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Multi Calculus
Assignment #1

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{Question No #1}

The Points A, B, C have Position vectors.

$4\hat{i}+4\hat{j}+\hat{k}$, $-4\hat{i}+3\hat{j}-4\hat{k}$, $4\hat{i}-\hat{j}-2\hat{k}$ respectively, relative to the origin O.

(a) Find the equation of the plan ABC, giving your answers in the form $ax+by+c=d$.

Sol

We have: $A(4,-4,1)$

$B(-4,3,-4)$

$C(4,-1,-2)$

$\vec{AB} = \vec{OB} - \vec{OA}$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$= -8\hat{i}+7\hat{j}-5\hat{k}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ +3 \\ -3 \end{bmatrix} \Rightarrow 3\hat{j}-3\hat{k}$$

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$n = \vec{AB} \times \vec{AC}$

$$\Rightarrow \begin{bmatrix} \hat{i} & \hat{j} & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$$

$= i(-4+15) - (24-0)j + (-24)\hat{k}$

$= -6\hat{i} - 24\hat{j} - 24\hat{k}$

$= \hat{i} + 4\hat{j} + 4\hat{k}$

$d = a.n = (4\hat{i} - 4\hat{j} + \hat{k}).(\hat{i} + 4\hat{j} + 4\hat{k})$

$= 4 - 16 + 4 = -8$

Equation of plane

$r.n = d$

$r(\hat{i} + 4j + 4k) = -8$

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 4\hat{k}) = -8$

$x + 4y + 4z = -8$

$x + 4y + 4z + 8 = 0$

b) Find the perpendicular distance from O to the plane ABC

Perp distance $= \frac{d}{(n)} = \frac{+8}{\sqrt{(1)^2 + (4)^2 + (4)^2}}$

$= \frac{8}{3.3} = \boxed{1.39}$ Ans

c) The point D has position vector $2\hat{i} + 3\hat{j} - 3\hat{k}$. Find the coordinates of the point of intersection of the line OD with the plane ABC

Line OD: $r = a + \lambda b$

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$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

as same value of λ

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$$= 2\lambda + \cancel{12\lambda} - \cancel{12\lambda} = -8$$

$$2\lambda = -8$$

$$\boxed{\lambda = -4}$$

$$r = \begin{bmatrix} 2(-4) \\ 2(-4) \\ -3(-4) \end{bmatrix} = (-8, -12, 12)$$

{ Question No # 3 }

Let t be a positive constant. The line L_1 passes through...

a) Find the value of t.

Sol

$L_1 = t\hat{i} + \hat{j}$ $\quad - 2\hat{i} - \hat{j}$

$L_2 = \hat{j} + t\hat{k}$ $\quad - 2\hat{j} + \hat{k}$

The shortest distance between L_1 and L_2 is $\sqrt{21}$

$r_1 = OA + \lambda AB$

$r_2 = OA + \lambda AB$

$r_1 = ti + \hat{j} + \lambda 2\hat{i} - \hat{j}$

$$\vec{L_1} = r_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -0 \\ 0 \end{bmatrix}$$

$$\vec{L_2} = r_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ -2 \\ t \end{bmatrix}$$

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$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(-2) + \hat{k}(4)$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1+4+16}$$

$$= \sqrt{21}$$

$$(a_2 - a_1) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}$$

$$= -t\hat{i} + t\hat{k}$$

b)

$$D = \frac{(-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})}{\sqrt{21}}$$

$$\sqrt{21} = t + 4 / \sqrt{21}$$

$$21 = t + 4t$$

$$21 = 5t$$

$$\boxed{t = \frac{21}{5}}$$

c) Part b

$$\vec{r_1} = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$$

$$\vec{r_2} = \hat{j} - \frac{21}{5}\hat{k} + 4(-2\hat{j} + \hat{k})$$

$$\pi_1 \Rightarrow r = a + \lambda \vec{AB} + \mu AC$$

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$$\vec{r} = -\frac{21}{5}\hat{i} + \hat{j} + \lambda(-2\hat{i} - \hat{j}) + \mu(-2\hat{j} + \hat{k})$$

Part C:

$\lambda_2 = 5x - 6y + 5z = 0$

$\lambda_2 = \frac{x-0}{0}$, $\lambda_2 = \frac{y-1}{-2}$

$\lambda_2 = \frac{z-4.2}{1}$

From L_2 direction vector is

$= \{0, -2, 1\}$

From Π_2 normal vector is

$= (5, -6, 7)$

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|a||b|}$$

$$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ -\frac{21}{5} \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$= -6 + 29.4 = 23.4$

$|a| = \sqrt{(1)^2 + (21/5)^2} = \sqrt{1 + 17.64}$

$\boxed{|a| = 4.3}$

$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2}$

$|b| = 10.49$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$$\theta = \cos^{-1}\frac{23.4}{45.11}$$

$\boxed{\theta = 59.34^\circ}$

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Part d

$n_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}$, $n_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$n_1 \cdot n_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$= -21-6 = -27$

$|a| = \sqrt{\left(\frac{-21}{5}\right)^2 + (1)^2} = \sqrt{17.64+1} = 4.3$

$|b| = \sqrt{5^2+(-6)^2+7^2} = 10.49$

$\theta = \cos^{-1} \frac{-27}{4.3 \times 10.49}$, $\theta = 126.78$

acute angle :-

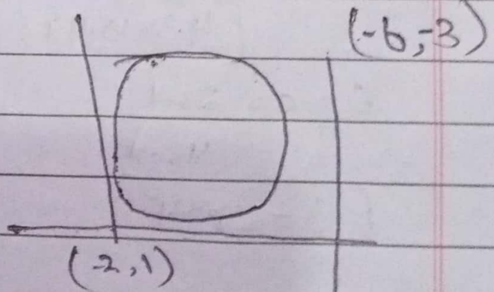
$\phi = 180 - 126.78$

$\boxed{\phi = 53.23^\circ}$ Ans.

{Question No # 05}

Part (a)

If $P=(-2,-1)$ and $Q=(-6,-3)$ are two end point of diameter of a circle, find the equation of the circle.



Mid point

$\left(\frac{-2-6}{2}\right) \left(\frac{-1-3}{2}\right)$

$\frac{-8}{2}, \frac{-4}{2}$

$(-4,-2)$

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Equation of circle $= (x+h)^2 + (y-k)^2 = r^2 - (1)$

$(x+4)^2 + (y+2)^2 = r^2$

$(x,y) = (-2,-1)$

$(-2+4)^2 + (-1+2)^2 = r^2$

$(2)^2 + (1)^2 = r^2$

$4+1 = r^2$

$5 = r^2$

$r^2 = 5$

Put the values

$(x+4)^2 + (y+2)^2 = 5$

Part (b)

If the circle pass through (4,0) and (0,2) and center at y-axis then find the radius of the circle.

Sol

Equation of circle

$(x-h)^2 + (y-k)^2 = r^2$

Let $x=0$, $y=b$

at point (4,0)

$(4)^2 + (-b)^2 = r^2$

$16 + b^2 = r^2 - (i)$

at point (0,2)

$(0)^2 + (2-b)^2 = r^2$

$(2-b)^2 = r^2 - (ii)$

Compare eq (i) and eq (ii)

$16 + b^2 = (2-b)^2$

$16 + b^2 = 4 - 4b + b^2$

$16 \cancel{+b^2} \cancel{-b^2} - 4 + 4b = 0$

$12 + 4b = 0$

$4b = -12$

$\boxed{b = -3}$

Put the value in eq (1)

So, $r^2 = (4)^2 + (-3)^2$

$r^2 = 16+9$

$r = \pm 5$

neglect -ve

$\boxed{r = 5}$

Part C :-

Find the equation of directive of parabola

$y^2 = 100x$

$y^2 = 100x$

Compare with

$y^2 = 4ax$

$4a = 100$

$\boxed{a = 25}$

equation of directrix = $x = -9$

$x = -25$

Part (d) :-

Find the equation of the axis of the Parabola $x^2 = 24y$

$x^2 = 24y$ Compare with $x^2 = 4ay$

$4a = 24$

$a = 8$

So, focus is $f(a,0) = f(0,0)$ and equation of directrix is

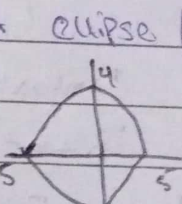
$x = -a \Rightarrow \boxed{x = -8}$

Part (e) :-

What is the major axis length for ellipse $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{4}\right)^2 =$

Compare with

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$



$a = 5, b = 4, c = \sqrt{a^2 - b^2} = \sqrt{25-16} = \pm 13$

$f_1 = (3,0)$ $f_2 = (-3,0)$

length of Major axis $= 2a$

$2(5) = 10$

Q2)

$A = (7i + 4j - k)$, $B(11i + 3j)$, $C(2i + 6j + 3k)$

$D(2i + 7j + \lambda k)$.

(a) $AB = +4i - 1j + k$.

$CD = 0i - j + (3 - \lambda)k$

l_1 : Line $AB = OA + \lambda AB$

l_2 : Line $CD = OC + \mu CD$.

$$l_1 : \vec{r}_1 := \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}.$$

$$l_2 : \vec{r}_2 = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -1 \\ 3 - \lambda \end{pmatrix}$$

(b) Let π_1 be plane ABD when $\lambda = 1$.
Let π_2 be plane ABD when $\lambda = 4$.

i Write equ of π_1 in form $r = a + sb + tc$.

ii Write equ of π_2 in form $ax + by + cz = d$.

Plane ABD when $\lambda = 1$.

so $OD = 8i + 7j + k$

$AB \times AD$.

$$= \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}$$

$$= i(-2-3) - j(8+5) + k(12-5).$$

$$= -5i - 13j + 7k$$

Equ: $-5(2) - 13(7)s + 7(1)t$

$= -10 - 91s + 7t.$

For Plane π_2, when $\lambda = 4$.

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 5 \end{vmatrix}.$$

$$= i(-5-3) - j(20-5) + k(12+5).$$

$$= -8i - 15j + 17k.$$

Equ $= -8(x-11) - 15(y-3) + 17(z-0).$

$= -8x + 88 - 15y + 45 + 17z$

$= 8x + 15y - 17z - 133$

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$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$b_1 \times b_2 = i(-3+\lambda) - j(12-4\lambda) + k(-4).$$

$$|b_1 \times b_2| = \sqrt{\lambda^2 + 9 - 2\lambda + 144 + 16\lambda^2 - 24\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 26\lambda + 169}$$

$$a_2 - a_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$9(17\lambda^2 - 26\lambda + 169) = (-15 + 5\lambda + 24 - 8\lambda - 8)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda + 24 - 23)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda - 1)^2$$

$$= 49\lambda^2 + 1 - 24\lambda$$

$$\lambda =$$

Then $\lambda^2 - 5\lambda + 4 = 0$