QSAT $\in$ PSPACE

Let the formulas we use be written $Q_i x_i Q_{i+1} x_{i+1} \ldots Q_n x_n \phi_i(x_i, \ldots, x_n)$.

QSAT-REK(ϕ)

if The first quantifier is $\exists x_i$

  if QSAT-REK($Q_{i+1} \ldots \phi(0, x_{i+1}, \ldots, x_n)$) = 1

  or

  QSAT-REK ($Q_{i+1} \ldots \phi(1, x_{i+1}, \ldots, x_n)$) = 1

    Erase all recursively active memory

    return 1

if The first quantifier is $\forall x_i$

  if QSAT-REK($Q_{i+1} \ldots \phi(0, x_{i+1}, \ldots x_n)$) = 1

  and

  QSAT-REK ($Q_{i+1} \ldots \phi(1, x_{i+1}, \ldots x_n)$) = 1

    Erase all recursively active memory

    return 1

if ϕ does not contain any quantifier

  Compute the value of ϕ and return it

When we have a formula with k variables we use $p(k)$ bits of memory for each variable. This shows that $p(n) + p(n-1) + \ldots p(1) \leq np(n)$ bits of memory are used and this shows that QSAT $\in$ PSPACE.

NSPACE

A non-deterministic algorithm decides a language L if

- $A(x) =$ Yes with probability $> 0 \Leftrightarrow x \in L$.
- $A(x) =$ No with probability 1 $\Leftrightarrow x \notin L$.

TIME($f(n)$) is the class of problems which can be decided in time $O(f(n))$ by a deterministic algorithm.

NTIME($f(n)$) is the class of problems which can be decided in time $O(f(n))$ by a non-deterministic algorithm.

It is possible to show that $A \in$ NTIME($f(n)$) $\Rightarrow$ $A \in$ TIME($c^{f(n)}$)

$A \in \mathsf{P} \Leftrightarrow A \in \mathsf{TIME}(n^k)$ for some k.

$A \in \mathsf{NP} \Leftrightarrow A \in \mathsf{NTIME}(n^k)$ for some k

In the same way we can define NPSPACE by

$A \in \mathsf{NPSPACE} \Leftrightarrow A \in \mathsf{NSPACE}(n^k)$ for some k

The Planning Problem

We have a set of *state variables* $c_1, c_2, \ldots, c_n$ with values 0 or 1. The values of $c_1, c_2, \ldots, c_n$ tells us what *state* we are in. We have *operators* $O_1, O_2, \ldots O_k$ which changes the state variables. The problem is:

Input : Lists $c_1, c_2, \ldots, c_n$ and $O_1, O_2, \ldots O_k$. A start state C_0 and a goal state C^*.

Goal: Is there a sequence $O_{i_1}, O_{i2}, \ldots O_{i_j}$ that transforms C_0 to C^*?

Planning $\in$ PSPACE

We use Savitch's Theorem. There can be at most 2^n different states in Planning. We want to know if there is a path $C_0 \to C^*$. Such a path has length $\leq 2^n - 1$. Use the algorithm in Savitch's Theorem. It uses $O(n)$ bits of memory.

PSPACE = NPSPACE

Sketch proof:

Let X be a problem in NPSPACE. Let M be a non-deterministic Turing Machine which decides X and uses $O(n^k)$ bits of memory. The computation graph contains at most $O(c^{n^k})$ vertices.

The algorithm in Savitch's Theorem finds an accepting computation in the computation graph (if there is one) and uses at most $O((\log c^{n^k})^2) = O(n^{2k})$.

So we get $X \in$ PSPACE.

GEOGRAPHY is PSPACE-Complete

We know that GEOGRAPHY $\in$ PSPACE.

It is possible to make a reduction QSAT $\leq_P$ GEOGRAPHY.