# Quantifiers

## Universal Quantifier

The phrase "for all", denoted by $\forall$. is called the universal quantifier.
for ex. "All human beings are mortal."

Let, U be the universal set of "all human beings". Consider the proposition P(x): "x is mortal". Then the above sentence can be written as:

$(\forall x \in U)\, P(x)$ or simply $\forall x\, P(x)$.

The expression P(x) by itself is an ~~one~~ open sentence & hence it has no truth value. However, "$\forall x\, P(x)$" does have a truth value & as such it is a statement & is called a universal statement.

The truth values of "$\forall x\, P(x)$" are assigned as follows:

- "$\forall x\, P(x)$" is true iff P(x) is ~~false for atleast one $x \in U$.~~

true for every $x$ in $U$, $U$ being the domain or universe of $x$.

- "$\forall x\, P(x)$" is false, iff $P(x)$ is false for at least one $x \in U$.

specifically,

If $\{x : x \in U, P(x)\} = U$, then "$\forall x\, P(x)$" is true else "$\forall x\, P(x)$" is false

## Existential quantifiers

The phrase "there exists", denoted by $\exists$, is called existential quantifier.

for ex. "There exists a real no. $x$ such that $x^3 = 2$".

Let, $\mathbb{R}$ be the set of all real nos. Then the above sentence can be expressed as $(\exists x \in \mathbb{R})\, P(x)$ or simply $\exists x\, P(x)$ where $P(x)$: "$x^3 = 2$".

The expression "$\exists x P(x)$" has truth value and as such it is a statement & is called existential statement.

The truth values of "$\exists x\, P(x)$" are assigned as follows:

- "$\exists x\, P(x)$" is true iff $P(x)$ is true for at least one $x \in U$, $U$ being the domain or universe of $x$.
- "$\exists x\, P(x)$" is false iff $P(x)$ is false for every $x \in U$.

specifically,

If $\{x : x \in P(x)\} \neq \phi$ then "$\exists x\, P(x)$" is true, otherwise, "$\exists x\, P(x)$" is false.

Note: $\forall x$ represents each of the following pharases:

"for all $x$", "for every $x$", "for each $x$", "for any $x$".

$\exists x$ represents each of the following pharases: "there exists an $x$", "there is an $x$", "there is some $x$", "there is at least one $x$", "for some $x$".