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Random Variable:

It is a ~~real~~ function that ~~of~~ associates a real no. which ~~is~~ each statement in sample

Discrete Prob Distribution

eg: if a coin is tossed 2 time ~~to~~ X = no. of Heads.

X		PD P(X) = P(F)	Cumulative Freq F(x)
0	TT	1/4	1/4
1	HT, ~~H~~TH	2/4	3/4
2	HH	1/4	4/4

DPD is also called Probability marks function

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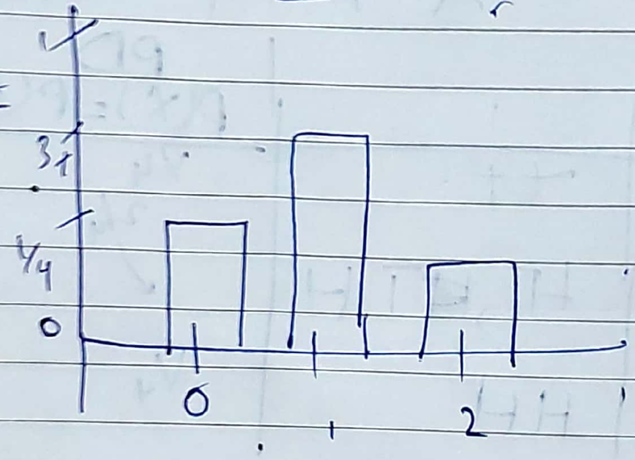
$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1/4 & x \geq 0 \ \& \ x \leq 1 \\ 3/4 & x \geq 1 \ \& \ x \leq 2 \\ 4/4 & x \geq 2 \end{cases}$$

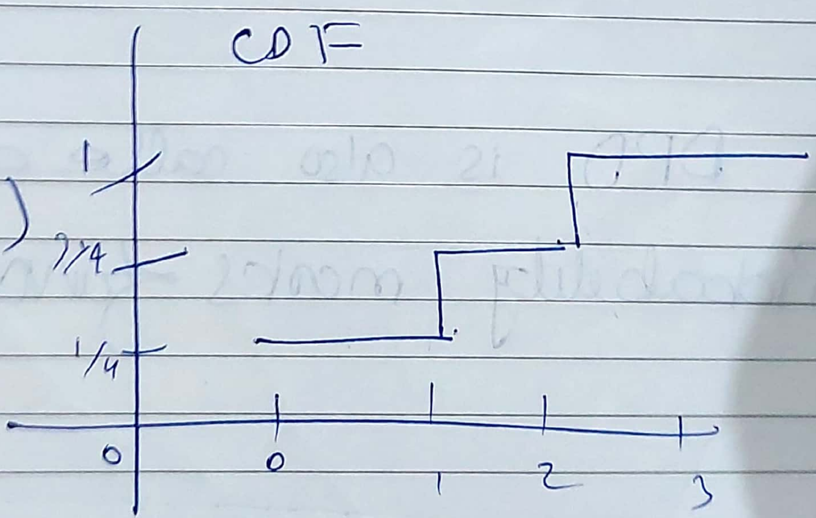
$F(x) = \text{CDF}$

Prob mass function

$f(x) =$



$F(x)$



Three time

X		f(x)	F(x)
0	TTT	1/8	1/8
1	HTH, ~~[illegible]~~ ~~[illegible]~~ THH, HHT	3/8	4/8
2	~~[illegible]~~		
3	THT, HTT HTHH	3/8	7/8
4	~~[illegible]~~, ~~[illegible]~~ THH	1/8	8/8
5	HHT ~~[illegible]~~		
6	TTH, ~~[illegible]~~ THT		
7	HTT, HTT		
8			

$$
Fx = \begin{cases} 0 & x < 0 \\ 1/8 & x \geq 0 \ \& \ x < 1 \\ 4/8 & x \geq 1 \ \& \ x \leq 2 \\ 7/8 & x \geq 2 \ \& \ x \leq 3 \\ 1/8 & x \geq 3 \end{cases}
$$

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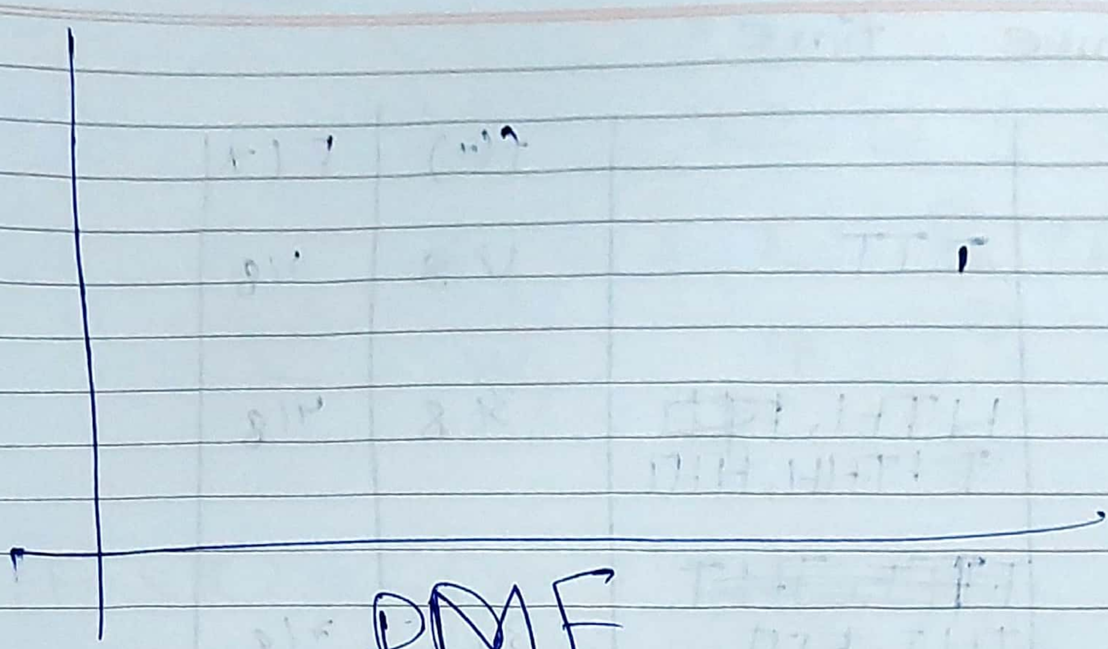
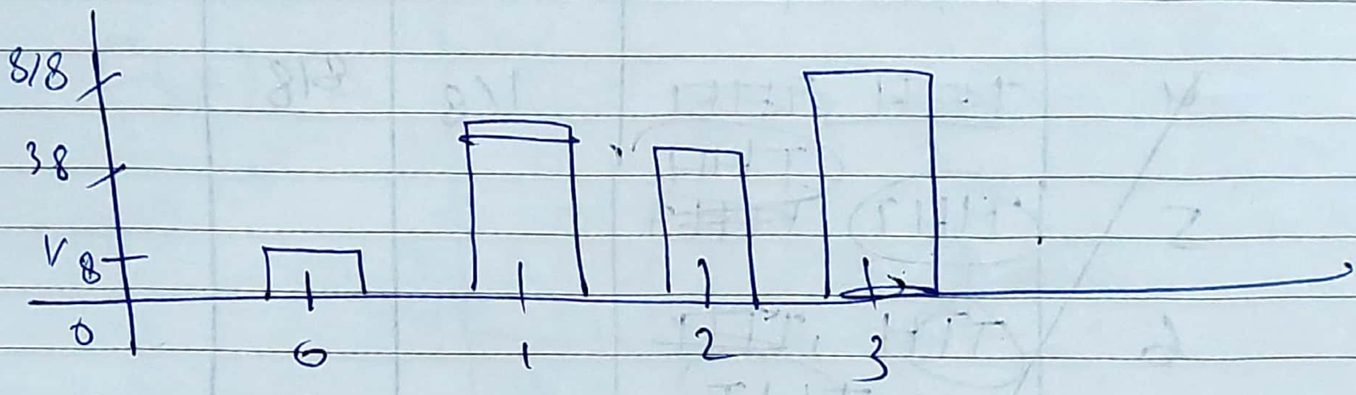
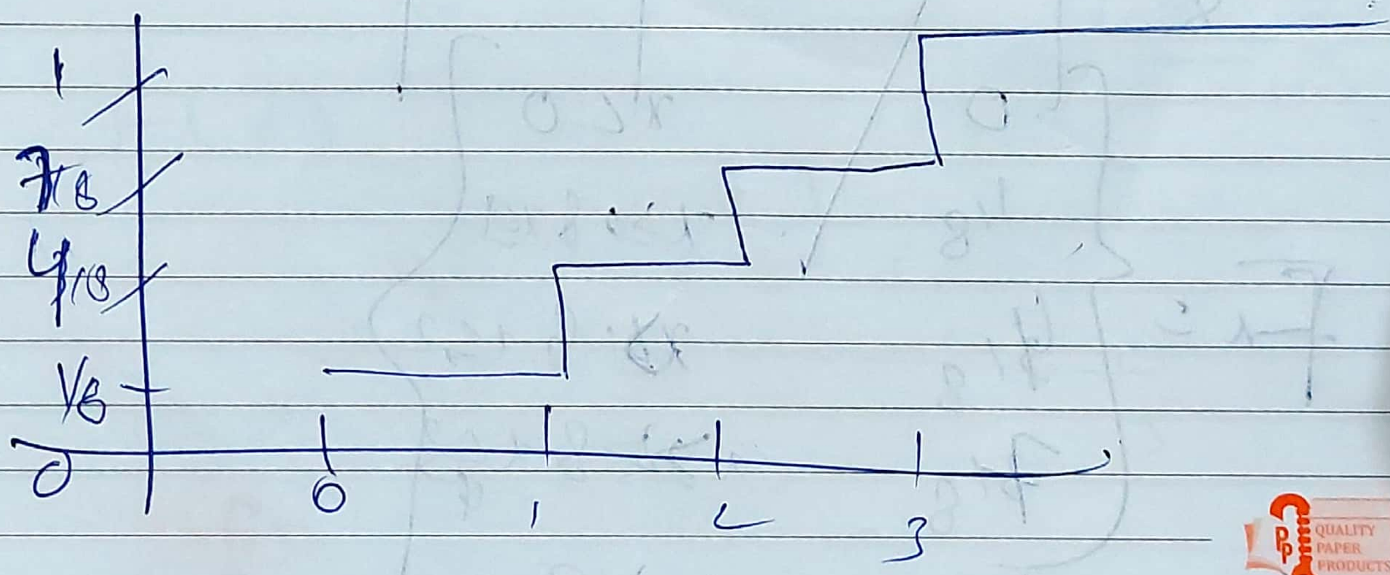


PMF



CDF



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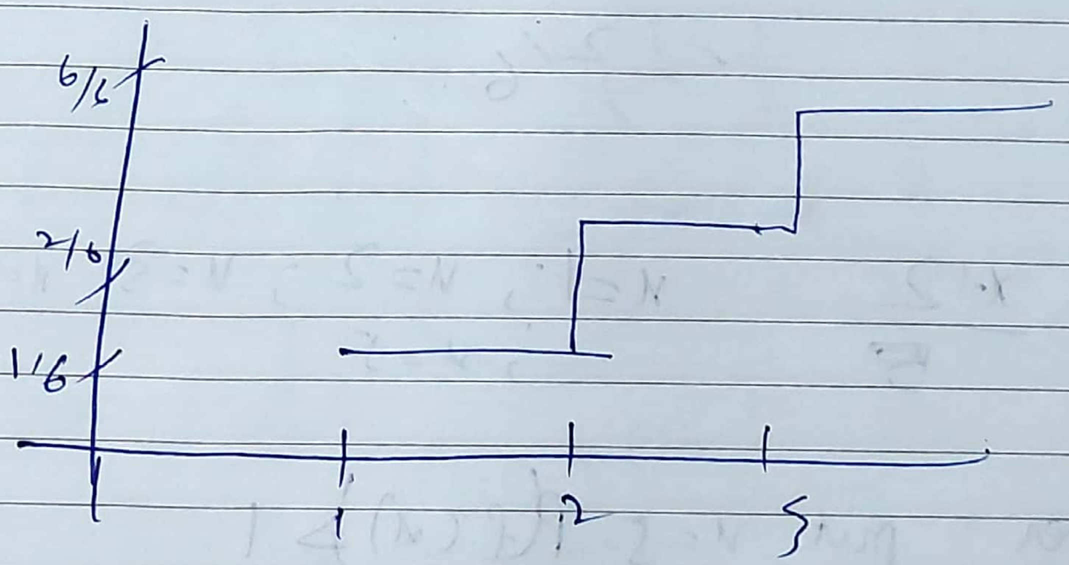
PMF $= f(x) = x/6$, $x = 1, 2, 3$.

$f(x) = 1/6, \; 2/6, \; 3/6$

x	$f(x)$	CDF
1	1/6	1/6
2	2/6	3/6
3	3/6	6/6



$$\text{CDF} = F(x) = \begin{cases} 0 & x < 0 \\ 1/6 & x \geq 1 \; \& \; x < 2 \\ 3/6 & x \geq 2 \; \& \; x < 3 \\ 6/6 & x \geq 3 \end{cases}$$

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$= F(4.5) - F(1.5)$

$= 1 - 1/6$

$= 5/6$

$P(x=2) = f(x=2) = 2/6$

$= F(2) - F(1)$

$= 3/6 - \frac{1}{6}$

$= 2/6$

$f(x) = \frac{x+2}{\cancel{5}\,6}$ $x=1; x=2; x=3; x=4; x=5$

when put $x=5$ $P(f(x)) > 1$

So not PMF

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CDF $F(x) = 1 - \frac{1}{2})^{x+1}$ $\quad x = 0, 1, 2$

(a) $P(x=3) = F(3) - F(2)$

$$= \left(1 - \frac{1}{16}\right) - \left(1 - \frac{1}{8}\right)$$

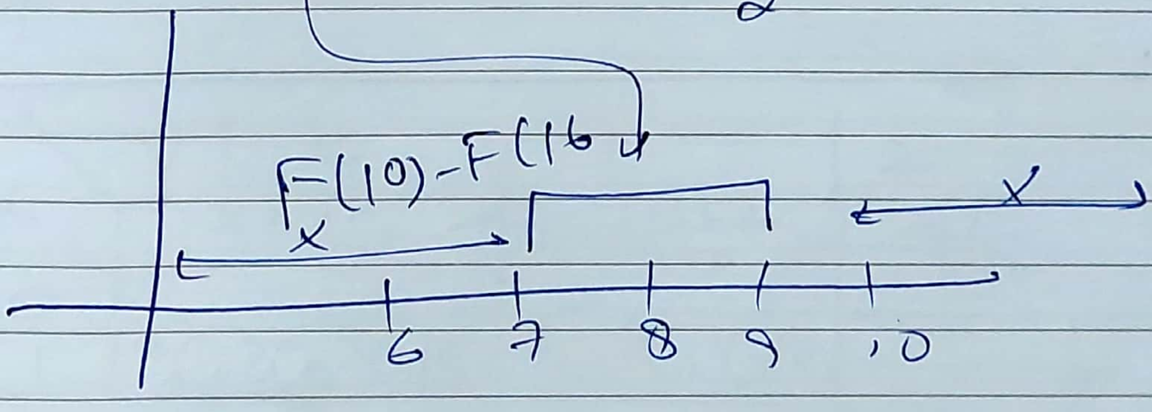
$$= \cancel{7/8 - 3/4} \quad \frac{15}{16} - \frac{7}{8}$$

$$= \frac{1}{16}$$

(b) $P[7 \le x \le 10] = F(9) - F(6)$

$$= \left(1 - \frac{1}{2^{10}} - 1 - \frac{1}{2^{7}}\right)$$

$$= \frac{7}{2^{10}}$$



PMF if CDF given:

(c) PMF = $F(x) - F(x-1)$
$f(x)$

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Example:
A coin has biased that head occurs if coin flips 3 time find PD of no. of head

$3H = 3 \quad w + 3w = 1 \Rightarrow w = \frac{1}{4}$

x	
~~0~~	~~HHH~~
1	~~HTH, THH~~
2	
3	

x		$P(x)$	$P(x)$
0	~~HHH~~ HHT	$(\frac{1}{4})(\frac{1}{4})(\frac{1}{4})$	$\frac{1}{64}$
2	HTH, THH	$\frac{9}{64} + \frac{9}{64} + \frac{9}{64}$	$\frac{27}{64}$
1	HTT, TTH, THT	$\cancel{(\frac{3}{4})(\frac{3}{4})}$ $\frac{3}{64} + \frac{3}{64} + \frac{3}{64}$	$\frac{9}{64}$
3	HHH	$\frac{27}{64}$	$\frac{27}{64}$

Joint Distribution

- $f(x, y) \rightarrow$ representation
- $f(x, y) \geq 0$
- $\Sigma \Sigma f(x) = 1$

Example:

let

3 B pen
2 B pen
3 G pen

Total = 8
2 → Random
x = no. of blue
y = no of red

find joint distribution

$x \backslash y$	0	1	2
0	$3C_2/8C_2$	$\frac{3C_0\,2C_1\,3C_1}{8C_2}$	$\frac{2C_2}{8C_2}$
1	$3C_1\,2C_0\,3C_1/8C_2$	$3C_1\,2C_1\,3C_0/8C_2$	—
2	$\frac{3C_2\,2C_0}{8C_2}$	—	—

$x \backslash Y$	0	1	2	Marginal dis of x $g(x)$
0	$\frac{^3C_2}{^8C_2} = \frac{3}{28}$	$\frac{^3C_0\,{^2C_1}\,{^3C_1}}{^8C_2} = \frac{6}{28}$	$\frac{^2C_2}{^8C_2} = \frac{1}{28}$	$\frac{10}{28}$
1	$\frac{^3C_1 \cdot {^2C_0}\,{^3C_1}}{^8C_2} = \frac{9}{28}$	$\frac{^3C_1 \cdot {^2C_1}\,{^3C_0}}{^8C_2} = \frac{6}{28}$	—	$\frac{15}{28}$
2	$\frac{^3C_2}{^8C_2} = \frac{3}{28}$	—	—	$\frac{3}{28}$
Total / Marginal dis of $y = h(y)$	$15/28$	$\frac{12}{28}$	$1/28$	$\frac{28}{28}$

Example 2: 2 Date ______ 20___

$P(Y=3 \mid x=2)$

$f(y|x) = \frac{\text{Joint}}{\text{Marginal}} = \frac{f(x,y)}{h(x)} = \frac{f(y=3, x=2)}{g(x=2)}$

= Not possible

$f(x|y) = \frac{f(x,y)}{f(y)}$

3

$P(x=0|y=1) = \frac{f(x,y)}{h(y=1)} = \frac{6/28}{12/28} = \frac{6}{12}$

$= 1/2$

$P(x \leq 2, y=1)$

$= f(0,1) + f(1,1) + f(\cancel{2,1})$

$= \frac{6}{28} + \frac{6}{28} = \frac{12}{28} = \frac{6}{14}$

$= \frac{3}{7}$

b) $P(x>2, y \le 1)$

$= 0$

(c) $P(x>y)$

$= f(1,0) + f(2,0)$

$= \frac{9}{28} + \frac{3}{28} = \frac{12}{28} = \frac{6}{14}$

$= \frac{3}{7}$

d) $(x+y=4)$

$\cancel{f(2,2)} = \; = 0$

Mathematic Expectation

$E[x] = \sum x f(x)$

$E[x^2] = \sum x^2 f(x)$

from Example earlier

x	0	1	2	Total
	$10/28$	$15/28$	$3/28$	1
$x f(x)$	0	$15/28$	$6/28$	$\frac{21}{28}$
$x^2 f(x)$	0	$15/28$	$12/28$	$\frac{27}{28}$

$\text{Variance} = E(x^2) - [E(x)]^2$

$Var(x) = \frac{27}{28} - \left(\frac{21}{28}\right)^2$

$= 0.401$

y	0	1	2	
	15/28	12/28	1/28	1
$yf(x)$	0	12/28	2/28	14/28
$y^2f(x)$	0	12/28	4/28	16/28

$$\mathrm{var}(x) = E(y^2) - (E(y))^2$$

$$= \frac{16}{28} - \left(\frac{14}{28}\right)^2$$

$$= 0.32$$

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x \ y	0	1	2
0	0	1/30	2/30
1	1/30	2/30	3/30
2	2/30	3/30	4/30
~~3~~	~~3/30~~	~~4/30~~	~~5/30~~
3	3/30	4/30	5/30

(a) $\frac{1}{30} + \frac{2}{30} + \frac{3}{30}$

$= \boxed{\frac{6}{30}}$

(b) $\frac{3}{30} + \frac{4}{30} = \boxed{\frac{7}{30}}$

(c) 1,0 , 2,0 , 3,0 + 2,1, ~~3,0~~

3,1, 3,2 ,

$\frac{1}{30} + \frac{2}{30} + \frac{3}{30} + \frac{3}{30} + \frac{4}{30} + \frac{5}{30}$ $\boxed{18/30}$

(d) 1, ~~3~~, 3, 1 , 2, 2

$\frac{4}{30} + \frac{4}{30} = \boxed{\frac{8}{30}}$

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Q 8 (a)

Mar(x) 0.10 + 0.35 + 0.55

1.00

(b)

Mar(y) = 0.20 + 0.5 + 0.3

= 1.00

(c) ~~$\frac{3,3}{x=2}$~~

$= \frac{0.10}{0.35}$

$= \frac{2}{7}$

~~Q 3 (a)~~

E(x) = 1

$E(y) = \frac{38}{30}$

Q 1 (c) Var X

Var(y)

x	0	1	2	3	
	3/30	6/30	9/30	12/30	
xf(x)	0	6/30	18/30	36/30	60/30
$x^2f(x)$	0	6/30	36/30	108/30	$\frac{32}{15}$

y	0	1	2
	6/30	10/30	14/30
Ey	0	10/30	28/30

$\frac{38}{30}$

$E(y) = \frac{38}{30}$

$5 - 2^2$

$\frac{32}{15} - \frac{(36)^2}{(30)^2} \boxed{1} = \frac{82}{...}$

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Covariance

Covariance $= E(xy) - E(x) \cdot E(y)$

$E(x,y) = \Sigma x \Sigma y \, (x,y) \, f(x,y)$

y \ x	0	1	2	3
0	0 0	1/30 0	2/30 0	3/30 0
1	1/10 0	2/30 2/30	3/30 6/30	6/30 12/30
2	2/30 0	3/10 6/30	6/30 16/30	5/30 30/30

$(X)(y)(f(x,y)) \rightarrow (\Sigma x \Sigma y) + (E(x) E(y))$

$\cancel{\frac{72}{30}}$

$= \frac{72}{30} - 2\left(\frac{48}{30}\right)$

$= -4/30$

$\frac{\text{Covariance}}{\text{std}}$ = ~~Co~~relation $= \frac{\text{Covar}}{\text{var}(x) \, \text{var}(y)}$

$= \frac{-4/30}{\sqrt{1} \sqrt{}}$

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$\rho \rightarrow \pm 1$ strong

$\rho \rightarrow \pm 0.5 \rightarrow$ Moderate

$\rho \rightarrow 0$ weak/negligible

= (Sum of multiply of x and y) + (Expected of x)(Expected of y)

CDF

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Constant Density function.

- $P(x=a) = 0$
- $P(a<x<b) = P(a \le x \le b) = P(a \le x < b)$
- $f(x) \Rightarrow$ Probability Density function

$$f(x) = \begin{cases} x^2/3 & ; -1 < x < 2 \\ 0 & ; \text{elesewhee} \end{cases}$$

(a) Verify $f(x)$ is PDF

$$= \int_{-1}^{2} f(x) \Rightarrow \int_{-1}^{2} \frac{x^2}{3}$$

$$= \left[\frac{x^3}{9} \right]_{-1}^{2}$$

$$= \frac{8}{9} + \frac{1}{9} = 1$$

<u>PDF</u> : $\int_{-\infty}^{\infty} f(x) = 1$

$f(x) \cancel{\ge 0} = 1$

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(b) $P(0 < x < 1)$

$$= \int_0^1 \frac{x^2}{3}$$

$$= \left[\frac{x^3}{9}\right]_0^1$$

$$= \frac{1}{9}$$

(c) find CDF $= F(x) = P(X \leq x)$

$$= \int_{-1}^{x} \frac{x^2}{3}$$

$$= \frac{x^3}{9} + \frac{1}{9}$$

(d) find prob $(0 < x \leq 1)$

using CDF

$$= F(x) - F(x-1)$$

$$= F(1) - F(0)$$

$= \frac{1}{9} \frac{-1}{9} - \frac{1}{9} = \frac{1}{9}$

(e) $f(x) = F'(x) = \frac{dx}{dx}$

PDF by CDF

from part (c)

$= \frac{x^3}{9} - \frac{1}{9}.$

$= \frac{3x^2}{9}$

$= \frac{x^2}{3}$

Q PDF $f(y) = \begin{cases} \frac{5}{8b} ; \frac{2b}{5} \le y \le 2b \\ 0 ; \text{elsewhere} \end{cases}$

(a) $F(y) = CDF = P(Y \le y) =$

$\int_{2b}^{2b/5} \frac{5}{8b} dy$

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$$= \frac{5}{8b} y$$

$$= \frac{5y}{8b}$$

$$= \frac{5y}{8b} - \frac{5\left(\frac{2}{5}b^2\right)}{8b}$$

$$= \frac{5y}{8b} - \frac{1}{4}$$

(b) $P(Y < b) = \int^{b} = F(b)$

$\cancel{\frac{2b}{5}}$

$$= \frac{5b}{8b} - \frac{1}{4} = \frac{3}{8}$$

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$k(3-x^2)$

$= \left[3k - \frac{x^3 k}{3}\right]$

$= \left[3kx - \frac{x^4 k}{12}\right]_{-1}^{1}$

$= \cancel{3k}\ 3k - \frac{1}{12} + 3k\ \frac{+1}{12}$

$= 6k =$

$\frac{6}{24}$

$= 3k - kx^2$

$= \left[3kx - \frac{kx^3}{3}\right]_{-1}^{1}$

$\frac{3}{16}$

$\frac{6}{32}$

$= 3k - \frac{k}{3} + 3k - \frac{k}{3}$

$= 6k - \frac{2k}{3} = 1$

$16k = 3$

$k = \frac{3}{16}$

$= k\left\{3 - x^2\right\}_{-1}^{1}$

$= k\left[3x - \frac{x^3}{3}\right]_{-1}^{1}$

$= k\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right] = 1$

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$k(3-x^2)$

$= \left[3k - \frac{x^2k}{.}\right]_{-1}^{1}$

$= \left[3kx - \frac{x^3k}{3}\right]_{-1}^{1}$

$= 3k - \frac{1k}{3} + 3k - \frac{1k}{3}$

$= 6k - \frac{2k}{3} = 1$

$= 6k =$

(c) $P(|x| < 0.8) = P(x < -.8) +$

$P(x > 0.8)$

(b) $(P \quad x < \frac{1}{2})$

$$= \int_{-1}^{1/2} \frac{3}{16}(3 - x^2)$$

$$= \left[\frac{9}{16} - \frac{3x^2}{16}\right]_{-1}^{1/2}$$

(c)

Joint Prob Density function

$$f(x,y) \geq 0$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

① = $f(x,y) = \frac{2}{5}(2x+3y)$ $\quad 0 < x < 1$, $0 < y < 1$

$$= \int_0^1\int_0^1 \frac{2}{5}(2x+y).$$

$$= \int_0^1 \frac{2}{5}\left(\cancel{\frac{2x^2}{2}}\right)\left[\frac{2x^2}{\cancel{2}} + 3xy\right]$$

$$= \int_0^1 \cancel{\frac{2}{5}\left[x^2 + 3xy\right]}. \quad \left[x^2+3xy\right].$$

$$\cancel{= \int_0^1\int_0^1 \frac{2}{5}\left[\right.}$$

$$= \int_0^1 \frac{2}{5}(1+3y)$$

$$= \int \frac{2}{5}\left[y + \frac{3y^2}{2}\right]$$

$$= \frac{2}{5}\left[1+\frac{3}{2}\right] = \frac{2}{5}\left(\frac{5}{2}\right) = 1$$

(b)

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Q $P\left[0 < x < 1 \; ; \; \frac{1}{4} < y < \frac{1}{2}\right]$

$$= \frac{2}{5}\int_{\frac{1}{4}}^{\frac{1}{2}} dy \int_0^{1/2} (2x + 3y)\, dx$$

$$= \frac{2}{5}\int_{1/4}^{1/2} dy \left[\frac{2x^2}{2} + 3xy\right]_0^{1/2}$$

$$= \frac{2}{5}\int_{1/4}^{1/2} \left[x^2 + 3xy\right]_0^{1/2}$$

$$= \frac{2}{5}\int_{1/4}^{1/2} \left[\frac{1}{4} + \frac{3}{2}y\right]$$

$$= \frac{2}{5}\int_{1/4}^{1/2} \left[\frac{y}{4} + \frac{3y^2}{4}\right]$$

$$= \frac{2}{5}\left[\frac{1/2}{4} + \frac{3/4}{4}\right] - \left[\frac{1/4}{4} + \frac{3(1/16)}{4}\right]$$

$$= \cancel{\frac{2}{5}\left[\frac{5}{2}\right]} = \frac{2}{5}\left[\frac{1}{8} + \frac{3}{16} - \frac{1}{16} + \frac{3}{64}\right]$$

$$= \frac{2}{5}\left[\frac{8+12+\cdot 4}{64} + 3\right] = \left(\frac{1}{64}\right)\left(\frac{1}{5}\right)$$

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$$= \left(\frac{18}{64}\right)\left(\frac{32}{5}\right)$$

$$= \frac{1\cancel{8}6}{(5)(32)}$$

$$= \frac{186}{160} \qquad 30$$

$$= \boxed{\frac{1\cancel{8}6}{160}}$$

$$= \frac{8}{30} = \frac{4}{15}$$

(c) Margral density for X and Y.

For X

$$g(x) = \frac{2}{5}\int_0^1 (2x+3y)\,dy.$$

$$= \frac{2}{5}\left[2xy + \frac{3y^2}{2}\right]_0^1$$

$$= \frac{2}{5}\left[2x \cdot 1 + \frac{3}{2}\right] = \frac{2}{5}\left[2x + \frac{3}{2}\right]$$

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for Y

$$h(y) = \frac{2}{5}\int_0^1 (2x+3y)\,dx$$

$$= \frac{2}{5}\left[x^2+3xy\right]_0^1$$

$$= \frac{2}{5}\left[1+3y\right]$$

(d) Correlation (x, y).

~~fE~~ $E(x) = \int_{-\infty}^{\infty} x\,g(x)\,dx$

$\cancel{x(2x+3}$ $\longleftarrow$ $= \frac{2}{5}\int_0^1 \left(2x^2+\frac{3x}{2}\right)dx$

$g(x) = \frac{2}{5}\left(2x+\frac{1}{3}\right)$

$x\,g(x) = \frac{2}{5}\left(2x^2+\frac{x}{3}\right)$

$$= \frac{2}{5}\left(\frac{2x^3}{3}+\frac{3}{4}x^2\right)_0^1$$

$$= \frac{2}{5}\left(\frac{2}{3}+\frac{3}{4}\right).$$

$$= \frac{2}{5}\left(\frac{8+9}{12}\right)$$

$$= \frac{2}{5}\left(\frac{17}{6}\right) = \frac{17}{30}$$

$= \frac{17}{\cancel{15}}$

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$$E(y) = \int_{-\infty}^{\infty} y\,(g(y))\,dy.$$

$$g(y) = \frac{2}{5}(1+3y)$$

$$= \int_0^1 \frac{2}{5}\left(y\,(1+3y)\right)$$

$$= \frac{2}{5}\int_0^1 y+3y^2)$$

$$= \frac{2}{5}\left[\frac{y^2}{2}+\frac{3y^3}{3}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{y^2}{2}+y^3\right]_0^1$$

$$= \frac{2}{5}\left[\frac{1}{2}+1\times 1\right]$$

$$= \frac{2}{5}\left[\frac{1}{2}+1\right]$$

$$= \frac{\cancel{2}}{5}\left(\frac{3}{\cancel{2}}\right)$$

$$g(y) = \frac{3}{5}$$

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$$E(x^2) = \int_{-\infty}^{\infty} x^2 (g(x))\,dx$$

$$= \frac{2}{5}\int_0^1 2x^3 + \frac{3x^3}{2})\,dx$$

$$= \frac{2}{5}\left[\frac{x^4}{2} + \frac{x^3}{2}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{1}{2} + \frac{1}{2}\right]$$

$$E(x^2) = \frac{2}{5}$$

$$E(y^2) = \int_0^1 y^2 \frac{2}{5}(1+3y)\cancel{y^2}$$

$$= \int_0^1 \frac{2}{5}(y^2 + 3y^3)$$

$$= \frac{2}{5}\left[\frac{y^3}{3} + \frac{3y^4}{4}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{1}{3} + \frac{3}{4}\right] = \frac{2}{5}\left(\frac{4+9}{12}\right)$$

$$= \frac{2}{5}\left[\frac{4+12}{12}\right] = \left(\frac{16}{12}\right)\left(\frac{2}{5}\right)$$

$$= \left(\frac{\cancel{2}}{5}\right)\left(\frac{13}{6\cancel{12}}\right) = \frac{13}{30} \qquad = \left(\frac{4}{3}\right)\left(\frac{2}{5}\right) = \frac{8}{15}$$

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$$V(x) = E(x^2) - [E(x)]^2$$
$$= \frac{2}{5} - \left(\frac{17}{30}\right)^2$$

$$V(y) = E(y^2) - [E(y)]^2$$
$$= \frac{8}{15} - \left(\frac{3}{5}\right)^2$$
$$= \frac{18}{90} - \frac{9}{25}$$

$$E(xy) = xy \int_0^1 \int_0^1 \frac{2}{5}(2x+3y)\,dx\,dy$$

$$= xy \int_0^1 \int_0^1 \frac{2}{5}[2x+3y]\,dx\,dy$$

$$= \int_0^1 \int_0^1 \frac{2}{5}[2x^2y + 3y^2x]\,dx\,dy$$

$$= \int_0^1 \left[\frac{2}{5}\left(\frac{2x^3}{3}y + \frac{3y^2x^2}{2}\right)\right]_0^1 dy = \int_0^1 \frac{2}{5}\left[\frac{2x^3y}{3} + \frac{3y^2x^2}{2}\right]_0^1$$

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$$= \int_0^1 \frac{2}{5}\left[\frac{2y}{3} + \frac{3y^2}{2}\right]$$

$$= \frac{2}{5}\left[\frac{2y^2}{6} + \frac{3y^3}{6}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{2}{6} + \frac{3}{6}\right]$$

$$= \frac{2}{5}\left[\frac{5}{6}\right]$$

$$\boxed{= \frac{1}{3}}$$

Covariance $= E(xy - E(x).E(y))$

$$= \frac{1}{3} - \frac{17}{30} \cdot \frac{3}{5} = -\frac{1}{150}$$

$$= \frac{1}{3} - \frac{51}{150}$$

$$= \frac{50 - 21}{150}$$

$$= \frac{37}{150} = \frac{13}{50}$$

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$= \frac{\cancel{17}}{80} \quad \frac{-1}{150}$

$\sqrt{} \quad \sqrt{} \quad \sqrt{}$

$= -0.0876$

(e) $P\left[\frac{1}{4} < x < \frac{1}{2} \middle| y = \frac{1}{3}\right] = \frac{f(x,y)}{h(y = 1/3)}$

$= \frac{\frac{2}{5}(2x + 3y)}{\frac{2}{5}(1 + 3y)}$

$= \frac{\cancel{2/5}\left[2x + 4/3\right.}{\cancel{2/5}\left(1 + 1/3 \cdot 3\right]}$

$= \int_{1/4}^{1/2} \frac{(2x + 1)}{2}$

$\left[x^2 + \frac{x^2}{4}\right]_{1/4}^{1/2} \Leftarrow \left[\frac{2x^2}{2} + \frac{x}{2}\right]_{1/4}^{1/2}$

$= \frac{7}{32}$

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$f(x,y) = g(x)h(y)$

Statistical independence

(a) $g(x) = \frac{1}{4}\{x(1+3y^2)$

$= \frac{1}{4}(x\cancel{y} + 3xy^2)$

$= \frac{1}{4}\left[xy + \cancel{3}xy^3\right]$

$= \frac{1}{4}\left[x + \cancel{1}x\right]$

$= \frac{2x}{4} = \frac{x}{2}$

$h(y) = \frac{1}{4}\{x + 3xy^2\}$

$= \frac{1}{4}\left[\frac{x^2}{2} + \frac{3x^2y^2}{2}\right]$

$= \frac{1}{4}\left[\frac{4^2}{2} + \frac{3(4)^2 y^2}{2}\right]$

$= \frac{1}{4}\left[2 + 6y^2\right]$

$= \frac{2+6y^2}{4} = \frac{1+3y}{2}$

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(c) $f(x|y) = \dfrac{f(x,y)}{h(y)}$

$h(y) = \dfrac{1+3y}{2}$

$= \dfrac{1+3\cdot 1/3}{2}$

$h(y) = 1$

$f(x,y)$

$f(\bar{x},y) = \int_0^2 \int_0^1 \dfrac{x(1+3y^2)}{4}$

$= \int_0^2 \left[\dfrac{xy+3xy^3}{4}\right]_0^1$

$= \int_0^{0}$

$= \int_0^2 \left[\dfrac{x+3xy^2}{4}\right]_0^1$

$= \int_0^2 \dfrac{1}{4}\left[xy + 3xy^3\right]_0^1$

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$= \int_0^2 \frac{1}{4}\left[x + x \right].$

$= \left[\frac{x}{2} \right]_0^2$

$= \frac{1}{2}\left[\frac{x^2}{2} \right]$

$= \frac{4}{4}$

$= 1$

(d) $E(x) = \int_0^2 x \left[\frac{x}{2} \right] dx.$

$= \int_0^2 \frac{x^2}{2} dx$

$= \int_0^2 \frac{1}{2}\left[\frac{x^3}{3} \right] dx$

$= \left[\frac{x^2}{6} \right]_0^2$

$= \frac{4}{6} = \frac{1}{3}$

(e) $E(y) = \int_0^1 y \left(\frac{1+3y}{2} \right)$

$= \int_0^1 \left[\frac{y+3y^2}{2} \right]$

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$$\frac{1}{2}\left[\frac{y^2 x}{2} + y^3\right]_0^1$$

$$= \frac{1}{2}\left[\frac{1}{2} + 1\right]$$

$$= \left[\frac{3}{2}\right]\left(\frac{1}{2}\right)$$

$$= \frac{3}{4}$$

(f) $f(x,y) = \dfrac{\left(x\cancel{\left(\frac{1+3y^2}{4}\right)}\right)}{\cancel{x\left(\frac{1+3y^2}{2}\right)}}$

$$= \frac{x}{2}$$

$\cancel{E(x/y) = x}$

(g) $\dfrac{x\left(\dfrac{1+3y^2}{4}\right)}{}$

$$= \frac{1}{4}\left(x + 3xy^2\right)$$

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$$= \frac{1}{4}\left[x + \cancel{3}x\left(\frac{1}{\cancel{3}x}\right)\right]$$

$$= \frac{1}{4}\left[x + \frac{x}{3}\right]$$

$$= \left(\frac{4x}{3}\right)\frac{1}{4}$$

$$= \left[\frac{x}{3}\right]_{1/4}^{1/2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\right] - \frac{1}{3}\left[\frac{1}{4}\right].$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{4}\right].$$

$$= \frac{1}{3}\left[\frac{2-1}{4}\right]$$

$$P\left(\frac{1}{4} < X < \frac{1}{2} \mid Y = \frac{1}{2}\right) = \frac{1}{12}$$

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$$E(xy) = xy \int_0^1 \int_0^2 \frac{x + 3xy^2}{4}$$

$$\frac{1}{4}\left[x + 3y^2\right]$$

$$= \cancel{xy} \int_0^1 \int_0^2 \frac{1}{4}\left[x^2y + 3xy^3\right]$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^2y^2}{2} + \frac{3x^2y^4}{4}\right]_0^1$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^2}{2} + \frac{3x^2}{4}\right]$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^3}{6} + \frac{3x^2}{12}\right]$$

$$= \frac{1}{4}\left[\frac{8}{6} + \frac{24}{12}\right]$$

$$= \frac{1}{4}\left[\frac{64 + 72}{48}\right]$$

$$= \frac{1}{4}\left[\frac{136}{48}\right]$$

$$= \left[\frac{34}{48}\right]$$

$$=$$

$$\begin{array}{r} 72 \\ 64 \\ \hline 136 \end{array}$$

$E(x^2)$

$$= \frac{1}{4}\left[\frac{8}{6} + \frac{24}{12}\right]$$

$$= \frac{1}{4}\left[\frac{16+24}{12}\right]$$

$$= \left[\frac{30}{12}\right]\left(\frac{1}{4}\right)$$

$$= \frac{15}{(12)(2)}$$

$$= \frac{15}{24}$$

$$= \frac{1}{4}\left[\frac{8}{6} + \frac{24}{4}\right]$$

$$= \frac{5}{6}$$

$$E(x^2) = x^2\left(\frac{x}{2}\right)$$

$$= \left[\frac{x^3}{2}\right]$$

$$= \left[\frac{x^4}{8}\right]_0^2 = \frac{16}{8} = \boxed{2}$$

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$$E(y^3) = \frac{y^2(1+3y^2)}{2}$$

$$= \frac{y^2+3y^3}{2}$$

$$= \frac{1}{2}\left[\frac{y^3}{3} + \frac{3y^4}{8}\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{3}{8}\right].$$

$$= \frac{17}{48}$$

Topic

Date ______ 20__

$f(x,y) = 24xy \quad 0 < x, y < 1$

$x + y < 1$

(a) $P(x > \frac{1}{2}) = \int_0^1 g(x)\,dx = \frac{5}{16}$

(b) $g(x) = \int_0^{1-x} 24xy\,dy = 12x(1-x)^2$

(b) $h(y) = \int_0^{1-y} 24xy\,d(x) = 12y(1-y)^2$

(c) $P(x < \frac{1}{8} \mid y = \frac{3}{4})$ $\int_0^{\cancel{1-x}} 12(x)(\frac{3}{4})$

$= \int_0^{1/8} 9(x)$

$= \cancel{\left[\frac{9x^2}{2}\right]_0^{1/8}}$

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$$= \frac{9(1/8)^2}{2}$$

$$= \frac{\frac{1}{2}\left(\frac{9}{64}\right)}{12\left(\frac{3}{4}\right)\left(1-\frac{3}{4}\right)}$$

$$= \frac{1}{4}$$

$$\int_0^{1/8} \frac{24(x)\left(\frac{3}{4}\right)}{12\left(\frac{3}{4}\right)\left(1-\frac{3}{4}\right)^2}$$

$$= \frac{1}{4}.$$

$$f(x,y) = \frac{x(1+3y)}{4} \qquad 0 < x < 2, \quad x+y < 1$$

$$E(Y/X) = \int_0^1 \int_0^2 \frac{y}{x}\left(\frac{x(1+3y)}{4}\right.$$

$$E(x+2) = E(x)+2$$

$$E(x,y) = \int_0^1 \int_0^2 f(x,y)$$

$$E(3x) = 3E(x)$$

\$	p(\$)	\$ p( )
1000	$(1-0.7)(0.4)$	$(0.7)(1-0.4) = 0.42$
1500	$(1-0.4)(0.7)$	$(0.4)(1-0.7) = 0.12$
2500		$(0.7)(0.4) = 0.28$
0		$(1-0.7)(1-0.4)$ =

Multinormal distribution Date______20___

$$p(x) = \frac{n!}{x_1!\, x_2! \ldots x_k!} p^{x_1} p^{x_2} \cdots p_k^{x_k}$$

. If outcomes are more and probality of success is same

Example

$$= \frac{5!}{3!\,1!\,1!} (0.5)^3 (0.3)^1 (0.2)^1$$

$$= 0.15$$

Examples

$$\frac{8!}{4!\,2!\,1!\,1!} (0.3)^4 (0.5)^2 (0.15)^1 (0.05)^1$$

4, 1, 3

$$= 0.0354$$

<u>Example 10</u>

$$= \frac{9!}{3!\,3!\,1!\,2!}$$

$$= 7.7 \times 10^{-4}$$

Hyperbolic

Date ____ 20 __

- Prob of success changes
- Trials are ~~inde~~dependent

Example = $\dfrac{\binom{5}{3}\binom{5}{0}}{\binom{10}{3}}$

Bernoli Experiment

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1. The experimet consist of repeated trials
2. Each trial in ~~relation~~ an outcome that maybe classified as a success or false
3. p(success) = p
4. Trials are indepedent.

The number X of success in on Bernoulli trial is called bernoulli

pmf = binomial dist = $b(x; n, p)$ =

$$= \binom{n}{x} p^x q^{n-x} \qquad x = 0, 1, 2.$$

two ~~pa~~ parameters n & p

$$p + q = 1$$

Mean = ~~p~~ np variance = n/q

$; q = 1 - p$

Imran Khan

Date ______ 20__

eg coin is tossed 3 times

find the probability of getting exactly 2 heads

$x = 2$; $n = 3$ $p = \frac{1}{2}$ $q = 1 - p$

$x = 2$ $n = 3$ $p = 1/2$ $q = \frac{1}{2} = 1/2$

$$p(2, 3, 1/2) = \binom{3}{2}(1/2)^2 (1/2)^{3-2}$$

$$= \frac{3!}{2!\,1!}(1/4)(1/2)$$

$$= \frac{3}{8}$$

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eg Probability that a certain kind of component will survive a shock test is $\frac{3}{4}$

find probability exactly 2 of next 4 component test survived

$p = \frac{3}{4}$ $\quad x = 2 \quad n = 4$

$q = 1 - \frac{3}{4} = \frac{1}{4}$

$$\binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2$$

$$= \frac{4!}{2!\,2!}\left(\frac{9}{16}\right)\left(\frac{1}{16}\right)$$

$$= \frac{\cancel{4}^{2} \times 3}{\cancel{2}} = \frac{6(9)}{16(16)}$$

$$= \frac{(3)(9)}{(4)(16)}$$

$$= \frac{27}{1\cancel{28}} = \frac{27}{1\cancel{28}} = \boxed{\frac{27}{128}}$$

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$p = 3\%$

$n = 20$

$x = 1$ $q = 7\%$

$q = 7\%$

$\binom{20}{1}\left(\frac{3}{100}\right)^{1}\left(\frac{7}{100}\right)^{19}$

(a) $P(x \geq 1) = 1 - P(x = 0)$

Example.

$= 1 - \binom{20}{0}\left(\frac{3}{100}\right)^{0}\left(\frac{7}{100}\right)^{20}$

$= 0.4562$

Example 4

$p = 0.4$

$n = 15$

(a) $P(x \geq 10) = P(x = 10) + \dots \dots P(x \leq 15)$

(2) $P[3 \leq x \leq 8] = P(x = 3) + \cancel{P(}$

$\dots P(x = 8)$

$= 0.877$

(3) $P(x = 5) = 0.1859$

Poisson distribution

Date______ 20__

- When data is frequency or count based dataset

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

- Average no. of succes know.
- Interval region known

Example

$\lambda = 10$ → mean = variance

$p(x > 15)$

$$1 - p(x \leq 15)$$

$$= 1 - \sum_{x=0}^{15} \frac{e^{-10} x^{x}}{x!}$$

Hyperbolic

Date______20___

. Prob of success changes

. Trials or ~~inde~~dependent

Example = $\dfrac{\binom{5}{3}\binom{5}{0}}{\binom{10}{3}}$

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Multinomial distribution

$$p(x) = \frac{n!}{x_1!\, x_2! \cdots x_k!}\, p_1^{x_1}\, p_2^{x_2} \cdots p_k^{x_k}$$

. If outcomes are more and probabilty of success is same

Example

$$= \frac{5!}{3!\,1!\,1!}(0.5)^3(0.3)^1(0.2)^1$$

$$= 0.15$$

Examples

$$\frac{8!}{4!\,2!\,1!\,1!}(0.3)^4(0.5)^2(0.15)^1(0.05)^1$$

, 1, 3

$$= 0.0354$$