The Asymmetry of Conditional Dependence Structures in the Cryptocurrency Market

Abstract

We design a quantile measure of conditional tail asymmetry and use it to examine dependence structures in the cryptocurrency and the equity market. We find that conditional tail asymmetry is significantly stronger in cryptocurrencies. This happens because cryptocurrencies have largely weaker right-tail correlations while their left-tail dependence is more pronounced, indicating a higher systemic risk in that market. We also find that conditional tail asymmetry in the equity market is mostly found in tails that only include very extreme returns. We show that this asymmetry substantially weakens as the quantile tail criterion rises. In the cryptocurrency market, however, tail asymmetry remains significantly large throughout the entire joint distribution of returns. We verify the robustness of our findings through a series of parametric bootstraps.

1 Introduction

In only 15 years since the introduction of Bitcoin, the global market capitalization of cryptocurrencies has already surpassed 2.5 trillion USD in 2024. This surge has captured the attention of many researchers. Though some studies merely consider cryptocurrencies as alternatives to traditional financial instruments -*e.g.,* equities- (Guesmi et al., 2019, Bouri et al., 2020, Wen et al., 2022), their rising importance has brought to the center their returns' dynamics as well as their relationship with one another (Liu, 2019, Li et al., 2022). From this literature, we know that, despite many similarities, there are consequential differences between equities and cryptocurrencies. For example, cryptocurrencies have heavier tails (Gkillas and Katsiampa, 2018) and are more volatile (Fung et al., 2022, Malek et al., 2023) than the equities. We highlight another difference between the equity market and the cryptocurrency market as related to the tail conditional dependence structures.

It is well known that correlations between equities are greater during extreme downside moves than upswings (Longin and Solnik, 2001, Ang and Chen, 2002). This so-called 'asymmetry' results in strong(er) left-tail correlations compared to right-tail correlations in the equity market. Overall, similar relations have been reported in cryptocurrencies (Feng et al., 2018, Li et al., 2022).[1] However, there are idiosyncrasies in the cryptocurrency market that have inspired considering nonlinear (Li et al., 2022) and time-varying (Aslanidis et al., 2019, Wang et al., 2022) correlation models. Understanding tail dependence structures is crucial for the investors as well as the regulators because they have important implications for portfolio management Sleire et al. (2022) and directional predictions Bekiros et al. (2021). We contribute to this literature by identifying two differences in the pattern of tail asymmetries between the cryptocurrency and the equity market.

Our sample, based on market capitalization, includes the ten largest cryptocurrencies and the five largest companies in each of the eleven sectors of the S&P 500. For each pair, we compute the difference between left-tail and right-tail correlations of assets conditional to a quantile tail criterion. A small quantile tail criterion implies that the tail only includes extreme returns while for larger values of this quantile tail criteria less extreme returns are also considered. Thus, by changing this criterion, one can capture the evolution of tail asymmetry in both markets. We call

[1] Ahn (2022) also provide empirical evidence for the asymmetry of conditional correlations between three cryptocurrencies and the S&P 500 index.

this measure 'Conditional Quantile Correlation Difference (CQCD)', and compute it for different tail criteria based on the Kendall's rank correlation in both markets.

We find that conditional correlation between assets on the lower-left tail of their joint distribution (in short, left-tail correlations) are by and large stronger in the cryptocurrency market compared to the equity market. On the contrary, conditional correlations between assets on the upper-right tail of their joint distribution (in short, right-tail correlations) are stronger in the equity market. As a result, tail asymmetry is significantly stronger between cryptocurrencies. For instance, when the tail criterion is such that only five percent of the data on each tail is considered, the average CQCD in cryptocurrencies, 0.30, is noticeably greater than its average between the equities, 0.02. This quantitative difference between the two markets is the first result of our paper for which we provide more evidence in the paper. We also show that the asymmetry between conditional correlations in the equity market substantially weakens as the tail criterion rises while in the cryptocurrency market, it remains significantly large over the entire joint distribution of assets. For instance, if we raise the tail threshold so that instead of five, up to fifteen percent of the data on each tail is considered, the proportion of equity pairs for which conditional tail asymmetry (CQCD) is less than 0.10, on average, increases from 57 to 79 percent as opposed to the cryptocurrency market where this ratio remains almost unchanged at about 0.4 percent. Even after doubling the tail threshold, this proportion in cryptocurrencies does not increase while in equities, it reaches to over 85 percent. After empirically establishing both patterns, we run a series of parametric bootstraps where we capture the joint distribution of each pair by fitting a bivariate copula to the data, and use that for random sampling. The resulting synthetic bootstrap-generated measures of tail asymmetry confirm both our findings.

The remainder of the paper is organized as follows. Section 2 reviews our methodology. Section 3 introduces the data. Section 4 presents our findings. Finally, Section 5 concludes.

2 Methodology

The conditional right-tail and left-tail correlations are usually defined as the correlation between two assets on a subspace of their mutual return distribution where *both* returns satisfy a tail criterion as formally given by 1:

$$\begin{aligned}\tau_{ij}^{+}(\alpha) &\equiv \mathrm{Corr}\Big(r_{i,t}, r_{j,t} \;\Big|\; r_{i,t} > F_{r_i}^{-1}(\alpha)\ ,\ r_{j,t} > F_{r_j}^{-1}(\alpha)\Big) \\ \tau_{ij}^{-}(\alpha) &\equiv \mathrm{Corr}\Big(r_{i,t}, r_{j,t} \;\Big|\; r_{i,t} < F_{r_i}^{-1}(\alpha)\ ,\ r_{j,t} < F_{r_j}^{-1}(\alpha)\Big)\end{aligned} \tag{1}$$

where the return series $r_t \in \mathbb{R},\ t = 1, \ldots, T$ has a CDF, $F_r(\cdot)$, and quantile function, $F_r^{-1}(\cdot)$. According to this definition, the correlations are computed when both returns are higher (or lower) than their α^{th} percentile of their respective distributions.[2]. The idea of asymmetric tail dependence structures, then, implies that the assets are more correlated when both are performing poorly compared to when both are performing well; *i.e.*, $\tau_{ij}^{-}(1) > \tau_{ij}^{+}(99)$.

We, however, use a slightly different definition. The main difference is that the second measure only requires one of the assets to satisfy the tail criterion and there are no requirements for the other as demonstrated in 2:

$$\begin{aligned}\tilde{\tau}_{ij}^{i,+}(\alpha) &\equiv \mathrm{Corr}\Big(r_{i,t}, r_{j,t} \;\Big|\; r_{i,t} > F_{r_i}^{-1}(\alpha)\Big) \\ \tilde{\tau}_{ij}^{i,-}(\alpha) &\equiv \mathrm{Corr}\Big(r_{i,t}, r_{j,t} \;\Big|\; r_{i,t} < F_{r_i}^{-1}(\alpha)\Big)\end{aligned} \tag{2}$$

[2]In this paper, We measure and report the correlation between assets using Kendall's τ. We have also computed all our measures of tail asymmetry using the Pearson's correlation coefficient. Our findings are not sensitive to this change. These results are available upon request.

We use this definition for both empirical and conceptual reasons.[3] First, the fact that no restrictions are imposed on the second asset implies that, in any sample, the number of observations that satisfy the second definition in 2 is greater than or equal to those that satisfy 1. This is an empirical advantage for studying tail structures where usually the sample size is small. Second, conceptually, the first definition aims to capture dependence structures in different market conditions as it requires both assets to meet the tail criterion. For instance, when both assets have high returns, the the overall market condition is likely to favorable. Similarly, when both are performing poorly, we are likely to be in a bear market. The second definition, however, simply captures asset j's response to unusually high or low returns of asset i, regardless of the overall market conditions. This is more intuitive for risk and portfolio management as the investors' interest in diversification is not limited to the times of crisis only. As a caveat, the second definition is not symmetric as there is no reason to conclude that $\tilde{\tau}_{ij}^{i,+}(\alpha) = \tilde{\tau}_{ij}^{j,+}(\alpha)$. This increases the computational burden of applying this definition to a large number of assets.

2.1 Conditional Quantile Correlation Difference

To quantify the asymmetry in tail dependence structures between two assets, we compute a relative measure of conditional dependence, which we call 'Conditional Quantile Correlation Difference (CQCD)'. The CQCD, for a pair of assets and a tail quantile criterion $\alpha \in (0, 50)$, is defined as the difference between two conditional correlations:

$$\mathbb{C}_{ij}^{i}(\alpha) \equiv \tilde{\tau}_{ij}^{i,-}(\alpha) \; - \; \tilde{\tau}_{ij}^{i,+}(100 - \alpha) \tag{3}$$

This measures the difference between the assets' correlation over the lowest α percentiles of the return of asset i, and their correlation over the highest α percentiles of the return of asset i. When α is close to zero, we are only considering a small segment of the joint distribution that contains extreme returns of asset i on each tail. As α increases, less extreme returns are also included so that $\alpha = 50$ covers the entire joint distribution. It is worth noting that this measure of conditional asymmetry is not symmetric; *i.e.*, $\mathbb{C}_{ij}^{i}(\alpha) \neq \mathbb{C}_{ij}^{j}(\alpha)$.

2.2 Parametric Bootstrap Algorithm

The steps, for each pair, include:

I) **Converting Returns to Pseudo-observations:** For each asset $i \in \{1, 2\}$, compute pseudo-observations from the return series $r_{i,t} \in \mathbb{R}$, $t = 1, \ldots, T$, such that each element is represented as $u_i = F_{r_i}(r_{i,t})$, where F_{r_i} is the empirical distribution function of returns and $u_i \sim U_{[0,1]}$.

II) **Fitting Copulas:** Utilize pseudo-observations $U = (u_1, u_2)$ to fit a bivariate copula $\mathcal{C}(u_1, u_2; \theta)$ to approximate the joint distribution of returns where parameters θ are estimated using the Maximum Likelihood Estimation (MLE) method (L denotes the likelihood function):

$$\theta^{*} = \underset{\theta \in \Theta}{\operatorname{argmax}}\, L(U; \theta)$$

III) **Sampling:** Generate a random sample, $U^{'}$, with the same size of the observed data s.t. $U^{'} \sim \mathcal{C}(\theta^{*})$.

IV) **Measurement and Iteration:** Compute measures of asymmetry and conditional tail dependence in the sample, and repeat these steps for n times .

[3]We have computed all the measures of tail asymmetry with the first definition as well. Our finding are robust to this change. These results are available upon request.

3 Data

We examine ten largest cryptocurrencies, and five largest equities in each of eleven sectors of the standard S&P classification, based on market capitalization.[4] We retrieved our data from Yahoo Finance from 9 Nov. 2017 to 15 May 2024.[5] Table 1 reports the tickers of assets in each sector, and Table 6 in the appendix summarizes the descriptive statistics of their daily returns.

Table 1: Sampled Assets

Sector	Tickers				
Cryptocurrency	BTC	ETH	BNB	XRP	ADA
	NEO	LINK	BCH	EOS	XLM
Communications	CMCSA	VZ	TMUS	DIS	NFLX
Consumer Discretionary	TSLA	AMZN	HD	NKE	MCD
Consumer Staples	PG	KO	PEP	COST	WMT
Energy	XOM	CVX	COP	EOG	OXY
Financials	V	JPM	MA	BAC	WFC
Health Care	LLY	JNJ	UNH	MRK	ABBV
Industrials	UPS	UNP	CAT	BA	HON
Materials	LIN	SHW	FCX	ECL	APD
Real Estate	PLD	AMT	CCI	SPG	EQIX
Technology	MSFT	AAPL	GOOG	NVDA	META
Utilities	NEE	DUK	SO	D	EXC

4 Empirical Results

We start by demonstrating our findings in a small set of assets with only three pairs: Bitcoin and Ethereum; Microsoft Corp. and Alphabet Inc.; and JPMorgan Chase and Bank of America. We measure rank correlations in all pairs on both tails according to definition 2.[6] Table 2 reports the measures of tail asymmetry for each pair for $\alpha = 3, 5$, and 10.[7]

Table 2: Left-tail and Right-tail Conditional Correlations in a 3-pair Example

Assets	$\alpha = 3$			$\alpha = 5$			$\alpha = 10$		
(i, j)	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$
BTC-ETH	0.4413	0.0548	0.3865	0.4719	0.1930	0.2789	0.4350	0.1902	0.2447
ETH-BTC	0.4382	0.2105	0.2277	0.4232	0.1671	0.2561	0.4590	0.1432	0.3158
LLY-JNJ	0.2343	0.0482	0.1861	0.1894	0.0039	0.1855	0.1782	0.0652	0.1130
JNJ-LLY	0.0416	0.2261	-0.1845	0.1984	0.2532	-0.0548	0.2583	0.1433	0.1149
JPM-BAC	0.4547	0.5641	-0.1094	0.3466	0.5525	-0.2060	0.4297	0.4974	-0.0676
BAC-JPM	0.3322	0.4824	-0.1502	0.4201	0.4616	-0.0416	0.4510	0.4673	-0.0163

Note: This table reports conditional correlations between three select pairs of assets according to def. 2.

[4] This brings the total number of assets in our sample to 65.

[5] The data for Bitcoin and many equities starts earlier. However, 9 Nov. 2017 is the first day for which prices of other cryptocurrencies are available. To maintain a balanced sample across markets, we start our analysis from this date.

[6] As discussed earlier, this measurement is not symmetric. Therefore, for each pair, we report two sets of results. In each set, the tail criterion is imposed on the first asset only.

[7] We conduct the same analysis for smaller value of tail criterion as well. The results are consistent with our reported findings. However, the statistical power of these analyses is too low. For instance, when $\alpha = 1$, there are only about 15 observations on each tail for the equities and about 20 observations for the cryptocurrencies. We start reporting our results from $\alpha = 3$, which results in approximately 45 observations on each tail for the equity pairs and 60 observations for the cryptocurrency pairs.

When $\alpha = 3$, the average CQCD between cryptocurrencies is about 0.31 while the average CQCD in four equity pairs is significantly smaller, -0.06. Similar results are found for $\alpha = 5$ and $\alpha = 10$ as well. This suggests that the asymmetry of tail dependence structures is stronger in the cryptocurrency market. As we will demonstrate in this section, this occurs because left-tail conditional correlations ($\tilde{\tau}_{ij}^{i;-}(\alpha)$) are largely stronger in the cryptocurrency market while right-tail correlations (($\tilde{\tau}_{ij}^{i;+}(\alpha)$)) tend to be stronger in the equity market. Therefore, the difference between them, which is our measure of tail asymmetry (CQCD), is almost universally larger in the cryptocurrency market. This is our first main finding.

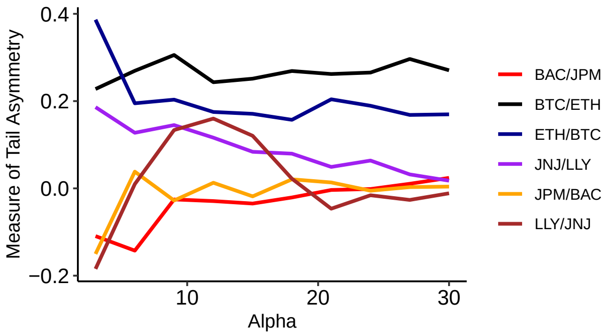
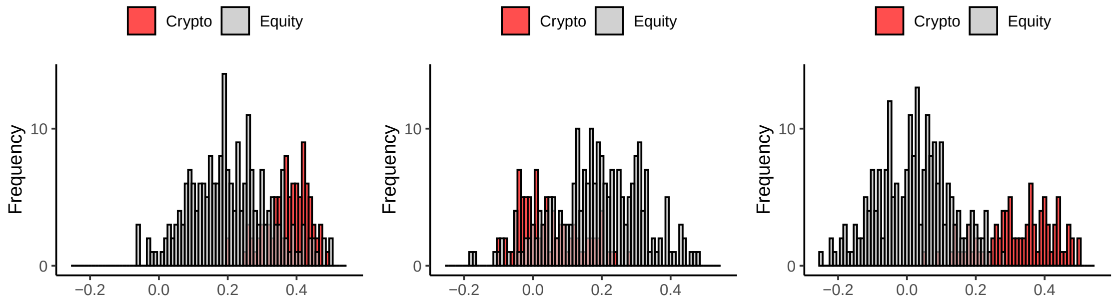


Figure 1: Changes in the Asymmetry of Dependence Structures over the Tail Size

Further examination also revealed that as the tail criterion, α, rises the absolute value of $\mathbb{C}_{ij}^{i}$ in the equity market falls strongly while this is not the case in cryptocurrencies. For instance, as table 2 shows, when α rises from 3 to 10, the average of the absolute value of CQCD, $|\mathbb{C}_{ij}^{i}|$, in equity pairs falls from 0.15 to 0.07. This pattern continues as the value of α rises. This is more evident in figure 1. With the rise in the threshold, α, tail asymmetry among equity pairs converges to zero while in cryptocurrency pairs, this asymmetry persists even for much higher values of α. In other words, in the equity market, the disparity between left-tail and right-tail correlations is mostly found when we only consider extreme events on each tail. The inclusion of less extreme events in the analysis, which happens by increasing the tail criterion, noticeably weakens this asymmetry. This is while tail asymmetry remains significant in both cryptocurrency pairs for larger values of α. This is our second finding in this paper.

4.1 Patterns of Asymmetry Across Markets

Building on the suggestive evidence in the previous section, this section establishes both patterns with the entire market data. Because our measures of tail dependence are not symmetric, we examine $2 \times \binom{5}{2} = 20$ pairs in each equity sector -a total of $20 \times 11 = 220$ pairs- and $2 \times \binom{10}{2} = 90$ pairs in the cryptocurrency market. Figure 2 shows the distribution of conditional correlations on each tail as well as the difference between them, which is our measure of tail asymmetry (CQCD), for all pairs in both markets when the tail criterion is $\alpha = 5$. Figure 2(c) confirms that the conditional tail asymmetry is stronger in cryptocurrencies. For instance, one can visually compare the mean of the distribution of CQCD in cryptocurrencies with its mean in the equity market. To further establish this pattern, table 3 reports the industry-specific averages of measures of tail dependence for different values of α. When $\alpha = 5$, the average CQCD between cryptocurrencies, 0.30, is markedly greater than its average in the equity market, nearly 0.02.



(a) Left-tail (b) Right-tail (c) CQCD

Figure 2: Histograms of Conditional Tail Correlations in Equities vs. Cryptocurrencies ($\alpha = 5$)

The mechanism that generates this difference may be seen in histograms 2(a) and 2(b), as well as in table 3. Overall, left-tail conditional correlations are stronger in the cryptocurrency market while right-tail correlations are stronger in the equity market. Thus, their difference, which is our measure of tail asymmetry, is significantly greater in the cryptocurrency market. Notably, in equity sectors where left-tail correlations are strong -*e.g.,* energy- right-tail correlations are also equally strong. As a results their difference remains relatively small.

Table 3: Asymmetry of Conditional Tail Dependence Structures Across Markets

Sector	$\alpha = 3$			$\alpha = 5$			$\alpha = 10$		
	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$	$\tilde{\tau}_{ij}^{i,-}(\alpha)$	$\tilde{\tau}_{ij}^{i,+}(\alpha)$	$\mathbb{C}_{ij}^{i}(\alpha)$
Cryptocurrency	0.3470	0.0421	0.3050	0.3610	0.0617	0.3000	0.3780	0.1170	0.2610
Communications	0.1220	-0.0058	0.1280	0.1030	0.0579	0.0454	0.1310	0.1150	0.0156
Consumer Discretionary	0.1390	0.0631	0.0762	0.1550	0.0538	0.1010	0.1600	0.0709	0.0888
Consumer Staples	0.2040	0.2140	-0.0091	0.2410	0.1670	0.0742	0.2050	0.1530	0.0523
Energy	0.3210	0.3570	-0.0361	0.3450	0.3140	0.0308	0.3160	0.2880	0.0281
Financials	0.2670	0.3130	-0.0461	0.3210	0.3300	-0.0093	0.3290	0.2940	0.0356
Health Care	0.1450	0.1560	-0.0105	0.1140	0.1630	-0.0494	0.1570	0.1570	-0.0009
Industrials	0.0659	0.1330	-0.0670	0.1220	0.1380	-0.0163	0.1600	0.1880	-0.0287
Materials	0.1240	0.1040	0.0198	0.1620	0.1350	0.0272	0.2010	0.1820	0.0192
Real Estate	0.1960	0.2420	-0.0464	0.2050	0.2230	-0.0178	0.2000	0.2240	-0.0231
Technology	0.1560	0.1690	-0.0128	0.2250	0.2030	0.0222	0.2660	0.2000	0.0656
Utilities	0.3220	0.3450	-0.0233	0.3130	0.3030	0.0099	0.2880	0.2850	0.0024

To illustrate the second pattern, table 4 shows how the share of pairs with a 'small' tail asymmetry changes with the tail criterion. More precisely, in each sector, we report the proportion of pairs where the absolute value of CQCD is less than 0.05 or 0.10. Consistent with our previous finding, the share of such pairs is significantly larger in the equity market. For instance, when $\alpha = 5$ the fraction of cryptocurrencies whose CQCD is less than 0.10 is nearly half of one percent while the average across the equities is more than 58%. More importantly, as α rises, this proportion among the equities noticeably increases so much so that when $\alpha = 30$, the value of CQCD in more than 85% of equity pairs is less than 0.10. On the contrary, the tail asymmetry in almost all of cryptocurrency pairs remains greater than 0.10 at $\alpha = 30$, or even beyond that at $\alpha = 50$. Thus, we conclude that conditional tail asymmetry in the equity market is mostly limited to the tails with extreme returns while in cryptocurrencies, it is found throughout the joint distribution of returns.

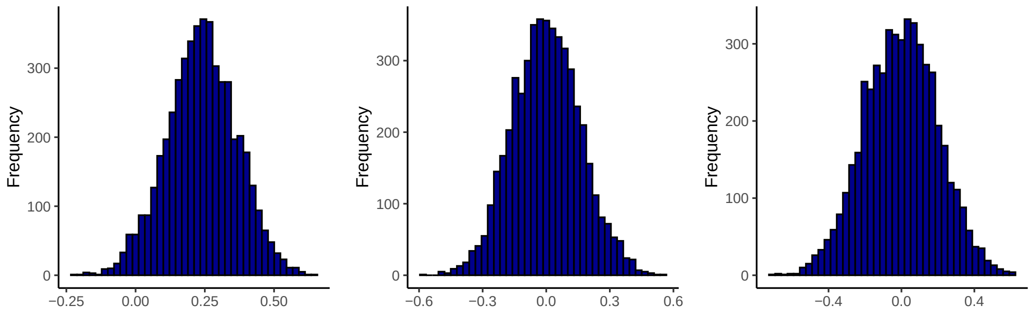
Table 4: Proportion of Small Tail Asymmetries in Equities vs. Cryptocurrencies (%)

Sector	$\alpha = 5$		$\alpha = 15$		$\alpha = 30$	
	$\lvert\mathbb{C}\rvert < 0.05$	$\lvert\mathbb{C}\rvert < 0.10$	$\lvert\mathbb{C}\rvert < 0.05$	$\lvert\mathbb{C}\rvert < 0.10$	$\lvert\mathbb{C}\rvert < 0.05$	$\lvert\mathbb{C}\rvert < 0.10$
Cryptocurrency	0.47	0.95	0	0.47	0	0
Communications	25	60	45	60	80	95
Consumer Discretionary	10	35	40	60	50	75
Consumer Staples	25	55	70	85	65	80
Energy	45	75	60	85	80	100
Financials	40	55	55	95	40	80
Health Care	25	55	50	85	60	95
Industrials	30	70	70	85	85	95
Materials	25	45	65	90	55	95
Real Estate	15	40	90	90	65	90
Technology	40	75	30	50	5	20
Utilities	45	70	35	80	85	100

Note: For each of α, the value of our measure of tail asymmetry is computed for all distinct pairs in each sector. This table reports the percentage of the pairs whose CQCD ($\mathbb{C}$) statistic is close to zero. It is evident that this share is rising with α in the equity market while it's negligible in the cryptocurrency market.

4.2 Parametric Bootstraps

Our parametric bootstraps follow the algorithm in section 2.2. We, first, fit a bivariate copula to capture the joint distribution of returns.[8] Then we construct random samples form the fitted copulas. For each pair, these simulated samples have the same size of the associated observed sample in the data. We compute measures of tail asymmetry in each simulated sample and repeat this process 5000 times. This results in bootstrap-generated distributions of CQCD in each pair. As an example, figure 3 shows this distribution at $\alpha = 5$ for three pairs. As evident, the mean of this distribution in the cryptocurrency pair (3(a)) is considerably greater than the mean of equity pairs (figures 3(b) and 3(c)).



(a) (BTC, ETH)　(b) (LLY, JNJ)　(c) (JPM, BAC)

Figure 3: Distributions of CQCD for Sample Pairs ($\alpha = 5$)

Table 5 compares the average of mean and standard deviations of bootstrapped CQCDs across all sectors, which confirm that tail asymmetry is consistently larger in cryptocurrencies. For each pair, we also compute the quantile of this distribution for which tail asymmetry fades away -*i.e.,* $\mathbb{C} = 0$. Table 5 reports industry averages of this quantile at $\alpha = 5$ and $\alpha = 10$. The distribution of CQCD in equities is almost perfectly centered at zero as this quantile is almost exactly in the middle of the distribution. On the contrary, in cryptocurrencies, the average quantile where tail asymmetry is negligible is noticeably smaller, which confirms our first finding.

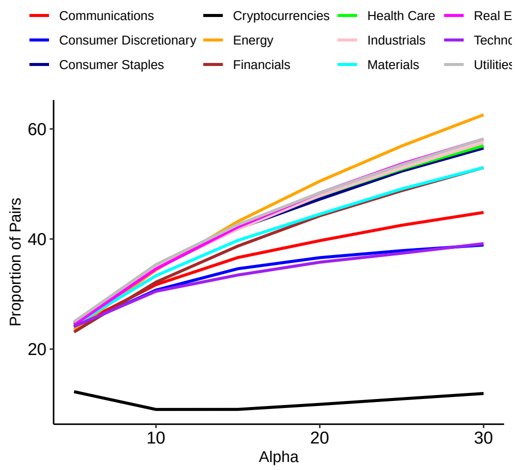
[8] We run this step for all 310 pairs individually. In the interest of brevity, we do not report the results in the paper but they are available upon request.

Table 5: Bootstraps of Conditional Tail Asymmetries

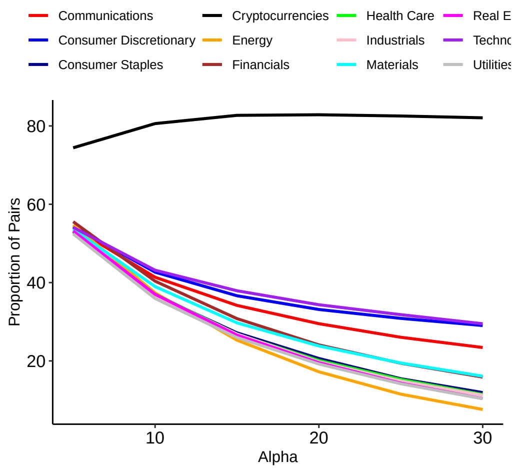
Sector	$\alpha = 5$			$\alpha = 10$		
	$\mathbb{E}[\mathbb{C}]$	$\text{Sd}(\mathbb{C})$	$\mathbb{E}\big[q(\mathbb{C}=0)\big]$	$\mathbb{E}[\mathbb{C}]$	$\text{Sd}(\mathbb{C})$	$\mathbb{E}\big[q(\mathbb{C}=0)\big]$
Cryptocurrency	0.156	0.123	0.139	0.178	0.084	0.103
Communications	0.027	0.156	0.426	0.031	0.110	0.393
Consumer Discretionary	0.043	0.153	0.389	0.049	0.109	0.331
Consumer Staples	0.003	0.157	0.491	0.006	0.110	0.478
Energy	0.000	0.165	0.500	0.000	0.112	0.501
Financials	0.009	0.167	0.472	0.012	0.115	0.452
Health Care	0.003	0.158	0.496	0.001	0.111	0.495
Industrials	0.000	0.159	0.501	0.000	0.111	0.499
Materials	0.011	0.157	0.471	0.014	0.110	0.452
Real Estate	0.002	0.159	0.500	0.000	0.111	0.500
Technology	0.036	0.152	0.402	0.044	0.107	0.342
Utilities	0.010	0.154	0.469	0.009	0.107	0.460

Note: This table presents a summary of our bootstrap simulations in each sector. After fitting bivariate copulas to each pair of assets, random sample was constructed and measures of tail asymmetry were computed. This process was repeated for $n = 5000$ time, which resulted in a distribution of CQCD's in each sector. This table reports some moments of this distribution in each sector for $\alpha = 5$ and $\alpha = 10$.

As for the second pattern, figure 4 shows the proportion of pairs with small (< 0.05) or large (> 0.10) tail asymmetries in each sector in our bootstraps. Figure 4(a) shows that as the tail criterion rises, the proportion of pairs with a small tail asymmetry (< 0.05) in the equity market steadily rises while in cryptocurrencies it all but falls. At the same time, the share of pairs with large CQCDs (> 0.10) unambiguously falls in all equity sectors while the rise of α has almost no impact on the portion of cryptocurrency pairs with large tail asymmetries (fig. 4(b)). In other words, including less extreme returns significantly weakens tail asymmetry in the equities while it doesn't affect the cryptocurrencies, which confirms our second finding.



(a) $|\mathbb{C}| < 0.05$



(b) $|\mathbb{C}| > 0.10$

Figure 4: Proportion of Pairs (%) with Small and Large Tail Asymmetries in Bootstraps

5 Conclusion

We study the asymmetry of tail dependence structures in both the equity and cryptocurrency market. While it is known that financial assets are more strongly correlated during the downfalls, our results uncover two differences in the nature of tail dependence structures in these two markets. We find that (1) the asymmetry of tail correlations is significantly stronger between cryptocurrencies.

We show this pattern by introducing a measure of asymmetry, which we call Conditional Quantile Correlation Difference (CQCD). What causes this disparity in the markets is that, overall, left-tail correlations tend to be stronger in cryptocurrencies while right-tail correlations are stronger in the equity market. We also show that (2) this asymmetry in the equity market is mostly limited to the tails that only include extreme events. We find that increasing the tail criterion leads to a noticeable fall in the value of CQCD in the equities. In cryptocurrencies, however, the asymmetry is observed throughout the entire joint distribution of returns.

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Appendix

Table 6: Descriptive Statistics of Return Series

Sector	Name	Ticker	N	Mean	std.	Min	Max	Skewness	Kurtosis
Cryptocurrency									
	Bitcoin	BTC	2379	0.001	0.038	-0.465	0.225	-0.793	15.800
	Ethereum	ETH	2379	0.001	0.047	-0.551	0.235	-0.926	14.000
	Binance Coin	BNB	2379	0.002	0.054	-0.543	0.529	0.407	20.400
	Ripple	XRP	2379	0.000	0.059	-0.551	0.607	1.140	23.400
	Cardano	ADA	2379	0.001	0.062	-0.504	0.862	1.970	30.000
	Chainlink	LINK	2379	0.002	0.065	-0.615	0.481	-0.066	11.000
	Bitcoin Cash	BCH	2379	0.000	0.061	-0.561	0.460	0.292	15.500
	EOS	EOS	2379	0.000	0.061	-0.504	0.440	-0.083	12.300
	Stellar	XLM	2379	0.000	0.057	-0.410	0.559	1.030	16.600
	Neo	NEO	2379	0.000	0.059	-0.466	0.346	-0.247	9.710
Communications									
	Comcast Corp.	CMCSA	1636	0.000	0.017	-0.096	0.118	-0.247	8.270
	Verizon Communications	VZ	1636	0.000	0.013	-0.078	0.089	0.022	8.840
	T-Mobile US Inc.	TMUS	1636	0.001	0.017	-0.119	0.111	0.130	11.300
	The Walt Disney Company	DIS	1636	0.000	0.020	-0.141	0.135	0.063	12.300
	Netflix Inc.	NFLX	1636	0.001	0.029	-0.433	0.156	-2.140	37.000
Consumer Discretionary									
	Tesla Inc.	TSLA	1636	0.001	0.040	-0.237	0.181	-0.120	6.720
	Amazon.com Inc.	AMZN	1636	0.001	0.022	-0.151	0.127	-0.131	7.080
	Home Depot	HD	1636	0.001	0.017	-0.221	0.129	-1.490	25.500
	Nike	NKE	1636	0.000	0.020	-0.137	0.144	-0.024	12.100
	McDonald's Corp	MCD	1636	0.000	0.014	-0.173	0.167	-0.288	35.500
Consumer Staples									
	Procter & Gamble	PG	1636	0.000	0.013	-0.091	0.113	0.020	14.600
	Coca-Cola Co.	KO	1636	0.000	0.013	-0.102	0.063	-0.939	13.200
	PepsiCo	PEP	1636	0.000	0.013	-0.121	0.122	-0.603	23.900
	Costco Wholesale	COST	1636	0.001	0.015	-0.133	0.095	-0.485	12.400
	Walmart Inc.	WMT	1636	0.000	0.014	-0.121	0.111	-0.197	19.200
Energy									
	Exxon Mobil Corp.	XOM	1636	0.000	0.020	-0.130	0.119	-0.179	8.450
	Chevron Corp.	CVX	1636	0.000	0.021	-0.250	0.205	-1.070	29.000
	ConocoPhillips	COP	1636	0.001	0.026	-0.286	0.225	-0.641	19.300
	EOG Resources	EOG	1636	0.000	0.028	-0.386	0.153	-1.450	26.500
	Occidental Petroleum	OXY	1636	0.000	0.037	-0.734	0.290	-4.130	97.800
Financials									
	Visa Inc.	V	1636	0.001	0.017	-0.146	0.130	-0.071	12.500
	JPMorgan Chase	JPM	1636	0.001	0.019	-0.162	0.166	-0.100	16.600
	Mastercard	MA	1636	0.001	0.019	-0.136	0.154	0.012	11.100
	Bank of America	BAC	1636	0.000	0.021	-0.167	0.164	-0.004	13.500
	Wells Fargo	WFC	1636	0.000	0.022	-0.173	0.136	-0.343	11.000
Healthcare									
	Eli Lilly and Co	LLY	1636	0.001	0.018	-0.105	0.146	1.010	13.300
	Johnson & Johnson	JNJ	1636	0.000	0.013	-0.106	0.077	-0.441	12.800
	UnitedHealth Group	UNH	1636	0.001	0.018	-0.190	0.120	-0.482	17.400
	Merck & Co.	MRK	1636	0.001	0.014	-0.104	0.080	-0.209	9.640
	AbbVie	ABBV	1636	0.000	0.017	-0.177	0.129	-1.280	19.500
Industrials									
	UPS	UPS	1636	0.000	0.018	-0.105	0.134	0.276	11.300
	Union Pacific Corp.	UNP	1636	0.001	0.017	-0.140	0.122	-0.295	13.400
	Caterpillar Inc.	CAT	1636	0.001	0.020	-0.154	0.098	-0.465	7.470
	Boeing	BA	1636	0.000	0.030	-0.272	0.218	-0.440	17.200
	Honeywell International	HON	1636	0.000	0.016	-0.129	0.140	-0.215	14.100
Materials									
	Linde	LIN	1636	0.001	0.016	-0.109	0.111	-0.091	8.830
	Sherwin-Williams	SHW	1636	0.001	0.018	-0.207	0.135	-0.757	18.700
	Freeport-McMoRan	FCX	1636	0.001	0.031	-0.199	0.260	-0.153	8.890
	Ecolab	ECL	1636	0.000	0.018	-0.125	0.200	0.122	20.600
	Air Products and Chemicals	APD	1636	0.000	0.018	-0.169	0.129	-1.190	18.200
Real Estate									
	Prologis	PLD	1636	0.000	0.018	-0.190	0.112	-0.727	14.300
	American Tower	AMT	1636	0.000	0.018	-0.164	0.115	-0.087	12.000
	Crown Castle	CCI	1636	0.000	0.017	-0.133	0.107	-0.185	9.200
	Simon Property Group	SPG	1636	0.000	0.027	-0.311	0.246	-1.010	30.900
	Equinix	EQIX	1636	0.000	0.018	-0.135	0.110	0.040	8.390
Technology									
	Microsoft Corp.	MSFT	1636	0.001	0.019	-0.159	0.133	-0.242	10.000
	Apple Inc.	AAPL	1636	0.001	0.020	-0.138	0.113	-0.217	8.190
	Alphabet Inc.	GOOG	1636	0.001	0.020	-0.118	0.099	-0.220	7.170
	NVIDIA Corporation	NVDA	1636	0.002	0.032	-0.208	0.218	-0.164	7.720
	Meta Platforms	META	1636	0.001	0.027	-0.306	0.209	-1.360	27.000
Utilities									
	NextEra Energy	NEE	1636	0.001	0.017	-0.144	0.128	-0.453	13.300
	Duke Energy	DUK	1636	0.000	0.014	-0.122	0.116	-0.195	17.000
	Southern Co.	SO	1636	0.000	0.015	-0.125	0.172	0.263	23.600
	Dominion Energy	D	1636	0.000	0.016	-0.131	0.159	-0.419	19.400
	Exelon Corp.	EXC	1636	0.000	0.017	-0.175	0.165	-0.285	24.000

Note: This table reports the descriptive statistics of log daily returns of assets in our sample from 9 Nov. 2017 to 15 May 2024.