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(1)

Q1. a) Yes True
b) Yes False
c) Yes True
d) Yes False
e) No
f) No

Q2

	RAM	ROM	CAM
A	256	32	8
B	288	64	4
C	128	32	5

a) Let p:- Smartphone B has the most RAM of these 3 smartphones (True)

p True

b) Let q:- Smartphone C has more ROM than B (False)
r:- Smartphone C has higher resolution camera than smartphone B (True)

q ∨ r True

c) Let s:- Smartphone B has more RAM than A (True)
t:- Smartphone B has more ROM than A (True)
u:- Smartphone B has more higher resolution than smartphone A. (False)

s ∧ t ∧ u False

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(2)

ï Let v:- Smartphone B has more RAM than C (True)
w:- Smartphone B has more RAM than C (True)
x:- Smartphone B has higher resolution camera than C. (False)

$(v \wedge w) \rightarrow x$ False

ë Let y:- Smartphone A has more RAM than B (False)
z:- Smartphone B has more RAM than A (True)

$y \leftrightarrow z$ False

Q3

Company	Revenue	Profit
Acme	138	8
Nadir	87	5
Quixote	111	13

ä Let p:- ~~The annual~~ Quixote Media had the largest annual revenue (False)

p False

b Let q:- Nadir software had the lowest net profit
r:- Acme Computer had the largest annual revenue.

$q \wedge r$ True

q = true
r = true

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(3)

c) Let s:- Acme computer had the highest net profit (False)

t:- Quixote Media had the highest net profit (True)

s ∨ t	true

d) Let u:- Quixote Media had the smallest net profit (False)

v:- Acme computer had the largest ~~net profit~~ annual revenue. (True)

u → v	~~False~~ True

e) Let w:- Nadir software had the smallest net profit (True)

x:- Acme computer had the largest annual revenue (True)

w ↔ x	True
T T	

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(4)

Q4 a) If you have flu then you missed the final examination.

b) You will not miss the final examination if and only if you pass the course.

c) If you miss the final examination, then you failed the course.

d) It is either the case that if you have the flu then you have failed the course, or if you have missed the final examination then you have failed the course.

e) You have the flu or missed the final examination or have passed the course

f) It is either the case that you have the flu and you missed your final examination or you did not miss your final examination and passed the course.

Q5 a) $r \vee \neg q$

b) $p \wedge q \wedge r$

c) $r \rightarrow p$

d) $p \wedge \neg q \wedge r$

e) $(p \wedge q) \rightarrow r$

f) $r \leftrightarrow (q \vee p)$

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(5)

Q6a If you send me an email message, then I will remember to send you the address

b. If you were born in United states, then you are a citizen of this country.

c. If you keep your textbook then it will be a useful reference in your future courses.

d. If their goalie plays well, then the red wings will win the Stanley cup.

e. If you get the job, then you had the best credentials.

f. If there is a storm, then the beach erodes

g. If you log on the server, then you have a valid password.

h. If you donot begin your climb too late, then you will reach the summit

Q7 P implies Q

① It is a sunny day tomorrow implies that I will go for a walk in the woods.

Q if P

② I will go for a walk in the woods if it is sunny tomorrow

Q unless not P

③ I will go for a walk in the woods unless tomorrow is not sunny day.

If P, Q

④ If it is sunny tomorrow, I will go for a walk in the woods.

Q when P

⑤ I will go for a walk in the woods when tomorrow is a sunny day.

(7)

b) $p \rightarrow q$, $q \rightarrow p$, converse

If I will go for a walk in the woods then it is sunny tomorrow

$p \rightarrow q$, $\neg p \rightarrow \neg q$, inverse

If it is not sunny tomorrow, then I will not go for a walk in woods.

$p \rightarrow q$, $\neg q \rightarrow \neg p$ contrapositive

If I will not go for a walk in woods then it is not sunny tomorrow.

c) $\neg p \rightarrow \neg q$ $p \rightarrow q$ Inverse of Inverse

If it is sunny tomorrow, then I will go for a walk in the woods.

$q \rightarrow p$ $\neg q \rightarrow \neg p$ Inverse of Converse

If I will not go for a walk in the woods then it ~~is~~ will not sunny tomorrow

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(8)

$\neg q \to \neg p$ $q \to p$ Inverse of Contrapositive

If I will go for a walk in the woods then it is sunny tomorrow.

Q8

a Jan is either not rich or not happy.
b Carlos will not bicycle and not run tomorrow.
c The fan is not slow ~~so~~ and it is not very hot
d Akram is not unfit or saleem is not injured

Q9

a Exclusive
b Inclusive
c Inclusive
d ~~Exclusive~~ Inclusive

Q10

a $(p \wedge (\neg(\neg p \vee q))) \equiv p \vee (p \wedge q) \equiv p$

$(p \wedge (p \wedge \neg q)) \vee (p \wedge q)$ De Morgan, Double negative
$(p \wedge \neg q) \vee (p \wedge q)$ Idempotent
$p \wedge (\neg q \wedge q)$ Distributive
p Negation

Hence Proved

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(9)

b. $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

$\neg[(p \to q) \wedge (q \to p)]$	Bi-implication
$\neg(p \to q) \vee \neg(q \to p)$	De Morgan
$\neg(\neg p \vee q) \vee \neg(\neg q \vee p)$	Implication
$(p \wedge \neg q) \vee (q \wedge \neg p)$	De Morgan
$[(p \wedge \neg q) \vee q] \wedge [(p \wedge \neg q) \vee \neg p]$	Distributive
$q \vee (p \wedge \neg q) \wedge (\neg p \vee (p \wedge \neg q))$	Commutative
$[(q \vee p) \wedge (q \vee \neg q)] \wedge [(\neg p \vee p) \wedge (\neg p \vee \neg q)]$	Distributive
$(q \vee p) \wedge (\neg p \vee \neg q)$	Negation
$\neg(\neg q \vee p) \wedge (p \to \neg q)$	Double Negation
$(\neg q \to p) \wedge (p \to \neg q)$	
$(\neg q \leftrightarrow p)$	
$(p \leftrightarrow \neg q)$	Commutative

c. $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

$(\neg p \to q) \wedge (q \to \neg p)$	
$(p \vee q) \wedge (\neg q \vee \neg p)$	
$(q \vee p) \wedge (\neg q \vee \neg p)$	Commutative
$(\neg\neg q \vee p) \wedge (\neg q \vee \neg p)$	Double Negation
$(\neg\neg q \vee p) \wedge (\neg p \vee \neg q)$	Commutative
$(\neg q \to p) \wedge (p \to \neg q)$	
$\neg q \leftrightarrow p$	
$p \leftrightarrow \neg q$	Commutative

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(10)

d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

$(p \wedge q) \rightarrow (\neg p \vee q)$

$\neg(p \wedge q) \vee (\neg p \vee q)$

$(\neg p \vee \neg q) \vee (\neg p \vee q)$

$(\neg p \vee \neg p) \vee (\neg q \vee q)$

$(\neg p \vee t)$

t

DeMorgan
Assosiative, ~~ED~~
Idemputent, Negation
Universal bound.

e) $\neg(p \vee \neg(p \wedge q)) \equiv F$

$\neg p \wedge (p \wedge q)$

$C \wedge q$

F

DeMorgan
Negation
Universal Bound

Q11 a)

p	q	r	p→r	q→r	X∧Y	pvq	pvq→r
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	T	F	F	T	F
F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F
T	F	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	T	T	T	T	T	T	T

X Y

Equivalent

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(11)

i) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

p	q	r	p→q (X)	p→r	X ∨ Y	q∨r	p→(q∨r) (Y)
F	F	F	F	T	T	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	T	T	T
T	T	T	T	T	T	T	T

ii) Equivalent

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(12)

i $(p \to q) \to (r \to s) \equiv (p \to r) \to (q \to s)$

				X	Y	A	B		
p	q	r	s	$p \to q$	$r \to s$	$p \to r$	$q \to s$	$X \to Y$	$A \to B$
F	F	F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	T	F	T	F	T	T	F	T
F	F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	F	T	F
F	T	F	T	T	T	T	T	T	T
F	T	T	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T	T
T	F	F	F	F	T	F	T	T	T
T	F	F	T	F	T	F	T	T	T
T	F	T	F	F	F	T	F	T	F
T	F	T	T	F	T	T	T	T	T
T	T	F	F	T	T	F	T	T	T
T	T	F	T	T	T	F	F	T	F
T	T	T	F	T	F	T	F	F	T
T	T	T	T	T	T	T	T	T	T

Not equivalent

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(13)

Q12

a) $5 = K(4)$

$\frac{n}{m} = \frac{5}{4}$ False

b) $4 = K(2)$

$K = 2$ True

c) FALSE

~~let m = 1 & let n = 2~~

Let $n = 1$ & $m = 2$

$\frac{n}{m} = \frac{1}{2}$

d) True $\frac{n}{m} = \frac{2}{2} = 1$

e) False $\frac{n}{m} = \frac{1}{1,2,3} = \frac{1}{2} = \frac{1}{3}$

f) True $\frac{n}{m} = \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$

Q13 a) True $x = \sqrt{2}$

(14)

b) False $\sqrt{-1}$ is not a real number

c) $x^2 + 2 \geq 1$ True
The least value 0 satisfies the ~~equation~~ proposition ; $2 \geq 1$ (L.H.S is at least 2)

d) ~~False~~ True Eq satisfies for $x=0, x=1$

Q14
a) $\forall x\, F(x, \text{Bob})$
b) $\forall y\, F(\text{Alice}, y)$
c) $\forall x \exists y\, F(x, y)$
d) $\neg \exists x \forall y\, F(x, y)$
~~e) $\forall y \exists y$~~

e) $\forall y \exists x\, F(x, y)$

Q15
a) $\exists x (P(x) \wedge Q(x))$

b) $\exists x (P(x) \wedge \neg Q(x))$

c) $\forall x (P(x) \vee Q(x))$

d) $\neg \exists x (P(x) \vee Q(x))$

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(15)

Q16

a. There is some student in your class who has sent an email message to some student in your class.

b. There is some student in your class who has sent an email message to ~~everyone~~ every student in your class.

c. Every student in your class has sent a message to atleast one student in your class.

d. ~~At le~~ There is some student in your class who has ~~taken by~~ been sent an email message by every student in your class.

e. Every student in your class has been sent an email by at least one student in your class.

f. Every student in your class has been sent an email to every student in your class.

Moh

Q17

a) There is some student in your class who has taken some class in computer science course at your school.

b) There is some student in your class who has taken ~~some~~ every class in computer science course at your school.

c) Every student in your class has taken at least one class in computer science course at your school.

d) At least one class of computer science at your school has been taken by every student in your class.

e) Every class of computer science course at your school has been taken by at least one student in your class.

f) Every student in your class has taken every class of computer science course at your school.

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(17)

Q18

a Addition
b Simplification
c Modus Ponens
d Modus Tollens
e Hypothetical Syllogism

Q19

i Let

A: Today is Tuesday
B: Test of Maths
C: Test of Economics
D: Economics professor is sick

$A \to (B \vee C)$
$D \to \neg C$
$A \wedge D$

B

① Associative Conjunction

$A \to (B \vee C)$
$D \to \neg C$
A
D

B

② Modus Ponens

$A \to (B \vee C)$
A

B

③ Disjunctive Syllogism

$B \vee C$
$\neg C$

B

$A \to (B \vee C) \wedge (D \to \neg C) \wedge (A \wedge D)$

Associative Conjunction

$A \to (B \vee C) \wedge (D \to \neg C) \wedge A \wedge D$

$A \to (B \vee C) \wedge A \wedge (D \to \neg C) \wedge D$

$B \vee C \wedge \neg C$

Modus Pollens
Disjunctive Syllogism

Conclusion: B

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(18)

b Let

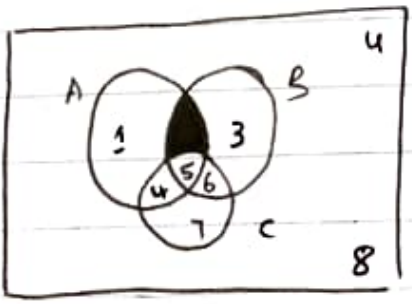
A: Ali is lawyer
B: He is ambitious
C: Ali is eaily user
D: Doesn't like chocolate

$A \to B$
$C \to D$
$B \to C$
$A \to D$

$(A \to B) \wedge (B \to C) \wedge (C \to D)$ Assosiative *
$(A \to C) \wedge (C \to D)$ Hypothetical Syllogism
$A \to D$ Hypothetical Syllogism
↑ conclusion

Q20

i $(A \cap B) \cap \bar{C} = \{2\}$



ii

$\bar{A} \cup (B \cup C) = \{$
$= \{2, 3, 4, 5, 6, 7, 8\}$

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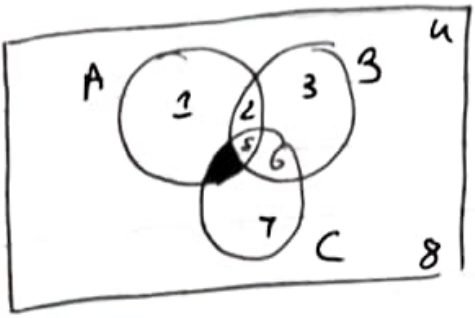
19

c) $(A-B) \cap C = \{4\}$



d) $(A \cap \bar{B}) \cup \bar{C} = \{1,2,3,4,8\}$

Q21 a) $(A-(A \cap B)) \cap (B-(A \cap B)) = \emptyset$

a) $[A \cap (\overline{A \cap B})] \cap [B \cap (\overline{A \cap B})]$ (Complement)

$(A \cap B) \cdot (\overline{A \cap B})$ Assosiative, Idempotent Law

$\emptyset = \emptyset$ Complement Law

b) $(A-B) \cup (A \cap B) \equiv A$

$(A \cap \bar{B}) \cup (A \cap B)$

$A \cap (\bar{B} \cup B)$ Distributive Law

$A \cap U$ Complement Law

A Identity Law

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i) $(A-B)-C \equiv (A-C)-B$

$(A \cap \bar{B}) \cap \bar{C}$

$(A \cap \bar{C}) \cap \bar{B}$ Associative Law

$(A-C)-B$

ii) $\overline{(\bar{B} \cup (\bar{B}-A))} \equiv B$

~~$\bar{\bar{B}} \cap \overline{(\bar{B}-A)}$~~

$\bar{\bar{B}} \cap \overline{(\bar{B}-A)}$ De Morgan

$B \cap (\overline{\bar{B} \cap \bar{A}})$ Complementation

$B \cap (\bar{\bar{B}} \cup \bar{\bar{A}})$ De Morgan

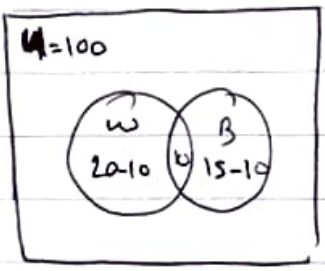
~~$(B \cap \bar{B}) \cup A$~~

$B \cap (B \cup A)$ Complementation

B Absorption Law.

Q22

a)



$|W - W \cap B| = 10 \rightarrow$ only worms

$|B - W \cap B| = 5 \rightarrow$ only bruises

$+ \quad |W \cap B| = +10 \rightarrow$ Both worms & bruises

25

$100 - 25 = 75$

$141 - 25 = 75$

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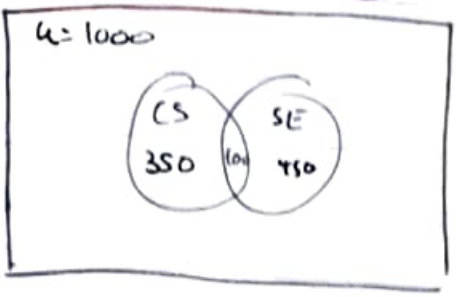
(21)

b)



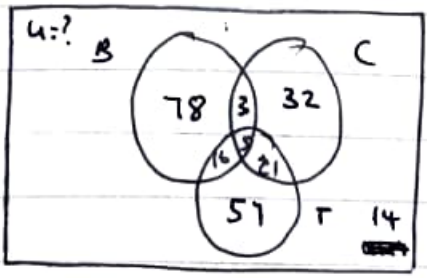
(350 - 100) + 100 + (450 - 100) = 700 (either)

1000 - 700 = 300 (neither)

→ Only CS students + Both CS & SE students + Only SE students

$|CS| + |CS \cap SE| + |SE|$

c)



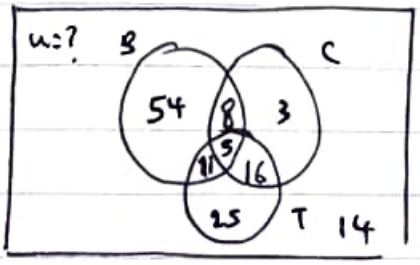
← Rough working

← Incorrect

78 - (8+5+11) = 54

32 - (8+5+16) = 3

57 - (11+5+16) = 24



$|U| = |B| + |C| + |T| + |B \cap C \cap T| + |\overline{B \cup C \cup T}| + |B \cap C \cap \bar{T}| + |B \cap T \cap \bar{C}| + |C \cap T \cap \bar{B}|$

$|U| = 54 + 8 + 5 + 11 + 16 + 25 + 14 + 3 = 136$

Q22 d Done on Last Page

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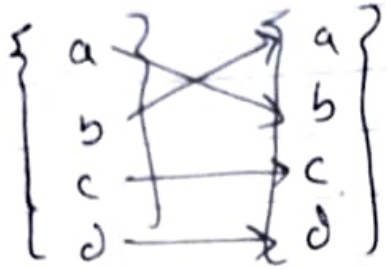
(22)

Q23

a)



Domain {a, b, c, d} — For all parts of Q23
Codomain {a, b, c, d}

a) i) Range {a, b, c, d}

ii) Bijective

Both one to one & unto function.

iii) Inverse

One to one function, $f^{-1}(a)=b$, $f^{-1}(b)=a$
$f^{-1}(c)=c$, $f^{-1}(d)=d$

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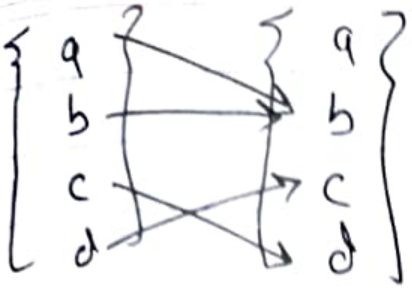
b)



i) Range $\{b, c, d\}$

ii) Not Bijective

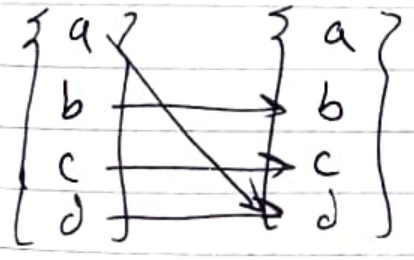
Not surjective because no element of domain maps to element "a" of codomain.

Not Injective because $f(a) = f(b)$, $a \neq b$

iii) Not Inverse

Not one to one function / Injective $f(a) = f(b)$, $a \neq b$

c)



i) Range $\{b, c, d\}$

ii) Not Bijective

iii) No Inverse

Not one to one function because $f(a) = f(d)$, $a \neq d$

Not surjective/onto because no element of domain maps to element "a" of co domain.

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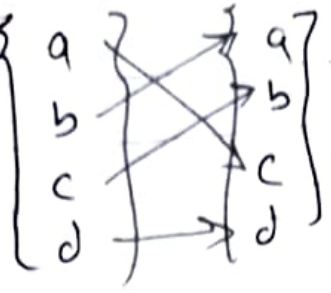
d 

i) Range:- {a, b, c, d}
ii) Bijective
iii) Both one to one & onto function

iii) Inverse

Because it is one to one function
$f^{-1}(c)=a$ $f^{-1}(a)=b$ $f^{-1}(b)=c$, $f^{-1}(d)=d$

Q24 a)
i) $f(S)=\{0,1,3\}$
ii) $f(S)=\{0,1,3,5,8\}$
iii) $f(S)=\{0,8,16,40\}$
iv) $f(S)=\{1,12,33,65\}$

b i) $\lceil 3/4 \rceil = 1$
ii) $\lfloor 7/8 \rfloor = 0$
iii) $\lceil -3/4 \rceil = 0$
iv) $\lfloor -7/8 \rfloor = -1$
v) $\lceil 3 \rceil = 3$
vi) $\lfloor -1 \rfloor = -1$

vii) $\lfloor \frac{1}{2} + 2 \rfloor = 2$

viii) $\lfloor \frac{1}{2} \cdot 2 \rfloor = 1$

$\therefore \lfloor -x \rfloor = -\lceil x \rceil \quad , \lceil -x \rceil = -\lfloor x \rfloor$

$\lfloor -x \rfloor = -\lceil x \rceil$

Let $x = n - \varepsilon$ where $0 \leq \varepsilon < 1$ & n is integer

$\lceil x \rceil = \lceil n - \varepsilon \rceil$

$\lceil x \rceil = n$

$-\lceil x \rceil = -n$

$\lfloor -(n - \varepsilon) \rfloor = -\lceil x \rceil$

$\lfloor -n + \varepsilon \rfloor = -\lceil x \rceil$

$\boxed{-n = -n}$ Proved

$\lceil -x \rceil = -\lfloor x \rfloor$

Let $x = n + \varepsilon$ where $0 \leq \varepsilon < 1$ & n is integer

$\lfloor x \rfloor = \lfloor n + \varepsilon \rfloor$

$\lfloor x \rfloor = n$

$-\lfloor x \rfloor = -n$

$\lceil -(n + \varepsilon) \rceil = -\lfloor x \rfloor$

$\lceil -n - \varepsilon \rceil = -\lfloor x \rfloor$

$\therefore \boxed{-n = -n}$ Proved.

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Q25

a) $f(a) = 2a+3$ $\quad g(a) = 3a+2$

$fog(a) = 2(3a+2)+3$

$= 6a+7$

$gof(a) = 3(2a+3)+2$

$= 6a+11$

b) ~~Both~~ Both one to one function and strictly increasing

$x > y \quad 2x+3 > 2y+3, \quad 3x+2 > 3y+2$

Not onto function because 0 ~~can't be~~ is ~~mapped by any domain~~ not in the range but is in the co-domain.

c)

No both f & g are not invertible

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Q22 i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(x, y) \in A \times (B \cap C)$

$\{(x,y) \mid x \in A \wedge y \in (B \cap C)\}$

$\{(x,y) \mid (x \in A) \wedge (y \in B) \wedge (y \in C)\}$ Distibutive

$\{(x,y) \mid (x \in A) \wedge (x \in A) \wedge (y \in B) \wedge (y \in C)\}$

$\{(x,y) \mid (x \in A) \wedge (y \in B) \wedge (x \in A) \wedge (y \in C)\}$

$\{(x,y) \mid (x,y) \in (A \times B) \wedge (x,y) \wedge (A \cap C)\}$

$(A \times B) \cap (A \times C)$ Hence Proved.