Binary Composition:

Let, A be a non-empty set. A binary composition (or binary operation) on A is a mapping f: $A \times A \rightarrow A$. This mapping f is generally denoted by the symbol $\odot$ or o. For a pair of elements a,b in A, the image of (a,b) under the binary composition(BC) o is denoted by a o b. The image of the element (b,a) is b o a.

The symbols like *, +, · are also used to denote a BC.

Ex.1. On the set **Z** let o stand for the BC 'addition'. Then 2 o 3 = 5, 4 o -4 = 0.

Ex.2. On the set **Z** let o stand for the BC 'multiplication'. Then 2 o 3 = 6, 4 o 0 = 0.

Contd.

Ex.3. On the set **Z** let o stand for the BC 'subtraction'. Then 3 o 2 = 1, 1 o 3 = -2.

Ex.4. On the set **Z** let o stand for the BC defined by a o b = a+2b, a,b∈**Z**. Then 2 o 3 = 8, 3 o 0 = 3

Ex.5. 'subtraction' is not a BC on the set **N** since 2,3∈**N** but 2-3 ∉ **N**.

A BC o is said to be defined on a non empty set A if a o b ∈ A ∀ a,b∈A. In this case the set A is said to be closed under the BC o. For ex. **N** is closed under 'addition' but not closed under 'subtraction'.

Def^{n}: Let, o be a BC on a set A. o is said to be commutative if a o b= b o a ∀ a,b∈A.

o is said to be associative if a o(b o c)=(a o b) o c ∀ a,b,c∈A.

Contd.

Ex.6. Addition on the set R is both commutative and associative but subtraction on set R is neither commutative nor associative.

Ex.7. Let, S be a non empty set & P(S) be the power set of S. Then U and ∩ are BC on P(S). Both are commutative and associative.

Ex.8. Let, M2(R) be the set of all 2×2 real matrices. Let o stand for multiplication of matrices.

Then

o is associative?

o is commutative?

Ans. o is associative but not commutative.

Contd.

Groupoid

Let G be a non empty set on which a BC o is defined. Some algebraic structure is imposed on G by the composition o and (G,o) become a algebraic system. The algebraic system (G,o) is said to be a groupoid. The groupoid (G,o) is comprised of two entities, the set G & the composition o on G.

Ex.1. (**Z**,+), (**Z**,-) are both groupoids. So same set make different groupoids.

Ex2. (**Q**,+), (**R**,+), (**Q**,.), (**R**,.) are groupoids.

Def^n: A groupoid (G,o) is said to be a commutative groupoid if the BC o is commutative.

An element e in G is said to be an identity element in the groupoid (G,o) if a o e = e o a = a ∀ a in G.

Ex1. (**Z**,+) is a commutative groupoid but (**Z**,-) is not a commutative groupoid. Zero(0) is an identity element in (Z,+).

There is no identity element in (**Z**,-).

Def^n: An element e in G is said to be a right identity in the groupoid (G,o) if a o e = a ∀ a in G.