Mapping

Let, A,B be two non empty sets. A mapping f from A to B is a rule that assigns to each element x of A a definite element y in B.

$A \rightarrow$ domain of f

$B \rightarrow$ co-domain of f

$f: A \rightarrow B$

Let, $f:A \rightarrow B$ be a mapping & $x \epsilon A$. Then the unique element y of B that correspond to x by the mapping f is called the f-image of x (f(x)). If f(x)=y we say that

'f maps x to y'.

The set of all f-images i.e. $\{f(x):x \epsilon A\}$ is denoted by f(A) & is said to be the image set of f (denoted by im f) or the range set of f.

Contd.

In some texts

$D(f) \rightarrow$ domain

$R(f) \rightarrow$ range

Ex1. Let, $S=\{1,2,3,4\}$, $T=\{a,b,c,d\}$

Let us consider following rl^ns bet^n S & T.

i) $f1=\{(1,a), (1,b), (2,c), (3,c), (4,d)\}$

ii) $f2=\{(1,a), (2,b), (3,c)\}$

iii) $f3=\{(1,b), (2,b), (3,c), (4,d)\}$

iv) $f4=\{(1,b), (2,c), (3,d), (4,a)\}$

Ans.

i) f1 is not a mapping since element 1 is related to 2 different elements.

Ans.

ii) f2 is not a mapping as element 4 is not related to any element of T by the rl^n.

iii) f3 is a mapping. Here image set is { b,c,d } & it is a proper subset of co-domain set T.

iv) f4 is a mapping from S to T. Here the image set is T.

Ex2. Let, $f=\{ (x,y)\in R\times R: y=1/x \}$

Check f is a mapping from R to R.

Sol^n: The element 0 in the domain set R is not related to an element of the co-domain set.

Therefore f is not a mapping from R to R.

Contd.

Let, $S=R-\{0\}$. Then $f=\{(x,y)\in S\times R: y=1/x\}$ is a mapping from S to R.

" $f:S\rightarrow R$ is defined by $f(x)=1/x$, $x\in S$".

Def^n:

1. A mapping $f : A\rightarrow B$ is said to be an **into mapping** if $f(A)$ is a proper subset of B. In this case we say that f maps A into B.
2. A mapping $f : A\rightarrow B$ is said to be an **onto mapping** if $f(A)=B$. In this case we say that f maps A onto B.

Ex1. Let, $f:Z\rightarrow Z$ be defined by $f(x)=2x$, $x\in Z$. Then f is an into mapping because $f(Z)$ (set of all even integers) is a proper subset of the co-domain set Z.

Contd.

Ex2. Let, $f:Z \to Z$ be defined by $f(x)=|x|$, $x \in Z$. The f is an into mapping because $f(Z)$ (set of all +ve integers) is a proper subset of the co-domain set Z.

Ex3. Let, $f:Z \to Z$ be defined by $f(x)=x+1$, $x \in Z$. Then every element y in the co-domain set Z has a pre-image y-1 in the domain set Z. Therefore $f(Z)=Z$ & f is an onto mapping.

In Ex2. 0 in the co-domain set Z has only one pre-image in the domain set, 1 in the co-domain set Z has 2 pre-images in the domain set, -2 in the co-domain set Z has no pre-image in the domain set.

Let, $f:R \to R$ be defined by $f(x)=2x$, $x \in R$. For an element y in the co-domain set R, $f^{-1}(y)=(1/2)\times y$, a single element in the domain set R.

Contd.

Def^n: A mapping f: A→B is said to be injective (or one-to-one) if for each pair of distinct elements of A their f-images are distinct.

A mapping f:A→B is said to be surjective (or onto) if f(A)=B.

A mapping f:A→B is said to be bijective if f is both injective & surjective.

Thus f:A→B is injective if $x1 \neq x2$ in A implies $f(x1) \neq f(x2)$ in B. In this case, each element of B has at most one pre image.

If f is surjective each element of B has at least one pre-image.

If f is bijective each element of B has exactly one pre-image.