Design a Turing machine that accepts the language denoted by RE 00^* on $\Sigma = \{0,1\}$.

Ex: 000

...	B	0	0	0	B	...
		$\uparrow$			$\uparrow$	
		q_0			Blank	

Tape

$B q_0 000B$
$\longrightarrow q_0, 0, R$

$B 0 q_0 00 B$
$\longrightarrow q_0, 0, R$

$B00 q_0 0 B$
$\longrightarrow q_0, 0, R$

$B000 q_0 B$
$\longrightarrow q_1, B, N$

Transition Table

Q \ Σ	0	1	B
q_0	$q_0, 0, R$	error	q_1, B, N
q_1	error	error	Accept

Design a TM that copies string of 1's.

I/p: 11 o/p: 1111

Ex:

...	B	1	1	1	B	-	...

$\uparrow q_0$

$B\, q_0\, 111\, B$ $\xrightarrow{q_0, X, R}$

$B\, X\, q_0\, 11\, B$ $\xrightarrow{q_0, X, R}$

$B\, XX\, q_0\, 1\, B$ $\xrightarrow{q_0, X, R}$

$B\, XXX\, q_0\, B$ q_1, B, L

$B\, XXX\, q_1\, B$ $q_2, 1, R$

$B\, XX1\, q_2\, B$ $q_1, 1, L$

$B\, XX1\, q_1\, 1B$ $\xleftarrow{q_1, 1, L}$

$B\, XX\, q_1\, 11B$ $q_2, 1, R$

$B\, X1\, q_2\, 11B$ $\xrightarrow{q_2, 1, R}$

$B\, X11\, q_2\, 1B$ $\xrightarrow{q_2, 1, R}$

$B\, X111\, q_2\, B$ $q_1, 1, L$

$B\, X111\, q_1\, 1B$ $\xleftarrow{q_1, 1, L}$

$B\, X11\, q_1\, 11B$ $\xleftarrow{q_1, 1, L}$

$B\, X1\, q_1\, 111B$ $\xleftarrow{q_1, 1, L}$

$B\, X\, q_1\, 1111B$ $q_2, 1, R$

$B1\, q_2\, 1111B$ $\xrightarrow{q_2, 1, R}$

$B11\, q_2\, 111B$ $\xrightarrow{q_2, 1, R}$

$B111\, q_2\, 11B$ $\xrightarrow{q_2, 1, R}$

$B1111\, q_2\, 1B$ $\xrightarrow{q_2, 1, R}$

$B11111\, q_2\, B$ $q_1, 1, L$

$B11111\, q_1\, 1B$ $\xleftarrow{q_1, 1, L}$

$B1111\, q_1\, 11B$ $\xleftarrow{q_1, 1, L}$

$B111\, q_1\, 111B$ $\xleftarrow{q_1, 1, L}$

$B11\, q_1\, 1111B$ $\xleftarrow{q_1, 1, L}$

$B1\, q_1\, 11111B$ $\xleftarrow{q_1, 1, L}$

$B\, q_1\, 111111\, B$ q_3, B, R

$B\, q_3\, 111111\, B$ $\xrightarrow{q_3, 1, R}$

$B\, 111111\, q_3\, B$

Accept

Transition Table

Q \ Γ	1	X	B
q_0	q_0, X, R	—	q_1, B, L
q_1	q_1, 1, L	q_2, 1, R	q_3, B, R
q_2	q_2, 1, R	—	q_1, 1, L
q_3	q_3, 1, R	--	Accept

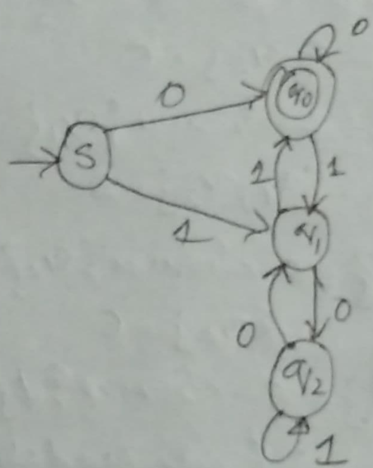
Design a DFA divisible by '3'.

10010 ~~10101~~

. 00
01
10
11
100
101
110
111
1000
1001
1010
1011
1100
1101
1110
1111

$$\begin{array}{r|ll} 2 & 21 & \\ 2 & 10 & 1 \\ 2 & 5 & 0 \\ 2 & 2 & 1 \\ & 1 & 0 \end{array}$$

10101



State	0 I/P	1
S	q_0	q_1
q_1	q_0	q_1
q_2	q_2	q_0
q_3	q_1	q_2

Let $q_0 \Rightarrow$ remainder 0
$q_1 \Rightarrow$ remainder 1
$q_2 \Rightarrow$ remainder 2

q_0 : 2×0 (rem 0) + 0 (next bit)
$= 0 + 0 = q_0$

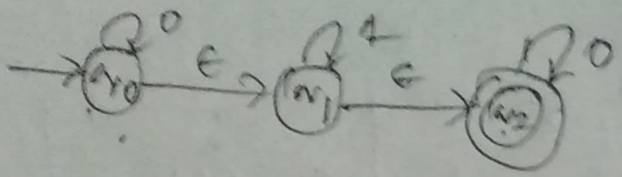
q_0 : 2×0 (rem 0) + 1 (next bit)
$= \cancel{0}\ 0 + \cancel{0}1 = \cancel{2}\ 1 \Rightarrow q_1$

q_1 : $2 \times \cancel{0}1$ (rem 1) + 0 (next bit)
$= 2 + 0 \Rightarrow q_2$

q_1 : 2×1 (rem 1) + 1 (next bit)
$= 2 + 1 = 3$ (divisible by 3) $\Rightarrow q_0$

q_2 : $2 \times \cancel{2}\,2$ (rem 2) + 0 (next bit)
$= 4 + 0 = 4 \% 3 = 1 \Rightarrow q_1$

q_2 : 2×2 (rem 2) + 1 (next bit)
$= 4 + 1 = 5 \% 3 = 2 \Rightarrow q_2$



Start state:

ϵ-closure$(q_0) = \{q_0, q_1, q_2\} = A$

~~(A, 0)~~

ϵ-closure$(A, 0) = \epsilon$-closure$\{q_0, q_2\}$

$= \{q_0, q_1, q_2\} = A$

ϵ-closure$(\text{move}(A, 1)) = \epsilon$-closure$\{q_1\}$

$= \{q_1, q_2\} = B$

ϵ-closure$(\text{move}(B, 0)) = \epsilon$-closure$\{q_2\}$

$= \{q_2\} = C$

ϵ-closure$\{\text{move}(B, 1)\} = \epsilon$-closure$\{q_1\}$

$= \{q_1, q_2\} = B$

ϵ-closure$\{\text{move}(C, 0)\} = \epsilon$-closure$\{q_2\}$

$= \{q_2\} = C$

ϵ-closure$\{\text{move}(C, 1)\} = \epsilon$-closure$\{\phi\}$

$= \phi \Rightarrow$ Dead state

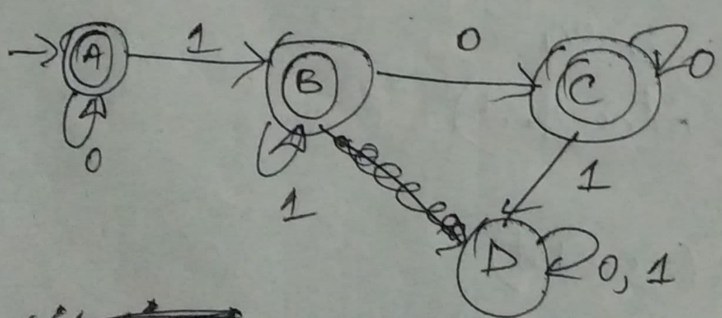
$= D$

State	I/P 0	1
$\rightarrow$ A	A	B
B	C	B
C	C	D
D	D	D

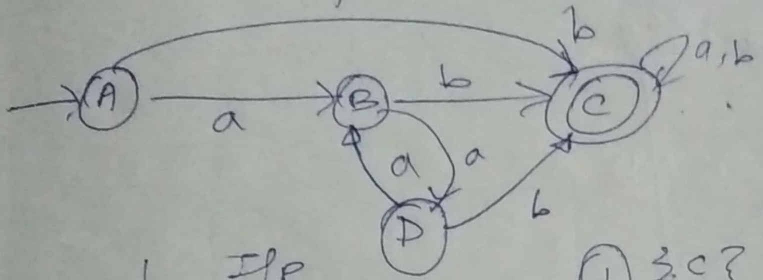
Final states

A, B, C

as they contain q_2



Minimization of DFA

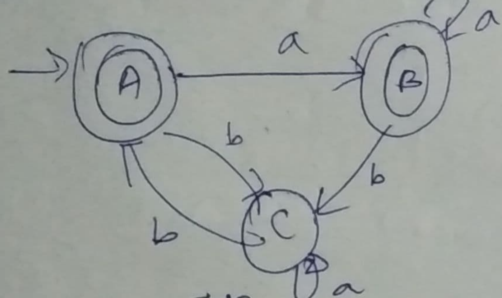


states	I/P a	b
A	B	C
B	D	C
C	C	C
D	B	C

① {C} {A, B, D}

② A & D are equivalent

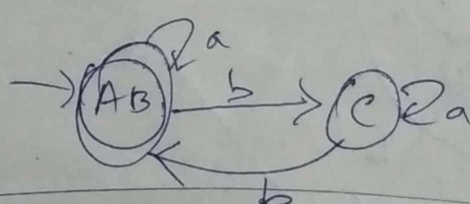
{C} {B} {A D}

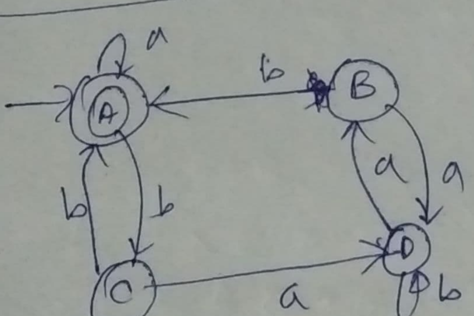


State	I/P a	b
A	B	C
B	B	C
C	C	A

State	I/P a	b
AB	AB	C
C	C	AB

{A, B} {C}



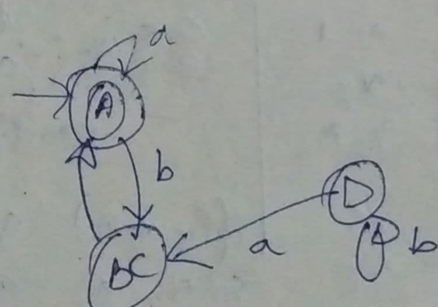


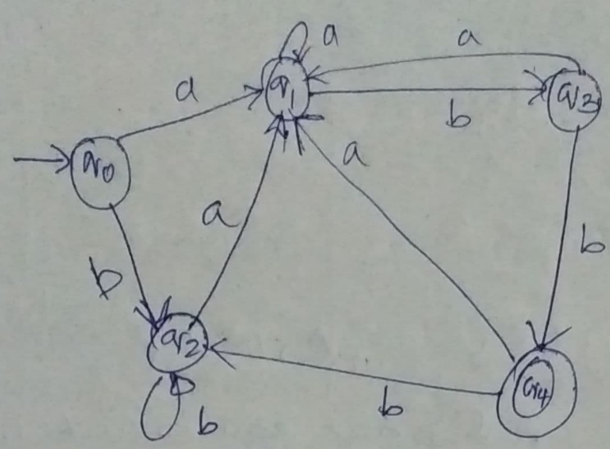
States	I/P a	b
A	A	C
B	D	A
C	D	A
D	B	D

① {A} {B C D}

② B & C are equivalent

{A} {D} {BC}



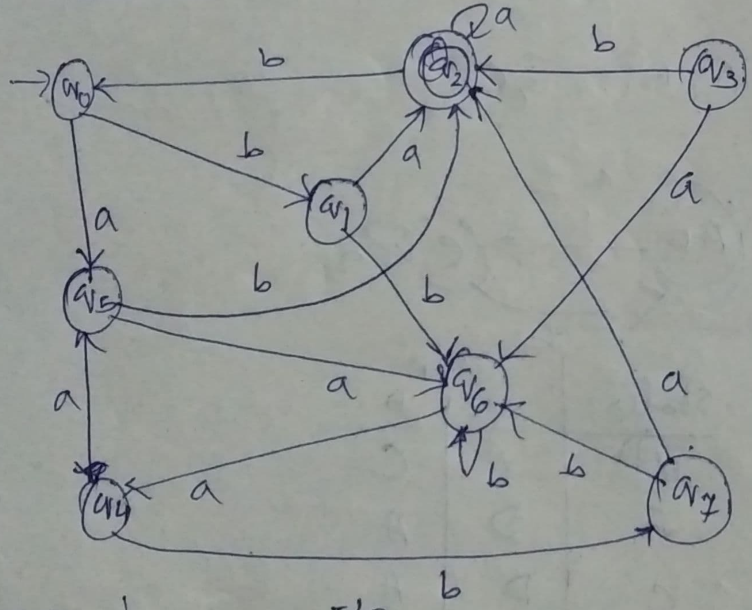


States	I/P a	b
→ q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
q_4	q_1	q_2

① $\{q_4\}\ \{q_0\ q_1\ q_2\ q_3\}$

② $\{q_4\}\ \{q_3\}\ \{q_0\ q_1\ q_2\}$

③ $\{q_4\}\ \{q_3\}\ \{q_1\}\ \{q_0\ q_2\}$



State	I/P a	b
→ q_0	q_5	q_1
q_1	q_2^*	q_6
q_2^*	q_2^*	q_0
q_3	q_6	q_2^*
q_4	q_5	q_7
q_5	q_6	q_2^*
q_6	q_4	q_6
q_7	q_2^*	q_6

① $\{q_2\}\ \{q_0\ q_1\ q_3\ q_4\ q_5\ q_6\ q_7\}$

② $\{q_2\}\ \{q_1\ q_3\ q_5\ q_7\}\ \{q_0\ q_4\ q_6\}$

③ $\{q_2\}\ \{q_1\ q_7\}\ \{q_3\ q_5\}\ \{q_0\ q_4\}\ \{q_6\}$

same states as o/p

$\delta(q_0, a)$ q_5

$\delta(q_4, a)$ q_5 q_5 & q_4

$\delta(q_6, a)$ q_4 not in same state

I/P

States	a	b
$q_0\ q_4$	$q_3\ q_5$	$q_1\ q_7$
$q_1\ q_7$	q_2^*	q_6
q_2^*	q_2^*	$q_0\ q_4$
$q_3\ q_5$	q_6	q_3^*
q_6	$q_0\ q_4$	q_6

Regular Expression - Pumping Lemma

$L = \{a^{2n} \mid n > 0\}$

$w = xy^iz$ $\Rightarrow$ $w = \Lambda(aa)^i\Lambda$

$y = aa$ $\quad w = (aa)^i$

$x = \Lambda$

$z = \Lambda$

$i = 1,\ n = aa$ in L

$i = 2,\ n = aaaa$ in L

$i = 3,\ n = aaaaaa$ in L

$\vdots$

$\therefore L = \{a^{2n} \mid n > 0\}$ is regular.

$L = \{a^{n^2} \mid n \geq 0\}$

$L = \{\Lambda, a, aaaa, \ldots\}$

$y = a$ $[\because y \neq \Lambda]$

$x = \Lambda$

$z = \Lambda$

$w = \Lambda(a)^i\Lambda$

$i = 0$ $\quad w = \Lambda$ $\quad$ in L

$i = 1$ $\quad w = a$ $\quad$ in L

$i = 2$ $\quad w = aa$ $\quad$ not in L $\qquad$ Not regular

$L = \{a^p \mid p$ is a prime$\}$

Prime No-s are 2, 3, 5, 7, 11, 13

$L = \{aa, aaa, aaaaa, \ldots\}$

$y = aa\ (\because y \neq \Lambda)$ $\qquad$ $i = 1$ $\quad w = aa$ $\quad$ in L

$x = \Lambda,\ z = \Lambda$ $\qquad$ $i = 2$ $\quad w = aaaa$ $\quad$ not in L

$w = \Lambda(aa)^i\Lambda$

$\therefore$ not regular

$L = \{0^i 1^i \mid i \geq 1\}$ is not regular.

$L = \{01, 0011, 000111, \ldots\}$

Let, $y = 1$ $\quad \therefore w = 0(1)^i \Lambda$

$x = 0$

$z = \Lambda$ $\quad i = 1 \quad w = 01$ in L

$i = 2 \quad w = 011$ not in L

Let, $y = 0$ $\quad w = (0)^i 1$

$x = \Lambda$

$z = 1$ $\quad i = 1 \quad w = 01$ in L

$i = 2 \quad w = 001$ not in L

Let, $y = 01$ $\quad w = \Lambda(01)^i \Lambda$

$x = \Lambda$ $\quad w = 01$ in L

$z = \Lambda$ $\quad i = 1$

$i = 2 \quad w = 0101$ not in L

$\therefore$ Not Regular

$L = \{a^n b^n c^n \mid n \geq 1\}$

$L' = \{ww \mid w \in (a, b)^*\}$