Mathematical Logic

Propositions:

A statement or proposition is a declarative sentence which is either true or false but not both. Thus it has only one truth value namely ‘true’ or ‘false’.

Ex. i) The Sun rises in the East.

ii) Do you know Tamil?

iii) $5+7=10$

iv) $x-5=6$

v) Give me the pen.

vi) The temperature of Darjeeling is 11^{o} C.

vii) The Sun will set at 5.45 p.m.

Contd.

i) The Sun rises in the East.

is declarative sentence & has truth value “true’. Hence it is statement.

ii) Do you know Tamil?

is an interrogative sentence so it is not statement.

iii) $5+7=10$

is a statement that happens to be false.

iv) $x-5=6$

is a declarative sentence but not a statement, depending on x value it may be true or false.

v) Give me the pen.

is not a statement since it is a command(not a declarative sentence).

Contd.

vi) The temperature of Darjeeling is 11^{o} C.

is a declarative sentence(statement) whose truth value or falsity we do not know at this moment. However we can in principle determine if it is true or false, so it is a statement.

vii) The Sun will set at 5.45 p.m.

is a statement since it is either true or false but not both, although we have to wait till 5.45 p.m. to find out if it is true or false.

Contd.

A proposition obtained by the combination of 2 or more elementary propositions (only a single propositional variable or a single propositional constant (either ‘true’ or ‘false’)) by means of logical connectives or operators is called a compound or composite proposition.

The words or phrases or symbols used to form a compound proposition are called logical connectives. There are 5 basic connectives namely negation, conjunction, disjunction, conditional & biconditional.

Contd.

Negation:

Let p be a proposition. The negation of p is denoted by ~p or ¬p (not p) is a proposition that is false when p is true & vice versa.

Let, p: “Jeff Bezos is intelligent”.

~p: “Jeff Bezos is not intelligent”.

Note: Negation of a negation is the affirmation.

Thus ~(~p)=p

Conjunction:

If p and q are two statements, then conjunction of p and q denoted by p ∧ q is a compound statements which is true only when p and q are both true.

Contd.

If r : “$5>3$” and s: “$5<10$” then

r ∧ s : “$3 < 5 < 10$”

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

p ∨ q is true only when

at least one of p or q is true.

Truth Table:

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Contd.

Conditional:

If p & q are two propositions then the conditional of p and q denoted by $p \rightarrow q$ or $p \Rightarrow q$ (read as " if p then q") is a compound proposition called conditional proposition or implication whose truth values are as follows:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Contd.

p is called antecedent or hypothesis

q is called consequent or conclusion

Let, p: “Tomorrow is Monday”

q: “Today is Sunday”

Then $p \rightarrow q$: “If tomorrow is Monday then Today is Sunday”.

Contd.

Converse, Inverse & Contrapositive of an Implication:

Let, $p \rightarrow q$ be an implication. Then

i) the converse of $p \rightarrow q$ is the implication $q \rightarrow p$

ii) the inverse of $p \rightarrow q$ is the implication $\sim p \rightarrow \sim q$

iii) the contrapositive of $p \rightarrow q$ is the implication $\sim q \rightarrow \sim p$

Truth Table of Converse:

p	q	Converse($q \rightarrow p$)
T	T	T
T	F	T
F	T	F
F	F	T

Contd.

Truth Table of Inverse:

p	q	~p	~q	Inverse(~p→~q)
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Truth Table of Contrapositive:

p	q	~q	~p	Contrapositive (~q→~p)
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Contd.

Truth Table for all these Implications:

p	q	Conditional (p→q)	Converse (q→p)	Inverse (~p→~q)	Contrapositive (~q→~p)
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**T**
F	**F**	**T**	**T**	**T**	**T**

Contd.

Let p: “The day is Sunny” and
q: “I go to the School”

$p \rightarrow q$: “ If the day is Sunny then I go to the School”

Converse($q \rightarrow p$)

$q \rightarrow p$: “If I go to the School then the day is Sunny”

Inverse($\sim p \rightarrow \sim q$)

$\sim p \rightarrow \sim q$: “If the day is not Sunny then I do not go to the School”

Contrapositive($\sim q \rightarrow \sim p$)

$\sim q \rightarrow \sim p$: “If I do not go to the School then the day is not Sunny”

Contd.

Biconditional:

Biconditional of p and q denoted by $p \leftrightarrow q$ (read as "p iff q"). Truth table of $p \leftrightarrow q$

p	q	$p\leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Thus $p \leftrightarrow q$ is true only when either both p and q are true or both p and q are false.

Let, p: "I will get wet"

q: " I swim"

Then $p \leftrightarrow q$: "I will get wet if and only if I swim".