E200353 Mohsin Ali Mirza Ex 4.4 Q11, 14

Ex4.1

Q2 a $u = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \; v = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

i) $u + v = \begin{bmatrix} 0+1+1 \\ 4-3+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

ii) $ku = k \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \; 2 \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$

b $u + 0 = u$

$\begin{bmatrix} 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+1 \\ 4+0+1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 5 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

c $\begin{bmatrix} 0 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0-\cancel{1}+\cancel{1} \\ 4-\cancel{1}+\cancel{1} \end{bmatrix}$

$\boxed{\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}}$

d $u + (-u) = 0$

$\begin{bmatrix} 0 \\ 4 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 4 \end{bmatrix} \cancel{- \begin{bmatrix} -4 \\ -4 \end{bmatrix}} \; \begin{bmatrix} 0-2+1 \\ 4-6+1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Q3 1

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$$(-u)+u=0$$

$$\begin{bmatrix}-2\\-6\end{bmatrix}+\begin{bmatrix}0\\4\end{bmatrix}=\begin{bmatrix}-2+0+1\\-6+4+1\end{bmatrix}=\begin{bmatrix}-1\\-1\end{bmatrix}$$

ë Anxiom 7

$$k(u+v)=ku+kv$$

$$2\left(\begin{bmatrix}0\\4\end{bmatrix}+\begin{bmatrix}1\\-3\end{bmatrix}\right)=2\begin{bmatrix}0\\4\end{bmatrix}+2\begin{bmatrix}1\\-3\end{bmatrix}$$

$$2\begin{bmatrix}2\\2\end{bmatrix}=\begin{bmatrix}0\\8\end{bmatrix}+\begin{bmatrix}2\\-6\end{bmatrix}$$

$$\begin{bmatrix}4\\4\end{bmatrix}\neq\begin{bmatrix}3\\3\end{bmatrix}$$ Anxiom 7 Fails

$K\neq m$

ë Anxiom 7:-

$$k\left(\begin{bmatrix}0\\4\end{bmatrix}+\begin{bmatrix}1\\-3\end{bmatrix}\right)=k\begin{bmatrix}0\\4\end{bmatrix}+k\begin{bmatrix}1\\-3\end{bmatrix}$$

$$\begin{bmatrix}2k\\2k\end{bmatrix}\neq\begin{bmatrix}k+1\\k+1\end{bmatrix}$$ $\boxed{\text{Anxiom 7 fails}}$.

Anxiom 8:-

$$(k+m)u=ku+mu$$

$$(k+m)\begin{bmatrix}0\\4\end{bmatrix}=k\begin{bmatrix}0\\4\end{bmatrix}+m\begin{bmatrix}0\\4\end{bmatrix}$$

$$\begin{bmatrix}0\\4(k+m)\end{bmatrix}\neq\begin{bmatrix}0\\4k+4m+1\end{bmatrix}$$ $\boxed{\text{Anxiom 8 Fails}}$

COPY

Q9 $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Anxiom 1:- $u+v \in V$

$$\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

Anxiom 2:-

$u+v = v+u$

$$\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} = \begin{bmatrix} a_2+a_1 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

Anxiom 3:- $u+v+w = u+(v+w)$

$$\begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \left(\begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix} \right)$$

$$\begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & 0 \\ 0 & b_2+b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix} = \begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix}$$

Anxiom 4:-

$0+u=u$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

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Axiom 5:- $u+(-u) = (-u)+u = 0$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} = \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Axiom 6:- $ku \in V$

$$k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix}$$

Axiom 7:- $k(u+v) = ku+kv$

$$k\begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} = k\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + k\begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

$$\begin{bmatrix} k(a_1+b_1) & 0 \\ 0 & k(b_1+b_2) \end{bmatrix} = \begin{bmatrix} ka_1 & 0 \\ 0 & kb_1 \end{bmatrix} + \begin{bmatrix} ka_2 & 0 \\ 0 & kb_2 \end{bmatrix}$$

$$\begin{bmatrix} k(a_1+b_1) & 0 \\ 0 & k(b_1+b_2) \end{bmatrix} = \begin{bmatrix} k(a_1+a_2) & 0 \\ 0 & k(b_1+b_2) \end{bmatrix}$$

Axiom 9:- $k(mu) = (km)u$

$$k\begin{bmatrix} ma & 0 \\ 0 & mb \end{bmatrix} = km\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\begin{bmatrix} kma & 0 \\ 0 & kmb \end{bmatrix} = \begin{bmatrix} kma & 0 \\ 0 & kmb \end{bmatrix}$$

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Anxiom 8:-

$(k+m)u = ku + mu$

$$(k+m)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + m\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\begin{bmatrix} (k+m)a & 0 \\ 0 & (k+m)b \end{bmatrix} = \begin{pmatrix} (k+m)a & 0 \\ 0 & (k+m)b \end{pmatrix}$$

Anxiom 10:

$1u = u$

$$1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

All the anxioms hold therefore u is a vector space.

Q11 $u = \begin{bmatrix} 1 \\ x \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ y \end{bmatrix}$

Anxiom1- $u+v = \begin{bmatrix} 1 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ x+y \end{bmatrix} \in V$

Anxiom6:- $k \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ kx \end{bmatrix} \in V$

Anxiom3:-

$u+v+w = (u+(v+w))$

$$\begin{bmatrix} 1 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ z \end{bmatrix} = \left(\begin{bmatrix} 1 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ y \end{bmatrix}\right) + \begin{bmatrix} 1 \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x+y+z \end{bmatrix} = \begin{bmatrix} 1 \\ x+y \end{bmatrix} + \begin{bmatrix} 1 \\ y+z \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x+y+z \end{bmatrix} = \begin{bmatrix} 1 \\ x+y+z \end{bmatrix}$$

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Anxiom 2:-

$u+v = v+u$

$u+v = v+u$

$$\begin{bmatrix} 1 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ x \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ x+y \end{bmatrix}$$

Anxiom 4:-

$0+u = u$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ x \end{bmatrix}$$

Anxiom 5:-

$u+(-u) = (-u)+u = 0$

$$\begin{bmatrix} 1 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ -x \end{bmatrix} + \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Anxiom 7:-

$k(u+v) = ku+kv$

$$k\begin{bmatrix} 1 \\ x+y \end{bmatrix} = k\begin{bmatrix} 1 \\ x \end{bmatrix} + k\begin{bmatrix} 1 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ k(x+y) \end{bmatrix} = \begin{bmatrix} 1 \\ k(x+y) \end{bmatrix}$$

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Axiom 8:-

$(k+m)u = ku + mu$

$$(k+m)\begin{bmatrix} \frac{1}{x} \end{bmatrix} = k\begin{bmatrix} \frac{1}{x} \end{bmatrix} + m\begin{bmatrix} \frac{1}{x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{(k+m)x} \end{bmatrix} = \begin{bmatrix} \frac{1}{\cancel{kx}\ k+m(x)} \end{bmatrix}$$

Axiom 9:-

$k(mu) = (km)u$

$$k\left[m\left(\frac{1}{x}\right)\right] = km\begin{bmatrix} \frac{1}{x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{kmx} \end{bmatrix} = \begin{bmatrix} \frac{1}{kmx} \end{bmatrix}.$$

Axiom 10:-

$1u = u$

$$1\begin{bmatrix} \frac{1}{x} \end{bmatrix} = \begin{bmatrix} \frac{1}{x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \end{bmatrix} = \begin{bmatrix} \frac{1}{x} \end{bmatrix}$$

All axioms hold, therefore u is a vector space.

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Q12

$(a_0+a_1x)+(b_0+b_1x)=(a_0+b_0)+(a_1+b_1)x$

taking polynomials as vectorized format.

Axiom 1:-

$u+v \in V$

$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+\begin{bmatrix} b_0 \\ b_1 \end{bmatrix}=\begin{bmatrix} a_0+b_0 \\ a_1+b_1 \end{bmatrix}\begin{bmatrix} 1 \\ x \end{bmatrix} \in V$

Axiom 2:-

$u+v=v+u$

$\begin{bmatrix} a_0+b_0 \\ a_1+b_1 \end{bmatrix}=\begin{bmatrix} b_0+a_0 \\ b_1+a_1 \end{bmatrix}$

Axiom 3:-

$u+v+w=u+(v+w)$

$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+\begin{bmatrix} b_0 \\ b_1 \end{bmatrix}+\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}=\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+\left(\begin{bmatrix} b_0 \\ b_1 \end{bmatrix}+\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}\right)$

$\begin{bmatrix} a_0+b_0+c_0 \\ a_1+b_1+c_1 \end{bmatrix}=\begin{bmatrix} a_0+b_0+c_0 \\ a_1+b_1+c_1 \end{bmatrix}$

Axiom 4:-

$u+0=u$

$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \end{bmatrix}=\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

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Axiom 5:-

$u+(-u)=(-u)+u=0$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+\begin{bmatrix} -a_0 \\ -a_1 \end{bmatrix}=\begin{bmatrix} -a_0 \\ -a_1 \end{bmatrix}+\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Axiom 6:-

$ku \in V$

$$k\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}=\begin{bmatrix} ka_0 \\ ka_1 x \end{bmatrix} \in V$$

Axiom 7:-

$k(u+v)=ku+kv$

$$k\begin{bmatrix} a_0+b_0 \\ a_1+b_1 \end{bmatrix}=k\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+k\begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$\begin{bmatrix} k(a_0+b_0) \\ k(a_1+b_1) \end{bmatrix}=\begin{bmatrix} k(a_0+b_0) \\ k(a_1+b_1) \end{bmatrix}$$

Axiom 8:-

$(k+m)u = ku+mu$

$$(k+m)\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}=k\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}+k\begin{bmatrix} \cancel{a_0} \\ \cancel{a_1} \end{bmatrix}$$

$$\begin{bmatrix} (k+m)a_0 \\ (k+m)a_1 \end{bmatrix}=\cancel{k+a}\begin{bmatrix} (k+m)a_0 \\ (k+m)a_1 \end{bmatrix}$$

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Anxiom 9:-

$(km)u = k(mu)$

$$km\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = k\left(m\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}\right)$$

$$\begin{bmatrix} kma_0 \\ kma_1 \end{bmatrix} = \begin{bmatrix} kma_0 \\ kma_1 \end{bmatrix}$$

All anxioms hold therefore u is a vector space.

Anxiom 10:-

$1u = u$

$$1\cdot\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Q4.2

Q3 a Anxiom1:-

$u+v = \in V$

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ 0 & b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{mn} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ 0 & a_{22}+b_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

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$$k\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} ka_{11} & 0 & \cdots & 0 \\ 0 & ka_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & ka_{mn} \end{bmatrix}$$ Axiom 6

It is a ~~vector~~ Subspace

b let $u = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ Axiom 1

$u+v \in V$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\det(u+v) \neq 0$, therefore it is not a subspace

c $A = [a_{ij}]$, $B = [b_{ij}]$

$tr(A) = a_{11} + a_{22} + \cdots + a_{mn} = 0$ Axiom 1:- $u+v \in V$

$tr(B) = b_{11} + b_{22} + \cdots + b_{mn} = 0$

$tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) \cdots + (a_{mn}+b_{mn}) = 0+0 = 0$

$k\,a_{mn} = k(a_{11} + a_{22} + \cdots + a_{mn}) = 0$, Axiom 6 $ku \in V$

therefore It is a subspace

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(i) $(\cancel{A+B})$ $A^T = A$, $B^T = B$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{12}+b_{12} & a_{22}+b_{22} \end{pmatrix}^T$$

$(A+B)^T = A+B \in V$

Axiom 1 holds $(u+v \in V)$

$$k \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{12} & ka_{22} \end{pmatrix}^T = \begin{matrix} ka_{11} & ka_{12} \\ ka_{12} & ka_{22} \end{matrix}$$

Axiom 6 hold's $(ku \in V)$

$\boxed{\text{It is a Subspace}}$

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Q4 a:- $\begin{bmatrix} a_{11} & \\ & a_{22} \end{bmatrix}$ $(A+B)$

$(A+B)^T = A^T + B^T = A+B$

$A^T = -A \quad B^T = -B$

$A^T + B^T = (A+B)^T$

$-A-B = -(A+B) \in V$

$k(A^T)^T = (kA^T) \in V$

It is a Subspace

c:- $AB = BA$

$AB + CB = AB+CB$

$= (A+C)B$

$BA + BC = BA+BC$

$= B(A+C)$

$B(A+C) = (A+C)B$

$k(AB) = kBA$

It is a subspace.

'b' and 'd' done on next page

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Q4 b) $Ax = 0$

$ku \in V$

$k = 0$, $k(Ax) = 0$, det

$k(A) = k$

$k(A) \notin V$, because $\det(A) \neq 0$

therefore Axiom 6 fails to hold and it is not a subspace.

c) A^{-1},

$k = 0$

$ku \in V$

$k(A) \notin V$ because $\det(A) \neq 0$

therefore Axiom 6 fails to hold and it is not a subspace.

Q14 a) $Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

st $ku \in V$ and $k = 0$

$k(x) = 0$ $Ax \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

therefore Axiom 6 fails to hold and it is not a subspace.

b) $A(x+y) - Ax + ay = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Anxiom 1 holds $u+v \in V$

$A(kx) = kAx = k\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Anxiom 6 holds $ku \in V$

$\boxed{\text{It is a subspace}}$

Q16 a) $\cancel{k=0}$ $k = -1/2$

$\cancel{ku}$ $ku \in V$

$k\begin{bmatrix} a_0 \\ a_1 x \end{bmatrix} \notin V$

no longer remains an even coefficient so Not a subspace

b) $\overset{1}{a_0} + \overset{-1}{a_1}x + \overset{1}{a_2}$

$u+v \in V$

$\cancel{\begin{bmatrix} a_0 \\ -a_1 \end{bmatrix} + \begin{bmatrix} b_0 \\ -b_1 \end{bmatrix} = \begin{bmatrix} a_0+b_0 \\ -a_0-b_0 \end{bmatrix}} \in V$

$\begin{bmatrix} a_0 \\ -a_0 \end{bmatrix} + \begin{bmatrix} b_0 \\ -b_0 \end{bmatrix} = \begin{bmatrix} a_0+b_0 \\ -(a_0+b_0) \end{bmatrix} \in V$

$ku \in V$

$k\begin{bmatrix} a_0 \\ -a_0 \end{bmatrix} = \begin{bmatrix} ka_0 \\ -ka_0 \end{bmatrix} \in V$ It is a subspace

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$ku \in V$

$$k\begin{bmatrix} a_0 \\ a_1x^2 \end{bmatrix} = \begin{bmatrix} ka_0 \\ ka_1x^2 \end{bmatrix}$$ Axiom 6 holds

$u+v \in V$

$$\begin{bmatrix} a_0 \\ a_1x^2 \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1x^2 \end{bmatrix} = \begin{bmatrix} a_0+b_0 \\ x^2(a_1+b_1) \end{bmatrix} \in V$$ Axiom 1 holds

It is a subspace.

Ex 4.3

S is a

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = R_2 - R_1 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$-R_2$

multiple $(R_1 - R_2)$ $$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, R_3 - R_2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} s \\ t \\ -s \\ -t \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$v_1 = u$ $\quad$ $v_2 = v$

therefore they span the solution w.

b $u = v_1$

$$V = v_1 + v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

therefore u & v span the solution W.

Q.6 a

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$$R_2 - 2R_1 \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$$R_3 - 3R_1 \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} r \\ s-t \\ s \\ t \end{bmatrix} = r \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{v_1} + s \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{v_2} + t \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{v_3}$$

Therefore they donot span the solution W, because

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

, no value of a and b can give $(1,0,0,0)$ so v_1 is not in the span.

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b'

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = a\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

, no value of a and b can give $(1, 0, 0, 0)$ so v_1 is not in the span of vectors

therefore they donot span the solution W.

Q17 a

$T_A(1,2) = (-1,4)$

$B.T_A(-1,1) = (-2,2)$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = k_1\begin{bmatrix} -1 \\ 4 \end{bmatrix} + k_2\begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\det = \begin{array}{cc} k_1 & k_2 \\ \end{array} \begin{vmatrix} -1 & -2 \\ 4 & 2 \end{vmatrix} = 6 \neq 0$$

therefore the system is consistent for all side vectors b and it spans R^2.

b) $T_A(1,2) = (-1,2)$ $T(-1,1) = (-2,4)$

$$\left(\begin{array}{cc|c} k_1 & k_2 & \\ -1 & -2 & b_1 \\ 2 & 4 & b_2 \end{array}\right)$$

$$\det = \begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} = 0$$ therefore the sytem is not consistent

and $T_A(u_1)$ and $T_A(u_2)$ donot span R^2

~~Q5~~ Ex 4.4

Q11

$$\det = \left|\begin{array}{ccc|cc} K_1 & K_2 & K_3 & & \\ \lambda & -1/2 & -1/2 & \lambda & -1/2 \\ -1/2 & \lambda & -1/2 & -1/2 & \lambda \\ -1/2 & -1/2 & \lambda & -\frac{1}{2} & -1/2 \end{array}\right|$$

$$\det \begin{array}{ccc|cc} k_1 & k_2 & k_3 & & \\ \lambda & -1/2 & -1/2 & \lambda & -1/2 \\ -1/2 & \lambda & -1/2 & -1/2 & \lambda \\ -1/2 & -1/2 & \lambda & -1/2 & -1/2 \end{array}$$

$\det = \lambda^3 - \frac{3}{4}\lambda - \frac{1}{4} = 0$ $\quad (\lambda + \frac{1}{2})(\lambda + \frac{1}{2})(\lambda - 1) = 0$

$\boxed{\lambda = -\frac{1}{2}}, \boxed{\lambda = 1}$

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Q#4 i) $T_A(1,0,0) = (1,1,2)$

$T_A(2,-1,1) = (3,-1,2)$

$T_A(0,1,1) = (3,-3,2)$

$$\begin{array}{ccc} k_1 & k_2 & k_3 \end{array}$$

$$\begin{vmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = -8 \neq 0$$

therefore the system only has trivial solution and $T_A(u_1), T_A(u_2), T_A(u_3)$ are linearly independent

ii) $T_A = (1,0,0) = (1,1,2)$, ~~7~~

$T_A = (2,-1,1) = (2,-2,2)$

$T_A = \cancel{(0,1,1) = (}$ $T_A(0,1,1) = (2,-2,2)$

$T_A(u_2) = 1\,T_A(u_3)$ therefore the system is linearly dependent.