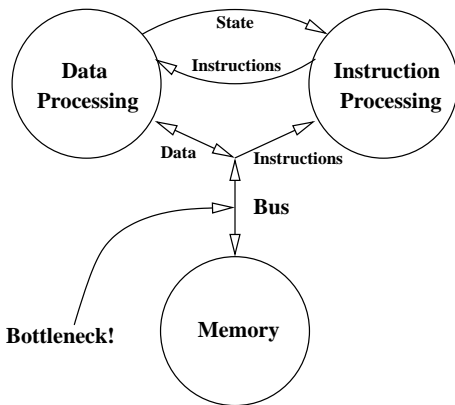
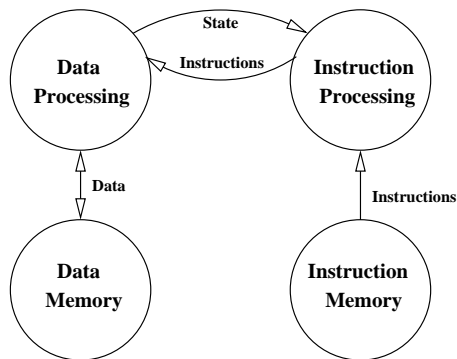


# Von Neumann Architecture



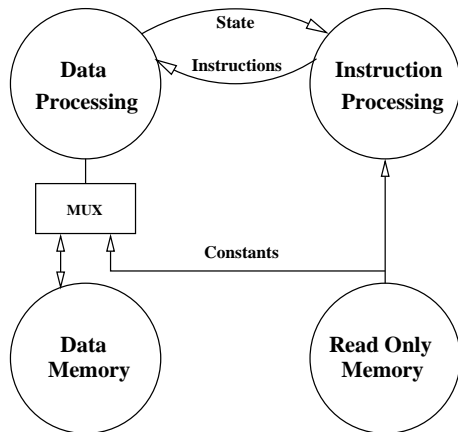
- A common bus is used for data as well as instructions.
- The system can become 'bus bound'.

# Harvard Architecture



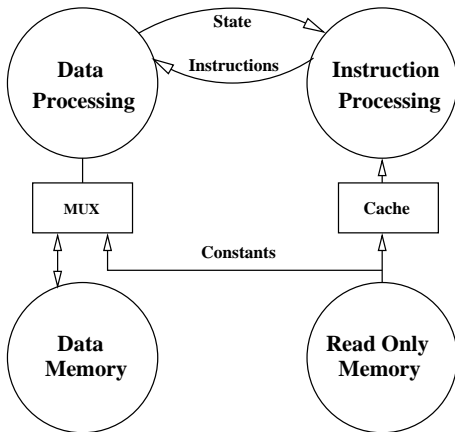
- Separate data and instruction paths
- Good performance
- Needs 2 buses → expensive!
- Traffic on the buses is not balanced.
- Instruction bus may remain idle.

# Modified Harvard Architecture



- Constants can be stored with Instructions in ROM.
- Better Bus balancing is possible.
- Typically, 1 instruction read, 1 constant read, 1 data read and 1 result write per instruction.
- 2 mem ops per bus.

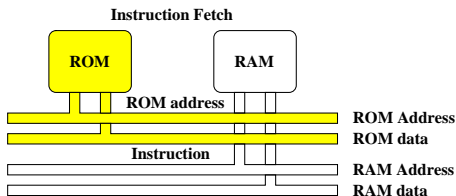
# Modified Harvard with Cache



- Cache allows optimum utilization of bus bandwidths.
- Each operation need not be balanced individually.



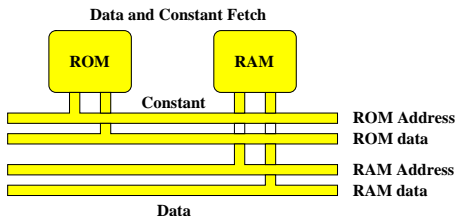
# A pipelined processor



Consider a Harvard architecture processor, which performs the following tasks repetitively:

## Fetch Op Code (ROM)

# A pipelined processor



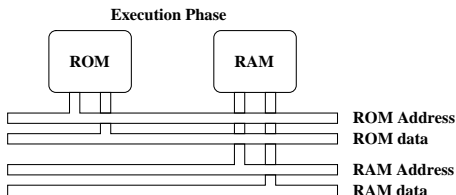
Consider a Harvard architecture processor, which performs the following tasks repetitively:

Fetch Op Code (ROM)

**Fetch variable (RAM)**

**Fetch constant (ROM)**

# A pipelined processor



Consider a Harvard architecture processor, which performs the following tasks repetitively:

Fetch Op Code (ROM)

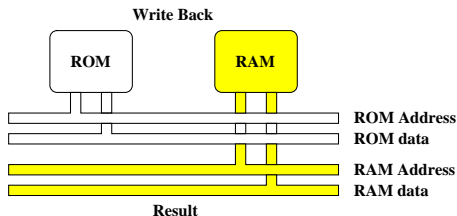
Fetch variable (RAM)

Fetch constant (ROM)

**Calculate result**



# A pipelined processor



Consider a Harvard architecture processor, which performs the following tasks repetitively:

Fetch Op Code (ROM)

Fetch variable (RAM)

Fetch constant (ROM)

Calculate result

**Store result (RAM)**

# Resource Reservation

We can keep track of which resource is doing what at any given time by a table as shown below:

Resource Reservation Table

	0	1	2	3	4
ROM	Instr Fetch	Const. fetch			
RAM		Var. Fetch		Write Back	
ALU			Compute		

This is called a reservation table.

Given this reservation table, It appears that we can launch a new instruction every 4 cycles.

# Overlapping Operations

However, we need not wait for the previous operation to be over before launching a new one.

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0									
RAM		0		0							
ALU			0								

When can we launch the next calculation?

# Pipelining

We can fetch the next instruction from ROM while we write back the result of the current one to the RAM.

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0		1	1		2	2			
RAM		0		0	1		1	2		2	
ALU			0			1			2		

This will enable us to launch a new calculation every third cycle.

# Overlapping Operations

Is this the best we can do?

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0		1	1		2	2			
RAM		0		0	1		1	2		2	
ALU			0			1			2		

None of the resources are utilized 100% in this scheme.  
The ROM and the RAM are busy for 2 out of 3 cycles, whereas the ALU is used for 1 cycle out of 3.

A new sample is handled every 3rd cycle now.

Can we get even better throughput?

# Improved Scheduling

If we store the result in a local register for 1 cycle, and write it to the RAM only in the 4th cycle, we get

Modified Resource Reservation Table

	0	1	2	3	4	5	6
ROM	0	0					
RAM		0			0		
ALU			0				
BUF				0			

By delaying the write back,  
we can launch the next instruction earlier!

# Improved Scheduling

If we store the result in a local register for 1 cycle, and write it to the RAM only in the 4th cycle, we get

Modified Resource Reservation Table

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0	1	1	2	2	3	3	4	4	5
RAM		0		1	0	2	1	3	2	4	3
ALU			0		1		2		3		4
BUF				0		1		2		3	

We can now launch a new operation every 2nd cycle.

Can this be further improved?

# Improved Scheduling

If we store the result in a local register for 1 cycle, and write it to the RAM only in the 4th cycle, we get

Modified Resource Reservation Table

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0	1	1	2	2	3	3	4	4	5
RAM		0		1	0	2	1	3	2	4	3
ALU			0		1		2		3		4
BUF				0		1		2		3	

The RAM and the ROM are now occupied 100% of the time, So the design is optimal and the throughput cannot be improved any further.



# How can we always find the optimum solution?

- Given a Resource Reservation Table, we would like to set up a **systematic** method which optimizes the throughput of the process using this table.
- For maximum throughput, we would like to launch new operations as frequently as possible.
- Thus, we want to minimize the time gap between launching two operations.
- This is called the **Sample Period** (SP).

What is the minimum possible value of SP?

# The minimum Sampling Period

- Consider an operation in which the busiest resource is used for  $n$  cycles.
- If we launch a new operation every  $n$  cycles, this resource will be used 100% of the time.
- If we launch operations any more frequently than this, the resource will not have enough time to do its work.
- Therefore, the minimum possible Sample Period is equal to the maximum number of cycles for which the busiest of the resource(s) is in operation.

# Sampling Period

- We want to minimize the sampling period.
- But the sampling period need not be a constant!
- SP can cycle through a finite set of values.
- We should therefore define an **Average** Sampling period ASP.
- The minimum value of this **average** Sampling Period (MASP) is given by the number of cycles for which the busiest resource is used in an operation.

# Cyclic Sampling Period

Consider the following reservation table:

	0	1	2	3	4	5	6	7	8
RSC1	0		0						
RSC2		0		0					
RSC3			0						

Now the next operation can be launched in cycle 1 itself. However, the following one can only be launched after a gap of 3 cycles in cycle 4.

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	1	0	1	2	3	2	3	4	5	4
RAM		0	1	0	1	2	3	2	3	4	5
ALU			0	1			2	3			4

Again, the next operation can be launched in the next cycle (in cycle 5) and after that, with a gap of 3 cycles in cycle 8.

# Average Sampling Period

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	1	0	1	2	3	2	3	4	5	4
RAM		0	1	0	1	2	3	2	3	4	5
ALU			0	1			2	3			4

- New operations can be launched in clock periods 0,1,4,5,8,9 ....
- Thus, the sample period cycles through the values {1,3}.
- The average of the cycle is called the Average Sampling Period (ASP).
- The Average Sampling period (ASP) is 2 here.
- The whole pattern repeats every 4 cycles. This is called the period (p).

# Minimum Average Sampling Period

- The minimum value of the Average Sampling Period (MASP) is given by the maximum number of cycles for which a resource is busy during an operation.
- Therefore, given a reservation table, MASP is known.
- If the actual average Sampling Period is equal to MASP, the system is already optimum and nothing needs to be done.
- If the actual average Sampling Period is greater than MASP, we can attempt to modify the reservation table, such that MASP is achieved.

# Pipeline Optimization

- 1 For a given reservation table, find the current average sample period (ASP).
- 2 Find the largest no. of cycles for which a resource is busy.
- 3 This is equal to the Minimum possible Average Sampling Time (MASP).
- 4 If  $ASP = MASP$ , there is nothing to be done.
- 5 Else, we should try to re-schedule events such that MASP is achieved.

# Method to achieve MASP

- We first consider various cycles whose average is the desired MASP.
- For example, if MASP is 2, we can have cycles of  $\{2\}$ ,  $\{1,3\}$  or  $\{1,1,4\}$  etc.
- The periods are 2, 4 and 6 in these three cases.



# The Generator Set

- For each cycle, we construct a generator set  $G$ , which contains elements of the cycle, their sums taken two at a time, three at a time etc., modulo periodicity  $p$ .
- In our example, cycles are  $\{2\}$ ,  $\{1,3\}$  and  $\{1,1,4\}$ 
  - For a cycle of  $\{2\}$ ,  $p = 2$ , so  $G = \{0\}$
  - For a cycle of  $\{1,3\}$ ,  $p = 4$ , so  $G = \{0,1,3\}$
  - For a cycle of  $\{1,1,4\}$ ,  $p = 6$ , so  $G = \{0,1,2,4,5\}$

# The Source Set

- For each selected cycle, We now construct the Source set S. This contains integers 0 through p-1, from which all members of G except 0 have been removed.
- In our example, cycles are {2}, {1,3} and {1,1,4}

Cycle	p	G	S
{2},	2	{0}	{0,1}
{1,3},	4	{0,1,3}	{0,2}
{1,1,4},	6	{0,1,2,4,5}	{0,3}

# Design Sets

- For each selected cycle, We construct Design sets  $D_i$  which have the property that:

if  $a \in D$  and  $b \in D$

then  $|a - b|$  also  $\in D$ .

- In our example,

Cycle	p	S	D sets
{2},	2	{0,1}	{0}, {1} and {0,1}
{1,3},	4	{0,2}	{0}, {2}, {0,2}
{1,1,4},	6	{0,3}	{0}, {3}, {0,3}

- Notice that Design sets do not depend on the reservation table.
- The sets  $G$ ,  $S$  and  $D_i$  are constructed from the repetition cycles whose average value is the MASP.
- Therefore we can make a library of these in advance for different combinations of MASP values and cycles - and use them when needed.

# Row Vectors

- We construct a row vector for each resource in the reservation table.
- The row vector is a set which contains the clock period in which a specific resource is busy.

Resource Reservation Table

	0	1	2	3
ROM	0	0		
RAM		0		0
ALU			0	

In this example, the row vector for ROM is {0,1}, for RAM is {1,3} and for ALU is {2}.

# Matching Rows with Design Sets

- Choose a particular cycle with the desired MASP.  
(Say  $MA SP = 2$ , cycle =  $\{2\}$ ).
- Pick the corresponding design sets.  
(In this example,  $D = \{0\}, \{1\}, \{0,1\}$ ).
- For each resource,  
take its row vector and take a design set with the same cardinality.
- Align these according to defined rules.

# Rules for Alignment of the First elements

- Compare  $R(1)$  and  $D(1)$ .  
If these are equal, nothing needs to be done.
- Else,
  - If  $R(1) < D(1)$ , add  $D(1)-R(1)$  to all members of  $R$
  - If  $R(1) > D(1)$ , add  $R(1)-D(1)$  to all members of  $D$
- This is equivalent to a rigid shift of  $R$  or  $D$  till their first members are aligned.

For Example, if  $R = \{1,3,4,6\}$  and  $D=\{0,2,5,6\}$

	0	1	2	3	4	5	6	7	
R		X		X	X		X		1,3,4,6
D	X		X			X	X		0,2,5,6

→

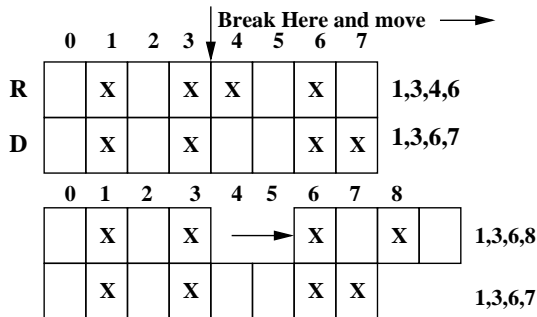
	0	1	2	3	4	5	6	7	
R		X		X	X		X		1,3,4,6
D		X		X			X	X	1,3,6,7

# Alignment of other elements

If  $R(i) = D(i)$  Nothing needs to be done.

If  $R(i) < D(i)$

Add  $D(i) - R(i)$  delays to all members of R at position  $i$  and beyond.



The  $i$ 'th elements are now aligned.



# Alignment of other elements

If  $D(i) < R(i)$

(for Example,  $p = 2$

$R = \{1,3,4,6\}$ ,  $D = \{1,2,5,6\}$ ).

Now  $D_2 < R_2$ )

- 1 Add sufficient multiples of  $p$  to  $D(i)$  such that it is  $\geq R(i)$ .
- 2 Add the same number to members of  $D$  beyond  $i$ .
- 3 Now if  $R(i) < D(i)$ , add  $D(i) - R(i)$  delays to all members of  $R$  at position  $i$  and beyond.

Periodicity  $p = 2$

	0	1	2	3	4	5	6	7	
R		X		X	X		X		1,3,4,6
D		X	X			X	X		1,2,5,6

↑ Break here and  
move forward by  $p (=2)$  steps

	0	1	2	3	4	5	6	7	8	9	
R		X		X	X		X				1,3,4,6
D		X			X			X	X		1,4,7,8

Now align R

	0	1	2	3	4	5	6	7	8	9	
R		X			X	X		X			1,4,5,7
D		X			X			X	X		1,4,7,8

# Alignment Example

Let  $R = 1,3,4,6$  and  $D = 0,1,4,5$ ; with periodicity  $p = 2$

	0	1	2	3	4	5	6	7	8
R		X		X	X		X		
D	X	X			X	X			
		X	X			X	X		

To align the first element, move all elements of D forward by 1 step.  
Now  $D = 1,2,5,6$ .

	0	1	2	3	4	5	6	7	8
R		X		X	X		X		
D		X	X			X	X		
		X			X			X	X
R		X			X	X		X	

For the second element, D is behind. Move D2 onwards fwd by  $p = 2$ , so  $D = 1,4,7,8$ .  
Move R2 onwards fwd by 1  
So  $R = 1,4,5,7$

# Alignment Example

$R = 1,4,5,7$  and  $D = 1,4,7,8$ .  $R3 < D3$

	0	1	2	3	4	5	6	7	8	9	10
D		X			X			X	X		
R		X			X	X		X			
		X			X			X		X	

Move R3 and beyond forward by 2

So  $R = 1,4,7,9$   
and  $D = 1,4,7,8$ .

	0	1	2	3	4	5	6	7	8	9	10
R		X			X			X		X	
D		X			X			X	X		
D		X			X			X			X
R		X			X			X			X

$D4 < R4$

Move D4 forward by 2 to 10.

Now  $R4 < D4$ .

Move R4 forward by 1 to 10

Vectors are now aligned at 1,4,7,10.

# Example System

we shall illustrate the method using our original example, whose reservation table is:

Resource Reservation Table

	0	1	2	3	4	5	6
ROM	0	0					
RAM		0		0			
ALU			0				

Since the ROM and the RAM are used for 2 cycles each in every operation,  $MASP = 2$ .

However, as we had seen before,  $ASP = 3$  in this case. Therefore, the schedule needs improvement.

# Example Application

## Aligning the ROM

	0	1	2	3
ROM	0	0		
RAM		0		0
ALU			0	

MASP = 2, Choose the cycle:{2}

Then  $D = \{0\}, \{1\}, \{0,1\}$

For ROM:  $R = \{0,1\}, D=\{0,1\}$

So no alignment is required.

# Adjusting the RAM Schedule

For RAM:  $R = \{1,3\}$ ,  $D=\{0,1\}$

Aligning the First Element:	
$R(1) > D(1)$ Add $(1-0)=1$ to D elements	$\Rightarrow D = \{1,2\}$
Aligning other elements:	
$R(2) > D(2)$ Add $p (=2)$ to $D(2)$	$\Rightarrow D = \{1, 4\}$
Now $R(2) < D(2)$ Add $(3-2)=1$ to $R(2)$	$\Rightarrow R = \{1, 4\}$
R and D are now aligned.	

# ALU Schedule

For ALU:  $R = \{2\}$ ,  $D = \{0\}$

Aligning first element: Add  $(2-0) = 2$  to  $D \Rightarrow D = \{2\}$   
R and D are now aligned.

ROM =  $\{0,1\}$ , RAM =  $\{1,4\}$ , ALU =  $\{2\}$

Modified Reservation Table

	0	1	2	3	4
ROM	0	0			
RAM		0			0
ALU			0		

As we have seen earlier, this is indeed the optimal schedule with  $ASP = 2$ .

# Optimized Reservation Table

Modified Resource Reservation Table

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0	1	1	2	2	3	3	4	4	5
RAM		0		1	0	2	1	3	2	4	3
ALU			0		1		2		3		4

- The ALU is idle 50% of the time.
- Rather than buffering its result to delay the write back, we can use a slower ALU which takes 2 cycles to compute.



# Using a Slower ALU

The reservation table with a slower ALU is:

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	0	1	1	2	2	3	3	4	4	5
RAM		0		1	0	2	1	3	2	4	3
ALU			0	0	1	1	2	2	3	3	4

- One can trade off power for speed when designing the ALU.
- By using optimization techniques, we are able to reach a higher throughput, even with a slower ALU!

# Alternative Choice of Cycle

	0	1	2	3
ROM	0	0		
RAM		0		0
ALU			0	

MASP = 2, Choose the cycle: {1,3}

Then  $D = \{0\}, \{2\}, \{0,2\}$

For ROM:  $R = \{0,1\}$ ,  $D = \{0,2\}$

$R(1) = D(1) = 0$ ,  $R(2) < D(2)$

Add  $D(2) - R(2)$  to all members of  $R$  at position 2 (and beyond)

$\Rightarrow R(2) = 2$ .

$R$  and  $D$  are now aligned at  $\{0,2\}$

# Alternative Cycle:RAM Schedule

For RAM:  $R = \{1,3\}$ ,  $D=\{0,2\}$

$R(1) > D(1)$

Add  $(1-0)=1$  to D elements:  $\Rightarrow D = \{1,3\}$

R and D are now aligned at  $\{1,3\}$ .

For ALU:  $R = \{2\}$ ,  $D = \{0\}$

Aligning first element: Add  $(2-0) = 2$  to D  $\Rightarrow D = \{2\}$

R and D are now aligned at  $\{2\}$ .

	0	1	2	3
ROM	0		0	
RAM		0		0
ALU			0	

# Time Ordering

	0	1	2	3	4	5	6	7	8	9	10
ROM	0	1	0	1	2	3	2	3	4	5	4
RAM		0	1	0	1	2	3	2	3	4	5
ALU			0	1			2	3			4

- As expected, the schedule is optimum.
- The sampling rate alternates between 1 and 3.
- **However** this schedule does not preserve time order.
- It asks for computation and constant fetch in the same cycle.
- If we pre-fetch the constant for the *next to next* calculation in this cycle and store it for 4 cycles, it may still work.

# Conclusions

- Pipeline can improve throughput of systems.
- A systematic procedure for optimizing pipeline throughput exists. It can create modified reservation tables which are optimal by delaying some operations.
- However, it does not guarantee that the time order of different operations will be preserved.
- Different cycles with the same Average Sampling Period may have to be tried before an acceptable time order is found.
- The procedure also allows us to identify non-critical components which can then be redesigned to be slower but at lower power consumption.