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Q1 $y'' + 4y' + 3y = 0$

Auxilary eq:

$m^2 + 4m + 3 = 0$

$m = -1, \; m = -3$

$$\boxed{y = C_1 e^{-x} + C_2 e^{-3x}}$$

Q2 $y''' - y'' + y' - y = 0$

Auxilary eq

$m^3 - m^2 + m - 1 = 0$

$m = 1, \quad m = \pm i$

$$\boxed{y = C_1 e^{x} + C_2 \cos x + C_3 \sin x}$$

Q3 $2x^2 y'' + 3x y' - 15 y = 0$

Let $y = x^m$ d.o.t $y' = m(x^{m-1})$

$y'' = m(m-1)(x^{m-2})$

$2x^2 \cdot x^{m-2} \cdot m(m-1) + 3x \cdot m(x^{m-1}) - 15x^m = 0$

$x^m (2m(m-1) + 3m - 15) = 0$

$x^m (2m^2 - 2m + 3m - 15) = 0$

$x^m (2m^2 + m - 15) = 0 \qquad \underset{\text{Rej}}{x^m = 0} \qquad m = \frac{5}{2}, \; m = -3$

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$y_1 = x^{5/2}$, $y_2 = x^{-3}$

$y = c_1 y_1 + c_2 y_2$

$$\boxed{y = c_1 x^{5/2} + c_2 x^{-3}}$$

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Q4 $y'' - 3y' + 2y = x^2 e^x$

Assosciated ~~Ho~~ form:-

$m^2 - 3m + 2 = 0$

$m = 2, \quad m = 1$

$y_c = C_1 e^{2x} + C_2 e^x$

$y_p = x^r (Ax^2 + Bx + C) e^x$ → Trial function for $r = 1$

$y = (Ax^3 + Bx^2 + Cx) e^x$

$2y = \left[(2A)x^3 + (2B)x^2 + (2C)x\right] e^x \rightarrow (1)$

$y' = (Ax^3 + Bx^2 + Cx)e^x + e^x(3Ax^2 + 2Bx + C)$

$-3y = -3e^x \left[x^3(A) + x^2(B+3A) + x(\overset{+2B}{C}) + C\right]$

$= e^x \left[x^3(-3A) + x^2(-3B-9A) + x(\overset{-6B}{-3C}) - 3C\right] \rightarrow (2)$

$y'' = e^x\left[x^3(A) + x^2(B+3A) + x(\overset{+2B}{C}) + C\right] + e^x\left[3Ax^2 + x(2B+6A) + \overset{+2B}{C}\right]$

$y'' = e^x\left[x^3(A) + x^2(B+6A) + x(\cancel{4}B + 6A + C) + \overset{+2B}{2C}\right] \rightarrow (3)$

Inserting eq (1), (2), (3)

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$e^{x}\left[x^3(A)+x^2(B+6A)+x(4B+6A+C)+2C+x^3(-3A)\right.$ $+2B$

$+x^2(-3B-9A)+x(-3C-6B)-3C+x^3(2A)+x^2(2\ldots$

$=\cancel{e^{x}}\cdot x^2$

$x^3(\cancel{A-3A+2A})=0 \to 0$

$0=0$

$\cancel{x^2}(\cancel{B}+6A-3\cancel{B}-9A+2\cancel{B})=\cancel{x^2}$

$-3A=1$

$\boxed{A=\frac{-1}{3}}$

$x(4B+6A+\cancel{C}-3\cancel{C}-6B+2\cancel{C})=0$

$6A-2B=0$

$3A-B=0$

$\boxed{B=-1}$

$-3C+2C+2B=0$

$-C-2=0$

$\boxed{C=-2}$

$y_p=\left(\frac{-1}{3}x^3-x^2-2x\right)e^{x}$

General Sol

$y=y_p+y_c$

$\boxed{y=\left(\frac{-x^3}{3}-x^2-2x\right)e^{x}+C_1e^{2x}+C_2e^{x}}$

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$y'' + 4y = xe^{x} + x\sin 2x$

Assosciated Form:-

$y'' + 4y = 0$

$m^2 + 4 = 0$

$m = \pm 2i$

$y_c = C_1\cos 2x + C_2 \sin 2x$

$y_p = x^r (Ax+B)e^x + x^q[(Cx+D)\cos 2x + (Ex+F)\sin 2x]$

$r=0,\ q=1$

$y_p: (Ax+B)e^x + (Cx^2+Dx)\cos 2x + (Ex^2+Fx)\sin 2x$

$y_p' = Ae^x + e^x(Ax+B) + (2Cx+D)\cos 2x - (2\sin 2x)(Cx^2+Dx) +$
$(2Ex+F)(\sin 2x) + (2\cos 2x)(Ex^2+Fx)$

$y_p'' = 2Ae^x + e^x(Ax+B) + 2\cos 2x - 4\sin 2x(\cancel{Ex^2}\ 2Cx+D)$
$- \cancel{4\cos 2x(Cx^2+Dx)} + (2Cx+D)(-2\sin 2x) + 2E\sin 2x$
$+ 2\cos 2x(2E+F) - \cancel{4\sin 2x(Ex^2+Fx)} + (2Ex+F)(2\cos 2x)$

$+4y_p = 4(Ax+B)e^x + \cancel{4\cos 2x(Cx^2+Dx)} + \cancel{4\sin 2x(Ex^2+Fx)}$

$2Ae^x + 5(Ax+B)e^x + 2C\cos 2x - 4\sin 2x(2Cx+D) +$
$2E\sin 2x + 4\cos 2x(2Ex+F) \equiv xe^x + x\sin 2x$

$e^x(2A+5B) + xe^x(5A) + \cos 2x(2C+4F) + x\cos x(8E)$
$+ \sin 2x(-4D+2E) + x\sin 2x(-8C) \equiv xe^x + x\sin 2x$

$A = \frac{1}{5}$, $C = -\frac{1}{8}$, $E = 0$, $D = 0$, $B = -\frac{2}{25}$, $F = \frac{1}{16}$

$2A+5B=0$, $2C+4F=0$, $-8E=0$, $-4D+2E=0$,

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$$\boxed{y = \frac{xe^x}{5} - \frac{2e^x}{25} - \frac{x^2}{8}\cos 2x + \frac{x\sin 2x}{16} + C_1\cos 2x + C_2\sin 2x}$$

Q6 $y'' - 2y' + y = xe^x \ln x$

Associated Form:

$y'' - 2y' + y = 0$

$m^2 - 2m + m = 0$

$m = 1$

$y_c = e^x(C_1 + C_2 x) \quad y_1 = e^x, \; y_2 = xe^x$

$y_p = u_1 y_1 + u_2 y_2$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & xe^x \\ xe^x \ln x & e^x + e^x(x) \end{vmatrix} = -x^2 e^{2x} \ln x$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & xe^x \ln x \end{vmatrix} = xe^{2x} \ln x$$

$$u_1 = -\int \frac{x^2 \cancel{e^{2x}} \ln x}{\cancel{e^{2x}}}$$

$u' = \frac{1}{x} \quad v = x^3/3$

$v' = x^2 \quad u = \ln x$

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$$= -\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2$$

$$= \frac{-x^3 \ln x}{3} + \frac{x^3}{9}$$

$$u_2 = \int \frac{x\cancel{e^{2x}}\ln x}{\cancel{e^{2x}}}$$

$$u_2 = \int x \ln x$$

$u' = 1/x$ $\quad v = x^2/2$

$v' = x$ $\quad u = \ln x$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x$$

$$= \frac{x^2}{2}\ln x - \frac{x}{4}$$

$$\cancel{y = \left(\frac{x^3 \ln x}{6} - \frac{5x^3}{36}\right)e^x + C_1 e^x + C_2 e^x \cdot x}$$

$$\boxed{y = C_1 e^x + x C_2 e^x + e^x\left(\frac{-5x^3}{36} + \frac{x^3 \ln x}{6}\right)}$$

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~~$3e^x$~~ $5e^x - 4\sin 2x + 8x\sin 2x = x(e^x + \sin 2x)$

$5e^x - 4\sin 2x = 0$

~~$8x = x$~~

$8\sin 2x = e^x + \sin 2x$

Q8 $y'' - 4y' - 12y = 2t^3 - t + 3$

Assosciated Form:-

$y'' - 4y' - 12y = 0$

$m^2 - 4m - 12 = 0$

$m_1 = 6, \quad m_2 = -2$

$y_c = C_1 e^{6t} + C_2 e^{-2t}$

$y_p = (At^3 + Bt^2 + Ct + D)\, t^r \; ; \quad r = 0$

$y_p = At^3 + Bt^2 + Ct + D$

$y_p' = 3At^2 + 2Bt + C$

$y_p'' = 6At + 2B$

$6At + 2B - 4(3At^2 + 2Bt + C) - 12(At^3 + Bt^2 + Ct + D)$

$= 2t^3 - t + 3$

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$-\cancel{12}^{6}A = \cancel{2}$

$\boxed{A = -\frac{1}{6}}$

$-12B - 12A = 0$

$B + A = 0$

$\boxed{B = \frac{1}{6}}$

$6A - 8B - 12C = -1$

$-\cancel{1} - \frac{8\cancel{4}}{\cancel{6}3} - 12C = -\cancel{1}$

$3 - \cancel{12}C = \frac{\cancel{4}1}{3}$

$\boxed{C = -\frac{1}{9}}$

$2B - 4C - 12D = 3$

$\frac{1}{3} - \frac{4}{9} - 12D = 3$

$\boxed{D = -\frac{5}{27}}$

Gen Sol:-

$$\boxed{y = C_1 e^{6t} + C_2 e^{-2t} - \frac{1}{6}t^3 + \frac{t^2}{6} - \frac{t}{9} - \frac{5}{27}}$$

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Q9 $y'' + 5y' + 6y = 2x$

Assosciated Form:-

$y'' + 5y' + 6y = 0$
$m^2 + 5m + 6 = 0$

$m = -2, \quad m = -3$

$y_c = C_1 e^{-2x} + C_2 e^{-3x}$

$y_p = x^r (Ax + B) \quad ; \quad r = 0$

$y_p = Ax + B$
$y_p' = A$

$y_p'' = 0$

$5A + 6Ax + 6B = 2x$

$3\cancel{6}A\cancel{x} = \cancel{2x}$

$\boxed{A = \frac{1}{3}}$

$5A + 6B = 0$

$\frac{5}{3} + 6B = 0$

$\boxed{B = \frac{-5}{18}}$

~~Gen Solu~~ :-

$\cancel{y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{1}{3}}$

Gen Sol

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-3x} + \left(\frac{1}{3}x - \frac{5}{18}\right)}$$

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Q10 $y'' + 5y' - 9y = e^{-2x} + 2 - x$

$m^2 + 5m - 9 = 0$

$m_1 = 1.40 \quad , \quad m_2 = -6.4$

$y_c = C_1 e^{1.4x} + C_2 e^{-6.4x}$

$y_p = x^r (A e^{-2x}) + x^q (Bx + C) \quad ; \quad r = 0, q = 0$

$y_p = A e^{-2x} + Bx + C$

$y_p' = -2A e^{-2x} + B$

$y_p'' = 4A e^{-2x}$

$4Ae^{-2x} - 10Ae^{-2x} + 5B - 9Ae^{-2x} - 9Bx - 9C \equiv e^{-2x} + 2 - x$

$\boxed{A = \frac{-1}{15}} \quad \boxed{B = \frac{1}{9}} \quad \boxed{C = \frac{-13}{81}}$

$$\boxed{y = C_1 e^{1.4x} + C_2 e^{-6.4x} - \frac{1}{15} e^{-2x} + \frac{1}{9} x - \frac{13}{81}}$$

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Q:: $y'' - 100y = 9t^2 e^{10t} + \cos t - t\sin t$

Associated Form:-

$m^2 - 100 = 0$

$m = \pm 10$

$y_c = C_1 e^{10t} + C_2 e^{-10t}$

$y_p = (At^2 + Bt + C)e^{10t} \cdot t^r + t^q(\cos t(A_2 + C_2 t) + \sin t(B_2 + D t))$

$r = 1, \; q = 0$

$y_p = (At^3 + Bt^2 + Ct)e^{10t} + A_2\cos t + C_2 t\cos t + B_2\sin t + Dt\sin t$

$y_p' = (3At^2 + 2Bt + C)e^{10t} + 10(At^3 + Bt^2 + Ct)e^{10t} - A_2\sin t + C_2\cos t - \sin t(C_2 t) + B_2\cos t + D_2\sin t + D_2 t\cos t$

$y_p'' = (6At + 2B)e^{10t} + 20(3At^2 + 2Bt + C)e^{10t} + \cancel{100e^{10t}(At^3 + Bt^2 + Ct)} - A_2\cos t - 2C_2\sin t - C_2 t\cos t - B_2\sin t$
$+ \cancel{D\sin t} + 2D\cos t - Dt\sin t$

$-100y = -100\cancel{(At^3 + Bt^2 + Ct)e^{10t}} - 100A_2\cos t - 100C_2 t\cos t$
$-100B_2\sin t - 100D_2 t\sin t$

$e^{10t}(6At + 2B + 60At^2 + 40Bt + 20C)$
$-101A_2\cos t - 2C_2\sin t - 101C_2 t\cos t - 101B_2\sin t$
$+ 2D\cos t - 101Dt\sin t \equiv 9t^2 e^{10t} + \cos t - t\sin t$

let $A_2 = D$ $\quad C_2 = F$
$B_2 = E$ $\quad D = G$

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$60A = 9$ $\qquad A = \frac{9}{60}$

$6A + 40B = 0$ $\qquad B = \frac{-9}{400}$

$2B + 20C = 0$ $\qquad C = \frac{9}{4000}$

$-101D + 2G = 1$ $\qquad D = -\frac{99}{10201}$

$-2F - 101E = 0$

$-101F = 0$

$-101G = -1$

$E = 0 \qquad G = \frac{1}{101}$

$F = 0$

$$\boxed{y = C_1e^{10t} + C_2e^{-10t} + \frac{3}{20}t^3e^{10t} - \frac{9}{400}t^2e^{10t} + \frac{9}{4000}te^{10t} - \frac{99}{10201}\cos + \frac{1}{101}t\sin t}$$

Q12 $y'' - 2y' + 2y = e^{x}\tan x$

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Associated Form:-

$y'' - 2y' + 2y = 0$

$m^2 - 2m + 2 = 0$

$m = 1 \pm i$

$y_c = e^{x}(c_1 \cos x + c_2 \sin x)$

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$y_p = u_1 y_1 + u_2 y_2 \qquad y_1 = e^x \cos x, \quad y_2 = e^x \sin x$

$$W = \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x \cos x - \sin x e^x & e^x \sin x + \cos x e^x \end{vmatrix} =$$

$$e^x \cos x \left(\cancel{e^x \sin x}\right) + (e^x \cos x)^2 - \left[(e^x \sin x)^2 + \cancel{e^x \sin x}\, e^x \cos x\right]$$

$$e^{2x} \cos^2 x + e^{2x} \sin^2 x$$

$$= e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & e^x \sin x \\ e^x \tan x & e^x \sin x + \cos x e^x \end{vmatrix} = -e^{2x} \sin x \tan x$$

$$W_2 = \begin{vmatrix} e^x \cos x & 0 \\ e^x \cos x - \sin x e^x & e^x \tan x \end{vmatrix} = e^{2x} \cos x \tan x$$

$$u_1 = \int \frac{W_1}{W} dx$$

$$u_1 = -\int \frac{\cancel{e^{2x}}}{\cancel{e^{2x}}} \frac{\sin^2 x}{\cos x} = -\int \frac{1 - \cos^2 x}{\cos x \quad \cancel{\cos x}} = +\int -\sec x + \cos x$$

$$= -\ln|\sec x + \tan x| + \sin x$$

$$u_2 = \int \frac{W_2}{W} dx = \int \frac{\cancel{e^{2x}} \sin x}{\cancel{e^{2x}}} dx = -\cos x$$

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$$y_p = \left(-\ln|\sec x + \tan x| + \cancel{\sin x}\right) e^x \cos x + \cancel{e^x \sin x (-\cos x)}$$

$$\boxed{y = e^x(c_1 \cos x + c_2 \sin x) - e^x \cos x \ln|\sec x + \tan x|}$$

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Q13 $x^2y'' - 4xy' + 6y = 2x^4 + x^2$

Assosciated Form:-

Let $y = x^m$
$y' = mx^{m-1}$
$y'' = m(m-1)x^{m-2}$

$m^2 - 5m + 6 = 0$

$m = 3,\ m = 2$

$\cancel{y_c = c_1 e^{3x} + c_2 e^{2x}}$ $y_c = C_1 x^3 + C_1 x^2$

$y'' - \frac{4y'}{x} + \frac{6y}{x^2} = 2x^2 + 1$

$W = \begin{vmatrix} x^3 & x^2 \\ 3x^2 & 2x \end{vmatrix} = 2x^4 - 3x^4 = -x^4$

$W_1 = \begin{vmatrix} 0 & x^2 \\ 2x^2+1 & 2x \end{vmatrix} = -(2x^4 + x^2)$

$W_2 = \begin{vmatrix} x^3 & 0 \\ 3x^2 & 2x^2+1 \end{vmatrix} = 2x^5 + x^3$

$u_1 = -\int \frac{2\cancel{x^4}}{\cancel{x^4}} + \frac{\cancel{x^2}}{x^{4\,2}} = +\int 2 + \frac{1}{x^2}\,dx = +2x - \frac{1}{x}$

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$$u_2 = -\int \frac{2x^{\cancel{5}}}{\cancel{x^4}} + \frac{\cancel{x^3}}{x^{\cancel{4}}} = -x^2 + \ln x$$

$$y_p = \left(+2x - \frac{1}{x}\right)x^3 + \left(-x^2 - \ln x\right)\left(x^2\right)$$

$$y_p = +2x^4 - x^2 - x^4 - x^2 \ln x$$

$$y_p = x^4 - x^2\left(\ln x + 1\right)$$

$$\boxed{y = C_1 x^3 + C_2 x^2 + x^4 - x^2 \ln x}$$

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Q14 $x^2y'' + 10xy' + 8y = x^2$

Assosciated Form:

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$x^m(m^2 - m + 10m + 8) = 0$

$x^m = 0$ $\quad m^2 + 9m + 8 = 0$

Rej $\quad m = -8, \; m = -1$

$y_c = c_1 x^{-8} + c_2 x^{-1}$

$y'' + 10x^{-1}y' + 8x^{-2}y = 1$

$W = \begin{vmatrix} x^{-8} & x^{-1} \\ -8x^{-9} & -x^{-2} \end{vmatrix} = x^{-8}(-x^{-2}) - x^{-1}(-8x^{-9}) = -x^{-10} + 8x^{-10}$

$= +7x^{-10}$

$W_1 = \begin{vmatrix} 0 & x^{-1} \\ 1 & -x^{-2} \end{vmatrix} = -x^{-1}$ $\quad u_1 = \int \frac{-x^{-1}}{7x^{-10}} = \frac{-1}{7}\int x^9 = \frac{-1}{70}x^{10}$

$W_2 = \begin{vmatrix} x^{-8} & 0 \\ -8x^{-9} & 1 \end{vmatrix} = x^{-8}$ $\quad u_2 = \int \frac{x^{-8}}{7x^{-10}} = \frac{x^3}{21}$

$y_p = \frac{x^3}{21}(x^{-1}) - \frac{1}{70}x^{10}(x^{-8})$ $\quad y_p = \frac{-x^2}{70} + \frac{x^2}{21}$ $\quad y_p = \frac{1}{30}x^2$

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$$\boxed{y = C_1 x^{-8} + C_2 x^{-1} + \frac{x^2}{30}}$$

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Q:5 $x^2 y'' - 3xy' + 13y = 4 + 3x \rightarrow (1)$

$x = e^t$

$\cancel{e^{2t} y'' - 3e^t y' + 13y = 4 + 3e^t}$

$\ln x = t$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{1}{x} \qquad y' = \frac{1}{x}\frac{dy}{dt} \rightarrow (2)$$

$$\cancel{\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}} \qquad \frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dt} + \frac{d}{dx}\left(\frac{dy}{dt}\right) \cdot \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dt} + \frac{d}{\cancel{dx}}\left(\frac{dy}{dt}\right) \cdot \frac{\cancel{dx}}{dt} \cdot \frac{dt}{dx} \cdot \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dt} + \frac{d^2y}{dt^2}\left(\frac{1}{x^2}\right)$$

$$y'' = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) \rightarrow (3)$$

inserting (2) & (3) in (1)

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$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = 4 + 3e^t$$

Associated form:-

$m^2 - 4m + 13 = 0$

$m = 2 \pm 3i$

$y_c = e^{2t}[c_1\cos(3t) + c_2\sin(3t)]$

$y_p = A + Be^t$

$y_p' = Be^t$

$y_p'' = Be^t$

$Be^t - 4Be^t + 13A + 13Be^t = 4 + 3e^t$

$10B\cancel{e^t} = 3\cancel{e^t}$ $\quad A = \frac{4}{13}$

$B = 0.3$

$y_p = \frac{4}{13} + 0.3e^t$

$y = e^{2t}[c_1\cos 3t + c_2\sin 3t] + \frac{4}{13} + 0.3e^t$

$$\boxed{y = x^2[c_1\cos(3\ln x) + c_2\sin(3\ln x)] + \frac{4}{13} + 0.3x}$$

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Q16 $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

$x = e^t$ $\quad y' = \frac{1}{x}\frac{dy}{dt}$ $\quad y'' = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$

$\ln x = t$

$$y''' = \frac{d}{dx}\left(\frac{1}{x^2}\left(\frac{d^2y}{dt^2}\right) - \frac{1}{x^2}\left(\frac{dy}{dt}\right)\right)$$

$$y''' = \left[\frac{-2}{x^3}\frac{d^2y}{dt^2} + \frac{d}{dt}\left(\frac{d^2y}{dt^2}\right)\cdot\frac{1}{x}\left(\frac{1}{x^2}\right)\right] - \left[\frac{-2}{x^2}\frac{d^2y}{dt^2} + \frac{d}{dt}\left(\frac{dy}{dt}\right)\right]$$

$$y''' = \cancel{-\frac{1}{x^3}}\frac{1}{x^3}\left[-3\frac{d^2y}{dt^2} + \frac{d^3y}{dt^3} + 2\frac{dy}{dt}\right]$$

$$-3\frac{d^2y}{dt^2} + \frac{d^3y}{dt^3} + 2\frac{dy}{dt} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 6\frac{dy}{dt} - 6y = 3 + 3t$$

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} - 6y = 3 + 3t$$

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Associated Form:-

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 1, 2, 3$

$y_c = C_1 e^t + C_2 e^{2t} + C_3 e^{3t}$

$y_p = A + Bt$

$y_p' = B$

$y_p'' = 0$

$y_p''' = 0$

$11B - 6A - 6Bt = 3 + 3t$

$11B - 6A = 3$, $-\cancel{6}B\cancel{t} = \cancel{3}\cancel{t}$

$11(-\frac{1}{2}) - 6A = 3$ $B = -\frac{1}{2}$

$A = -\frac{17}{12}$

$$\boxed{y = C_1 x + C_2 x^2 + C_3 x^3 - \frac{17}{12} - \frac{1}{2}\ln x}$$

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Q17 $y'' - 2y' + y = \frac{1}{x}e^x$; $y(1) = 0,\ y'(1) = 1$

Assosociated form

$m^2 - 2m + 1 = 0$

$m = 1$

$y_c = e^x(C_1 + C_2 x)$

$y_p = u_1 y_1 + u_2 y_2$

$w = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^x(e^x + \cancel{xe^x}) - e^x(\cancel{xe^x})$
$= e^{2x}$

$w_1 = \begin{vmatrix} 0 & xe^x \\ \frac{e^x}{x} & e^x + xe^x \end{vmatrix} = -e^{2x}$

$w_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{x} \end{vmatrix} = \frac{e^{2x}}{x}$

$u_1 = \int \frac{-\cancel{e^{2x}}}{\cancel{e^{2x}}}\, dx = -x$

$u_2 = \int \frac{e^{2x}}{x \cdot e^{2x}} = \ln x$

$y = e^x(C_1 + C_2 x) - xe^x + xe^x \ln x$

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$$y = e^x(c_1 + c_2 x) + xe^x \ln x$$

$$y(1) = 0$$

$$0 = e(c_1 + c_2) + e\cancel{\ln(1)}^{\,0}$$

$$c_1 + c_2 = 0$$

$$y' = e^x(c_1 + c_2 x) + c_2 e^x + (xe^x + x)\ln x + \frac{1}{\cancel{x}}(\cancel{x}e^x)$$

$$y'(1) = 1$$

$$1 = e(c_1 + c_2) + c_2 e + \cancel{(e+1)\ln(1)}^{\,0} + e$$

$$\frac{1-e}{e} = c_1 + 2c_2$$

$$\frac{1-e}{e} = c_2 \quad , \quad c_1 = \frac{e-1}{e}$$

$$y = e^x\left(\frac{e-1}{e} + \left(\frac{1-e}{e}\right)x\right) + xe^x \ln x$$

$$y = e^x(1 - e^{-1} + (e^{-1} - 1)x) + xe^x \ln x$$

$$y = e^x[(1-e^{-1}) - 1(1-e^{-1})x) + xe^x \ln x$$

$$y = e^x[(1-e^{-1})[1-x]]) + xe^x \ln x$$

$$\boxed{y = e^{x-1}(e-1)(1-x) + xe^x \ln x}$$

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Q18 $y'' + 4y = \sin^2 2x$

Assosciated Form:-

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

$y_p = u_1 y_1 + u_2 y_2$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x$$
$$= 2$$

$$W_1 = \begin{vmatrix} 0 & \sin 2x \\ \sin^2 2x & 2\cos 2x \end{vmatrix} = -\sin^3 2x$$

$$W_2 = \begin{vmatrix} \cos 2x & 0 \\ -2\sin 2x & \sin^2 2x \end{vmatrix} = \sin^2 2x (\cos 2x)$$

$u_1 = \int \frac{-\sin^3 2x}{2} dx$, $u_1 = -\frac{1}{2} \int \sin^2 2x \cdot \sin 2x \, dx$

$u_1 = \frac{-1}{2} \int \sin 2x - \cos^2 2x . \sin 2x \, dx$

$u_1 = +\frac{1}{4} \cos 2x - \frac{\cos^3 2x}{12}$

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$$u_2 = \frac{1}{2}\int \sin^2 2x \cos 2x \, dx$$

$$u_2 = \frac{\sin^3 2x}{12}$$

$$y = C_1\cos 2x + C_2\sin 2x + \frac{1}{4}\cos^2 2x - \frac{\cos^4 2x}{12} + \frac{\sin^4 2x}{12}$$

$$0 = C_1\cos 2 + C_2\sin 2 + \frac{(\cos 2)^2}{4} - \frac{(\cos 2)^4}{12} + \frac{(\sin 2)^4}{12}$$

$$y' = -C_1\sin 2x(2) + 2C_2\cos 2x + \frac{\cancel{2}\cos 2x(-\sin 2x)(\cancel{2})}{\cancel{4}\cdot 2} + \frac{4\cos^3 2x(+\sin 2x)(\cancel{2})}{\cancel{6}\;3\cdot 2} + \frac{\cancel{4}\sin^3 2x\cos 2x\,(2)}{\cancel{12}\;3}$$

y' $0 = 2C_2$

$$\boxed{C_2 = 0}$$

$$0 = C_1\cos 2 + \frac{(\cos 2)^2}{4} - \frac{(\cos 2)^4}{12} + \frac{(\sin 2)^4}{12}$$

$$C_1 = \left(-\frac{(\cos 2)^2}{4} + \frac{(\cos 2)^4}{12} - \frac{(\sin 2)^4}{12}\right)\cdot\frac{1}{\cos 2}$$

$$\boxed{y = \left(-\frac{\cos 2}{4} + \frac{(\cos 2)^3}{12} - \frac{(\tan 2)(\sin(2))^3}{12}\right)\cos 2x + \frac{1}{4}\cos^2 2x - \frac{\cos^4 2x}{12} + \frac{\sin^2 2x}{12}}$$

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Q 19 $y'' - 6y' - 7y = -9e^{-2x}$

Assosciated Forms

$m^2 - 6m - 7 = 0$

$m = 7, \ m = -1$

$y_c = C_1 e^{7x} + C_2 e^{-x}$

$y_p = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y_p'' = +4Ae^{-2x}$

$4A\cancel{e^{-2x}} + 12A\cancel{e^{-2x}} - 7A\cancel{e^{-2x}} = -9\cancel{e^{-2x}}$

$\boxed{A = -1}$

$y = C_1 e^{7x} + C_2 e^{-x} - e^{-2x}$

$y' = 7C_1 e^{7x} - C_2 e^{-x} + 2e^{-2x}$

$1 - 2 = C_1 + \cancel{C_2}$

$-2 - 13 = 7C_1 - \cancel{C_2}$

$\frac{-16}{8} = C_1$

$C_1 = -2$

$-2 + C_2 = -1$

$C_2 = 1$

$\boxed{\cancel{y = e^{7x} - 2e^{-x} - e^{-2x}}}$

$\boxed{y = -2e^{7x} + e^{-x} - e^{-2x}}$

1

K200383 Mohsin Ali Mirza 2E-BSCS

Q~~18~~/ ~~$y'' + 4y = \sin^2$~~

Q20 $y'' - 4y' + 4y = 2e^{2x} - 12\cos 3x - 5\sin 3x$

Assosciated Form:>

$m^2 - 4m + 4 = 0$

$m = 2$

$y_c = e^{2x}(C_1 + C_2 x)$

$y_p = x^r(Ae^{2x}) + B\cos 3x + C\sin 3x \;;\; r = 2$

$y_p = Ax^2e^{2x} + B\cos 3x + C\sin 3x$

$y_p' = \cancel{2xAe}\; 2xAe^{2x} + 2Ax^2e^{2x} - 3B\sin 3x + 3C\cos 3x$

$y_p'' = 2Ae^{2x} + 4xAe^{2x} + 4Axe^{2x} + 4Ax^2e^{2x} - 9B\cos 3x - 9C\sin 3x$

$2Ae^{2x} + 8x\cancel{Ae^{2x}} + \underline{4A\cancel{x^2e^{2x}}} - 9B\cos 3x - 9C\sin 3x - \cancel{8xAe^{2x}}$
$\underline{-8A\cancel{x^2e^{2x}}} + 12B\sin 3x - 12C\cos 3x + \underline{4A\cancel{x^2e^{2x}}} + 4B\cos 3x$
$+ 4C\sin 3x \equiv 2e^{2x} - 12\cos 3x - 5\sin 3x$

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$-5B - 12C = -12$ $\boxed{A = 1}$

$-5C + 12B = -5$

$B = 0$

$C = 1$

$$\boxed{\cancel{y = x^2 e^{2x} + \sin 3x + C_1 e^{2x} + x C_2 e^{2x}}}$$

$y = x^2 e^{2x} + \sin 3x + C_1 e^{2x} + x C_2 e^{2x}$

$y' = 2x e^{2x} + 2x^2 e^{2x} + 3\cos 3x + 2C_1 e^{2x} + C_2 e^{2x} + 2x C_2 e^{2x}$

$y'' = 2e^{2x} + 4x e^{2x} + 4x e^{2x} + 4x^2 e^{2x} - 9\sin 3x + 4C_1 e^{2x} +$
$2C_2 e^{2x} + 2C_2 e^{2x} + 4x C_2 e^{2x}$

$-2 = C_1$ $1 = -4 + C_2$

$4 = 3 + 2C_1 + C_2$ $C_2 = 5$

$$\boxed{y = x^2 e^{2x} + \sin 3x - 2e^{2x} + 5x e^{2x}}$$