Mohsin Ali Mirza K200353 2E-BSCS (1)

Q1 $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$

$$\lim_{h \to \infty} \int_h^2 e^t e^{-st} dt + \lim_{a \to \infty} \int_2^a 3e^{-st} dt$$

$$\lim_{h \to \infty} \int_h^2 e^{t(1-s)} dt + \lim_{a \to \infty} \int_2^a 3e^{-st} dt.$$

$$\lim_{h \to -\infty} \left| \frac{e^{t(1-s)}}{1-s} \right|_h^2 + \lim_{a \to \infty} \left| -\frac{3e^{-st}}{s} \right|_2^a$$

$$\boxed{\frac{e^{2(1-s)}}{1-s} + \frac{3e^{-2s}}{s}}$$

Mohsin Ali Mirza K200353 2E-BSCS (2)

Q2 $f(t) = 3+2t^2$

$$\mathcal{L}\{f(t)\} = \lim_{h\to\infty} \int_0^h (3+2t^2)e^{-st}\,dt$$

$$F(s) = $$

$3+2t^2$ (+)	e^{-st}
$4t$ (−)	$-\frac{e^{-st}}{s}$
4 (+)	$\frac{e^{-st}}{s^2}$
0	$-\frac{e^{-st}}{s^3}$

$$\lim_{h\to\infty} \left| (3+2t^2)\left(\frac{-e^{-st}}{s}\right) - \frac{4t\,(e^{-st})}{s^2} - \frac{4e^{-st}}{s^3} \right|_0^h$$

$$\lim_{h\to\infty} \left[\cancel{(3+2h^2)\frac{(e^{-hs})}{s}}^{0} - \cancel{\frac{4h\,(e^{-hs})}{s^2}}^{0} - \cancel{\frac{4e^{-hs}}{s^3}}^{0} - \left[(3)\frac{(e^{-s(0)})}{s} - \frac{4e^{-s(0)}}{s^2}\right.\right.$$

$$\boxed{F(s) = \frac{3}{s} + \frac{4}{s^3}}$$

Mohsin Ali Mirza K200353 2E-BSCS (3)

Q3 $f(t) = 5\sin 3t - 17e^{-2t}$

$$\lim_{h\to\infty} \int_0^h (5\sin 3t - 17e^{-2t})(e^{-st})dt$$

$$\lim_{h\to\infty} \int_0^h 5\sin 3t\, e^{-st} dt - 17\int_0^h e^{-2t-st} dt$$

$u' = 15\cos 3t$ $\quad v = \frac{-e^{-st}}{s}$

$v' = e^{-st}$ $\quad u = 5\sin 3t$

$$\int 5e^{-st}\sin 3t = -\frac{5e^{-st}}{s}\sin 3t + \int 15\cos 3t \frac{e^{-st}}{s}$$

$u' = -45\sin 3t$ $\quad v = -e^{-st}/s^2$

$v' = \frac{e^{-st}}{s}$ $\quad u = 15\cos 3t$

$$\int 5e^{-st}\sin 3t = -\frac{5e^{-st}}{s}\sin 3t - 15\cos 3t\frac{e^{-st}}{s^2} - \frac{45\sin 3t e^{-st}}{s^2}$$

$$\int 5e^{-st}\sin 3t + \frac{9}{s^2}\int 5e^{-st}\sin 3t = \left| -\frac{5e^{-st}}{s}\sin 3t - \frac{15\cos 3t e^{-st}}{s^2}\right|_0^h$$

$$\int 5e^{-st}\sin 3t\left(\frac{s^2+9}{\cancel{s^2}}\right) = \left(\frac{+15}{\cancel{s^2}}\right)$$

$$\int_0^h 5e^{-st}\sin 3t = \frac{+15}{s^2+9}$$

$$-17\int_0^h e^{t(-2-s)}dt$$

$$17\lim_{h\to\infty}\left|\frac{e^{t(-2-s)}}{-2-s}\right|_0^h$$

$$= \frac{-17}{2+s}$$

$$\boxed{= \frac{15}{s^2+9} - \frac{17}{2+s}}$$

Mohsin Ali Mirza K200353 2E-BSCS (4)

Q4 $f(t) = te^{4t}$

$$L\{f(t)\} = \lim_{h\to\infty} \int_0^h (te^{4t})e^{-st}$$

$$\lim_{h\to\infty} \int_0^h te^{t(4-s)}\,dt$$

t	$e^{t(4-s)}$
1	$\frac{e^{t(4-s)}}{4-s}$
0	$\frac{e^{t(4-s)}}{(4-s)^2}$

$$\lim_{h\to\infty} \left[\frac{te^{t(4-s)}}{4-s} - \frac{e^{t(4-s)}}{(4-s)^2} \right]_0^h$$

$$\lim_{h\to\infty} \left[\cancel{\frac{he^{h(4-s)}}{4-s} - \frac{e^{h(4-s)}}{(4-s)^2}} \right]^0 + \frac{1}{(4-s)^2}$$

$$\boxed{F(s) = \frac{1}{(4-s)^2}}$$

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Q5 $L^{-1}\left\{ \frac{1}{s(s^2+2s+5)} \right\}$

$$\frac{A}{s} + \frac{Bs+C}{s^2+2s+5} = 1$$

$$A(s^2+2s+5) + (Bs+C)s = 1$$

$A+B=0$, $A = 1/5$, $B = -1/5$, $C = -2/5$

$2A+C=0$

$5A=1$

$$\frac{1}{5}\left(\frac{1}{s} - \frac{(s+2)}{s^2+2s+5} \right)$$

$$L^{-1}\left\{ \frac{1}{5}\left(\frac{1}{s} - \frac{(s+1)}{(s+1)^2+(2)^2} - \frac{1}{(s+1)^2+(2)^2} \right) \right\}$$

$$\cancel{L^{-1}\left\{\frac{1}{5}\right.} \boxed{\frac{1}{5}\left(1 - e^{-t}\cos 2t - \frac{1}{2}\sin 2t\, e^{-t} \right)}$$

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Q6 $L^{-1}\left\{\dfrac{7s-1}{(s+1)(s+2)(s-3)}\right\}$

$$\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s-3}=7s-1$$

$$A(s+2)(s-3)+B(s+1)(s-3)+C(s+1)(s+2)=7s-1$$

Let $s=-2$, $B=-3$
$s=+3$, $C=1$
$s=-1$, $A=2$

$$\frac{2}{s+1}-\frac{3}{s+2}+\frac{1}{s-3}$$

$$L^{-1}\left\{\frac{2}{s+1}-\frac{3}{s+2}+\frac{1}{s-3}\right\}$$

$$\boxed{f(t)=2e^{-t}-3e^{-2t}+e^{3t}}$$

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Q7 $\mathcal{L}^{-1}\left\{\dfrac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$

$$\frac{(As+B)}{(s-1)^2} + \frac{C}{(s+3)} \equiv s^2+9s+2$$

$$(As+B)(s+3) + C(s-1)^2$$

$$As^2+3As+Bs+3B+Cs^2-2Cs+C \equiv s^2+9s+2$$

$$s^2(A+C)+s(3A+B-2C)+3B+C \equiv s^2+9s+2$$

$A+C=1$, $A=2$, $B=1$, $C=-1$

$3A+B-2C=9$

$3B+C=2$

$$\frac{2s+1}{(s-1)^2} - \frac{1}{s+3}$$

$$\cancel{\frac{2s}{(s-1)^2}}\ \mathcal{L}^{-1}\left\{2\left(\frac{1}{s-1}\right)+\frac{3}{(s-1)^2}-\frac{1}{s+3}\right\}$$

$$\boxed{f(t) = 2e^t + 3te^t - e^{-3t}}$$

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Q8

$$\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{s^2-2s+5\,(s+1)}\right\}$$

$$\cancel{\frac{A}{s^2-2s+5}} \quad \frac{A}{s+1}+\frac{Bs+C}{s^2-2s+5} \equiv 2s^2+10s$$

$$As^2-2As+5A+Bs^2+Bs+Cs+C \equiv 2s^2+10s$$

$$s^2(A+B)+s(-2A+B+C)+5A+C \equiv 2s^2+10s$$

$A+B=2$ $A=-1,\ B=3,\ C=5$

$-2A+B+C=10$

$5A+C=0$

$$\mathcal{L}^{-1}\left\{\frac{-1}{s+1}+\frac{3s-3}{(s-1)^2+(2)^2}+\frac{8}{(s-1)^2+(2)^2}\right\}$$

$\cancel{-e^{-t}+3\cos t\, e^{t}+4\sin 2t\, e^{t}}$

$$\boxed{-e^{-t}+3\cos 2t\, e^{t}+4\sin 2t\, e^{t}}$$

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Q9 $y' - 5y = e^{5x}$ $y(0) = 0$

$\mathcal{L}\{y' - 5y\} = \mathcal{L}\{e^{5x}\}$

$sY(s) - y(0) - 5Y(s) = \dfrac{1}{s-5}$

$Y(s)[s-5] = \dfrac{1}{s-5}$

$Y(s) = \dfrac{1}{(s-5)^2}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\dfrac{1}{(s-5)^2}\right\}$

$\boxed{y = xe^{5x}}$

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$y' - 5y = e^{5x}$

$I = e^{\int -5\,dx}$

$= e^{-5x}$

$ye^{-5x} = \int \cancel{e^{5x}} \cdot \cancel{e^{-5x}}\,dx + C$

$ye^{-5x} = x + C$; $y(0) = 0$

$0 = 0 + C$

$\boxed{C = 0}$

$\boxed{y = xe^{5x}}$ Proved.

mohsin Ali Mirza K200353 2E+3SCS (1)

Q:10 $y' + y = \sin x$ $y(0) = 1$

$\cancel{sY(s) + y(0) + Y(s)}$

$L\{y' + y\} = L\{\sin x\}$

$sY(s) - y(0) + Y(s) = \frac{1}{s^2+1}$

$Y(s)[s+1] - 1 = \frac{1}{1+s^2}$

$Y(s) = \frac{1}{s+1}\left[\frac{1}{1+s^2} + 1\right]$

$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{(s^2+1)(s+1)} + \frac{1}{s+1}\right\}$

$\frac{As+B}{s^2+1} + \frac{C}{s+1} = 1$

$(As+B)(s+1) + C(s^2+1) = 1$

$As^2 + As + Bs + B + Cs^2 + C = 1$

$s^2(A+C) + s(A+B) + B + C = 1$

$A + C = 0$ $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$

$A + B = 0$

$B + C = 1$

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$$y = L^{-1}\left\{\frac{-\frac{1}{2}s+\frac{1}{2}}{s^2+1} + \frac{0.5}{s+1}\right\} + e^{-x}$$

$$y = L^{-1}\left\{0.5\frac{(-s+1)}{s^2+1} + \frac{0.5}{s+1}\right\} + e^{-x}$$

$$y = L^{-1}\left\{0.5\left[\left(\frac{-s}{s^2+1}\right) + \frac{1}{s^2+1}\right] + \frac{0.5}{s+1}\right\} + e^{-x}$$

$$\cancel{y = L^{-1}\left\{ \frac{1}{c} \right.}$$

$$y = -\frac{1}{2}\cos x + \frac{1}{2}\sin x + \frac{1}{2}e^{-x} + e^{-x}$$

$$\boxed{y = -\frac{1}{2}\cos x + \frac{1}{2}\sin x + \frac{3}{2}e^{-x}}$$

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$$y' + y = \sin x$$

+	e^x	$\sin x$
−	e^x	$-\cos x$
+	e^x	$-\sin x$

$$I = e^{\int 1\,dx}$$
$$= e^x$$

$$y e^x = \int \sin x \, e^x \, dx + C$$

$$\int \sin x \, e^x = \frac{1}{2}\left(-e^x \cos x + e^x \sin x\right) + C$$

$$y \cancel{e^x} = \frac{\cancel{e^x}}{2}\left(-\cos x + \sin x + \frac{C}{e^x}\right)$$

$$y = \frac{-\cos x}{2} + \frac{\sin x}{2} + \frac{C}{e^x} \quad ; \quad y(0) = 1$$

$$1 = -\frac{1}{2} + \cancel{\frac{\sin(0)}{2}} + C \qquad \boxed{C = \frac{3}{2}}$$

$$\boxed{y = \frac{1}{2}\left(-\cos x + \sin x + 3e^{-x}\right)}$$

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Q3 $y'' - y' = 2x$ $y(0) = 1,\ y'(0) = 2$

$L\{y'' - y'\} = L\{2x\}$

$\cancel{s^2 Y(s)} - y(0) - sy$

$s^2 Y(s) - s y(0) - y'(0) - sY(s) + y(0) = \frac{2}{s^2}$

$Y(s)[s^2 - s] - s + 2 + 1 = \frac{2}{s^2}$

$Y(s)[s^2 - s] = \frac{2}{s^2} + s - 3$

$Y(s) = \frac{2}{s^3(s-1)} + \frac{s}{s(s-1)} - \frac{3}{s(s-1)}$

$Y(s) = \frac{2}{s^3(s-1)} - \frac{3}{s(s-1)} + \frac{1}{s-1}$

$\cancel{\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s-1}} \equiv$

$\cancel{Y(s) = \frac{1}{s(s-1)}\left[\frac{2}{s^3} - 3\right] + \frac{1}{s-1}}$

$\cancel{Y(s) = \frac{2 - 3s^3}{s^3(s)(}}$

$$y(s) = \frac{1}{s(s-1)}\left[\frac{2}{s^2} - 3\right] + \frac{1}{s-1}$$

$$y(s) = \frac{2 - 3s^2}{s^3(s-1)} + \frac{1}{s-1}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1} \equiv 2-3s^2$$

$$As^3 - As^2 + Bs^2 - Bs + Cs - C + Ds^3 \equiv 2-3s^2$$

$A+D=0$ $\qquad A=1,\ B=-2,\ C=-2,\ D=-1$

$-A+B=-3$

$-B+C=0$

$-C=2$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{2}{s^2} - \frac{2}{s^3} \cancel{- \frac{1}{s-1}} + \cancel{\frac{1}{s-1}}\right\}$$

$$\boxed{y = 1 - 2x - x^2}$$

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Q3 $y'' - y' = 2x$

$m^2 - m = 0$

$m = 0, \quad m = 1$

$y_c = C_1 e^{0x} + C_2 e^x$

$y_c = C_1 + C_2 e^x$

$y_p = x^r(Ax + B), \quad r = 1$

$y_p = Ax^2 + Bx$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A - 2Ax - B = 2x$

$2A - B = 0, \quad -\cancel{2}A = \cancel{2}$

$2(-1) - B = 0 \quad \boxed{A = -1}$

$\boxed{B = -2}$

$y = C_1 + C_2 e^x - x^2 - 2x \; ; \; y(0) = 1$

$y' = C_2 e^x - 2x - 2 \; ; \; y'(0) = -2$

$1 = C_1 + \cancel{C_2}^{0} \qquad -\cancel{2} = C_2 - \cancel{2}$

$\boxed{C_1 = 1} \qquad \boxed{C_2 = 0}$

$\boxed{y = 1 - x^2 - 2x}$

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Q4

$L\{y'' - 2y' + 5y\} = L\{8e^{7-x}\}$ $y(7) = 2,\ y'(7) = 12$

$s^2 Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = \frac{-8e^7}{s+1}$

$Y(s)[s^2 - 2s + 5] - 2s - 12 + 4 = -\frac{8e^7}{s+1}$

$Y(s)[s^2 - 2s + 5] = \frac{-8e^7}{s+1} + 2s + 8$

$Y(s) = \frac{1}{s^2 - 2s + 5}\left[\frac{-8e^7}{(s+1)} + 2s + 8\right]$

$Y(s) = \frac{-8e^7}{s^2 - 2s + 5(s+1)} + \frac{2s + \cancel{4} - 2}{(s-1)^2 + (2)^2} + \frac{8+2}{(s-1)^2 + (2)^2}$

$Y(s) = \frac{-8e^7}{s^2 - 2s + 5\,(s+1)} + \frac{2(s-1)}{(s-1)^2 + (2)^2} + \frac{10}{(s-1)^2 + (2)^2}$

$\frac{A}{s+1} + \frac{Bs + C}{s^2 - 2s + 5} = 1$

$A = \frac{1}{8}$, $B = -\frac{1}{8}$, $C = \frac{3}{8}$

$-e^7\left(\frac{1}{s+1} - \frac{s+3}{(s-1)^2 + (2)^2}\right) + 2\left[\frac{(s-1)}{(s-1)^2 + (2)^2}\right] + 10\left[\frac{1}{(s-1)^2 + (2)^2}\right]$

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$$Y(s) = -e^7\left(\frac{1}{s+1} - \overset{\cos}{\frac{s-1}{(s-1)^2+(2)^2}} + 2\,\overset{\sin}{\frac{2}{(s-1)^2+(2)^2}}\right) + 2\left[\overset{\cos}{\frac{s-1}{(s-1)^2+(2)^2}}\right] + 5\left[\overset{\sin}{\frac{2}{(s-1)^2+(2)^2}}\right]$$

$$L^{-1}\{Y(s)\} = -e^7 L^{-1}\left\{\frac{1}{s+1} - \frac{s-1}{(s-1)^2+2^2} + 2\left[\frac{2}{(s-1)^2+(2)^2}\right]\right\} +$$

$$2\left(\frac{s-1}{(s-1)^2+(2)^2}\right) + 5\left(\frac{2}{(s-1)^2+(2)^2}\right)$$

$$y = -e^7\left(e^{-x} - \cos 2x\, e^x + 2\sin 2x\, e^x\right) +$$

$$2\cos 2x\, e^x + 5\sin 2x\, e^x$$

$$\cancel{y = -e^{7-x} - e^{7+x}\left(-\cos 2x + 2\sin x + 2\cos 2x + 5\sin 2x\right)}$$

$$\cancel{y = -e^{7-x} - e^{7+x}\left(\cos 2x + 7\sin 2x\right)}$$

$$\boxed{y = -e^{7-x} + \cos 2x\, e^x\left(2+e^7\right) + e^x \sin 2x\left(5-e^7\right)}$$

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Q4 $y'' - 2y' + 5y = -8e^7(e^{-x})$

$m^2 - 2m + 5 = 0 \quad m = 1 \pm 2i$

$y_c = e^x(A\cos 2x + B\sin 2x)$

$y_p = Ae^{-x}$
$y_p' = -Ae^{-x}$
$y_p'' = Ae^{-x}$

$Ae^{-x} + 2Ae^{-x} + 5Ae^{-x} = -8e^7(e^{-x})$

$A + 2A + 5A = -8e^7$
$\cancel{8}A = -\cancel{8}e^7$
$\boxed{A = -e^7}$

$y = e^x(A\cos 2x + \cancel{B\sin 2x}^{0}) - e^7e^{-x} \quad y(0)=2,\ y'(0)=12$

$y' = e^x(A\cos 2x + \cancel{B\sin 2x}^{0}) + (\cancel{-2A\sin 2x}^{0} + 2B\cos 2x)e^x$
$+ e^7e^{-x}$

$y'' = e^x(A\cos 2x + B\sin 2x) + 2(-2A\sin 2x + 2B\cos 2x)e^x +$
$(-4A\cos 2x - 4B\sin 2x)e^x - e^7e^{-x}$ (Not required)

$2 = A - e^7 \qquad 12 = A + 2B + e^7$
$A = 2 + e^7 \qquad 12 = 2 + e^7 + 2B + e^7$
$5\ \cancel{\frac{10}{2}} = B + e^7 \qquad \boxed{B = 5 - e^7}$

$\boxed{y = e^x((2+e^7)\cos 2x + (5-e^7)\sin 2x) - e^{7-x}}$