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Multi Calculus

Assignment No 2

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Question No: 1

(Part a)

Sol

$$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$$\lim_{(x-2y)\to(2-1)} \frac{(2)^2-2(2)(1)}{(2)^2-4(1)^2}$$

$$\frac{4-4}{4-4} = 0$$

$x^2 = 4y^2$

Squaring on b/s

$x = 4y \rightarrow y = \frac{4}{x}$

$= x^2/16 - 2(x)(x/4)$

$= x^2 - 4\left(\frac{x^2}{16}\right)$

$$\frac{x^2}{16} - \frac{x^2}{2} = \frac{(x^2-8x^2)}{4\,16(4x^2-x^2)}$$

$$= \frac{-7x^2}{4(3x^2)} \Rightarrow \frac{-7x^2}{12x^2} = \frac{-7}{12} \text{ Ans}$$

$= \cancel{4(3x^2)}$

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(Part B)

$$\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x} = 0$$

$6y = 7x$

$y = \frac{7x}{6}$

$= \frac{x-4(7x/6)}{7x+7x}$

$= \frac{7 - 28x/6}{14x}$

$= \frac{6x-28x}{6(14x)} = \frac{-22x}{6(14x)}$

$= -\frac{11}{52}$

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Part "c"

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$$

let $y = mx$

$$\frac{x^2 - m^6x^6}{xm^3x^3}$$

$$\frac{x^2(1 - m^6x^4)}{x^4m^3}$$

$$\frac{1 - m^6x^4}{x^2m^3}$$

Part "d"

$$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 4e^2y}{6x+2y-3z}$$

$$\Rightarrow \frac{(-1)^3 - 4(0)e^2}{6(-1)+2(0)-3(4)}$$

$$\Rightarrow \frac{-1-0}{-6+2-12}$$

$$\Rightarrow \frac{1}{16} \quad \Rightarrow 1/16$$

Question No: 2

Determine $D_{\vec{u}} f$ for the given function in the indicated direction:.

a) $f(x,y) = \cos\left(\frac{x}{y}\right)$ in the direction of $\vec{v} = (3, -4)$

Sol

$$\vec{v} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$\vec{v} = \hat{i}\frac{\partial\left(\cos\left(\frac{x}{y}\right)\right)}{\partial x} + \hat{j}\,\partial f \cos\left(\frac{x}{y}\right)$$

$$\vec{v} = \hat{i}\,\frac{\left(-\sin\frac{x}{y}\right)}{y} + \hat{j}\left[\left(-\sin\left(\frac{x}{y}\right)\right)x \cdot -y^{-2}\right]$$

$$\vec{v} = \left[\frac{-\sin\left(\frac{x}{y}\right)}{y}\right]\hat{i} + \hat{j}\left[\frac{x\sin\left(\frac{x}{y}\right)}{y^2}\right]$$

$$\vec{v} = \left[\frac{-\sin\left(\frac{3}{-4}\right)}{-4}\right]\hat{i} + \hat{j}\left[\frac{3\sin\left(\frac{3}{-4}\right)}{(-4)^2}\right]$$

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$$= \frac{1}{4}\left[-0.0131\hat{i} + \left(\frac{3}{16}(-0.0131)\hat{j}\right)\right]$$

$$\vec{v} = \frac{-0.0131}{4}\hat{i} - \frac{0.0393}{16}\hat{j}$$

part b:-

$$f(x,y,z) = x^2y^3 - 4xz \qquad \vec{v} = (-1, 2, 0)$$

$$\vec{v} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\vec{v} = \hat{i}\frac{\partial}{\partial x}(x^2y^3 - 4xz) + \hat{j}\frac{\partial}{\partial y}(x^2y^3 - 4xz) + \hat{k}\frac{\partial}{\partial z}(x^2y^3 - 4xz)$$

$$= \hat{i}(2xy^3 - 4z) + \hat{j}(3x^2y^2) + \hat{k}(-4x)$$

$$\vec{v} = \hat{i}\left[2(-1)(2)^3 - 4(0)\right] + \hat{j}\left[3(-1)^2(2)^2\right] + \hat{k}\left[-4(-1)\right]$$

$$\vec{v} = \hat{i}(-2 \times 8) + (+3 \times 4)\hat{j} + 4\hat{k}$$

$$\vec{v} = -16\hat{i} + 12\hat{j} + 4\hat{k}$$

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Question NO "3"

$f(x,y,z) = 4xy^2 e^{3xz}$

$\nabla f = 4y^2 e^{3xz}(3z)i - 2ye^{3x0}j + 4y^2 3^{xz} \; 3xk$

$= (4-0)i - 2j - 9k$

$v = (-1, 4, 2)$

$\hat{v} = \dfrac{-1i + 4j + 2k}{\sqrt{1+16+4}} \Rightarrow \dfrac{-1}{\sqrt{21}}i + \dfrac{4}{\sqrt{21}}j + \dfrac{2k}{\sqrt{21}}$

$\dfrac{-1(4)}{\sqrt{21}} \; \dfrac{-2(4)}{\sqrt{21}} \; \dfrac{-9(2)}{\sqrt{21}}$

$\Rightarrow \dfrac{-4}{\sqrt{21}} \; \dfrac{-8}{\sqrt{21}} \; \dfrac{-18}{\sqrt{21}}$

$\Rightarrow \dfrac{-4-8-18}{\sqrt{21}}$

$\Rightarrow \dfrac{-32}{\sqrt{21}}$

Question NO 4

$f(x,y) = \sqrt{x^2+y^3}$ at $(-2, 3)$

$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)j$

$= \dfrac{-2}{\sqrt{13}}i + \dfrac{27}{2\sqrt{13}}j$

Part "b"

$f(x,y,z) = e^{2x}\cos(y \cdot 2x)$ $\vec{a} (4,-2,0)$

$\nabla f \; (e^{2x} \cdot 2\cos(y-2x))i + e^{2x}(-\sin(y-2x)(1))j + e^{2x}(-\sin(y-2x))(-2)k$

$\nabla f(4,-2,0) = e^{4x} \; 2\cos(-2,0)i + e^{4x}(-\sin(-2))j + e^{4x}(-\sin(-2))(-2)$

$\nabla f = e^{4x}(-2\cos 2i - \sin(-2)j + 2\sin(-2)k$

Question No: 04

Find maximum rate of change of the function at the indicated point and direction in which this rate of change occurs.

(a) $f(x,y) = \sqrt{x^2+y^2}$ at $(-2,3)$

$\nabla f(x,y) = |\text{grad} f|$

$\text{grad} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y}$

$= \hat{i}\frac{\partial}{\partial x}(x^2+y^2)^{1/2} + \hat{j}\frac{\partial(x^2+y^2)^{1/2}}{\partial y}$

$= \hat{i}\left[\frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}x\right] + \hat{j}\left[\frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}y\right]$

$= \hat{i}\left[\frac{x}{\sqrt{x^2+y^2}}\right] + \hat{j}\left[\frac{y}{\sqrt{x^2+y^2}}\right]$

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$= \frac{-2\hat{i}}{\sqrt{4+9}} + \hat{j}\frac{3}{\sqrt{4+9}}$

$= \frac{-2\hat{i}}{\sqrt{13}} + \hat{j}\frac{3}{\sqrt{13}}$

$|\text{grad} f| = \sqrt{\left(\frac{-2}{\sqrt{13}}\right)^2 + \left(\frac{3}{\sqrt{13}}\right)^2}$

$= \sqrt{\frac{4}{13} + \frac{9}{13}} \Rightarrow \sqrt{\frac{13}{13}}$

$= \sqrt{1} = \underline{1}$

direction at which the rate of change occur $= \frac{\nabla \text{grad} f}{|\nabla \text{grad} f|} = \frac{\frac{-2\hat{i}}{\sqrt{13}} + \frac{3\hat{j}}{\sqrt{13}}}{1}$

$= \frac{-2\hat{i}}{\sqrt{13}} + \frac{3\hat{j}}{\sqrt{13}}$

part b:.

$f(x,y,z) = e^x \cos(y-2z)$ at $(4,-2,0)$

$\nabla f(x,y,z) = |\text{grad} f|$

$\text{grad} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

$= \hat{i}\frac{\partial}{\partial x}[e^x \cos(y-2z)] + \hat{j}\frac{\partial}{\partial y}[e^x \cos(y-2z)]$

$+ \hat{k}\frac{\partial}{\partial z}[e^x \cos(y-2z)]$

$= \hat{i}\,[e^x \cos(y-2z)] + \hat{j}\,[e^x \cdot -\sin(y-2z)$
$+ \hat{k}\,[e^x \cdot -\sin(y-2z) \cdot -2]$

$= \hat{i}\,[e^x \cos(y-2z)] + \hat{j}\,[-e^x \sin(y-2z)] +$
$\hat{k}\,[e^x\, 2\sin(y-2z)]$

$e^x\,[\hat{i}\,(\cos(y-2z)) + \hat{j}\,[-\sin(y-2z)]$
$+ \hat{k}\,(2\sin(y-2z))]$

$= e^4\,[\hat{i}\,[\cos(-2-2(0)] + \hat{j}\,(\sin(-2-2(0)]$
$+ \hat{k}\,[2\sin(-2-2(0)]$

$= 5.4.6\,[\hat{i}\,(\cos(-2)] + \hat{j}\,(\sin(-2) + \hat{k}\,(2\sin(-2)]$

$= 5.46\,[(1\hat{i}) + \hat{j}(-0.035) + \hat{k}(-0.07)]$
$= 54.6\hat{i} - 1.911\hat{j} - 3.8\hat{k}$

$|\nabla \text{grad} f| = \sqrt{(54.6)^2 + (-1.911)^2 + (-3.8)^2}$

$= \sqrt{\ }$

$\boxed{\nabla \text{grad} f = 54.8}$

The direction at which this rate of change occur $= \dfrac{\nabla \text{grad} f}{|\nabla \text{grad} f|}$

$= \dfrac{54.6\hat{i} - 1.911\hat{j} - 3.8\hat{k}}{54.8}$

$= \dfrac{54.6}{54.8}\hat{i} - \dfrac{1.911}{54.8}\hat{j} - \dfrac{3.8}{54.8}\hat{k}$

Question NO "5"

Part "A"

$F = x^2 y i - (z^3 - 3x) j + 4y^2 k$

Div $f = \nabla f \cdot f$

$= (2xy) (x^2 y i - (z^3 - 3x) j + 4y^2 k$

curl $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2 y & -(z^3 - 3x) & 4y^2 \end{vmatrix}$$

$(8y + 3z^2) i - 0 j + (2x^2) k$

Part "b"

$F = (2x + 2z^2) i + \frac{x^3}{2} y^2 j - (2z x) k$

Div $\cdot \nabla f = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k \right) (2x + 2z^2 + \frac{x^3}{2} y^2 j - (2zx) k$

$= 2 + \frac{2x^3 y}{2} - 2$

curl $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3x + 2z^2 & \frac{x^3 y^2}{2} & -(2 - 2x) \end{vmatrix}$$

$\frac{x^3 y^2}{2} i - (2 - 4z) j + \left(\frac{3x^2 y^2}{4} \right)$

$\frac{dy}{dx} = \frac{d}{dx} \sin(x^2) = \cos x^2 \cdot 2x$

$= 2x \cos^2$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$= (4x^2 y^3 - z)(2x \cos x^2)$

Question No: 06

Determine if the vector field is
(a) conservative.

a $\vec{F} = x^2 y \hat{i}$ -

$\vec{F} = \left(4x^2 + \frac{3x^2y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)\hat{k}$

The vector field is conservative if and only if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

$\vec{F} = \underbrace{\left(4y^2 + \frac{3x^2y}{z^2}\right)}_{M}\hat{i} + \underbrace{\left(8xy + \frac{x^3}{z^2}\right)}_{N}\hat{j}$

$+ \left(11 - \frac{2x^3y}{z^3}\right)\hat{k} \Rightarrow P$

$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}$, $\frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$

$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = \frac{-2x^3}{z^3}$

$\frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$

$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2y}{z^2} = 3x^2y(-2)z^{-3}$

$= \frac{-6x^2y}{z^3}$

$\frac{\partial P}{\partial x} = \frac{\partial\left(11 - \frac{2x^2y}{z^3}\right)}{\partial x} = \frac{-6x^2y}{z^3}$

Hence:-

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} , \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} = \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

Since the vector field is conservative.

(b) $\vec{F} = 6x\hat{i} + (2x-y^2)\hat{j} + (6z-x^3)\hat{k}$

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} , \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} , \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\vec{F} = \underbrace{6x}_{M}\hat{i} + (\underbrace{2x-y^2}_{N})\hat{j} + (\underbrace{6z-x^3}_{P})\hat{k}$$

$$\frac{\partial M}{\partial y} = \frac{\partial (6x)}{\partial y} = 0 , \quad \frac{\partial N}{\partial x} = \frac{\partial (2x-y^2)}{\partial x} = 2$$

$$\frac{\partial N}{\partial z} = \frac{\partial (2x-y^2)}{\partial z} = 0 , \quad \frac{\partial P}{\partial y} = \frac{\partial (6z-x^3)}{\partial y} = 0$$

$$\frac{\partial M}{\partial z} = \frac{\partial\, 6x}{\partial z} = 0 , \quad \frac{\partial P}{\partial x} = \frac{\partial (6z-x^3)}{\partial x} = -3x^2$$

So,

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} , \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} , \frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$$

Since, the vector field is not conservative

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Question No: 07

a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos(2t)$ · $w = 4t$

$\frac{dz}{dt} = ?$

$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$

$\frac{dz}{dt} = \frac{2x}{y^4}$, $\frac{\partial z}{\partial y} = (x^2 - w) y^{-4}$

$\frac{\partial z}{\partial y} = (x^2 - w) - 4y^{-5}$

$\frac{dx}{dt} = 3t^2$, $\frac{\partial z}{\partial y} = \frac{-4(x^2 - w)}{y^5}$

$\frac{dy}{dt} = \frac{d}{dt} \cos 2t = -\sin 2t \cdot 2$

$= -2 \sin 2t$

$\frac{\partial z}{\partial w} = \frac{-1}{y^4}$ $\frac{dw}{dt} = 4$

$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$

$= \frac{2x^2}{y^4} \cdot 3t^2 + \left(\frac{-4(x^2 - w)}{y^5} \cdot -2 \sin 2t \right)$

$+ \left(\frac{-1}{y^4} \right) \cdot 4$

$= \frac{6x^2 t^2}{y^4} + \frac{8(x^2 - w) \sin 2t}{y^5} - \frac{4w}{y^4}$

b) $z = x^2y^4 - 2y \quad , \quad y = \sin(x^2)$

$\frac{dz}{dx} = ?$

$\frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$

$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$

$= (4y^3x^2 - 2)$

$= (4x^2y^3 - 2)$

$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2) = \cos x^2 \cdot 2x$

$= 2x\cos x^2$

$\frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$

$= (4x^2y^3 - 2)(2x\cos x^2)$

$= 8x^3y^3\cos x^2 - 4x\cos x^2$

part (c)

Compute $\frac{dy}{dx}$ for the following equation

$x^2y^4 - 3 = \sin(xy)$

differentiate w.r.t with x

$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}\sin(x \cdot y)$

$2xy^4 + x^2 4y^3\frac{dy}{dx} = y\cos(xy)$

$2xy^4 + 4x^2y^3\frac{dy}{dx} = y\cos(xy)$

$4x^2y^3\frac{dy}{dx} = y\cos(xy) - 2xy^4$

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$$\frac{dy}{dx} = \frac{y(\cos(x,y)) - 2xy^3}{4x^3y^3}$$

$$\frac{dy}{dx} = \frac{(\cos(xy) - 2xy^3)}{4x^2y^2}$$