Set operations: Union

A U B

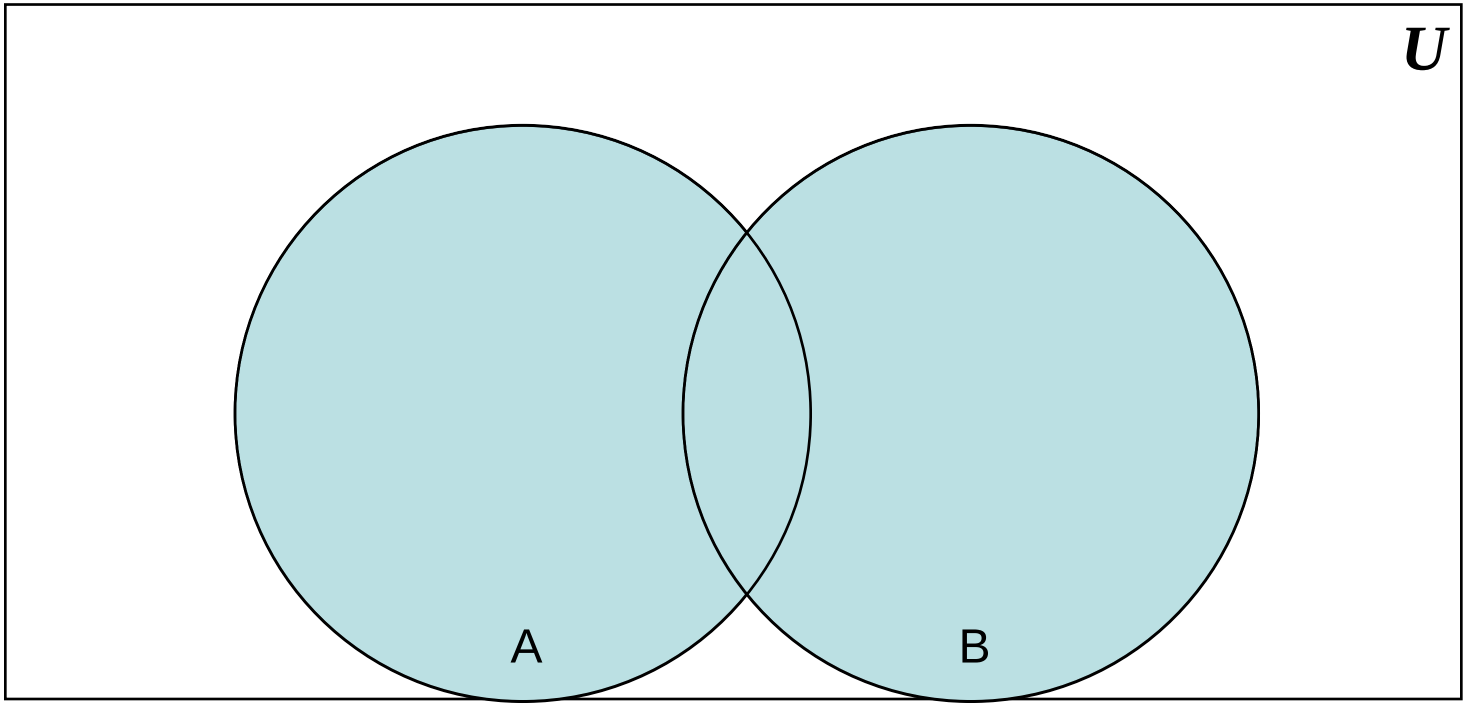


Fig. Venn Diagram representing AUB

Set operations: Union

- Formal definition for the union of two sets:
  $A \cup B = \{ x \mid x \in A \textbf{ or } x \in B \}$
- Examples
  - {1, 2, 3} U {1,3, 4, 5}
    = {1, 2, 3, 4, 5}
  - {New York, Washington} U {3, 4}
    = {New York, Washington, 3, 4}
  - {1, 2} U ∅
    = {1, 2}

[The empty set is a special set. It contains no elements. It is usually denoted as { } or ∅. The empty set is always considered a subset of any set.]

Set operations: Union

- Properties of the union operation
  - A U ∅

    = A — Identity law
  - A U ***U***

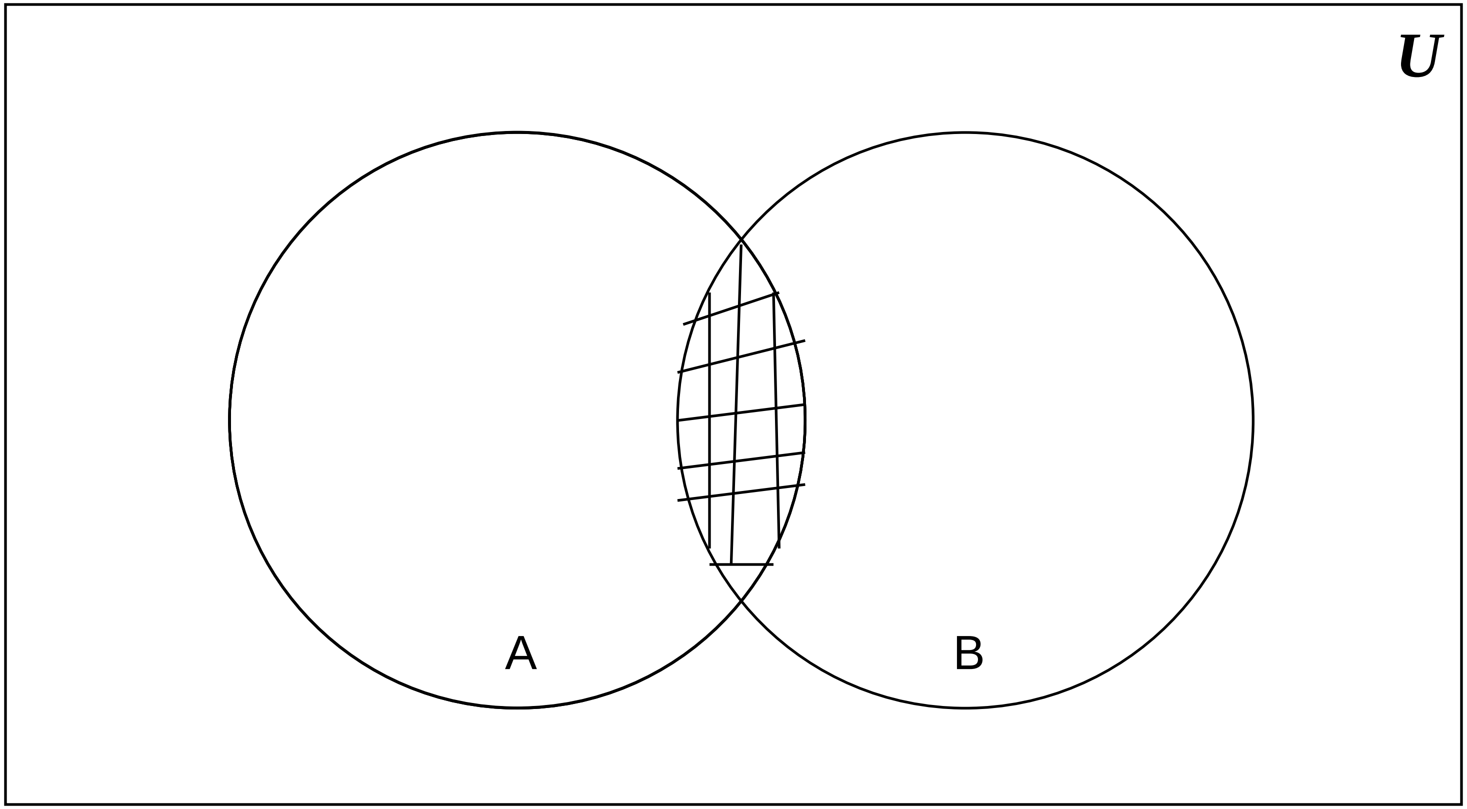
    = ***U*** — Domination law
  - A U A

    = A — Idempotent law
  - A U B

    = B U A — Commutative law
  - A U (B U C) = (A U B) U C — Associative law

Set operations: Intersection

$A \cap B$



Set operations: Intersection

- Formal definition for the intersection of two sets: $A \cap B = \{ x \mid x \in A \textbf{ and } x \in B \}$
- Examples
  - $\{1, 2, 3, 4, 5\} \cap \{3, 4, 6, 7, 9\} = \{3, 4\}$
  - {New York, Washington} $\cap$ {3, 4}

    = $\emptyset$ [No elements in common]
  - $\{1, 2\} \cap \emptyset$

    = $\emptyset$ [Any set intersection with the empty set yields the empty set]

Set operations: Intersection

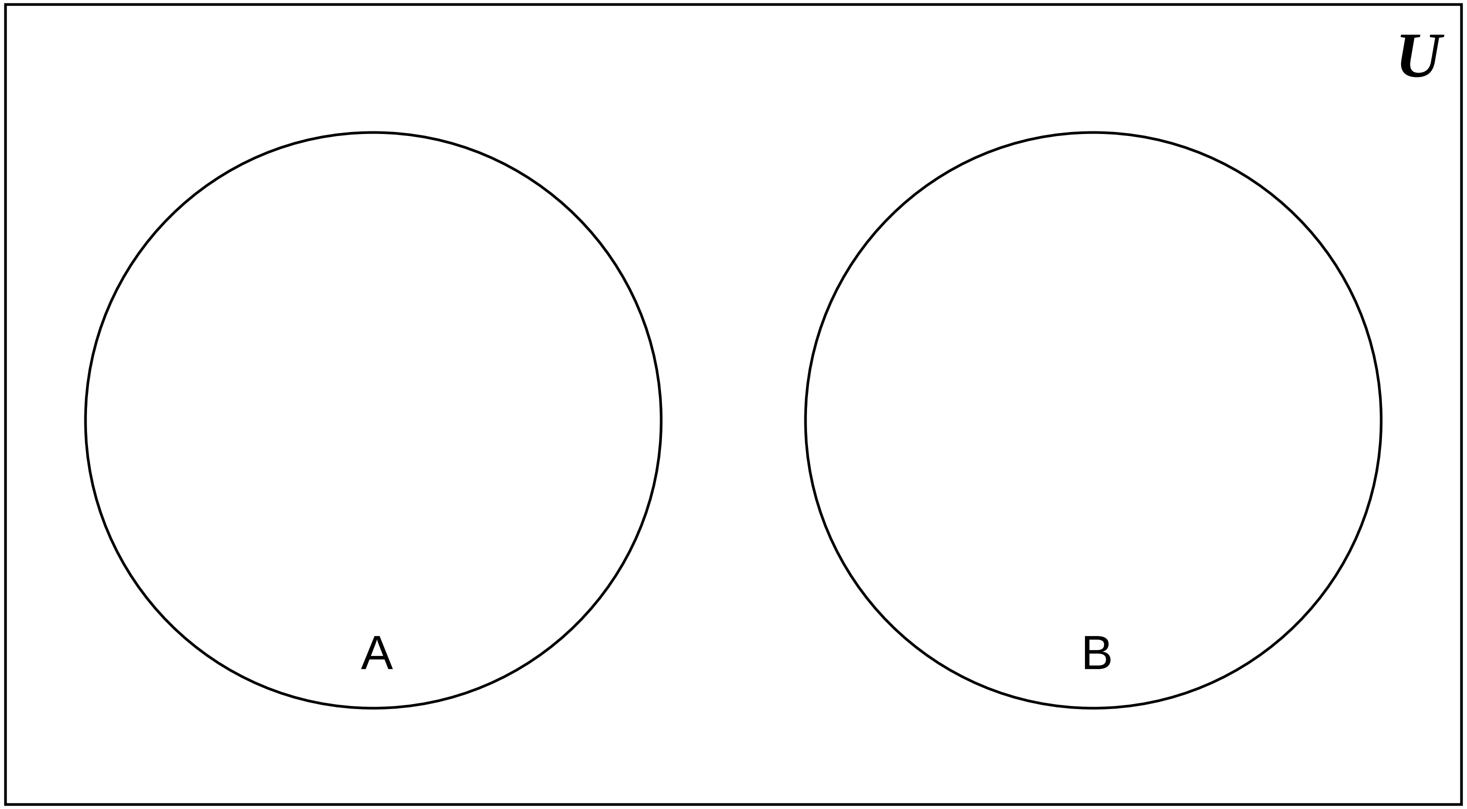
- Properties of the intersection operation
  - $A \cap \boldsymbol{U}$

  $= A$ Identity law
  - $A \cap \varnothing$

  $= \varnothing$ Domination law
  - $A \cap A$

  $= A$ Idempotent law
  - $A \cap B = B \cap A$ Commutative law
  - $A \cap (B \cap C) = (A \cap B) \cap C$ Associative law

Disjoint sets

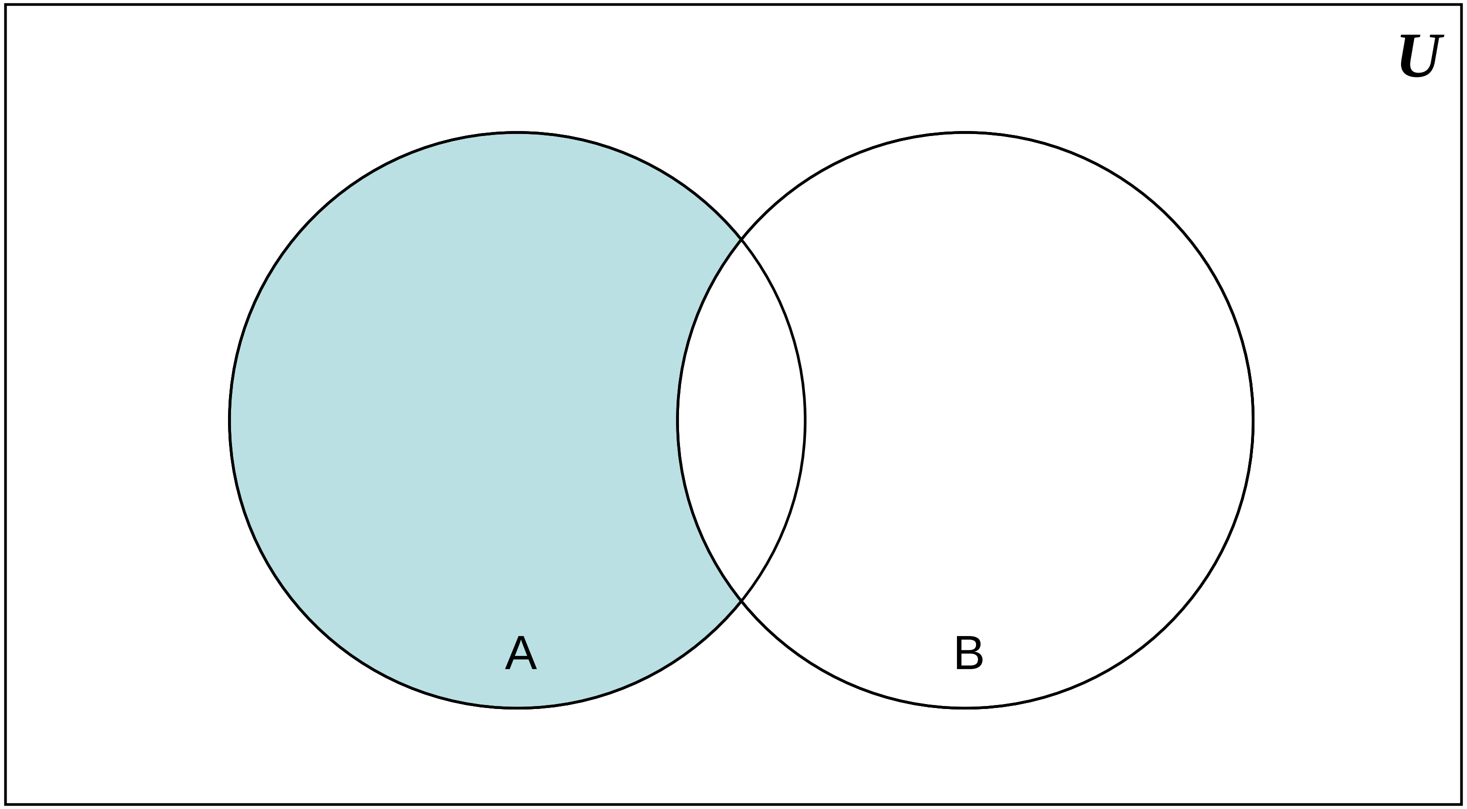


Disjoint sets

- Formal definition for disjoint sets: two sets are disjoint if their intersection is the empty set
- Examples
  - $\{1, 2, 3\}$ and $\{3, 4, 5\}$ are not disjoint
  - {New York, Washington} and $\{3, 4\}$ are disjoint
  - $\{1, 2\}$ and $\varnothing$ are disjoint
    - Their intersection is the empty set
  - $\varnothing$ and $\varnothing$ are disjoint ?
    - Their intersection is the empty set

Set operations: Difference

A - B



Set operations: Difference

- Formal definition for the difference of two sets:

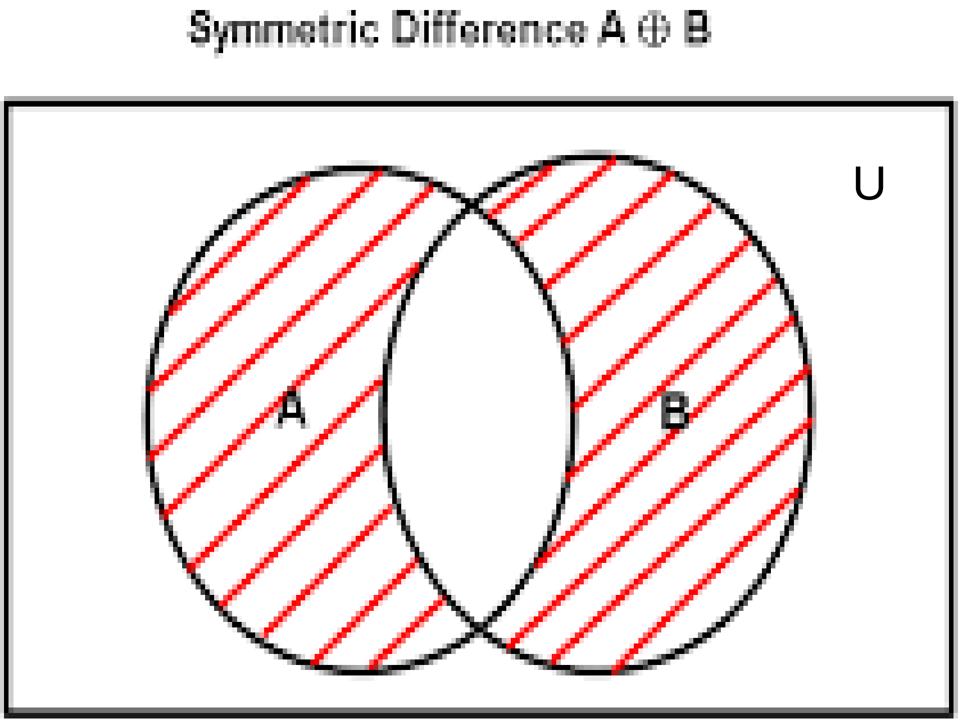
  $A - B = \{ x \mid x \in A \textbf{ and } x \notin B \}$

  $A - B = A \cap \overline{B}$ ← Important!

- Examples
  - $\{1, 2, 3\} - \{3, 4, 5\} = \{1, 2\}$
  - {New York, Washington} - {3, 4} = {New York, Washington}
  - $\{1, 2\} - \emptyset = \{1, 2\}$

    [The difference of any set S with the empty set will be the set S]

Set operations: Symmetric Difference



$A \oplus B$

Set operations: Symmetric Difference

- Formal definition for the symmetric difference of two sets:

  $A \oplus B = \{ x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$

  $A \oplus B = (A \cup B) - (A \cap B)$ ← Important!

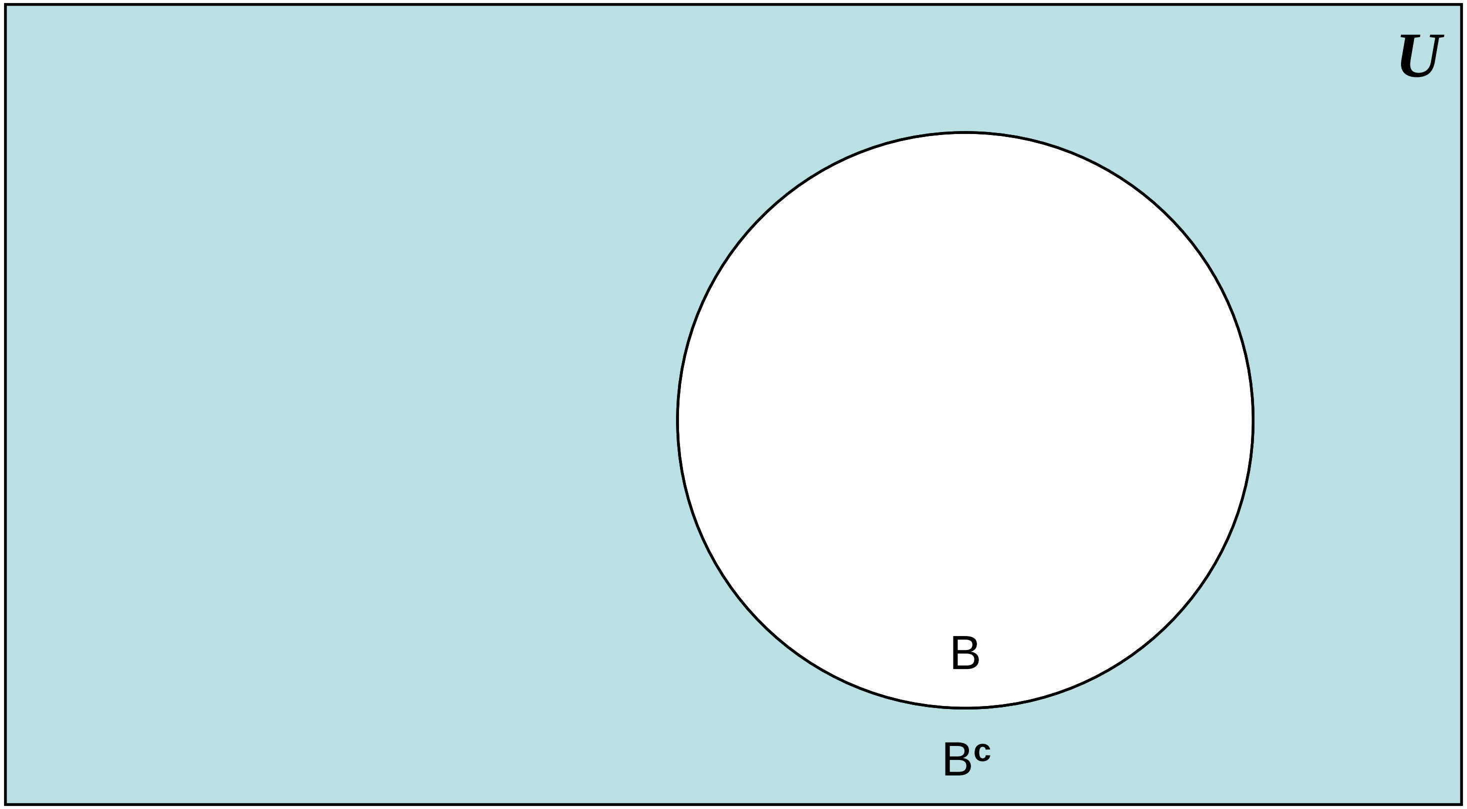
- Examples
  - $\{1, 2, 3,5,7\} \oplus \{3, 4, 5\} = \{1, 2, 4, 7\}$
  - {New York, Washington} $\oplus$ {3, 4}

  = {New York, Washington, 3, 4}
  - $\{1, 2\} \oplus \varnothing$

  $= \{1, 2\}$

  [The symmetric difference of any set S with the empty set will be the set S]

Complement sets



Complement sets

- Formal definition for the complement of a set: $\overline{A} = \{ x \mid x \notin A \}$
  - Or $\boldsymbol{U}$ – A, where $\boldsymbol{U}$ is the universal set
- Examples (assuming $\boldsymbol{U}$ = **Z,** A**=** {1, 2, 3})
  - $A^c = \{ \ldots, -2, -1, 0, 4, 5, 6, \ldots \}$

Complement sets

- Properties of complement sets

  - $\bar{\bar{A}} = A$ Complementation law
  - $A \cup \bar{A}$

  $= U$ Complement law

  - $A \cap \bar{A}$

  $= \varnothing$ Complement law

  # Let U be the universal set & A is its subset where U={ x : x϶N and x<=10}

  A= { y:y is a prime no. <10 }, Find A^c

  $A^c = \{ 1, 4, 6, 8, 9, 10 \}$

Cartesian Product of 2 sets

Let, $A=\{1,2\}$ $B=\{3,4,5\}$

Set of all ordered pairs of elements of A and B is

$A\times B=\{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$

$B\times A=\{(3,1),(3,2),(4,1),(4,2),(5,1),(5,2)\}$

$\Rightarrow A\times B\neq B\times A$

Note: if $A=\Phi$ or $B=\Phi$ or $A,B=\Phi$

then $A\times B = B\times A$

Contd.

\# Let, $A=\{a,b,c\}$, $B=\{d,e\}$, $C=\{a,d\}$

Find i) $A\times B$ ii) $B\times A$ iii) $A\times(B\cup C)$

iv) $(A\cap C)\times B$ v) $(A\cap B)\times C$ vi) $A\times(B-C)$

Sol^n: i) $A\times B=\{(a,d),(a,e),(b,d),(b,e),(c,d),(c,e)\}$

ii) $B\times A=\{(d,a),(d,b),(d,c),(e,a),(e,b),(e,c)\}$

iii) $A=\{a,b,c\}$, $B\cup C=\{a,d,e\}$

$A\times(B\cup C)=\{(a,a), (a,d),(a,e),(b,a),(b,d),(b,e),(c,a),(c,d),(c,e)\}$

iv) $A\cap C=\{a\}$, $B=\{d,e\}$

$(A\cap C)\times B=\{(a,d),(a,e)\}$

v) $A\cap B= \Phi$, $C=\{a,d\}$

$(A\cap B)\times C= \Phi$

vi) $A=\{a,b,c\}$, $B-C=\{e\}$

$A\times(B-C)=\{(a,e),(b,e),(c,e)\}$

Relations

Let A={Mohan,Soham,David,Karim}

B={Rita,Marry,Fatima}

Suppose Rita has two brothers(Mohan & Soham), Marry has one brother(David), Fatima's brother is Karim.

If we define a relation R " is a brother of " between the elements of A and B then

Mohan R Rita, Soham R Rita, David R Marry, Karim R Fatima.

Contd.

It can be written in the form of ordered pairs as:

R={(Mohan,Rita), …….,(Karim,Fatima)}

Clearly $R \subseteq A \times B$ i.e. $R=\{(a,b): a \epsilon A, b \epsilon B \;\&\; aRb\}$

If i) $R=\Phi$, R is called a void relation

ii) $R= A \times B$, R is called a universal relation

iii) If R is a relation defined from A to A, it is called a relation defined on A.

iv) $R=\{(a,a)$ for all $a \epsilon A\}$ is called the identity relation.

A is domain & B is range of the given relation.

Contd.

Let, S be a non empty set. A binary relation(rl^n) ρ on S is a subset of the cartesian product S×S. If (a,b) be an element of S×S & $(a,b)\epsilon\rho$ then it is represented by $a \rho b$.

Equivalence Rl^n:

Let, S be a non empty set & ρ be a binary rl^n on S. The rl^n ρ is said to be reflexive if $(a,a)\epsilon \rho$ for all a in S i.e. $a \rho a$ holds for all a in S.

The rl^n ρ is to be symmetric if for any two elements a,b in S

$(a,b)\epsilon\rho \Rightarrow (b,a)\epsilon\rho$ i.e. $a \rho b \Rightarrow b \rho a$

Contd.

The rl^n transitive if for any three elements a,b,c in S, (a,b)$\epsilon\rho$ and (b,c)$\epsilon\rho$ => (a,c)$\epsilon\rho$

i.e. $a\rho b$ and $b\rho c$ => $a\rho c$

The rl^n ρ on S is said to be an equivalence rl^n on S or an RST rl^n on S if ρ is reflexive, symmetric & transitive.

Let, the rl^n ρ is defined on the set **Z** by " $a\rho b$ iff (a-b) divisible by 5" for a,bϵ**Z**. Examine ρ is equivalence or not.

i) Let, aϵ **Z.** Then (a-a) is divisible by 5.

Therefore $a\rho a$ holds for all a in **Z** and ρ is reflexive.

Contd.

ii) Let, a,bϵ**Z** and aρb hold. Then (a-b) is divisible by 5 and therefore (b-a) is divisible by 5.

Thus aρb => bρa & therefore ρ is symmetric.

iii) Let, a,b,c ϵ**Z** & aρb, bρc both hold. Then (a-b) and (b-c) are both divisible by 5. Therefore a-c=(a-b)+(b-c) is divisible by 5.

Thus aρb and bρc => aρc & therefore ρ is transitive.

Since ρ is RST, so ρ is an equivalence rl^n on **Z.**

Contd.

\# A rl^n ρ on the set **N** is given by ρ={(a,b)ϵN×N : a is divisor of b}. Check ρ is RST?

i) Let, mϵ**N**. Then m is a divisor of m. Hence (m,m)ϵρ for all mϵ **N.** So ρ is reflexive.

ii) Let, m,nϵ**N** and (m,n)ϵρ. Then m is a divisor of n. This does not always imply that n is a divisor of m. So ρ is not symmetric.

iii) Let, m,n,yϵ**N** and (m,n)ϵρ, (n,y)ϵρ. Then m is divisor of n & n is a divisor of y & this implies m is a divisor of y.

Therefore (m,n)ϵρ and (n,y)ϵρ => (m,y)ϵρ.

So ρ is transitive.

Contd.

A rl^n ρ is defined on the set Z by "aρb iff ab>0" for all a,bϵ**Z**. Examine if ρ is

i) Reflexive ii) Symmetric iii) Transitive

Sol^n: i) Let,aϵ**Z**, then a.a>0 provided a≠0.

Therefore aρa does not hold for all a in **Z**.

So ρ is not reflexive.

ii) Let a,bϵ**Z** & aρb hold. Then ab>0 & therefore ba>0. So ρ is symmetric.

iii) Let, a,b,cϵ**Z** and aρb, bρc both hold. Then ab>0 and bc>0. We have (ab)(bc)>0.

$\Rightarrow$ac>0 since b^2>0.

$\Rightarrow$Thus aρb and bρc => aρc. So ρ is transitive.

Partial Order Rl^n:

Let S be a non-empty set. A rl^n ρ on the set S is said to be anti-symmetric

if aρb and bρa => a=b for a,bϵS.

Ex. i) The rl^n ρ defined on **R** by "xρy iff x<=y" for x,yϵR is antisymmetric.

ii) Let **X** be a non empty set. The rl^n ρ defined on P(X)(power set of X) by "AρB iff A is a subset of B" for A,BϵP(X) is antisymmetric.

Contd.

The power set is a set which includes all the subsets including the empty set and the original set itself. If set A = {x,y,z} is a set, then all its subsets {}, {x}, {y}, {z}, {x,y}, {y,z}, {x,z} and {x,y,z} are the elements of powerset.

Power set of A

P(A) = {{}, {x}, {y}, {z}, {x,y}, {y,z}, {x,z}, {x,y,z} }

Where P(A) denotes the powerset.

The number of elements of a power set is written as |A|, If A has n elements then it can be written as $|P(A)| = 2^n$

Def^n: Let S be a non empty set. A rl^n ρ on S is said to be a partial order rl^n if ρ is reflexive, antisymmetric & transitive.

A rl^n of partial order is often denoted by '**≤**' even if it is not "less than".

Poset: A non empty set S together with a rl^n of partial order **≤** on S is called a Poset(Partially Order Set) & is defined by (S, **≤**).

Ex1. (R, **≤**) is a poset where x**≤**y means " x is less than or equal to y" for x,y in R.

Ex2. Let X be a non empty set and P(X) be the power set of X (P(X), **≤**) is a poset where A **≤** B means “A is a subset of B” for A,BϵP(X).

Ex3. Let, X be a non-empty set & S be the set of all proper subsets of X. (S, **≤**) is a poset where A **≤** B means “A is a subset of B” for A,BϵS.

Ex4. (N, **≤**) is a poset where m **≤** n means “ m is a divisor of n” for m,nϵN.

Ex5. Let, S be the set of all +ve divisors of 72. (S, **≤**) is a poset where a **≤** b means “a is a divisor of B” for a,b in S.