[0] * 2;
                                       i1 = 0, i2 = 1 both writing to 0
for (i = 0; i < 128; i++) {
  a[i\%64] = a[i]*2;
```

```
for (i = 0; i < 128; i++) {
   a[i] = a[i] *2;
for (i = 0; i < 128; i++) {
                                       for (i = 1; i < 128; i++) {
   a[i] = a[0] *2;
                                          a[i] = a[0] * 2;
for (i = 0; i < 128; i++) {
                                       for (i = 0; i < 128; i++) {
  a[i\%64] = a[i]*2;
                                          a[i\%64] = a[i+64]*2;
```

Consider the following program:

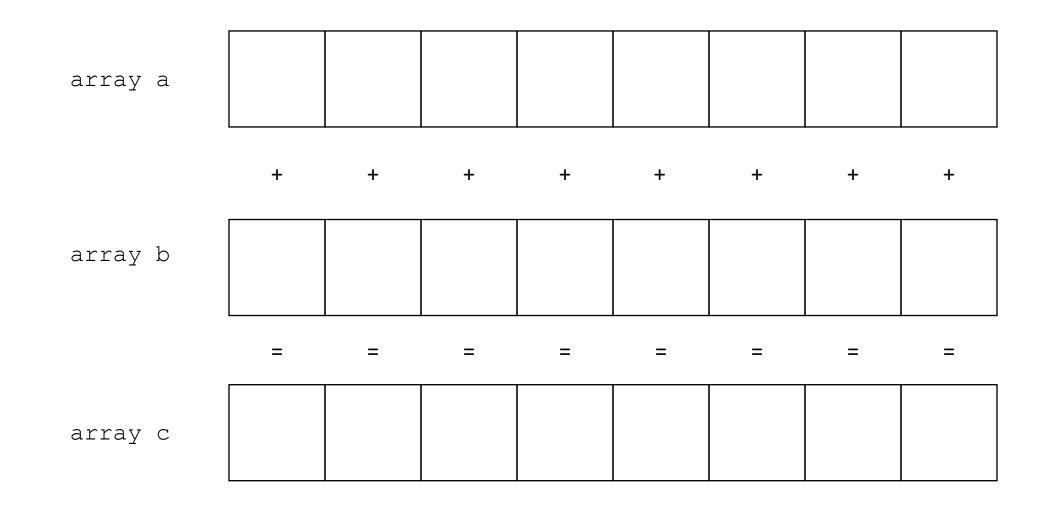
```
There are 3 arrays: a, b, c. We want to compute:
```

```
for (int i = 0; i < SIZE; i++) {
  c[i] = a[i] + b[i];
}</pre>
```

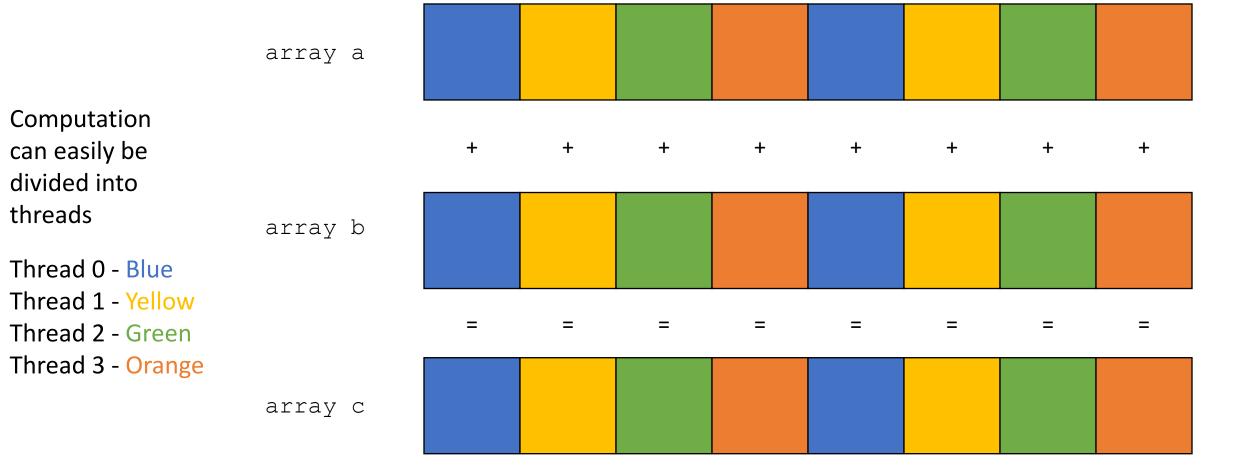
Consider the following program:

```
There are 3 arrays: a, b, c. We want to compute:
```

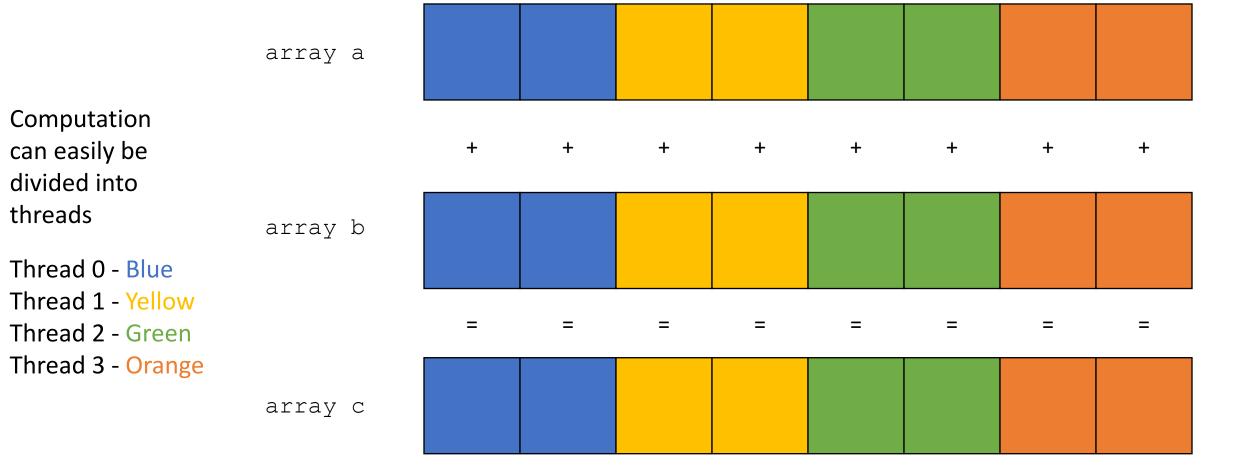
```
for (int i = 0; i < SIZE; i++) {
  c[i] = a[i] + b[i];
}</pre>
```



	array a								
Computation can easily be divided into		+	+	+	+	+	+	+	+
threads	array b								
Thread 0 - Blue Thread 1 - Yellow Thread 2 - Cross		=	=	=	=	=	=	=	=
Thread 2 - Green Thread 3 - Orange									
	array c								



	array a								
Computation can easily be divided into		+	+	+	+	+	+	+	+
threads	array b								
Thread 0 - Blue Thread 1 - Yellow Thread 2 - Cross		=	=	=	=	=	=	=	=
Thread 2 - Green Thread 3 - Orange									
	array c								



• Which one is more efficient?

• Which one is more efficient? Avoid false sharing.

• Which one is more efficient?

- These are called Parallel Schedules for DOALL Loops
- We will discuss several of them.

Schedule

• DOALL Loops

- Static
- Global Worklists
- Local Worklists

Works well when loop iterations take similar amounts of time

```
void foo() {
...
  for (int x = 0; x < SIZE; x++) {
    // Each iteration takes roughly
    // equal time
  }
...
}</pre>
```

0	1	2	3	4	5	6	7	SIZE -1

• Works well when loop iterations take similar amounts of time

```
void foo() {
...
  for (int x = 0; x < SIZE; x++) {
    // Each iteration takes roughly
    // equal time
  }
...
say SIZE / NUM_THREADS = 4
}</pre>
```

0	1	2	3	4	5	6	7		SIZE -1
---	---	---	---	---	---	---	---	--	---------

Works well when loop iterations take similar amounts of time

```
void foo() {
  for (int x = 0; x < SIZE; x++) {
  // Each iteration takes roughly
  // equal time
                                               say SIZE / NUM_THREADS = 4
    Thread 0
                          Thread 1
                                               Thread N
                                                SIZE -1
```

Works well when loop iterations take similar amounts of time

```
void foo() {
...
  for (int x = 0; x < SIZE; x++) {
    // Each iteration takes roughly
    // equal time
  }
...
}</pre>
```

make a new function with the for loop inside. Pass all needed variables as arguments. Take an extra argument for a thread id

Works well when loop iterations take similar amounts of time

```
void foo() {
...
    for (int x = 0; x < SIZE; x++) {
        // Each iteration takes roughly
        // equal time
        }
...
}</pre>
```

```
void parallel_loop(..., int tid, int num_threads)
{
   for (int x = 0; x < SIZE; x++) {
      // work based on x
   }
}</pre>
```

make a new function with the for loop inside. Pass all needed variables as arguments. Take an extra argument for a thread id

Works well when loop iterations take similar amounts of time

```
void foo() {
...
    for (int x = 0; x < SIZE; x++) {
        // Each iteration takes roughly
        // equal time
        }
...
}</pre>
```

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  for (int x = 0; x < SIZE; x++) {
    // work based on x
  }
}</pre>
```

Works well when loop iterations take similar amounts of time

```
void foo() {
...
    for (int x = 0; x < SIZE; x++) {
        // Each iteration takes roughly
        // equal time
        }
...
}</pre>
```

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

Works well when loop iterations take similar amounts of time

```
void foo() {
...
  for (int t = 0; t < NUM_THREADS; t++) {
    spawn(parallel_loop(..., t, NUM_THREADS))
  }
  join();
...
}</pre>
```

You will need to adapt the thread spawn, join to C++

Spawn threads

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

• Example, 2 threads/cores, array of size 8

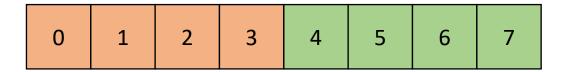
0	1	2	3	4	5	6	7	
---	---	---	---	---	---	---	---	--

```
chunk_size = ?
0: start = ? 1: start = ?
0: end = ? 1: end = ?
```

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

• Example, 2 threads/cores, array of size 8



```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

```
0: end = 4 1: end = 8
```

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

Example, 2 threads/cores, array of size 9

0	1	2	3	4	5	6	7	8
	_	_		•			•	

```
chunk size = ?
```

```
0: start = ? 1: start = ?
```

```
0: end = ? 1: end = ?
```

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

Example, 2 threads/cores, array of size 9



```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

```
0: end = 4 1: end = 8
```

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    // work based on x
  }
}</pre>
```

Example, 2 threads/cores, array of size 9



```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

0: end = 4 1: end = 8

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size = SIZE / NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   if (tid == num_threads - 1) {
      end = SIZE;
   }
   for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

last thread gets more work

Example, 2 threads/cores, array of size 9

```
0 1 2 3 4 5 6 7 8
```

```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

0: end = 4 1: end = ?

thread 0

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size = SIZE / NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   if (tid == num_threads - 1) {
      end = SIZE;
   }
   for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

• Example, 2 threads/cores, array of size 9



```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

0: end =
$$4$$
 1: end = 9

thread 0

thread 1

last thread gets more work

What is the worst case?

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size = SIZE / NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   if (tid == num_threads - 1) {
      end = SIZE;
   }
   for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

Example, 2 threads/cores, array of size 9



```
chunk size = 4
```

0: start = 0 1: start = 4

0: end = 4 1: end = 9

thread 0

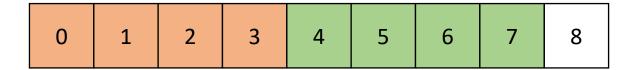
thread 1

last thread gets more work

What is the worst case? 3 items

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size = SIZE / NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   if (tid == num_threads - 1) {
      end = SIZE;
   }
   for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

Example, 2 threads/cores, array of size 9



```
chunk size = 4
```

```
0: start = 0 1: start = 4
```

0: end =
$$4$$
 1: end = 8

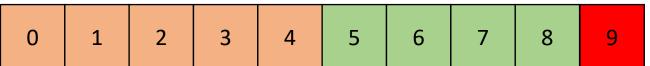
thread 0

thread 1

ceiling division, this will distribute uneven work in the last thread to all other threads

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size =
    (SIZE+(NUM_THREADS-1))/NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   for (int x = start; x < end; x++) {
        // work based on x
   }
}</pre>
```

• Example, 2 threads/cores, array of size 9



out of bounds

```
chunk_size = 5

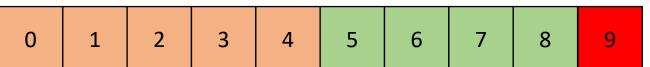
0: start = 0   1: start = 5

0: end = 5   1: end = 10

thread 0  thread 1
```

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size =
    (SIZE+(NUM_THREADS-1))/NUM_THREADS;
   int start = chunk_size * tid;
   int end = start + chunk_size;
   for (int x = start; x < end; x++) {
        // work based on x
   }
}</pre>
```

• Example, 2 threads/cores, array of size 9



out of bounds

```
chunk_size = 5

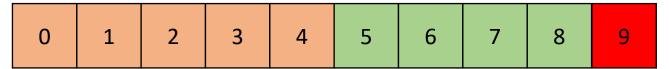
0: start = 0   1: start = 5

0: end = 5   1: end = 10

thread 0  thread 1
```

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size =
    (SIZE+(NUM_THREADS-1))/NUM_THREADS;
   int start = chunk_size * tid;
   int end =
      min(start+chunk_size, SIZE)
for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

Example, 2 threads/cores, array of size 9



most threads do equal amounts of work, last thread may do less.

Which one is better/worse?
Approach 1:
Max slowdown for last thread does all the extra work? 3 items
Approach 2: (with ceiling)
Max slowdown is 1 item.

```
void parallel_loop(..., int tid, int num_threads)
{
   int chunk_size =
    (SIZE+(NUM_THREADS-1))/NUM_THREADS;
   int start = chunk_size * tid;
   int end =
      min(start+chunk_size, SIZE)

for (int x = start; x < end; x++) {
      // work based on x
   }
}</pre>
```

Schedule

• DOALL Loops

• Parallel Schedules:

- Static
- Global Worklists
- Local Worklists

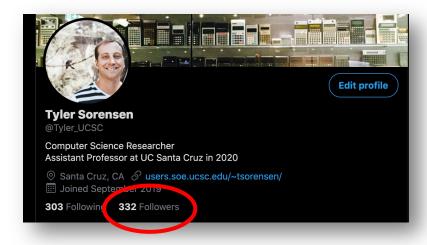
Tasks are not balanced

Appears in lots of emerging workloads

Tasks are not balanced

Appears in lots of emerging workloads

social network analytics where threads are parallel across users





Independent iterations have different amount of work to compute

Threads with longer tasks take longer to compute.

• Threads with shorter tasks are under utilized.

```
for (x = 0; x < SIZE; x++) {
  for (y = x; y < SIZE; y++) {
    a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

irregular (or unbalanced) parallelism: each x iteration performs different amount of work.

- Calculate imbalance cost if x is chunked:
 - Thread 1 takes iterations 0 SIZE/2
 - Thread 2 takes iterations SIZE/2 SIZE

```
for (x = 0; x < SIZE; x++) {
  for (y = x; y < SIZE; y++) {
    a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

- Calculate imbalance cost if x is chunked:
 - Thread 1 takes iterations 0 SIZE/2
 - Thread 2 takes iterations SIZE/2 SIZE

Calculate how much total work:

$$total_work = \sum_{n=0}^{SIZE} n$$

```
for (x = 0; x < SIZE; x++) {
  for (y = x; y < SIZE; y++) {
    a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

- Calculate imbalance cost if x is chunked:
 - Thread 1 takes iterations 0 SIZE/2
 - Thread 2 takes iterations SIZE/2 SIZE

for (x = 0; x < SIZE; x++) {
 for (y = x; y < SIZE; y++) {
 a[x,y] = b[x,y] + c[x,y];
 }
}</pre>

Calculate how much total work:

$$total_work = \sum_{n=0}^{SIZE} n$$

Calculate work done by first thread:

$$t1_work = \sum_{n=0}^{SIZE/2} n$$

- Calculate imbalance cost if x is chunked:
 - Thread 1 takes iterations 0 SIZE/2
 - Thread 2 takes iterations SIZE/2 SIZE

for (x = 0; x < SIZE; x++) {
 for (y = x; y < SIZE; y++) {
 a[x,y] = b[x,y] + c[x,y];
 }
}</pre>

Calculate how much total work:

$$total_work = \sum_{n=0}^{SIZE} n$$

Calculate work done by first thread:

$$t1_work = \sum_{n=0}^{SIZE/2} n$$

Calculate work work done by second thread:

Example: SIZE = 64

total_work = 2016 t2_work = 496 t1_work = 1520

t1 does ~3x more work than t2

Only provides ~1.3x speedup

Potential solution:

Have T1 do only ¼ of the iterations Gives a better speedup of 1.77x

Not a feasible solution because often times load imbalance is not given by a static equation on loop bounds!

Calculate how much total work:

$$total_work = \sum_{n=0}^{SIZE} n$$

Calculate work done by first thread:

$$t1_{\text{work}} = \sum_{n=0}^{SIZE/2} n$$

Calculate work work done by second thread:

Work stealing

• Tasks are dynamically assigned to threads.

- Pros
 - Simple to implement
- Cons:
 - High contention on global counter
 - Potentially bad memory locality.

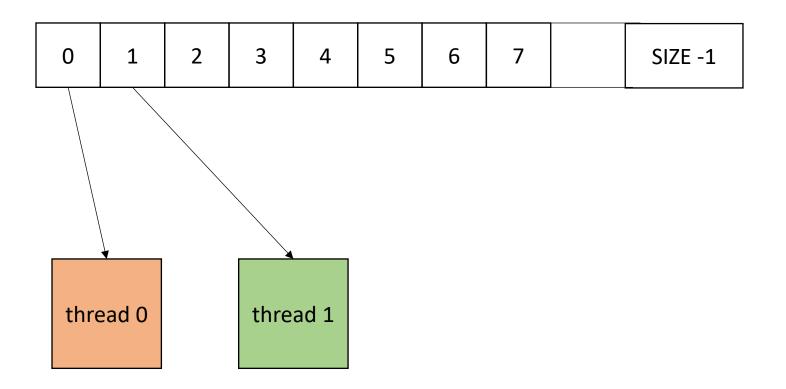
Global worklist: threads take tasks (iterations) dynamically

0	1	2	3	4	5	6	7		SIZE -1
---	---	---	---	---	---	---	---	--	---------

cannot color initially!

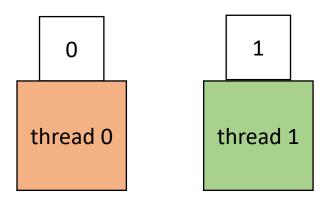
thread 0

Global worklist: threads take tasks (iterations) dynamically



Global worklist: threads take tasks (iterations) dynamically



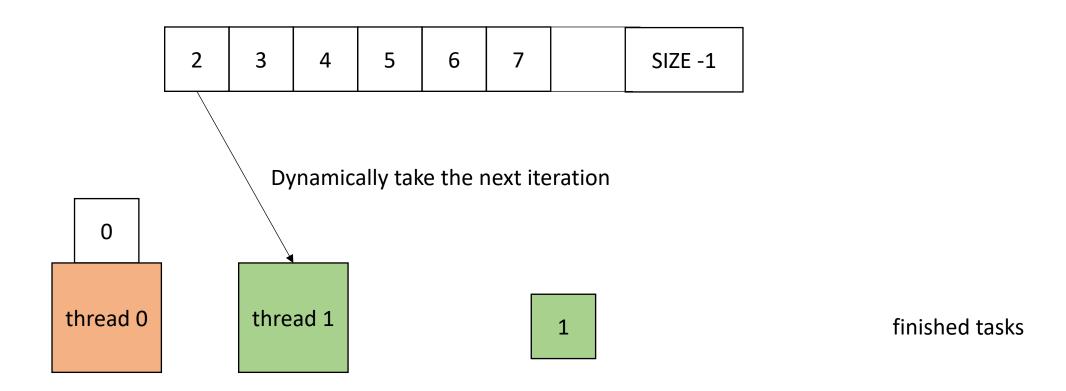


Global worklist: threads take tasks (iterations) dynamically

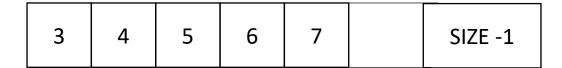


thread 0 thread 1 1 finished tasks

• Global worklist: threads take tasks (iterations) dynamically



• Global worklist: threads take tasks (iterations) dynamically

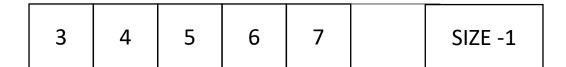


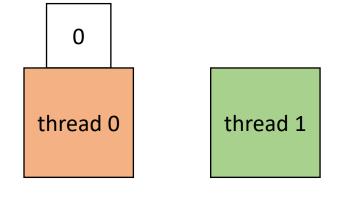
finished tasks

thread 0

thread 1

• Global worklist: threads take tasks (iterations) dynamically

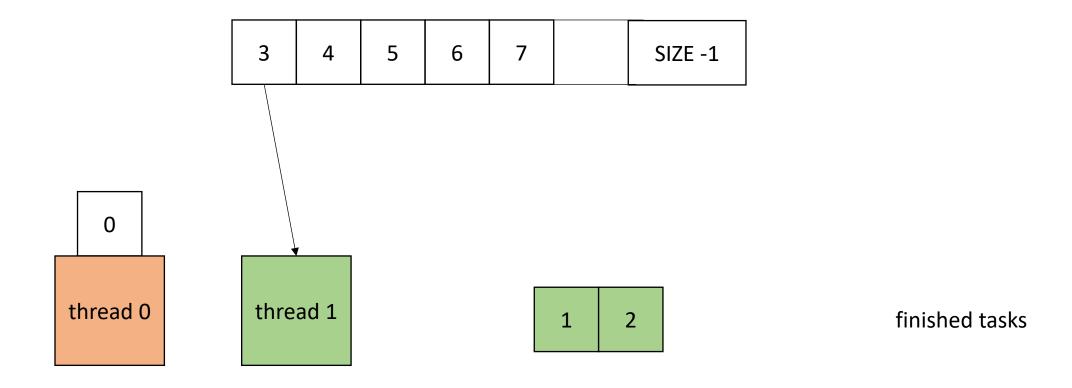




1 2

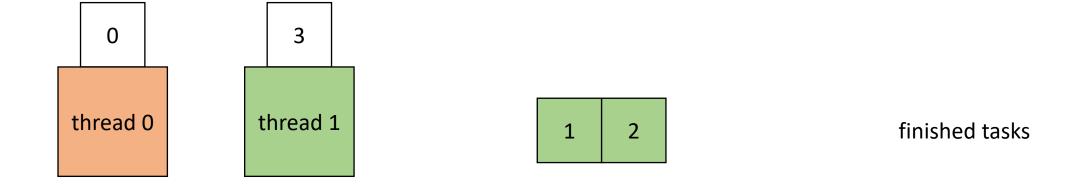
finished tasks

Global worklist: threads take tasks (iterations) dynamically



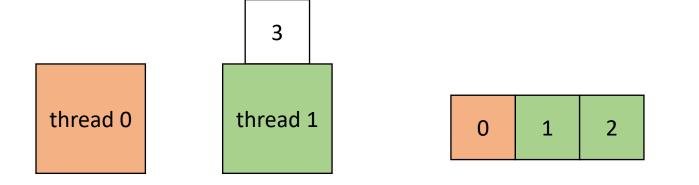
• Global worklist: threads take tasks (iterations) dynamically





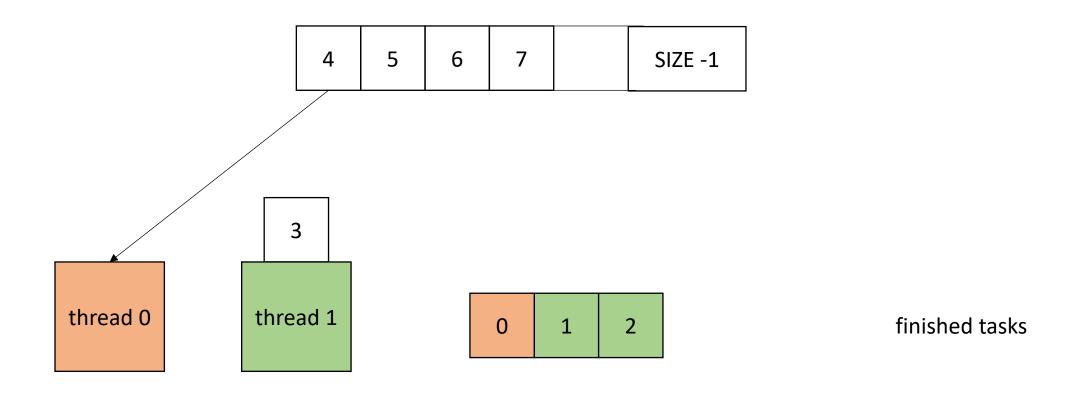
• Global worklist: threads take tasks (iterations) dynamically





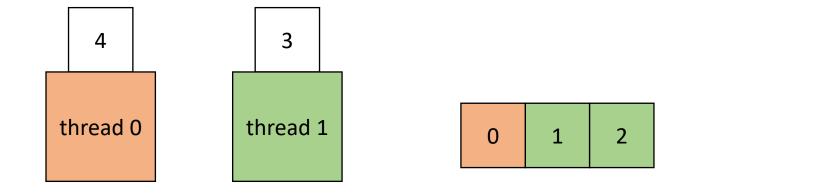
finished tasks

• Global worklist: threads take tasks (iterations) dynamically



• Global worklist: threads take tasks (iterations) dynamically





finished tasks

How to implement

```
void foo() {
    ...
    for (x = 0; x < SIZE; x++) {
        // dynamic work based on x
    }
    ...
}</pre>
```

How to implement

```
void parallel_loop(...) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

Replicate code in a new function. Pass all needed variables as arguments.

How to implement

```
atomic_int x(0);
void parallel_loop(...) {

for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

How to implement

change loop bounds in new function to use a local variable using global variable.

How to implement

These must be atomic updates!

change loop bounds in new function to use a local variable using global variable.

How to implement

```
void foo() {
    ...
    for (t = 0; x < THREADS; t++) {
        spawn(parallel_loop);
    }
    join();
    ...
}</pre>
```

Global worklist: threads take tasks (iterations) dynamically

0	1	2	3	4	5	6	7		SIZE -1	
---	---	---	---	---	---	---	---	--	---------	--

```
x: 0

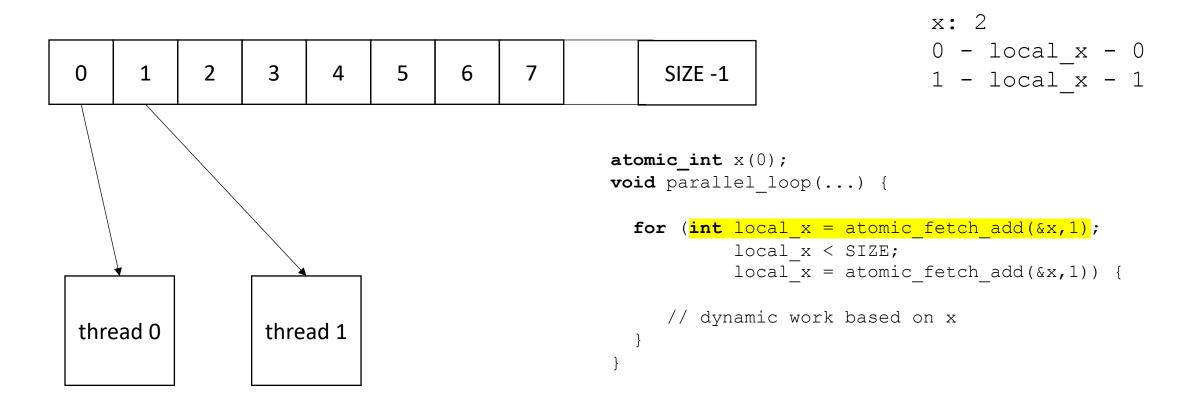
0 - local_x - UNDEF

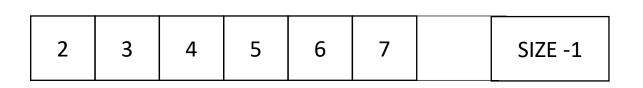
1 - local_x - UNDEF
```

atomic int x(0);

thread 0

thread 1



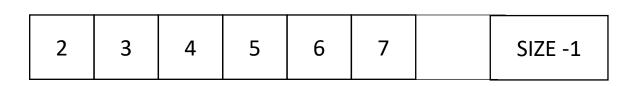


```
x: 2

0 - local_x - 0

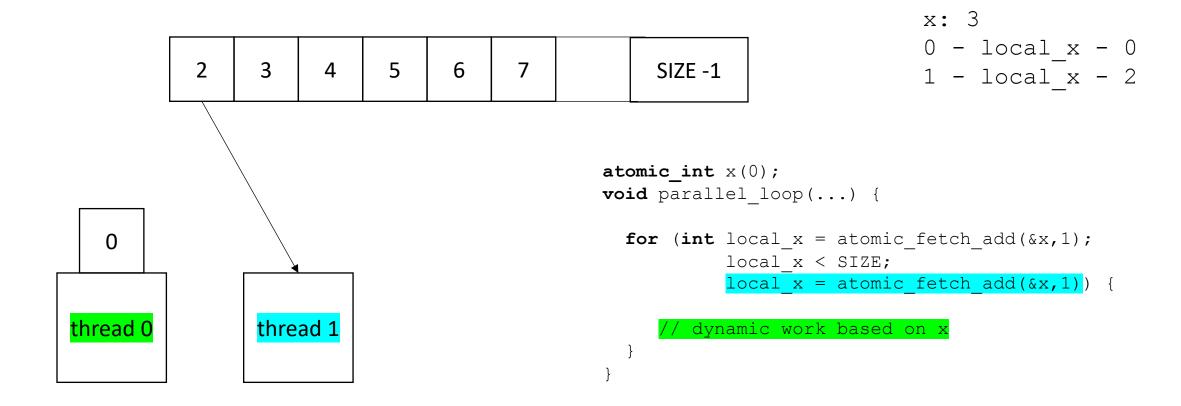
1 - local_x - 1
```

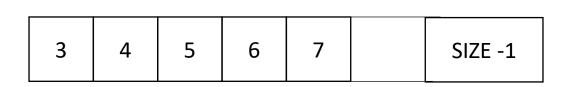
```
0 1 thread 1
```



```
x: 2
0 - local_x - 0
1 - local_x - 1
```

```
thread 0 thread 1
```



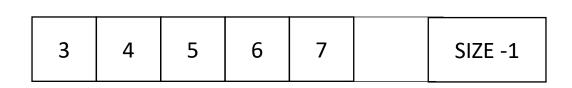


```
x: 3

0 - local_x - 0

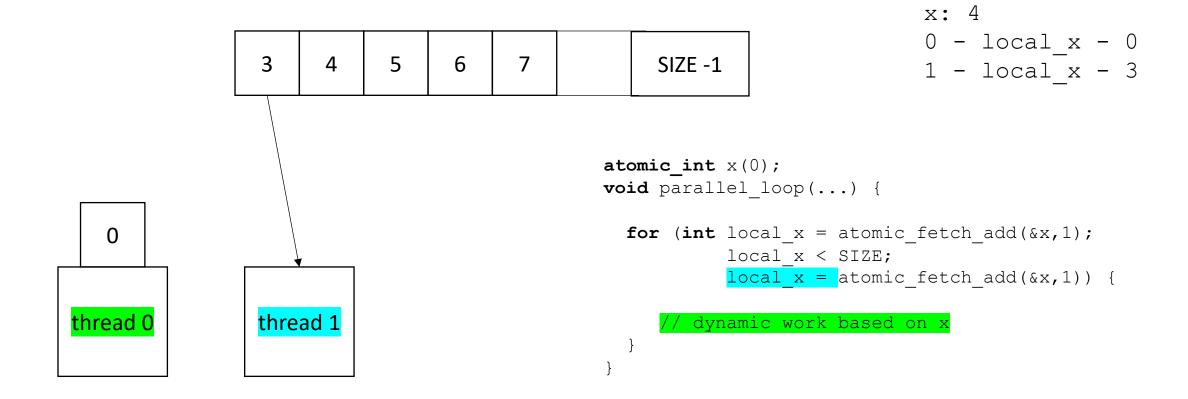
1 - local_x - 2
```

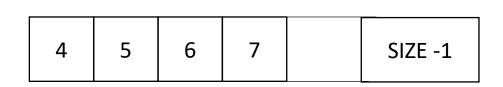
```
0 2 thread 1
```



```
x: 3
0 - local_x - 0
1 - local_x - 2
```

```
thread 0 thread 1
```



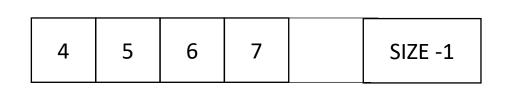


```
x: 4

0 - local_x - 0

1 - local_x - 3
```

```
0 3 thread 1
```

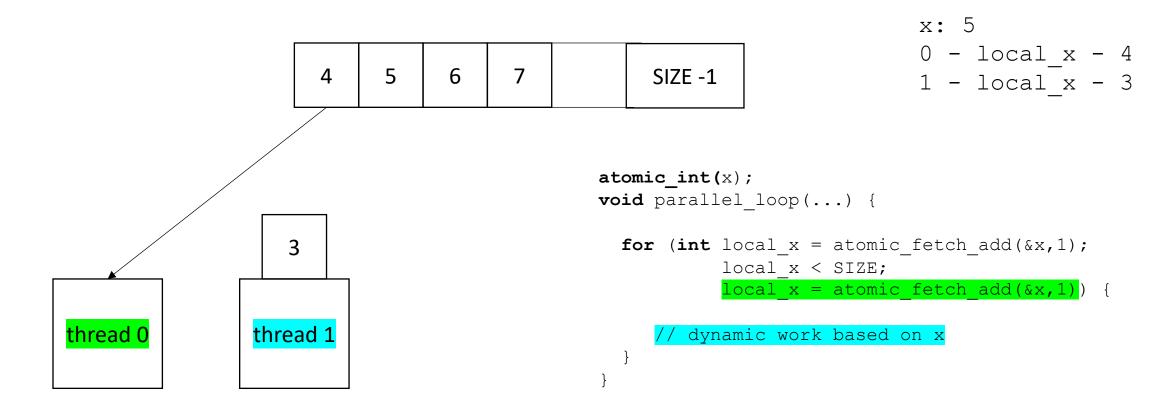


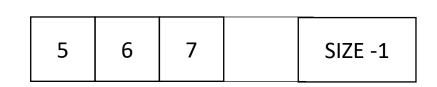
```
x: 4

0 - local_x - 0

1 - local_x - 3
```

```
thread 0 thread 1
```





```
x: 5

0 - local_x - 4

1 - local_x - 3
```

```
4 3 thread 1
```

Schedule

• DOALL Loops

• Parallel Schedules:

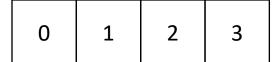
- Static
- Global Worklists
- Local Worklists

More difficult to implement

low contention on local data-structures

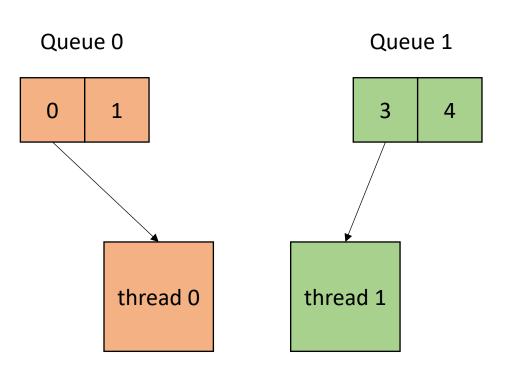
potentially better cache locality

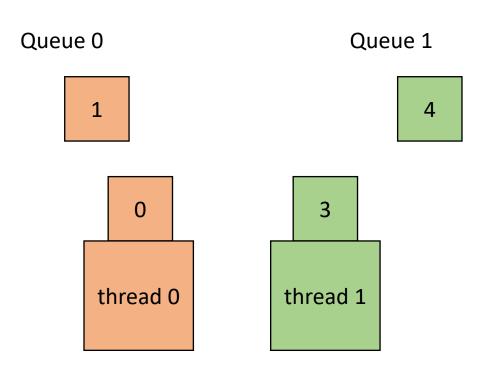
local worklists: divide tasks into different worklists for each thread

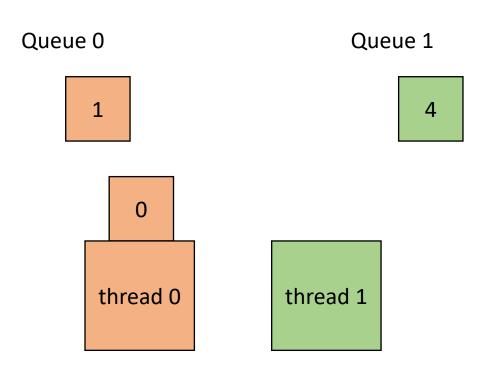


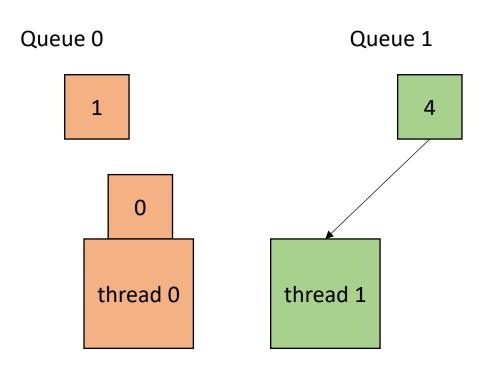
thread 0

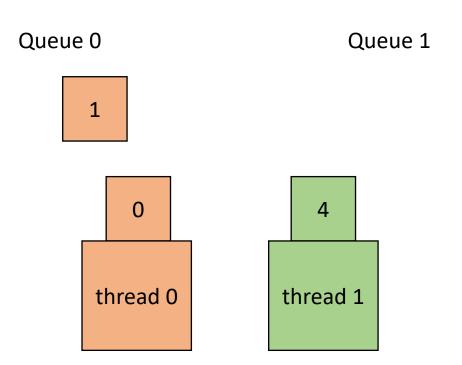
thread 1

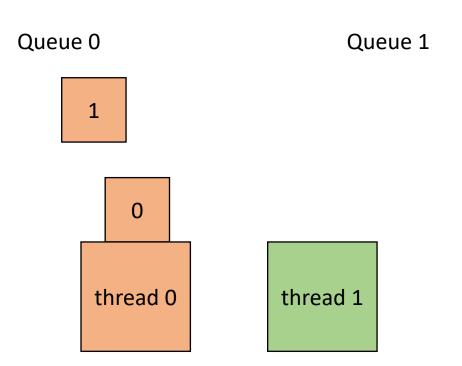


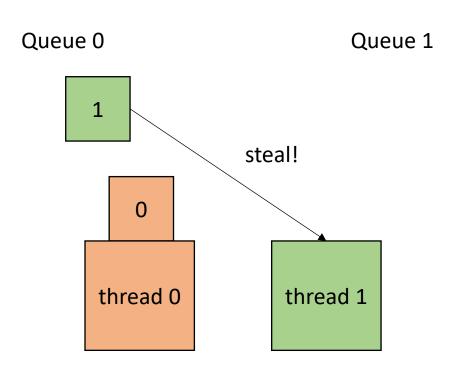












Queue 1

local worklists: divide tasks into different worklists for each thread

0 1 thread 1

Queue 0

How to implement:

```
void foo() {
    ...
    for (x = 0; x < SIZE; x++) {
        // dynamic work based on x
    }
    ...
}</pre>
```

How to implement:

```
void parallel_loop(..., int tid) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

Make a new function, taking any variables used in loop body as args. Additionally take in a thread id

How to implement:

```
concurrent_queue cq[NUM_THREADS];
void foo() {
    ...
    for (x = 0; x < SIZE; x++) {
         // dynamic work based on x
         ...
}</pre>
```

```
void parallel_loop(..., int tid) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

How to implement:

```
void parallel_loop(..., int tid) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

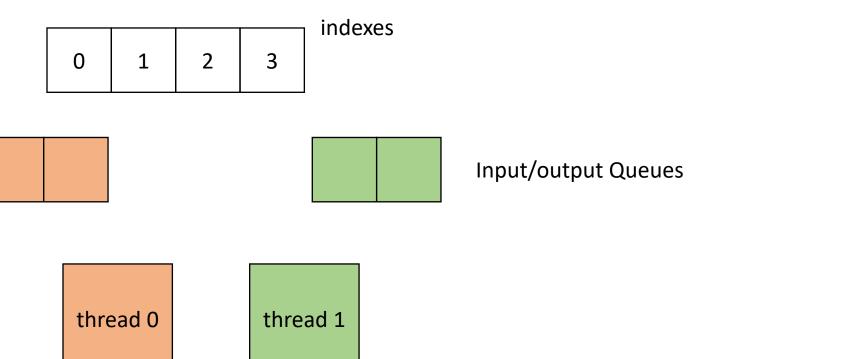
What type of queues?

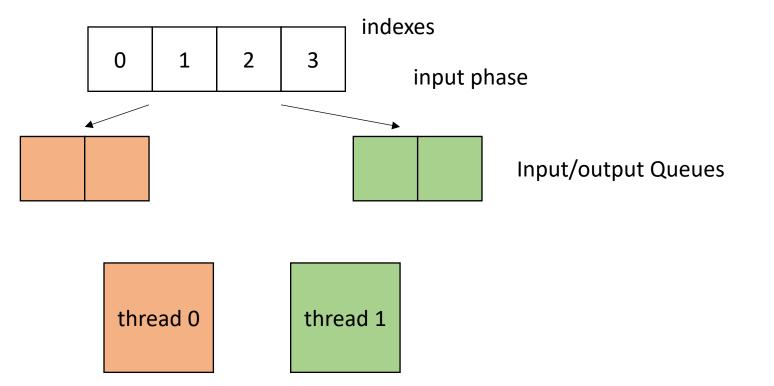
How to implement:

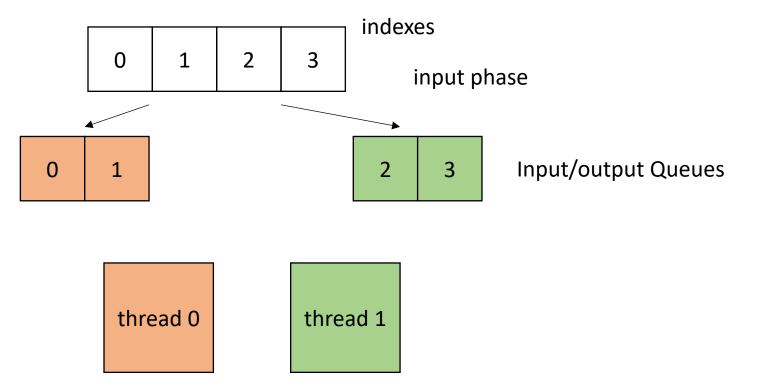
```
concurrent_queue cq[NUM_THREADS];
void foo() {
    ...
    for (x = 0; x < SIZE; x++) {
         // dynamic work based on x
         ...
}</pre>
```

```
void parallel_loop(..., int tid) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

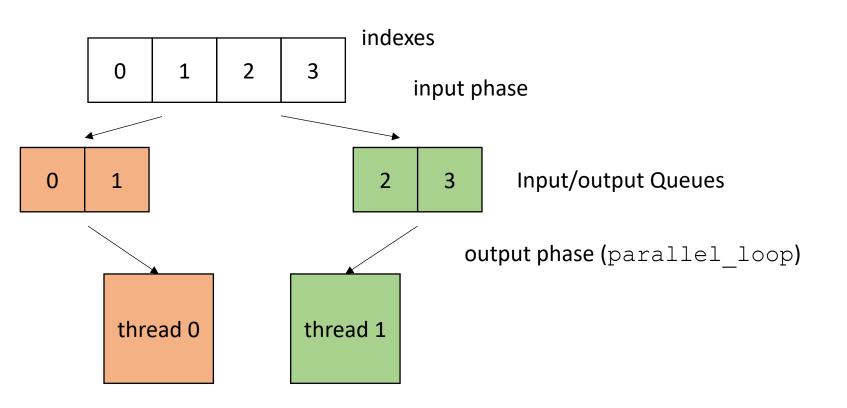
What type of queues? We're going to use InputOutput Queues!







local worklists: divide tasks into different worklists for each thread



```
concurrent_queues cq[NUM_THREADS];
void foo() {
   ...
}
```

First we need to initialize the queues

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // Spawn threads to initialize
    // join initializing threads
    ...
}
```

```
void parallel_enq(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    cq[tid].enq(x);
  }
}</pre>
```

Just like the static schedule, except we are enqueuing

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // Spawn threads to initialize
    // join initializing threads
    ...
}
```

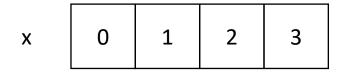
Make sure to account for boundary conditions!

```
void parallel_enq(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    cq[tid].enq(x);
  }
}</pre>
```

Just like the static schedule, except we are enqueuing

How to implement in a compiler:

```
NUM_THREADS = 2;
SIZE = 4;
CHUNK = 2;
```



```
tid 0 0 1 1
```

Make sure to account for boundary conditions!

```
void parallel_enq(..., int tid, int num_threads)
{
  int chunk_size = SIZE / NUM_THREADS;
  int start = chunk_size * tid;
  int end = start + chunk_size;
  for (int x = start; x < end; x++) {
    cq[tid].enq(x);
  }
}</pre>
```

Just like the static schedule, except we are enqueuing

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

// launch loop function
    ...
}
```

```
void parallel_loop(..., int tid, int num_threads) {
  for (x = 0; x < SIZE; x++) {
    // dynamic work based on x
  }
}</pre>
```

How do we modify the parallel loop?

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

    // launch loop function
    ...
}
```

```
void parallel_loop(..., int tid, int num_threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
  {
    // dynamic work based on task
  }
}
```

loop until the queue is empty

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

    // launch loop function
    ...
}
```

```
void parallel_loop(..., int tid, int num_threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
  {
     // dynamic work based on task
  }
}
```

loop until the queue is empty Are we finished?

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

// launch loop function
    ...
}
```

```
atomic_int finished_threads(0);
void parallel_loop(..., int tid, int num_threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
  {
    // dynamic work based on task
  }
  atomic_fetch_add(&finished_threads,1);
}
```

Track how many threads are finished

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

    // launch loop function
    ...
}
```

```
atomic_int finished_threads(0);
void parallel_loop(..., int tid, int num_threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
  {
    // dynamic work based on task
  }
  atomic_fetch_add(&finished_threads,1);
  while (finished_threads.load() != num_threads) {
  }
}
```

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

    // launch loop function
    ...
}
```

```
atomic_int finished_threads(0);
void parallel_loop(..., int tid, int num_threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
  {
    // dynamic work based on task
  }
  atomic_fetch_add(&finished_threads,1);
  while (finished_threads.load() != num_threads) {
    int target = // pick a thread to steal from
    int task = cq[target].deq();
  }
}
```

```
concurrent_queues cq[NUM_THREADS];
void foo() {
    ...
    // initialize queues
    // join threads

    // launch loop function
    // join loop threads
    ...
}
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic_fetch_add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
      // perform task
```

```
concurrent queues cq[NUM THREADS];
void foo() {
  // initialize queues
  // join threads
     launch loop function
     join loop threads
        join the threads
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = // pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

IOQueue 0

) | 1

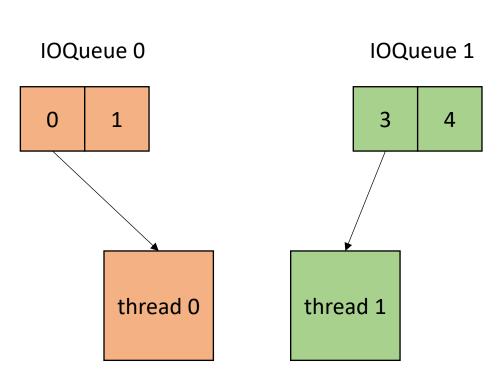
IOQueue 1

3 4

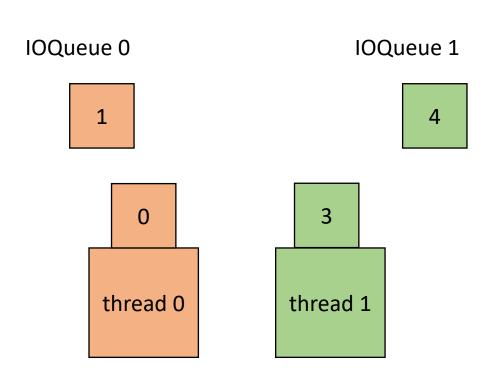
thread 0

thread 1

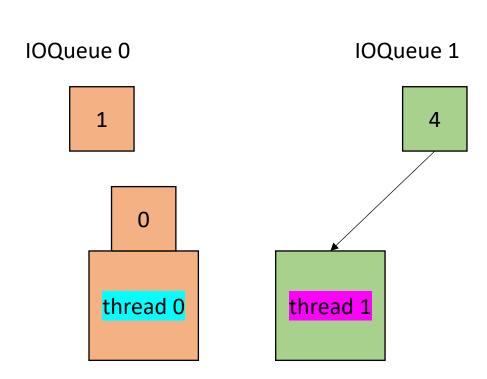
```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```



```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```



```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```



```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
IOQueue 1

1

thread 0

IOQueue 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

IOQueue 0

IOQueue 1

o
thread 0

thread 1

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
finished threads: 1
IOQueue 0
                            IOQueue 1
      thread 0
                      thread 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads, 1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

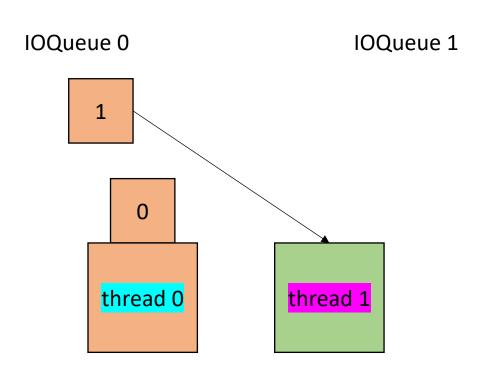
```
finished threads: 1
IOQueue 0
                            IOQueue 1
      thread 0
                      thread 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = // pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
finished threads: 1
IOQueue 0
                            IOQueue 1
      thread 0
                      thread 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = // pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

finished threads: 1

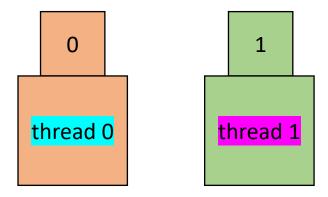


```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num_threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
finished_threads: 1
```

IOQueue 0

IOQueue 1



```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
finished_threads: 1

IOQueue 0

IOQueue 1

thread 0

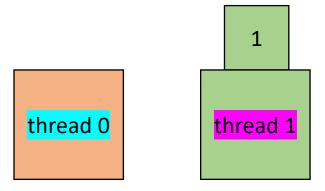
thread 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != -1) {
       // perform task
```

```
finished threads: 2
```

IOQueue 0

IOQueue 1



```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads, 1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

```
finished threads: 2
IOQueue 0
                             IOQueue 1
finished!
      thread 0
                       thread 1
```

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

finished threads: 2

IOQueue 0

IOQueue 1

thread 0

thread 1

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = // pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

finished threads: 2

IOQueue 0

IOQueue 1

thread 0

thread 1

```
atomic int finished threads(0);
void parallel loop(..., int tid, int num threads) {
  int task = 0;
  for (x = cq[tid].deq(); x != -1; x = cq[tid].deq())
    // dynamic work based on task
  atomic fetch add(&finished threads,1);
  while (finished threads.load() != num threads) {
    int target = \overline{//} pick a thread to steal from
    int task = cq[target].deq();
    if (task != −1) {
       // perform task
```

Next topic

General concurrent set