SUPPLEMENT TO

CHAPTER 7

Learning Curves

SUPPLEMENT OUTLINE

LEARNING OBJECTIVES

After completing this supplement, you should be able to:

LO 1 Explain the concept of learning curves.

LO 2 Make time estimates based on a learning curve.

LO 3 Determine the learning percentage.

LO 4 List and briefly describe some of the main applications of learning curves. Also, determine the minimum number of repetitions to achieve a given standard.

LO 5 Outline some of the cautions and criticisms of learning curves.

Learning usually occurs when humans are involved; this is a basic consideration in the design of work systems. It is important to be able to predict how learning will affect task times and costs. This supplement addresses those issues.

LO 1 THE CONCEPT OF LEARNING CURVES

Human performance of activities typically shows improvement when the activities are done on a repetitive basis: The time required to perform a task decreases with increasing repetitions. *Learning curves* display this phenomenon. The degree of improvement and the number of tasks needed to realize the major portion of the improvement is a function of the task being done. If the task is short and somewhat routine, only a modest amount of improvement is likely to occur, and it generally occurs during the first few repetitions. If the task is fairly complex and has a longer duration, improvements will occur over a larger number of repetitions. Therefore, learning curves have little relevance for planning or scheduling of routine short activities, but they do have relevance for complex long repetitive activities.

Figure 7S-1 illustrates the basic relationship between units produced and time per unit: time per unit decreases as the number of units produced increases. In fact, the most common learning curve model, which is studied here, assumes that the rate of decrease in unit time remains constant as the number of units produced doubles (this is illustrated on the next page).

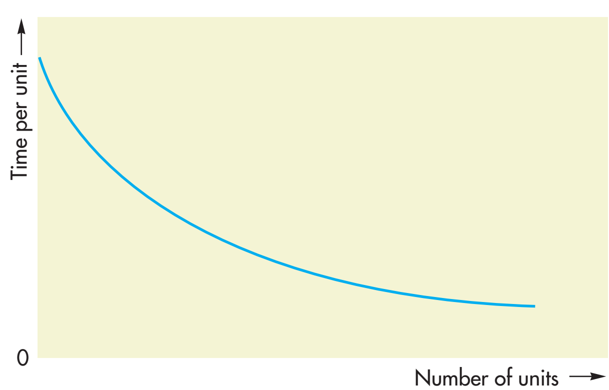
The general relationship is alternatively referred to as an improvement curve. When unit cost is used instead of unit time, the relationship is usually referred to as experience curve or progress function. Experts agree that the learning effect is the result of other factors in addition to actual worker learning (which results in increased dexterity, reduced rework, etc). Some of the improvement can be traced to preproduction factors, such as selection of tooling and equipment, product design and test, personnel training, and, in general, the amount of effort expended prior to the start of the work. Other contributing factors may involve changes after production has begun, such as changes in methods, layout, support services, tooling, design, and lot size increases. In addition, management input can be an important factor through improvements in planning, scheduling, motivation, and control.

Major changes that are made once production is under way, such as product redesign or new equipment, can cause a temporary *increase* in time per unit until workers adjust to the change. If a number of major changes are made during production, the learning curve would be more realistically described by a series of scallops instead of a smooth curve, as illustrated in Figure 7S-2. Nonetheless, it is convenient to work with a smooth curve, which can be interpreted as the average effect.

From an organizational standpoint, what makes the learning effect more than an interesting curiosity is its *predictability*, which becomes readily apparent if the relationship is

Figure 7S-1

The learning effect: time per unit decreases as the number of units produced increases



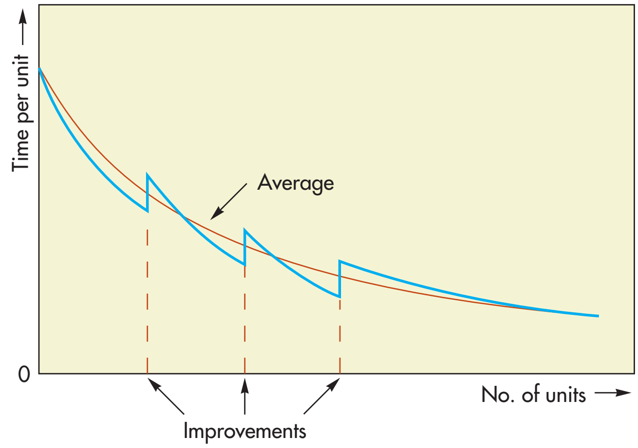


Figure 7S-2

Improvements may create a scallop effect in the learning curve

plotted on a log-log scale (see Figure 7S-3). The straight line that results reflects a constant learning percentage, which is the basis of learning curve estimates: Every *doubling* of units produced results in a *constant percentage* decrease in the time per unit. Typical decreases range from 10 percent to 20 percent. By convention, learning curves are referred to in terms of the *complements* of their decrease rates. For example, an 80-percent learning curve denotes a 20-percent decrease in unit time with each doubling of units produced and a 90-percent curve denotes a 10-percent decrease rate. The 80 or 90 percent learning percentage in the above examples is referred to as "slope" of the learning curve. Note that a 100-percent curve would imply no decrease in unit time at all (i.e., no learning).

For some examples of learning curves in industry see Figure 7S-4, and for slopes of typical industrial activities see Figure 7S-5.[1] Note the log-log scales and the fact that the unit time curves are approximately linear.

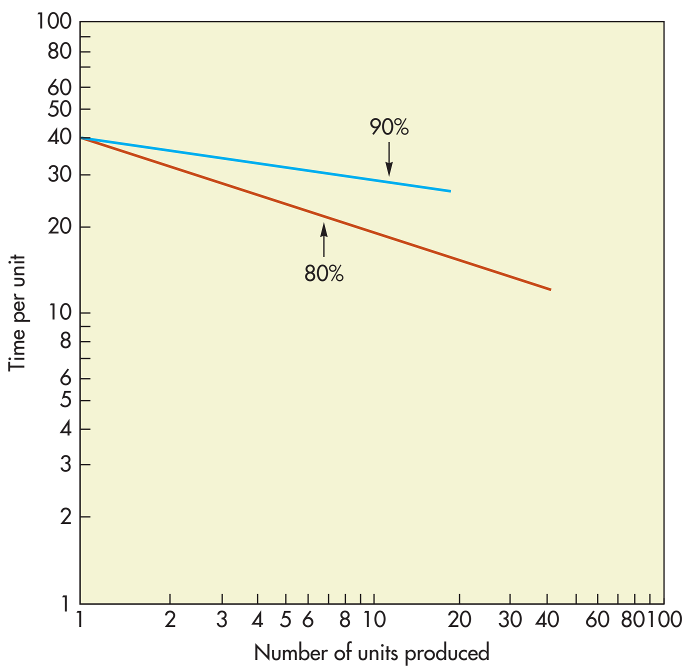


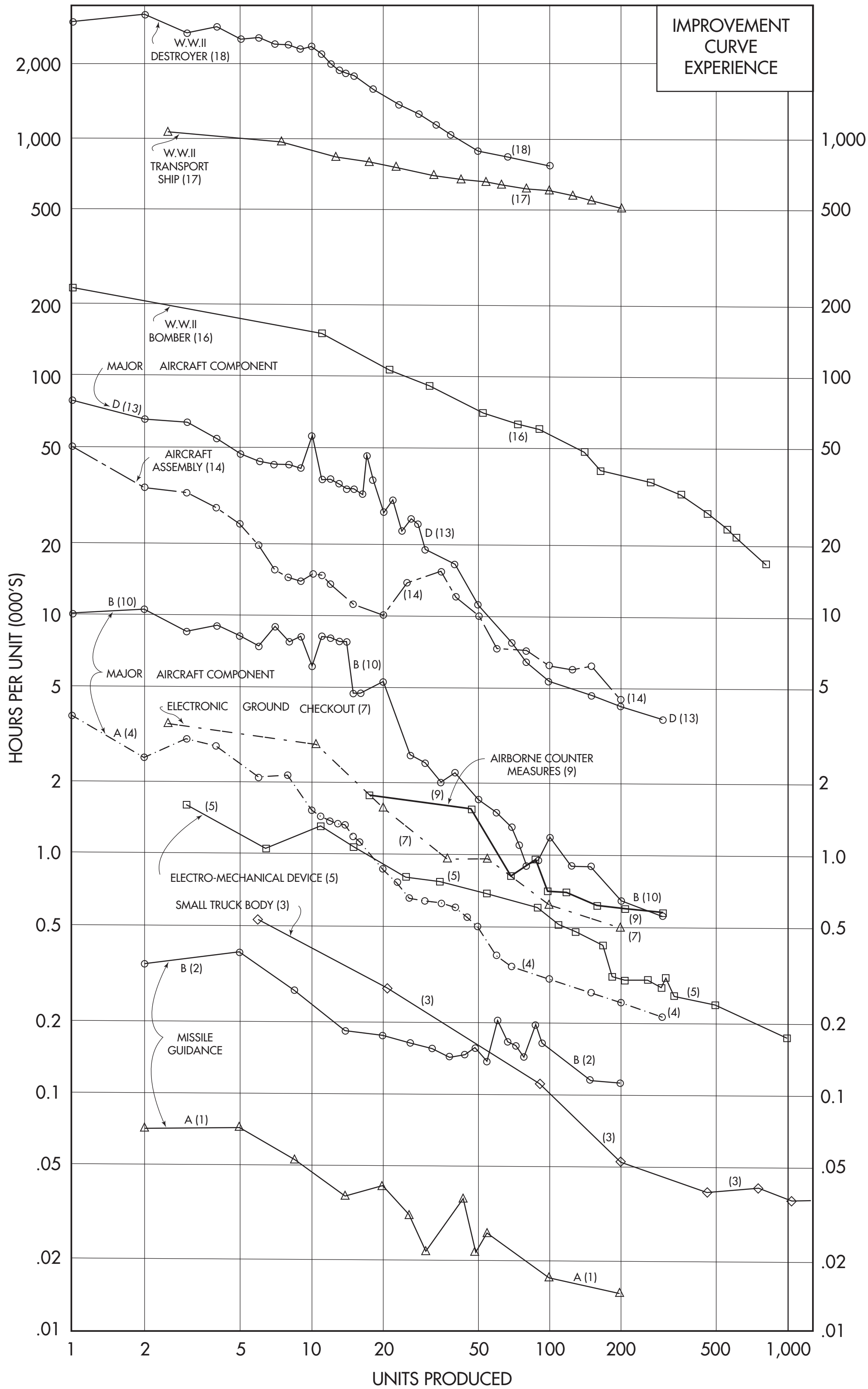
Figure 7S-3

On a log-log graph, learning curves are straight lines

[1]Both figures are from E. B. Cochran, *Planning Production Costs: Using the Improvement Curve*, San Francisco: Chandler Publishing co, 1968.

Figure 7S-4

Some examples of learning curves in industry



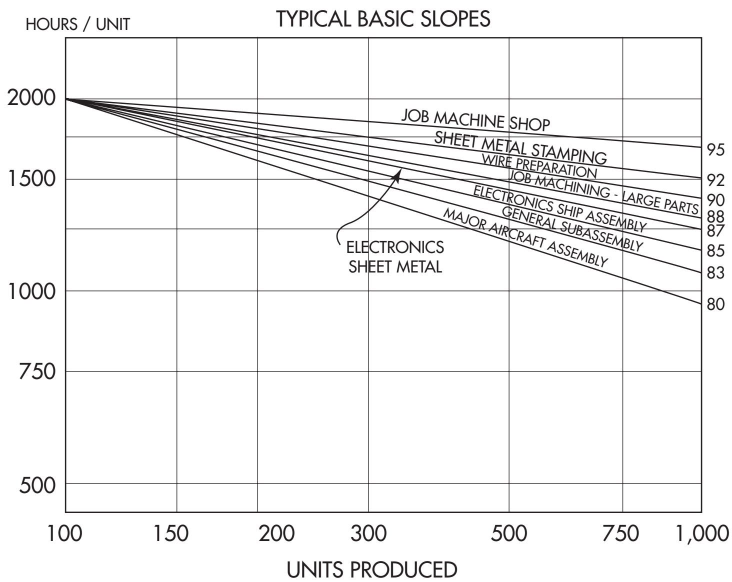


Figure 7S-5

Slopes of typical industrial activities

Example S-1

An activity is known to have an 80-percent learning curve. It has taken a worker 10 hours to produce the first unit. Determine expected completion times for these units: the 2nd, 4th, 8th, and 16th (note successive doubling of units).

Solution

Each time the cumulative output doubles, the time per unit for that quantity should be approximately equal to the previous time multiplied by the learning percentage (80 percent in this case). Thus:

Unit	Unit Time (hours)	
1		= 10
2	0.8(10)	= 8
4	0.8(8)	= 6.4
8	0.8(6.4)	= 5.12
16	0.8(5.12)	= 4.096

Example S-1 illustrates an important point and also raises an interesting question. The point is that the time reduction *per unit* becomes smaller and smaller as the number of units produced increases. For example, the second unit required two hours less time than the first, but the decrease from the 8th to the 16th unit was only slightly more than one hour. The question raised is: How are times calculated for units such as three, five, six, seven, and other units that don't fall into the doubling pattern?

LO 2 DETERMINING UNIT TIMES

There are two ways to obtain the unit times. One is to use a formula; the other is to use a table of values.

First, consider the formula approach. The formula is based on the existence of a linear relationship between the time per unit and the number of units produced when these two variables are expressed in logarithms.

The unit time (i.e., the number of direct labour hours required) for the nth unit can be calculated using the formula

$$T_n = T_1 \times n^b \qquad \text{(7S-1)}$$

where

T_n = Time for the nth unit

T_1 = Time for the first unit

$$b = \frac{\ln\left(\dfrac{\textit{learning percentage}}{100}\right)}{\ln(2)}$$; ln stands for the natural logarithm

To use the formula, you need to know the time for the first unit T_1 and the learning percentage. For example, for an 80-percent learning curve with $T_1 = 10$ hours, the time for the third unit would be calculated as

$$T_3 = 10(3^{\ln .8/\ln 2}) = 10(3^{-.223/.693}) = 10(3^{-.322}) = 7.02$$

The second approach is to use a "learning factor" from Table 7S-1. The table shows two things for some selected learning percentages. One is the unit time factor for the number of units in the sequence (unit number). This enables us to easily determine how long any unit in the sequence should take to produce. The other is the total (cumulative) time factor. The calculation for both times is a relatively simple operation: Multiply the learning factor by the time required for the first unit.

To find the time for unit n (e.g., $n = 10 \rightarrow$ the 10th unit), use the formula

$$T_n = T_1 \times \text{unit time factor} \tag{7S-2}$$

Thus, for an 85-percent learning curve with $T_1 = 4$ hours, the time for the 10th unit would be $4 \times .583 = 2.33$ hours. To find the time for all units up to a specified unit n (e.g., $n = 10 \rightarrow$ the first 10 units), use the formula

$$\Sigma T_n = T_1 \times \text{total time factor} \tag{7S-3}$$

Thus, for an 85-percent curve with $T_1 = 4$ hours, the total time for all first 10 units (including the time for unit 1) would be $4 \times 7.116 = 28.464$ hours.

Example S-2

An airplane manufacturer is negotiating a contract for the production of 20 small jets. The initial jet required 400 days of direct labour. The learning percentage is 80 percent. Estimate the expected number of days of direct labour for:

a. The 20th jet.

b. All 20 jets.

c. The average time for 20 jets.

Solution

Using Table 7S-1 with $n = 20$ and an 80-percent learning percentage, you find these factors: Unit time = .381. Total time = 10.485.

a. Expected time for 20th jet: 400(.381) = 152.4 days.

b. Expected total time for all 20 jets: 400(10.485) = 4,194 days.

c. Average time for 20 jets: 4,194 ÷ 20 = 209.7 days.

Use of Table 7S-1 requires a time for the first unit. If for some reason the completion time of the first unit is not available, or if the manager believes that the completion time for some later unit is more accurate, the table can also be used to obtain an estimate of the time for the first unit.

Example S-3

The manager in Example S-2 believes that some unusual problems were encountered in producing the first and second jets and would like to revise the time for the first unit based on the completion time of 276 days for the third jet.

Solution

The learning factor for $n = 3$ and an 80-percent curve is .702 (Table 7S-1). Divide the actual time for unit 3 by the learning factor to obtain the revised estimate for the time of first unit: 276 days ÷ .702 = 393.2 days.

Table 7S-1

Learning factors

	70%		75%		80%		85%		90%	
Unit Number	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	.700	1.700	.750	1.750	.800	1.800	.850	1.850	.900	1.900
3	.568	2.268	.634	2.384	.702	2.502	.773	2.623	.846	2.746
4	.490	2.758	.562	2.946	.640	3.142	.723	3.345	.810	3.556
5	.437	3.195	.513	3.459	.596	3.738	.686	4.031	.783	4.339
6	.398	3.593	.475	3.934	.562	4.299	.657	4.688	.762	5.101
7	.367	3.960	.446	4.380	.534	4.834	.634	5.322	.744	5.845
8	.343	4.303	.422	4.802	.512	5.346	.614	5.936	.729	6.574
9	.323	4.626	.402	5.204	.493	5.839	.597	6.533	.716	7.290
10	.306	4.932	.385	5.589	.477	6.315	.583	7.116	.705	7.994
11	.291	5.223	.370	5.958	.462	6.777	.570	7.686	.695	8.689
12	.278	5.501	.357	6.315	.449	7.227	.558	8.244	.685	9.374
13	.267	5.769	.345	6.660	.438	7.665	.548	8.792	.677	10.052
14	.257	6.026	.334	6.994	.428	8.092	.539	9.331	.670	10.721
15	.248	6.274	.325	7.319	.418	8.511	.530	9.861	.663	11.384
16	.240	6.514	.316	7.635	.410	8.920	.522	10.383	.656	12.040
17	.233	6.747	.309	7.944	.402	9.322	.515	10.898	.650	12.690
18	.226	6.973	.301	8.245	.394	9.716	.508	11.405	.644	13.334
19	.220	7.192	.295	8.540	.388	10.104	.501	11.907	.639	13.974
20	.214	7.407	.288	8.828	.381	10.485	.495	12.402	.634	14.608
21	.209	7.615	.283	9.111	.375	10.860	.490	12.892	.630	15.237
22	.204	7.819	.277	9.388	.370	11.230	.484	13.376	.625	15.862
23	.199	8.018	.272	9.660	.364	11.594	.479	13.856	.621	16.483
24	.195	8.213	.267	9.928	.359	11.954	.475	14.331	.617	17.100
25	.191	8.404	.263	10.191	.355	12.309	.470	14.801	.613	17.713
26	.187	8.591	.259	10.449	.350	12.659	.466	15.267	.609	18.323
27	.183	8.774	.255	10.704	.346	13.005	.462	15.728	.606	18.929
28	.180	8.954	.251	10.955	.342	13.347	.458	16.186	.603	19.531
29	.177	9.131	.247	11.202	.338	13.685	.454	16.640	.599	20.131
30	.174	9.305	.244	11.446	.335	14.020	.450	17.091	.596	20.727

LO3 DETERMINING THE LEARNING PERCENTAGE

If the learning percentage of the activity cannot be estimated based on similar previous activities or the industry learning slope, given a few observations of unit times one can estimate the learning percentage of the activity by fitting the power function of 7S-1 to the chart of the data. For simplicity, we will change notation to more familiar symbols: $y = T_n$, $a = T_1$, and $x = n$. The equation for the power function is $y = ax^b$. The fitted power function will provide the value for a = time of the first unit, and $b = \ln$ (learning percentage/100) / ln 2. Solving the b equation gives:

$$\text{Learning percentage} = 100 \times 2^b \tag{7S-4}$$

The power function can easily be obtained from Excel by charting the data (enter the data, then click Insert, and finally click Line) and using "Trendline" under the Layout menu.

Example S-4

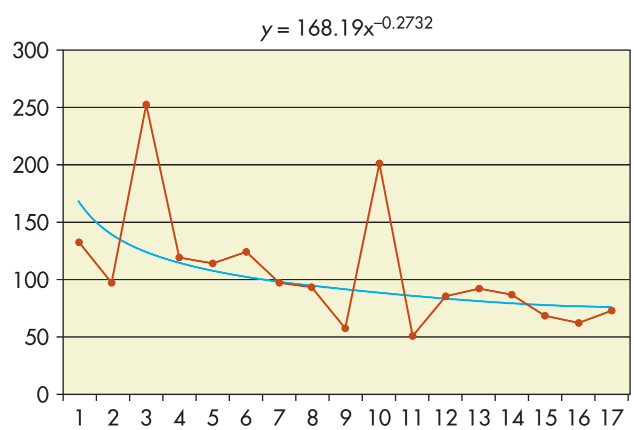
The cost per patient for heart transplant in Temple University Hospital in Philadelphia[2] for the first 17 patients is displayed below.

Heart Transplant Patient	Cost (in $1,000)
1	133
2	97
3	250
4	120
5	115
6	125
7	98
8	94
9	57
10	201
11	52
12	86
13	93
14	89
15	70
16	64
17	75

Chart the line plot of the data, fit the power function to the chart, and estimate the learning percentage.

Solution

Note that in this case, the data given are unit costs instead of unit times, and cost is directly related to the times of surgeons and nurses. After charting the data in Excel as a line plot, the "Trendline" command under Layout menu was used to fit the power function to the data.



Substituting $b = -0.2732$ into 100×2^b, the learning percentage = 82.75 percent.

[2]Adapted from D. B. Smith and J. L. Larsson, "The Impact of Learning on Cost: The Case of Heart Transplantation," *Hospital & Health Services Administration* 34(1), Spring 1989, pp. 85–97.

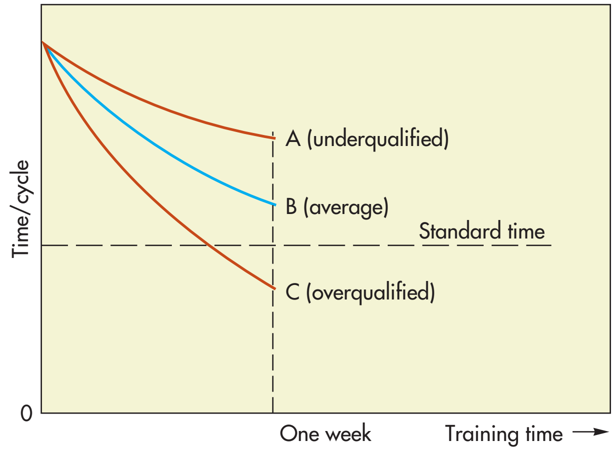


Figure 7S-6

Worker learning curves can help guide personnel decisions

LO4 APPLICATIONS OF LEARNING CURVES

Learning curves have useful application in a number of management activities, including:

1. Labour planning and scheduling
2. Negotiated selling/purchasing
3. Assessing labour training needs and performance

Knowledge of output projections in learning situations can help managers make better decisions about how many workers they will need. Of course, managers recognize that improvement will occur; what the learning curve contributes is a method for quantifying expected future improvements.

Negotiated selling/purchasing often involves contracting for specialized items that may have a high degree of complexity. Examples include aircraft, ships, and special-purpose equipment. The direct labour cost per unit of such items can be expected to decrease as more units are produced. Hence, negotiators negotiate cost/price on that basis. For contracts that are terminated before delivery of all units, suppliers can use learning curve data to argue for an increase in the unit price for the smaller number of units. Conversely, the customer can use that information to negotiate a lower price per unit on follow-on orders on the basis of projected additional learning gains.

Determining the Minimum Number of Repetitions to Achieve a Given Standard

Learning curves can be used to determine the length of training for new workers doing complex-long-cycle jobs. The progress of each worker can be evaluated by measuring each worker's performance, graphing the results, and comparing the learning to an expected rate of learning. The comparison will reveal if a worker is underqualified, average, or overqualified for a given type of work (see Figure 7S-6). Also, learning curves can be used to determine the minimum number of repetitions to achieve a given standard.

Example S-5

Use learning curves to predict the number of units that a trainee needs to produce to achieve a unit time of 6 minutes if the trainee took 10 minutes to produce the first unit and learning curve of 90 percent is expected. Use both

a. Formula 7S-2 and Table 7S-1 in reverse.

b. Logarithm version of Formula 7S-1:

$$\ln(T_n) = \ln(T_1) + b\ln(n) \qquad (7S\text{-}5)$$

Rewriting (where $e = 2.718282$):

$$n = e^{[\ln(T_n) - \ln(T_1)]/b} \qquad (7S\text{-}6)$$

Solution

a. Formula 7S-2:

$T_n = T_1 \times$ unit time factor

Setting T_n equal to the specified time of 6 minutes, T_1 to 10 minutes, and solving for the unit time factor:

6 min = 10 min × unit time factor → unit time factor = 6min ÷ 10 min = .600.

From Table 7S-1, under 90 percent in the Unit Time column, we find .599 at 29 units is the closest to .600. Hence, approximately 29 units will be required to achieve the specified time.

b. Using the Formula 7S-6:

(1) Calculate b, $b = \ln(\text{learning percentage}/100) \div \ln(2) = \ln(.90) \div \ln(2) = -0.1054 \div 0.6931 = -0.152$.

(2) $n = e^{[\ln(T_n) - \ln(T_1)]\,/\,b} = e^{[\ln(6) - \ln(10)]\,/\,-.152} = e^{[1.792 - 2.303)]\,/\,-.152} = e^{3.36} = 28.8$

Round to 29. Hence, 29 units will be needed to achieve a time of 6 minutes.

LO 5 CAUTIONS AND CRITICISMS

Managers/staff using learning curves should be aware of their limitations and pitfalls. This section briefly outlines some of the major cautions and criticisms of learning curves.

1. Learning percentage may differ from organization to organization and by type of work. Therefore, it is best to base learning percentage on empirical studies rather than assumed percentage.
2. Projections based on learning curves should be regarded as *approximations* of actual times and treated accordingly.
3. If time estimates are based on the time for the first unit, considerable care should be taken to ensure that this time is valid. The first unit time (or even several units after that) may not be accurate due to time compression, design changes, equipment problems, etc. It may be desirable to revise the first unit time as later times become available. Since it is often necessary to estimate the time for the first unit prior to production, this caution is very important.
4. Learning curves do *not* apply to mass production (which have short cycle times) because the decrease in time per unit is imperceptible.
5. Users of learning curves sometimes fail to include carryover effects; previous experience with similar activities can reduce unit times. In this case, instead of Formula 7S-1, the following model could be used: $T_n = T_1 \times (n + n_p)^b$ where n_p is the number of units produced previously.

Summary

Unit time tends to decrease at a constant rate as number of units produced doubles. This effect is called learning curve and is stronger for complex new products. Learning percentage (or slope) is defined as (100 – percentage reduction in unit time as units double). For example, 90 percent learning percentage means that production time decreases by 10 percent if number of units produced is doubled. Unit time can be calculated using Formula 7S-1 or Table 7S-1. Learning percentages for various industries/activities are available, but it is better to determine it empirically by fitting a power function to the plot of unit time drawn against the number of units produced.

Solved Problems

Problem 1

An assembly operation has a 90-percent learning curve. The line has just begun work on a new item. The initial unit required 28 hours. Estimate the time that will be needed to complete:

a. The first five units.

b. Units 20 through 25.

Solution

Use the total time factor in the 90-percent column of Table 7S-1.

a. Total time factor: 4.339.

Estimated time for the first five units: $T_1 \times$ Total time factor $= 28(4.339) = 121.492$ hours.

b. The total time for units 20 through 25 can be determined by subtraction, using total time factor for 90% learning of Table 7S-1.

	Hours
Total time for 25 units:	28(17.713) = 495.964
– Total time for 19 units:	28(13.974) = 391.272
Total time for 20 through 25	104.692 hours

Problem 2

A manager wants to determine an appropriate learning percentage for a new type of work his company will undertake. He has obtained unit times for the initial six repetitions (see below).

i. Estimate the value of *b* in Formula 7S-1 using the power function of Excel.

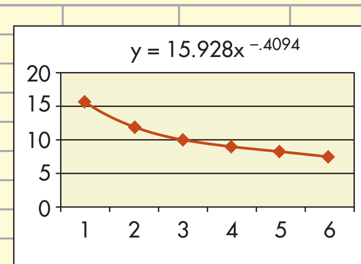
ii. What learning rate is appropriate?

Unit	Completion Time (hours)
1	15.9
2	12.0
3	10.1
4	9.1
5	8.4
6	7.5

Solution

(i)

Unit	Completion Time (hrs)
1	15.9
2	12.0
3	10.1
4	9.1
5	8.4
6	7.5



i. $b = -.4094$

ii. The learning percentage $= 100(2^{-.4094}) = 75.3\%$

Discussion and Review Questions

1. What is a learning curve and why is it important? (LO1)
2. Under what circumstances is most learning possible? (LO1 & 5)
3. What would a learning of 80 percent imply? (LO1)
4. List the factors that create the learning effect. (LO1)
5. Give an example of learning effect from your own education. Note: It must be a new complex task that had to be repeated many times, and you got better and faster in it, with an initial sharp drop in the unit time. (LO1)
6. Why should the learning percentage be higher (i.e., less learning) for a machine shop or an automated process than a manual assembly line? (LO1)
7. Compare using Formula 7S-1 and Table 7S-1. (LO2)

8. Explain how, instead of using the power function, one can use linear regression to estimate the learning percentage. (LO3 & 4)
9. In what management activity are learning curves most useful? (LO4)
10. Users of learning curves sometimes fail to include carryover effect. What is meant by this? (LO5)
11. If the learning phenomenon applies to all human activities, why isn't the effect noticeable in short-cycle mass production? (LO5)

Internet Exercise

Verify one of the values in Table 7S-1 using the learning curve calculator in http://cost.jsc.nasa.gov/learn.html (scroll down). Choose Crawford's method. Note: Crawford's method measures learning by the unit time whereas Wright's method measures learning by the average time. (LO2)

Problems

1. An aircraft services company has an order to refurbish the electronics of 18 planes. The work has a learning percentage of 80 percent. The first plane has required 300 hours to refurbish. Estimate the time needed to complete: (LO2)
   - **a.** The fifth plane.
   - **b.** The first five planes.
   - **c.** All 18 planes.
2. Estimate the time it will take to complete the 4th unit of a multi-unit job involving a large assembly if the initial unit required 80 hours and the learning percentage is: (LO2)
   - **a.** 72 percent.
   - **b.** 87 percent.
   - **c.** 95 percent.
3. A contractor intends to bid on installing 30 in-ground swimming pools. Because this will be a new line of work for the contractor, he believes there will be a learning effect. After reviewing time records from a similar type of activity, the contractor is convinced that an 85-percent learning curve is appropriate. He estimates that the first pool will take his crew eight days to install. How many days should the contractor schedule for: (LO2)
   - **a.** The first 10 pools?
   - **b.** The second 10 pools?
   - **c.** The final 10 pools?
4. A job is known to have a learning percentage of 82 percent. If the first unit took 20 hours, estimate the times for the third and fourth units. (LO2)
5. A manager wants to determine an appropriate learning percentage for a certain activity. Times of the first six units are given below: (LO3)

Units	Time (minutes)
1	46
2	39
3	35
4	33
5	32
6	30

   - **a.** Determine the learning percentage using Excel.
   - **b.** Using your answer from part *a*, estimate the average time if a total of 30 units are planned.
6. Students in an operations management class have been assigned four similar homework problems. One student took 50 minutes to complete the first problem. Assume that a 70-percent

learning curve is appropriate. How much time can this student plan to spend solving all the remaining problems? (LO2)

7. A subcontractor is responsible for outfitting six ships with new electronics. Four of the six ships have been completed in a total of 600 hours. If the task has 75-percent learning curve, how long should it take to finish the last two units? (LO2)

8. The 5th unit of a 25-unit job took 14.5 hours to complete. If a 90-percent learning curve is appropriate: (LO2)

a. How long should it take to complete the last unit?

b. How long should it take to complete the 10th unit?

c. Estimate the average time for the 25 units.

9. A lot of 20 units is to be produced. Labour cost is $8.50 per hour. Setup cost is $50 and material cost is $20 per unit. The learning percentage is expected to be 90 percent. Overhead is charged at the rate of 50 percent of total labour, material, and setup cost. Determine the average unit cost for the lot, given that the first unit took 5 hours to complete. (LO2)

10. A firm has a training program for a certain operation. The progress of trainees is carefully monitored. An established standard requires a trainee to be able to complete the sixth repetition of the operation in six hours or less. Those who are unable to do this are assigned to other jobs. Currently, three trainees have each completed two repetitions. Trainee A had times of 9 hours for the first and 8 hours for the second repetition; trainee B had times of 10 hours and 8 hours for the first and second repetitions; and trainee C had times of 12 and 9 hours. Which trainee(s) do you think will make the standard? Explain your reasoning. (LO2)

11. The first unit of a job took 40 hours to complete. The work has a learning percentage of 88 percent. Determine time estimates for units 2, 3, 4, and 5. (LO2)

12. Estimate the remaining time that will be needed to complete a five-unit job. The initial unit required 12 hours, and the work has a learning percentage of 77 percent. (LO2)

13. A job is supposed to have a learning percentage of 82 percent. Times for the first four units were 30.5, 28.4, 27.2, and 27.0 minutes. Does a learning percentage of 82 percent seem reasonable? Fit a power function to the line plot of the data in Excel. (LO3)

14. The 5th unit of a 10-unit job took five hours to complete. The 6th unit has been worked on for two hours, but is not yet finished. Estimate the *additional* amount of time needed to finish the 10-unit job if the work has a 75-percent learning percentage. (LO2)

15. Estimate the number of repetitions each of the workers listed below will require to reach a time of 7 hours per unit. Times of the first two repetitions (in hours) are given below. (LO4)

Trainee	T_1	T_2
Art	11	9.9
Sherry	10.5	8.4
Dave	12	10.2

16. Estimate the number of repetitions that a new worker will require to achieve the “standard” if the standard is 18 minutes per unit. She took 30 minutes to do the initial unit and 25 minutes to do the next unit. (LO4)

17. Estimate the number of repetitions each of the workers listed below will require to achieve a standard time of 25 minutes per unit. Times of the first two repetitions (in minutes) are given below. (LO4)

Trainee	T_1	T_2
Tracy	36	31
Darren	40	36
Lynn	37	30

18. A research analyst performs database searches for clients. According to her, a new search requires approximately 55 minutes. Repeated requests on the same or similar topic take less and less time, as shown below: (LO4)

Request no.	1	2	3	4	5	6	7	8
Time (min.)	55.0	41.0	35.2	31.0	28.7	26.1	24.8	23.5

How many more searches will it take until the search time gets down to 19 minutes?

19. The following data are the hours it took to assemble 20 identical sections of an aircraft fuselage.[3] Use Excel to chart the line plot of the data, fit a power function to the chart, and determine the learning percentage. (LO3)

Units Produced	**Actual Unit Time**
1	2,122
2	1,512
3	1,283
4	848
5	755
6	798
7	697
8	825
9	759
10	798
11	788
12	771
13	774
14	770
15	778
16	786
17	777
18	785
19	781
20	764

20. The following data are the cumulative number of Model T cars produced by Ford and their unit manufacturing cost for the period 1910 to 1926.[4] Chart the line plot of the data, fit a power function to the chart, and determine the learning percentage, assuming that the cost of material remained the same over these years. (LO3)

Ford's Model T

Year	**Cumulative Number of Cars Produced (1,000s)**	**Manufacturing Cost per Car ($)**
1910	50	2,578
1911	120	2,243
1912	290	1,765
1913	493	1,431

[3] C. J. Waterworth, "Relearning the Learning Curve: A Review of the Derivation and Applications of Learning-Curve Theory," *Project Management Journal* 31(1), March 2000, pp. 24–31.
[4] L. E. Yelle, "Adding Life Cycles to Learning Curves," *Long Range Planning* 16(6), 1983, pp. 82–87.

(*continued*)

Year	Cumulative Number of Cars Produced (1,000s)	Manufacturing Cost per Car ($)
1914	801	1,458
1915	1,302	1,602
1916	2,037	1,258
1917	2,701	1,273
1918	3,199	1,059
1919	4,140	941
1920	4,603	862
1921	5,574	831
1922	6,881	769
1923	8,900	804
1924	10,829	709
1925	12,749	661
1926	14,312	641

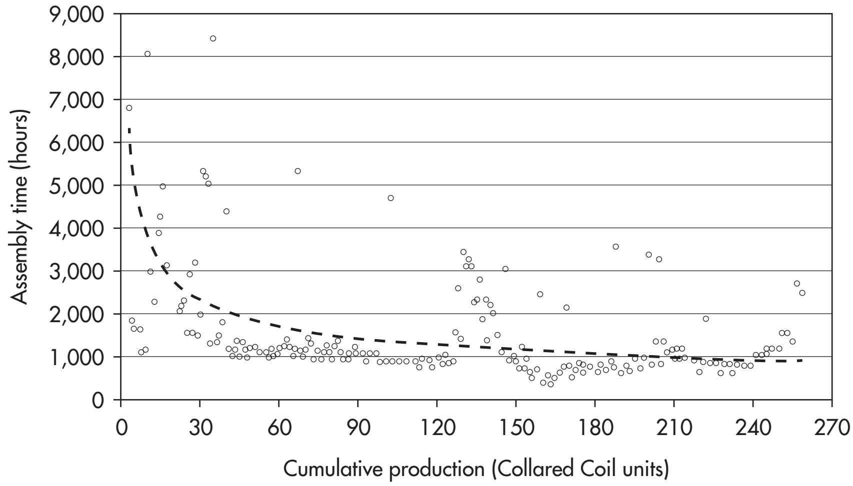
21. In a study of forgetting and relearning, 31 subjects individually built an Erector Set, dismantled it, and rebuilt it again several times, then went away and returned after a few months, and repeated the above process.[5] For one subject, the following 7 building times (in minutes) were observed (in order): 33, 25, 22, 20, 19, 16, and 15. After a 101-day break, the subject had the following 8 building times (in order): 21, 17, 20, 15, 17, 15, 15, and 13. Fit a power function to each set of data, and calculate the learning percentage and relearning percentage for the subject. Explain your results. (LO3)

22. The following data are the man-hours it took to build the concrete frame of each floor of four buildings in Porto, Portugal.[6] Chart the line plot of each set of data, fit a power function to each chart, and calculate the learning percentage for each building. Interpret your results. (LO3)

	Floor Level												
Building	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
SGL 3A	11	8	8	8	7	8	6	7					
SGL 3B	12	11	11	11	9	9	8	9	6	7	6	5	4
SGL 2A	11	8	9	6	6	8	10	6					
SGL 2B	12	11	8	9	10	9	10	8	7	8	6	7	9

***23.** A study was performed to determine the efficacy of laser to extract leads of pacemakers that attach themselves to heart and veins of patients after a few years.[7] Patients were divided in 3 groups of 25 each. Average surgery times fell from 19 minutes for the first 25, to 11 minutes for the next 25, and to 8 minutes for the final 25, without any major complications. Using midpoints of each group and the average times, chart a scatter plot of the three points, fit a power function to the chart, and calculate the learning percentage. Is there learning? (LO3)

[5] C. D. Bailey, "Forgetting and the Learning Curve: A Laboratory Study," *Management Science*, 35(3), March 1989, pp. 34–352.
[6] J. P. Couto and J. C. Teixeira, "Using Linear Model for Learning Curve Effect on Highrise Floor Construction," *Construction Management and Economics*, 23(4), 2005, pp. 355–364.
[7] N. Ghosh et al, "Laser Lead Extraction: Is There a Learning Curve?," *Pacing and Clinical Electrophysiology*, March 2005, pp. 180–184.



***24.** The manufacture of collared coils (see photo below) for CERN Particle Accelerator in Geneva, Switzerland, was performed by three suppliers.[8] The man-hours used by the third supplier for each unit are displayed in the chart above by a circle. The dashed line is the learning curve fitted. Check 2 points on the curve to verify that the learning percentage is 74 percent. (LO2 & 3)

25. A study was done to show that using a new equipment (Nucletron's FIRST system) will reduce the learning time for implanting radioactive seeds for treating prostate cancer.[9] For an explanation of the procedure, see e.g., http://www.prostate-cancer.com/brachytherapy/cancer-treatments/treatment-brachy-ldr.html. Two institutions were used for the study: Tom Baker Cancer Center (T.B.) and Center Hospitalier Universitaire de Quebec (CHUQ). T.B. had no previous experience in this procedure but CHUQ had treated 740 patients before. The results of the study were similar in terms of outcome, but CHUQ had faster procedure times (in minutes; see below). Chart the line plot of each set of data, fit a power function to each chart, and calculate the learning percentage for each institution. Explain the differences between the two results. (LO3)

Case Number	Time T.B.	Time CHUQ
1	260	130
2	340	120
3	250	80
4	100	100

[8]P. Fessia et al, "Application of the Learning Curve Analysis to the LHC Main Dipole Production: First Assessment," *IEEE Transactions on Applied Superconductivity*, 16(2), June 2006, pp. 242–247.
[9]L. Beaulieu et al, "Bypassing the Learning Curve in Permanent Seed Implants using State-of-the-art Technology," *International Journal of Radiation Oncology * Biology * Physics* 67(1), 2007, pp. 71–77.

(Continued)

Case Number	Time T.B.	Time CHUQ
5	220	70
6	180	90
7	220	80
8	200	90
9	160	100
10	170	130
11	200	90
12	210	70
13	200	75
14	130	90
15	230	80
16	190	85
17	140	70
18	110	80
19	130	75
20	140	80

26. A study was performed to determine the minimum number of tracheal intubation necessary for an intern to achieve "expert" status.[10] The objective was an intubation time of two minutes or less without any complications. Four residents with no prior experience intubated 15 patients each under supervision. A learning curve was generated of the mean time (in seconds) for intubation of patients 1 to 15 for all residents combined (see below). Chart the line plot of the data, fit a power function to the chart, and determine how many repetitions are necessary for a resident to attain the "expert" status. (LO3 & 4)

Repetition Number	Average Procedure Time
1	240
2	265
3	199
4	143
5	117
6	117
7	152
8	107
9	139
10	95
11	98
12	70
13	79
14	82
15	78

27*. An airplane component manufacturer has been approached by its customer (the airplane manufacturer) to make a specific component. Based on similar components produced before, the cost estimator has determined that the breakeven (standard) unit will be the 24th unit

[10] C. Johnson and J. T. Roberts, "Clinical Competence in the Performance of Fiberoptic Laryngoscopy and Endotracheal Intubation: A Study of Resident Instruction," *Journal of Clinical Anesthesia* 1(5), 1989, pp. 344–349.

made and the standard time for the 24th unit will be 161 hours. Also, she expects a learning percentage of 80 percent. The order is for 30 units. (LO2)

a. If a quotation is to be given to the customer for the total expected labour hours, what should it be?

b. The quotation was accepted by the customer and the component manufacturer has started making the component. Although the first few units took longer than expected, per-unit labour hours are now approaching the estimates. However, the customer has asked for a design change to be implemented after the 15th unit. The design change is estimated to result in 30 hours of new work for the standard (24th) unit, and is expected to have the same learning percentage (80 percent). The new work replaces the same amount of original work (30 hours) for the standard (24th) unit. What should the revised total labour hours (for all 30 units) be? Hint: The new and the original work behave differently at the mid-point of production of this batch because the original work has already been learned to a large degree whereas the new work is yet to be learned.

28. A study was performed on the new tube hydroforming line of a GM component plant to determine the future cost prospects of this new technology.[11] One of the characteristics measured was the cycle time to make one tube. The monthly averaged cycle time (in minutes) over a period of 53 months of production (approximately 42,000 units per month) are displayed below. Chart the line plot of the data, fit a power function to the chart, and calculate the learning percentage. Why is the learning effect small in this case? (LO3)

Month	Cycle Time
1	1.09
2	1.1
3	1.36
4	1.16
5	1.13
6	0.9
7	1
8	0.93
9	0.79
10	0.74
11	0.91
12	0.8
13	0.77
14	0.71
15	0.75
16	0.71
17	0.81
18	0.91
19	0.8
20	0.89
21	0.78
22	0.91
23	0.8
24	0.81
25	1.12
26	1.03
27	0.73

Month	Cycle Time
28	0.83
29	0.79
30	0.7
31	0.75
32	0.79
33	0.82
34	0.81
35	0.85
36	1.03
37	0.79
38	0.68
39	0.71
40	0.74
41	0.69
42	0.78
43	0.77
44	0.81
45	0.79
46	0.77
47	0.78
48	0.91
49	0.82
50	0.83
51	0.88
52	0.84
53	0.65

[11] M-C Nadeau et al, "A Dynamic Process-based Cost Modeling Approach to Understand Learning Effects in Manufacturing," *International Journal of Production Economics* 128, 2010, pp. 223–234.

***29.** An airplane component manufacturer has been asked to quote for 30 units of a new component. The following data are available: two departments are involved—fabrication with 90 percent learning slope and assembly with 85 percent learning slope; the breakeven (standard) unit for both departments is the 20^{th} unit; and the direct labour of the 20^{th} unit is estimated to be 100 hours for fabrication and 75 hours for assembly. What should the quoted total labour hours be? (LO2)

***30.** A machine shop processes a component in batches (lots) through its operations. Suppose three lots of the component have already been processed with the following lot sizes and lot average hours, and the shop is about to process the fourth lot that consists of 40 units[12] (LO3)

Lot No.	Lot Size	Lot Avg. Hours
1	6	6,800
2	9	4,500
3	15	3,500
4	40	

Determine the appropriate lot plot points (the *x*-axis values of each lot), chart the scatter plot of the data, fit a power function to the chart, and estimate the total hours for the fourth lot.

***31.** There has recently been a controversy over the expected cost of 65 F35 Joint Strike Fighter jets that Government of Canada wants to purchase. F35's design phase has taken several years longer than expected and costs are billions of dollars over budget. Also, Canada and the manufacturer, Lockheed Martin, have not yet signed a contract, so the price has not been determined yet. In addition, accurate cost data for the first few prototypes are not available publicly, while the prototypes are constantly being changed to improve them. This makes estimating the costs of jets very difficult. The first lot of 32 production-grade jets are expected to be delivered to the U.S. armed forces in 2012 at the cost of $207.6 million each (excluding the research and development costs; see e.g., http://comptroller.defense.gov/defbudget/fy2012/FY2012_Weapons.pdf p15). Israel has negotiated to buy 20 jets to be delivered at the end of 2015 at the cost of $137.5 million each (see e.g., http://www.aviationweek.com/aw/generic/story_channel.jsp?channel=defense&id=news/awst/2010/08/23/AW_08_23_2010_p32-249396.xml). According to the production schedule, 330 jets will have been made by the end of 2015 (see e.g., the last page of http://www.parl.gc.ca/PBO-DPB/documents/F-35_Cost_Estimate_EN.pdf).(LO3)

a. Using these two data points (midpoint of 16 for the U.S. purchase and midpoint of 340 for the Israeli purchase, chart the scatter plot of the two data points, fit a power function to the chart, and calculate the learning rate. Does it seem reasonable? Briefly explain.

b. Canada's jets are expected to be delivered between 2016 and 2020 in lots of approximately 13. The middle jet is expected to be the 848^{th} jet made. Estimate the cost of the 848^{th} jet using the power function of part a, and then estimate the total cost of Canada's purchase.

MINI-CASE

Product Recall

An automobile manufacturer is conducting a product recall after it was discovered that a possible defect in the braking mechanism could cause loss of braking in certain cars. The recall covers a span of three model years. The company sent out letters to car owners promising to repair the defect at no cost at any dealership.

The company's policy is to pay the dealer a fixed amount for each repair. The repair is somewhat complicated, and the company expects learning to be a factor. In order to set a reasonable rate for repairs, company engineers conducted a number of repairs themselves. It was then decided that a rate of $88 per repair would be appropriate, based on a flat hourly rate of $22 per hour and 90-percent learning.

[12]http://www.acq.osd.mil/dpap/cpf/docs/contract_pricing_finance_guide/vol2_ch7.pdf.

Shortly after dealers began making repairs, the company received word that several dealers were encountering resistance from workers who felt the flat rate was much too low and who were threatening to refuse to work on those jobs. One of the dealers collected data on job times and sent that information to the company: three mechanics each completed two repairs. Average time for the first unit was 9.6 hours and for the second unit was 7.2 hours. The dealer has suggested a rate of $110 per repair.

You have been asked to investigate the situation and to prepare a report.

Questions

1. Prepare a list of questions that you will need to have answered in order to analyze this situation.
2. Comment on the information provided in the case.
3. What preliminary thoughts do you have on solutions/partial solutions to the points you have raised?

MINI-CASE

Learning Curve in Surgery

New surgery techniques require a long learning curve on the part of a surgeon. Random complications may arise due to patients' conditions. Therefore, it is important to know the number of operations required to stabilize operating times and complication rates.

Dr. Voitk of Salvation Army Scarborough Grace Hospital reported the results of 100 consecutive operations for laparoscopic hernia repair on 98 patients. Approximately two-thirds of surgeries were uni-lateral (right/left) and the remaining one-third were bilateral (involving contra-lateral defects; many unsuspected before surgery). The average surgery time (from skin incision to skin closure) was 46 minutes for unilateral and 62 minutes for bilateral. Surgery times for the unilateral procedure began to level off after 50 operations. Dr. Voitk reported the average surgery times (in minutes) for each quartile of the 100 operations, classified by type of operation, as shown on the right.

At the end of the study the times had levelled off at 58 minutes (operating time), including 37 minutes (surgical time), for unilateral type, which approached the historical times for open repair. Learning also reduced the complication rates, which fell in an approximately exponential manner, beginning to level off at 50 operations and becoming stable after 75.

	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Unilateral	59	45	38	37
Bilateral	69	67	58	52

Questions

1. Determine the learning percentage for the unilateral laparoscopic hernia surgery. (Hint: Assume that the 8th, 24th, 40th, and 58th observations had unilateral surgeries.)
2. Determine the learning percentage for the bilateral laparoscopic hernia surgery. (Hint: Assume that the 4th, 12th, 20th, and 28th observations had the bilateral surgeries). Compare your answers.

Source: Adapted from Dr. Andrus Voitk, "The Learning Curve in Laparoscopic Inguinal Hernia Repair for the Community General Surgeon," *Canadian Journal of Surgery* 41(6) December 1998, pp. 446–450.

CASE

Renovating the Lions Gate Bridge

Dr. William C. Wedley

Ten sections completed, 44 more to go. The repairs are controversial, behind schedule, over budget, and subject to penalties. American Bridge/Surespan staff is contemplating how long it will take to install the remaining sections of Lions Gate Bridge.

Lions Gate Bridge, the scenic entranceway to Vancouver's Burrard Inlet, began service in 1938. At that time, its two lanes of traffic appeared more than ample. By the 1950s however, commuters to and from Vancouver's north shore were causing rush-hour line-ups. In 1952 two lanes were converted to three narrow lanes. But continued expansion on the north shore, growth of the Whistler ski community, and increased ferry service to Vancouver Island caused the bridge to reach its capacity.

Over the years, Lions Gate Bridge had undergone regular maintenance. Although its suspension cables and steel superstructure have never been changed, the bridge has undergone periodic painting and resurfacing. On the viaduct section in the north, the cantilevering of sidewalks allowed increased lane widths and separation of pedestrians from traffic. This accentuated the need for change on the rest of the bridge.

By the 1990s, the annual cost of maintenance made the existing situation uneconomical. The bridge either had to be repaired to "good as new" or an alternate crossing had to be built. The debate became extreme. Some proposed doubling capacity with a twin bridge or a new four-lane bridge. Other plans included various locations for a tunnel or combinations of bridges and tunnels. At the same time, the Parks Board and environmentalists opposed cutting of trees or any encroachment on Stanley Park. Consensus seemed impossible.

Complicating the issue was money. Highway construction is a provincial responsibility, but deficit financing already overwhelmed the government. User tolls were a possibility, but north shore communities claimed that tolls would be discrimination when not applied to other bridges. Private ownership, while a possibility, suffered from criticisms of tolls and monopoly. In the end, money ruled. In May 1999, the provincial government chose to replace only the deck of the existing bridge.

The outcry was instantaneous. While the bridge would be made safer, the same capacity problems would remain. Public meetings were held, citizens signed petitions, and "stop work" injunctions were sought through the court system. When tempers cooled, a detailed news article looked back and pronounced "Lions Gate Renewal Defies Reason."[13]

A team headed by American Bridge (AB) of Pittsburgh was selected to rehabilitate the bridge. In order to carry out the work, they assembled a group of other professionals for construction design.

The total budget for the upgrade was slightly over $100 million. The most complex and challenging component of the project is the requirement to replace the decking while the bridge is still in use. American Bridge's plan is to replace the bridge in sections—cutting out and lowering an old section and raising and inserting a wider prefabricated section during weekend or overnight shutdowns. In the end, the bridge would be wider, sleeker, and modernized.

In total, 54 sections must be removed and replaced. Removing and replacing deck panels requires tricky technology. Since the bridge is suspended, removing a section releases the structural integrity that holds it together. In addition, without some large clamping device to keep the bridge apart, it is possible that the pieces would not fit.

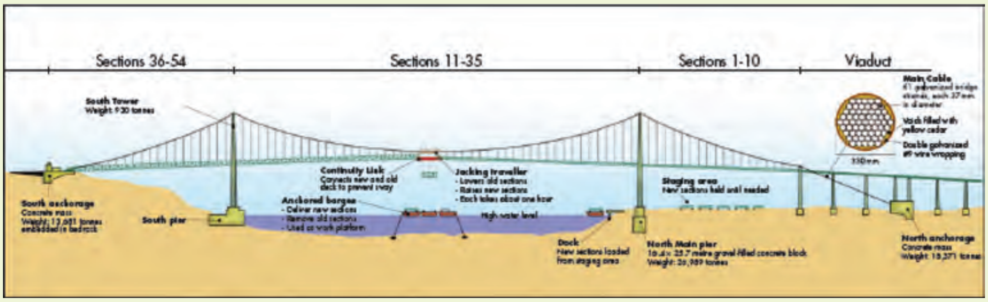


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Exhibit 1—Schematic Diagram of Lions Gate Bridge

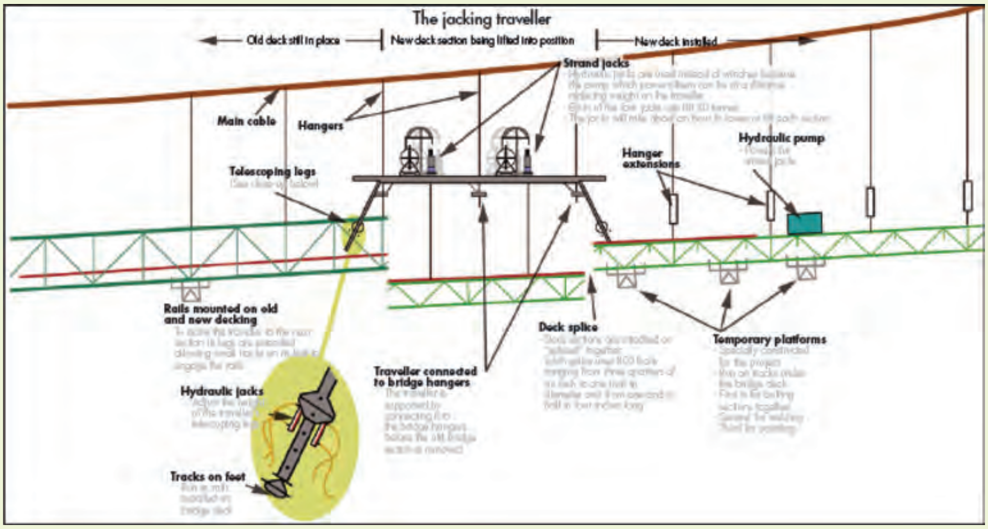

Exhibit 2—Schematic Diagram of the "Jacking Traveller"

[13] Peter Lambur, *The Vancouver Sun*, October 5, 1999.

American Bridge designed and developed the "jacking traveller." This device travels along the bridge from section to section as the work is done. Its essential function is twofold: (1) to clamp the bridge in place while a section is removed and (2) to position the jacks for lowering and raising sections. Once a new section is securely in place, the jacking traveller moved along to the next section, where the process is repeated.

Initially, AB had proposed to use the jacking traveller as a car ramp as well, so that the bridge did not have to be closed during panel replacement. However, later it was discovered that the bridge may not be able to safely hold all the weight during construction. This change in plan resulted in a six-month delay in the start of construction, which was initially supposed to start in January 2000 and finish in one year.[13]

The first panels were installed on the weekend of September 8–10, 2000. The plan was ambitious—to replace two panels in one weekend closure.

The bridge was closed at 11 p.m. on Friday and work began. At about 6 a.m., the first of the 54 panels was cut out and lowered to the ground using the jacking traveller. Shortly later, at daybreak, the new wider panel was lifted approximately 40 metres up to the bridge deck. Five hours later, after lateral jackeying, bolting, and welding, the panel was in place. This was the first time that an entire piece of a suspension bridge of this magnitude has been replaced at one time.

Q1

Exhibit 3—Raising Panel 1, Daybreak, Sept 9, 2000 Photo from the former project Website, courtesy of Stuart McCall/North Light

About noon Sunday, after 24 hours used for moving the jacking traveller, installation started on the second panel. Difficulty was encountered removing rusted suspension pins and dealing with a "frozen" expansion joint. In total, the second panel took 26 hours, and the bridge reopened at 2 p.m. Monday, well after the scheduled time for completion.

The traffic jams on Monday morning were horrendous. Vehicles were backed up for kilometres, roads became grid-locked, people were late for work, and economic activity was hindered.

From the first two panels, the project staff learned some lessons. As the work at this stage was still above land, the old section had been lowered 40 metres to dollies where it was moved out of the way. Then a new panel was moved into place and raised to the bridge deck. This lowering and raising process took 2.5 hours. Toward the centre of the bridge and over water, the vertical lifts will become longer.

The second lesson was that panels adjacent to towers (like the first panel) are more difficult to replace. The detached old section must be first swung sideways and then lowered without striking the tower. Then, the replacement panel has to be raised away from the tower and swung sideways once it gets to the top. Since space is tight and limited, replacement of panels next to towers takes longer.

They also discovered that moving and setting up the jacking traveller between lifts takes a considerable time. In total, bridge closure for the first two panels was 63 hours and 48 minutes. Of this time, approximately 24 hours was used for repositioning the jacking traveller and setting up for the second section. Hence, actual removal and insertion time, exclusive of setup, was about 40 hours for the first two sections. Approximately 14 hours of that time was for Panel 1. The remaining time, 26 hours, was for Panel 2, which required extra time to free a frozen expansion joint.

Except for special circumstances at the towers and the south abutment, bridge closures for future deck replacements were planned for one deck section at a time. That way, the setup procedure for the jacking traveller between lifts could be done during nighttime closures or even when traffic was flowing. (The jacking traveller did not obstruct the traffic, which just passed under it.) The total disruption and closure time could thereby be kept to a minimum.

To plan and monitor how long closures would take, management estimated replacement times. The time estimates and the actual times for the first ten panels are shown in Table 1.

The actual times for the first three panels were well above the team's estimates. The times for Panels 4 to 9 were much better, being within two hours of the estimate. As well, the times to complete Panels 4 to 8 improved as the work crews gained experience. Panel 10, however, took much longer than expected. Being adjacent to the north side of the North Tower, it was more difficult to insert. As well, expansion problems complicated its installation.

By looking at the replacement times for sections that were adjacent to towers, the project staff could get an idea of the extra

[13]A. Cho, "Lions Gate Bridge Rehabilitation Proves a Lion-sized Challenge," *ENR* 248(3), January 28, 2002, pp. 22–25.

Table 1
Bridge Closures

Date From–To	Time Closed	Time Opened	Time Work Completed	Comments	Estimate (hours)
Sept 8–11	22:12	14:00	63 hrs 48 mins	Panels #1 & #2 (three nights)	15 each
Sept 16–17	20:05	13:30	17 hrs 25 mins	Panel #3	12
Sept 23–24	22:06	12:26**	13 hrs 50 mins**	Panel #4 (** adjusted for 30 minute delay caused by a person trying to jump from the bridge)	13
Sept 30–Oct 1	22:05	10:42	12 hrs 37 mins	Panel #5	13
Oct 14–15	22:03	8:27	10 hrs 24 mins	Panel #6	10.5
Oct 21–22	22:07	8:07	10 hrs	Panel #7	10
Oct 28–29	22:07	6:07**	9 hrs**	Panel #8 (** adjusted for the end of daylight savings time)	10
Nov 2–3	9:21	5:58	10 hrs 37 mins	Panel #9	9.75
Dec 2–3	22:30	17:30	19 hrs	Panel #10 (North of North Tower)	10

time required for working close to towers. The two panels adjacent to towers (1 and 10) required 14 and 19 hours, respectively. Panel 10, however, was installed on a warm sunny day when expansion of metal caused the expected tolerances to disappear; this compounded the space problem and lengthened the installation process by four hours. Accordingly, the installation time for panels next to towers is 14 or 15 hours—about 5 hours longer than sections that are clear of the towers.

With the experience of the first ten sections, the project managers contemplated how much time would be required for the remaining 44 sections. They knew that Panel 11 would be difficult, because it is adjacent to a tower. To enable manoeuvrability around towers, Panels 10 and 11 are smaller in size (10-metre sections).

Panels 12 to 34 are full-sized sections (20 metres) that cover the main span between the north and south towers. Except for the first two of those sections, all are delivered by sea. Two barges with accompanying tugboats jockey into place below the opening, one to receive the old section and the other to deliver the new panel. The required vertical lift from the barges depends upon the tides at the time of the lift. The typical lift from the water level is expected to be 55 metres, as opposed to 40 metres over land. The increased distance and the jockeying of barges are expected to increase the removal and lifting time by one hour (i.e., 3.5 hours over water vs. 2.5 hours over land).

After Panel 34, the size changes to 10 metres but with no change in the lifting, bolting, and welding times. Panels 35 and 36, next to the south tower, are expected to take longer. Panels 37 and 38 are spliced together beforehand, delivered on a narrow barge at high tide, and lifted and installed as one piece. This splicing to form a double panel avoids one bridge closure, but the horizontal jockeying of a larger section adds about two hours to the replacement.

Panels 39 to 53 are installed differently. Since the land below is steep cliff, delivery and installation is done from above. First, the 10-metre section cut from the bridge is raised above the deck level and turned 90 degrees. A truck with extended girders backs onto the bridge from the south end so that its girders extend over the gap. The old section is then lowered onto the girders, moved onto the truck, and driven away. In the meantime, the new section is backed over the entire bridge from the north end on a similar truck. The new section is raised from the truck's girders, rotated 90 degrees, and lowered into the opening after the truck is removed. Although this process does not require as much time jacking sections up and down, it requires the tricky 90-degree turn. In this regard, Panels 39 to 53 are new learning experiences.

The final panel, number 54, is unique. Unlike Panels 39 to 53, it is longer (12 metres) and requires a different installation procedure. Being longer, it cannot be backed over the bridge or swung 90 degrees before installation. As a result, both removal of the old section and delivery of the new panel are done from the south abutment.

As a consequence of engineering delays and adverse weather conditions in December and January, the replacement schedule was cancelled for those months. This provided a time for reflection for the project managers. Given their experience to date, they had to estimate how long each remaining section would take for removal and replacement. With those estimates, they would be able to determine a closure schedule that would minimize disruption to the community, but ensure safety for workers and the public.

To help in this process, staff needed to prepare an estimate sheet for panel installation times (see Table 2). With the actual and estimated times for the first 10 panels and their knowledge of challenges ahead, they agreed to map out a plan of action. Their meeting is scheduled for next Monday. After that meeting, they will present their plan to their client, the B.C. Transportation Financing Authority. The client is insisting on an accurate and reliable closure schedule.

Table 2

Estimate Sheet for Panel Installation Times

Panel #	Actual Time (hours)	Lift Conditions	Panel Size	Estimate (hours)
1	14.0	Land, At Tower	20 m	15.0
2	26.0	Over Land	20 m	15.0
3	17.4	Over Land	20 m	12.0
4	13.8	Over Land	20 m	13.0
5	12.6	Over Land	20 m	13.0
6	10.4	Over Land	20 m	10.5
7	10.0	Over Land	20 m	10.0
8	9.0	Over Land	20 m	10.0
9	10.6	Over Land	20 m	9.75
10	19.0	Land, At Tower	10 m	10.0
11		Land, At Tower	10 m	
12		Land and Water	20 m	
13		Over Water	20 m	
14		Over Water	20 m	
15		Over Water	20 m	
15		Over Water	20 m	

Questions

1. Given the data on the replacement of the first 10 panels, is there any evidence that the work team is learning how to make installations faster?
2. What techniques can be used to estimate the length of time needed to perform each of the remaining installations?
3. How long will it take to perform each of the remaining installations?
4. What allowances will have to be made for factors such as (1) installations next to towers, (2) longer vertical lifts over water, and (3) different installation procedures for Panels 37 to 54?