Negation of a Suantified Statement
Parts (3) & (4) of the following theorem show the equivalences of negated quantific statements.
Theorem:  Let the universe of discourse be a finite set S= Sai, az,, and & P(2) be a propositional for defined on S. Then a propositional for defined on S. Then  1) (xx) P(2) = P(ai) AP(az) A AP(an)
1) $(\forall x) \Gamma(x) = \Gamma(a) \vee \Gamma(a_2) \vee \cdots \vee \Gamma(a_n)$ 2) $(\exists x) \Gamma(x) = \Gamma(a_1) \vee \Gamma(a_2) \vee \cdots \vee \Gamma(a_n)$ 3) $(\forall x) \Gamma(x) = (\exists x) \wedge \Gamma(x)$ 4) $(\exists x) \Gamma(x) = (\forall x) \wedge \Gamma(x)$ 4) $(\exists x) \Gamma(x) = (\forall x) \wedge \Gamma(x)$
Proof: (1) & (2) follows from the def of quantified (3) By (1), $\sim (\forall x P(x)) \equiv \sim (P(a_1) \land P(a_2) \land \cdots \land P(a_n))$
= NP(ai) VNP(a2) VVNP(an)
Eby De Morgan's law ]  = (3x) ~ P(x) [by (2)]
Hence, proved.

(4) By (2), ~ (3xp(x)) = ~ (p(ai) vp(a2) v ··· vp(am))  $= \sim P(a_1) \wedge \sim P(a_2) \wedge \cdots \wedge \sim P(a_n)$ [by De Morgan's Law] = (xx P(x)) [by (1)] Hence proved. Illustration consider the statement: "All students of B. Tech have taken the course of Discrete Malhematics! It can be written as " You P(x)" where domain of x -> set of all students of B. Tech
P > is the predicate "have taken. The course of Discrete Mathematic The negation of the above statement is" 9t is not the case that all students of B. Tech. howe taken the course of Discrete. Maltie matics 1. or equivalently,

"There exists a student of B. Tech. who has not taken the course of Discrete Mathematics." Hence the negation of " $\forall x P(x)$ " is  $\exists x \sim P(x)$  i.e.,  $\sim ((\forall x)) P(x) = ((\exists x) \sim P(x))$ which verifies part (3) theorem. Again consider the statement: "There is a student of B. Tech. who has taken the course of the Discoule Malhematics". This can be wrillen as 32 P(x) where domain of 20 set of all students of B. Tech The negation of the above statement is 19t is not the case that there is a student of B. Tech who has taken the course of Discoele Malhematics". or equivalently "All students of B. Tech have not taken the eourse of Discrete Malbematics!

Hence the negation of (3x) P(x) is (4x) ~ P(x) i.e., N((12) P(2)) = / Frx NP(2) which verifies part (4) od theorem. Negaled Quantified Statements Quantified Stalements i) (32) ~P(xi) (at least 1 is false 1). (xx) P(x) (all are true) 2) (4x) ~ P(x) (all are false) 2) (7x) P(x) (at least 1 is four (72) P(x) (at least 1 is tone) 3) (4x) NP(x) (all are false) 4) (4x) P(x) (all are true) 4) (7x) NP(x) (at least 1 is fatse) Table: Equivalences of quantified statements 2, their negations