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Q1 i) $\frac{dy}{dx} = \frac{x}{y}$

$\int y \cdot dy = \int x \cdot dx$

$\boxed{\frac{y^2}{2} = \frac{x^2}{2} + C}$

ii) $x\frac{dy}{dx} + y = x^2y^2$

$y' + \frac{y}{x} = xy^2$

$y^{-2}y' + \frac{y^{-1}}{x} = x$

Let

$u = y^{-1}$

$\cancel{u' = -y^{-2}y'}$

$\frac{u'}{-1} + \frac{u}{x} = x$

$u' - \frac{u}{x} = -x$

$I.F = e^{-\int \frac{1}{x} dx}$

$= e^{\ln x^{-1}}$

$= \frac{1}{x}$

$\frac{u}{x} = -\int \frac{1}{\cancel{x}} \cdot \cancel{x} dx + C$

$\frac{u}{x} = -x + C$

$u = -x^2 + xC$

$\frac{1}{y} = -x^2 + xC$

$y = \frac{1}{-x^2 + xC}$

$\boxed{y = \frac{1}{x(-x+C)}}$

iii) $(x^2+y^2)dx + xydy = 0$

$(x^2+y^2) + xy\,y' = 0$

$y' = -\frac{(x^2+y^2)}{xy}$.

$y = vx$

$y' = -\left(\frac{x^2}{xy} + \frac{y^2}{xy}\right)$

$y' = -\left(\frac{1}{v} + v\right)$

$v'x + v = -\left(\frac{1}{v} + v\right)$

$v'x = -\left(\frac{1}{v} + 2v\right)$

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$$v'x = -\left(\frac{1+2v^2}{v}\right)$$

$$\frac{dv}{dx}\cdot x = -\left(\frac{1+2v^2}{v}\right)$$

$$\frac{1}{4}\int \frac{4v\,dv}{1+2v^2} = -\int \frac{dx}{x}$$

$$\frac{1}{4}\ln|1+2v^2| = -\ln x + \ln c$$

$$\left(1+\frac{2y^2}{x^2}\right)^{\frac{1}{4}\cdot\cancel{4}} = \left(\frac{c}{x}\right)^{\cdot 4}$$

$$1+\frac{2y^2}{x^2} = \frac{c}{x^4}$$

$$\boxed{y^2 = \frac{c}{x^2} - \frac{x^2}{2}}$$

iv) $(x-y^2)dx + 2xy\,dy = 0$

$$(x-y^2) + 2xy\,y' = 0$$

$$y'\cdot y + \frac{\cancel{x}}{2\cancel{x}} - \frac{y^2}{2x} = 0$$

$$y'\cdot y - \frac{y^2}{2x} = \frac{-1}{2}$$

Let $u = y^2$

$$\frac{u'}{\cancel{2}} - \frac{u}{\cancel{2}x} = \frac{-1}{\cancel{2}}$$

$$u' - \frac{u}{x} = -1$$

$$I.F = e^{\int \frac{1}{x}dx}$$

$$= \cancel{e}^{\ln x^{-1}}$$

$$= \frac{1}{x}$$

$$\frac{u}{x} = \int -1\cdot\frac{1}{x}\,dx + c$$

$$\frac{u}{x} = \ln x^{-1} + \ln c$$

$$u = x\ln\left(\frac{c}{x}\right)$$

$$\boxed{y^2 = x\ln\left(\frac{c}{x}\right)}$$

(v) $e^{y}\left(\frac{dy}{dx}-1\right)=e^{x}$

$$\boxed{y=\ln(x+c)+x}$$

$e^{y}\frac{dy}{dx}-\cancel{e^{y}}=\cancel{e^{x}}$

$e^{y}\frac{dy}{dx}=e^{x}+e^{y}$

$\frac{dy}{dx}-1=e^{x-y}$

Let $v=x-y$

$-\frac{dv}{dx}=-1+\frac{dy}{dx}$

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$-\frac{dv}{dx}=e^{v}$

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$-\int\frac{dv}{e^{+v}}=\int dx$

$e^{-v}=x+c$

$-v=\ln(x+c)$

$y-x=\ln(x+c)$

'vi' $\sin y \frac{dy}{dx} = \cos x(2\cos y - \sin x)$

$\underbrace{\sin y}_{N} dy + \underbrace{\cos x(\sin x - 2\cos y)}_{M} dx = 0$

$M_y = +2\sin y \cos x$ $\quad M_y \neq N_x$ Not exact

$N_x = 0$

$\frac{M_y - N_x}{N} = P(x)$

$\frac{2\cancel{\sin y}\cos x}{\cancel{\sin y}} = P(x)$

$I.F = e^{2\int \cos x\, dx}$

$= e^{2\sin x}$

$\overbrace{e^{2\sin x} \sin y}^{N} dy + \overbrace{e^{2\sin x} \cos x(\sin x - 2\cos y)}^{M} dx = 0$

$M_y = 2\sin y \cdot e^{2\sin x} \cos x$ $\quad M_y = N_x$ exact Form

$N_x = 2\cos x \cdot e^{2\sin x} \cdot \sin y$

$M = f_x = e^{2\sin x} \cos x(\sin x - 2\cos y)$

$N = f_y = e^{2\sin x} \sin y$

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$f = \int e^{2\sin x} \sin y\, \partial y + h(x)$

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$f = -e^{2\sin x}\cos y + h(x)$ *

$\Rightarrow f_x = -2e^{2\sin x} \cdot \cancel{\sin} \cos x \cos y + h'(x)$

$M = +\cancel{2\sin y}\, e^{2\sin x}\cos x \sin x - 2e^{2\sin x}\cancel{\cos x}\cos y$

$\int h'(x) = \int e^{2\sin x}\cos x \sin x \, dx$

$h(x) = \int e^{2u}\cdot u \cdot \frac{du}{\cancel{\cos x}} \cdot \cancel{\cos x}$ Let $\sin x = u$

$\cos x = \frac{du}{dx}$

$h(x) = \int e^{2u}\cdot u$

u +	e^{2u}
1 −	$\frac{1}{2}e^{2u}$
0	$\frac{1}{4}e^{2u}$

$\frac{1}{2}ue^{2u} - \frac{1}{4}e^{2u} + C = h(x)$

$C = -e^{2\sin x}\cos y + \frac{1}{2}\sin x \, e^{2\sin x} - \frac{1}{4}e^{2\sin x}$

$Ce^{-2\sin x} = -\cos y + \frac{1}{2}\sin x - \frac{1}{4}$

$\boxed{y = \cos^{-1}\left(-Ce^{-2\sin x} + \frac{1}{2}\sin x - \frac{1}{4}\right)}$

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vii) $x(3x+2y^2)dx + 2y(1+x^2)dy = 0$

$(3x^2+2y^2x)\partial x + (2y+2yx^2)\,dy = 0$

$M_y = 4yx$ $\quad M_y = N_x$ exact

$N_x = 4yx$

$f = \int 3x^2 + 2y^2x\,\partial x = x^3 + x^2y^2$

$f = \int 2y + 2yx^2\,\partial y = y^2 + y^2x^2$

$c = x^3 + y^2 + x^2y^2$

$\boxed{y^2 = \frac{c - x^3}{1+x^2}}$

viii) $e^{-y}\sec^2 y\,dy = dx + x\,dy$

$(e^{-y}\sec^2 y - x)\frac{dy}{dy} - \frac{dx}{dy} = 0$

$(e^{-y}\sec^2 y) - x - x' = 0$

$x' + x = e^{-y}\sec^2 y$

Linear w.r.t x

$I.F = e^{\int dy}$

$= e^{y}$

$xe^{y} = \int \cancel{e^{y}}.\cancel{e^{-y}}\sec^2 y\,dy + c$

$\boxed{xe^{y} = \tan y + c}$

ix) $(x^2+y^2)\,dx + (x^2-xy)\,dy = 0$

$$(x^2+y^2) + (x^2-xy)\,y' = 0$$

$$y' = -\left(\frac{x^2+y^2}{x^2-xy}\right)$$

Let $y = vx$

$$\cancel{y}\; y' = -\left(\frac{x^2+(vx)^2}{x^2-x(vx)}\right)$$

$$y' = -\frac{\cancel{x^2}}{\cancel{x^2}}\left(\frac{1+v^2}{1-v}\right)$$

$$y' = +\left(\frac{1+v^2}{v-1}\right)$$

$$v'x+v = \left(\frac{1+v^2}{v-1}\right)$$

$$v'x = \left(\frac{1+v^2}{v-1} - v\right)$$

$$v'x = \left(\frac{1+\cancel{v^2}-\cancel{v^2}+v}{v-1}\right)$$

$$\frac{dv}{dx}\cdot x = \left(\frac{1+v}{v-1}\right)$$

$$\int \frac{v-1+1-1}{v+1}\,dv = \int \frac{dx}{x}$$

$$\int \frac{v\cancel{+1}}{v\cancel{+1}} - \frac{2}{v+1}\,dv = \int \frac{dx}{x}$$

$$v - 2\ln|v+1| = \ln x + c$$

$$v - \ln|(v+1)^2| - \ln x = c$$

$$\frac{y}{x} - \ln\left(x\left(\left(\frac{y}{x}\right)^2 + \frac{2y}{x} + 1\right)\right) = c$$

$$\boxed{\frac{y}{x} - \ln\left(\frac{y^2}{x} + 2y + x\right) = c}$$

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$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$y - xy' = ay^2 + ay'$$

$$y - ay^2 = ay' + xy'$$

$$y(1-ay) = y'(a+x)$$

$$\int \frac{dy}{y(1-ay)} = \int \frac{dx}{a+x}$$

$$\int \frac{1}{y} + \frac{a}{1-ay}\, dy = \int \frac{dx}{a+x}$$

$$\ln y - \ln(1-ay) = \ln(a+x) + \ln c$$

$$\frac{y}{(1-ay)} = (a+x)\cdot c$$

$$y = (1-ay)(ac+xc)$$

$$y = ac + xc - a^2yc - ayxc$$

$$y = ac + xc - y(a^2c + axc)$$

$$y(1 + a^2c + axc) = ac + xc$$

$$y = \frac{\cancel{c}(a+x)}{\cancel{c}\left(\frac{1}{c} + a^2 + ax\right)}$$

$$\boxed{y = \frac{a+x}{K+ax}}$$

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xi) $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$

$$(x+1)\,dy = (2e^{-y}-1)\,dx$$

$$\int \frac{dy}{2e^{-y}-1} = \int \frac{dx}{(x+1)}$$

$$\int \frac{dy}{\frac{2}{e^{y}}-1} = \int \frac{dx}{(x+1)}$$

$$\int \frac{e^{y}\,dy}{2-e^{y}} = \int \frac{dx}{(x+1)}$$

$$-\ln(2-e^{y}) = \ln(x+1) + \ln c$$

$$(2-e^{y})^{-1} = c(x+1)$$

$$(2-e^{y}) = \frac{c}{x+1}$$

$$e^{y} = 2 - \frac{c}{x+1}$$

$$\boxed{y = \ln\left(2 - \frac{c}{x+1}\right)}$$

$$-\ln(2-e^{y}) = \ln(x+1) + c$$

$$-\ln(2-e^{y}) - \ln(x+1) = c$$

$$-\ln\left(e^{y}(2e^{-y}-1)\right) - \ln(x+1) = c$$

$$-\ln e^{y} - \ln(2e^{-y}-1) - \ln(x+1) = c$$

$$\boxed{c = -y - \ln\left((2e^{-y}-1)(x+1)\right)}$$

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xii) $x^2y' + xy + y^2 = 0$

$y'x^2 = -(y^2 + xy)$

$y' = -\left(\frac{y^2}{x^2} + \frac{\cancel{x}y}{\cancel{x}}\right)$

$y' = -(v^2 + v)$

$v'x + v = -(v^2 + v)$

$v'x = -(v^2 + 2v)$

$x.\frac{dv}{dx} = -(v^2 + 2v)$

$\int \frac{dv}{v(v+2)} = -\int \frac{dx}{x}$

$\frac{1}{2}\int \frac{1}{v} + \frac{(-1)}{v+2} = -\int\frac{dx}{x}$

$\frac{1}{2}\left(\ln v - \ln(2+v)\right) = -\ln x + \ln c$

$\cancel{\ln}\left(\frac{v}{2+v}\right)^{1/2} = \cancel{\ln}\left(\frac{c}{x}\right)$

$\left(\frac{v}{2+v}\right)^{1/2} = \frac{c}{x} \rightarrow eq_1$

$\frac{v}{2+v} = \frac{c^2}{x^2}$

$vx^2 = K(2+v)$

$vx^2 = 2K + vK$

$vx^2 - vK = 2K$

$v(x^2 - K) = 2K$

$v = \frac{2K}{x^2 - K}$

$\boxed{y = \frac{2Kx}{x^2-K}}$

From eq i

$\left(\frac{2+v}{v}\right)^{1/2 \cdot 2} = (Kx)^2$

$\frac{2+v}{v} = Kx^2$

$2+v = vKx^2$

$2 = v(Kx^2 - 1)$

$v = \frac{2}{-1+Kx^2}$

$\boxed{y = \frac{2x}{-1+Kx^2}}$

$\frac{2x}{-1+Kx^2} \equiv \frac{2Kx}{x^2-K}$

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xiii) $\underbrace{(\sec x \tan x \tan y - e^x)}_{M} dx + \underbrace{(\sec x \sec^2 y)}_{N} dy = 0$

$M_y = \sec^2 y \sec x \tan x$
$N_x = \sec^2 y \sec x \tan x$

$M_y = N_x$ exact

$\cancel{f = \int \sec^2 y \sec x \tan x \, \partial}$

$f = \int \sec x \tan x \tan y - e^x \, \partial x$, $f = \int M \partial x$
$f = \sec x \tan y - e^x$ → eq₁

$f = \int \sec x \sec^2 y \, \partial y$, $f = \int N \partial y$

$f = \tan y \sec x$ → eq₂

merging eq₁ & eq₂

$c = \sec x \tan y - e^x$
$c + e^x = \sec x \tan y$

$$\boxed{y = \tan^{-1}\left(\frac{c + e^x}{\sec x}\right)}$$

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xiv) $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$

$y' x\cos x + y(x\sin x + \cos x) = 1$

$y' + y \frac{(x\sin x + \cos x)}{x\cos x} = \frac{1}{x\cos x}$

$y' + y\left(\frac{\cancel{x}\sin x}{\cancel{x}\cos x} + \frac{\cancel{\cos x}}{x\cancel{\cos x}}\right) = \frac{\sec x}{x}$

$y' + y\left(\tan x + \frac{1}{x}\right) = \frac{\sec x}{x}$

$I.F = e^{\int \tan x + \frac{1}{x} dx}$

$= e^{\ln \sec x} \cdot e^{\ln x}$

$= x\sec x$

$y x\sec x = \int \cancel{x}\sec x \cdot \frac{\sec x}{\cancel{x}} dx + c$

$y x\sec x = \int \sec^2 x \, dx + c$

$y x\sec x = \tan x + c$

$$\boxed{y = \frac{\tan x + c}{x\sec x}}$$

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xv) $x\ln x\, y' + y = 2\ln x$

$$y' + \frac{y}{x\ln x} = \frac{2\cancel{\ln x}}{x\cancel{\ln x}}$$

$$I.F = e^{\int \frac{1}{x\ln x}dx}$$
$$= e^{\ln(\ln x)}$$
$$= \ln x$$

$$y\ln x = \int \frac{2}{x}\cdot\ln x\, dx + C$$

$$y\ln x = \frac{\cancel{2}(\ln x)^2}{\cancel{2}} + C$$

$$y\ln x = (\ln x)^2 + C$$
$$\boxed{y = \ln x + \frac{C}{\ln x}}$$

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xvi) $y' + \frac{4}{x}y = x^3 y^2$

$y'y^{-2} + \frac{4y^{-1}}{x} = x^3$

Let $u = y^{-1}$

$-u' + \frac{4u}{x} = x^3$

$u' - \frac{4u}{x} = -x^3$

$I.F = e^{-4\int \frac{1}{x}}$

$= \cancel{e}^{\cancel{\ln} x^{-4}}$

$= x^{-4}$

$ux^{-4} = \int -\cancel{x^3} \cdot \cancel{x^{-4}}^{1} dx + C$

$\frac{x^{-4}}{y} = -\ln x + C$

$$\boxed{y = \frac{1}{x^4(-\ln x + C)}}$$

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Q2i $P \propto \frac{dP}{dt}$

$$\cancel{\int \frac{dP}{dt} = \int kP}$$

$$\frac{dP}{dt} = kP$$

$$\int \frac{dP}{P} = k \int dt$$

$$\ln P = kt + C$$

$$P = C_1 e^{kt}$$

$$P = P_0, \quad t = 0$$

$$P_0 = C_1 e^{k(0)}$$

$$P_0 = C_1$$

$$P = P_0 e^{kt}$$

$$2\cancel{P_0} = \cancel{P_0} e^{(5k)}$$

$$\frac{\ln(2)}{5} = k$$

$$3\cancel{P_0} = \cancel{P_0} e^{kt}$$

$$\ln(3) = kt$$

$$\ln(3) = \frac{\ln(2)t}{5}$$

$$t = \frac{5\ln(3)}{\ln(2)}$$

$$\boxed{t = 7.92 \text{ years}}$$

$$4\cancel{P_0} = \cancel{P_0} e^{kt}$$

$$4 = e^{kt}$$

$$\ln(4) = kt$$

$$\cancel{\ln(4) = \frac{\ln(3)t}{\ln(2)}}$$

$$\ln(4) = \frac{\ln(2)t}{5}$$

$$t = \frac{5\ln(4)}{\ln(2)}$$

$$\boxed{t = 10 \text{ years}}$$

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ii) $m \propto \frac{dm}{dt}$

$\frac{dm}{dt} = -km$

$\int \frac{dm}{m} = -k \int dt$

$\ln m = -kt + c$

$m = c_1 e^{-kt}$

$m = m_0$, $t = 0$, $m_0 = 1$

$m = e^{-kt}$

half life $= 3.3$, $m = 0.5 m_0 = m = 0.5(1)$

$0.5 = e^{-k(3.3)}$

$\ln(0.5) = -k(3.3)$

$-k = \frac{\ln(0.5)}{3.3}$

$m = e^{-kt}$

$(1 - 0.9) m_0 = 0.1 m_0 = 0.1$

$\ln 0.1 = e^{kt}$

$\ln(0.1) = kt$

$t = \frac{\ln(0.1) \cdot 3.3}{\ln(0.5)}$

$\boxed{t = 10.96 \approx 11 \text{ hrs}}$

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Q2 iii) $\frac{dT}{dt} \propto T - T_m$ $\quad t=0,\ T=70$

$\frac{dT}{dt} = k(T - T_m)$ $\quad t=\frac{1}{2},\ T=110$

$\int \frac{dT}{T - T_m} = k\int dt$ $\quad t=1,\ T=145$

$\ln(T - T_m) = kt + C$

$\ln(70 - T_m) = C$

$2\ln(110 - T_m) = \left[\frac{1}{2}k + \ln(70 - T_m)\right] \times 2$

$\ln(145 - T_m) = k + \ln(70 - T_m)$

$\ln\left(\frac{145 - T_m}{70 - T_m}\right) = k$

$2\ln\left(\frac{110 - T_m}{70 - T_m}\right) = \ln\left(\frac{145 - T_m}{70 - T_m}\right)$

$(110 - T_m)^2 = (145 - T_m)(70 - T_m)$

$12100 - 220T_m + (T_m)^2 = 10150 - 145T_m - 70T_m + (T_m)^2$

$5T_m = 1950$

$\boxed{T_m = 390°F}$

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(iv) $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = E(t)$

$L = 0.1$
$R = 50$
$i(t),\ i(0) = 0$
$E(t) = 30$

$L\frac{di}{dt} + Ri = E(t)$

$0.1\frac{di}{dt} + 50i = E(t)$

$0.1\frac{di}{dt} + 50i = 30$

$\cancel{0.1}\,i' + \frac{50i}{0.1} = \frac{30}{0.1}$

$\cancel{I.F = e^{50\int i\,di} = e^{\frac{50 i^2}{2}} = e^{250 i^2}}$

$I.F = e^{\int dt}$
$= e^{500t}$

$ie^{500t} = \int e^{500t} \cdot 300 + C$

$ie^{500t} = \frac{3}{5}e^{500t} + C$

$\frac{3}{5}e^{500(0)} + C = 0$

$C = -\frac{3}{5}$

$ie^{500t} - \frac{3}{5}e^{500t} = -\frac{3}{5}$

$\boxed{i(t) = \frac{+3}{5} - \frac{3}{5}e^{-500t}}$

$e^{500t}\left(i - \frac{3}{5}\right) = -\frac{3}{5}$

$\left(i - \frac{3}{5}\right) = -\frac{3}{5} \cdot e^{-500t}$

$t \to \infty,\ e^{-500t} \to 0$

$i - \frac{3}{5} = 0$

$\boxed{i = \frac{3}{5}}$