

Predicate Calculus

In a declarative sentence the part which describes the properties of an object or relation among objects is called a predicate.
For ex. consider the statements:

Mr. Fox is clever.

Mr. Jackle is clever.

Here "is clever" is the predicate. Both Mr. Fox and Mr. Jackle have the same property of being clever. In proposition calculus there is no symbolic representation of "is clever", because predicates are not declarative sentences. The above statements can be replaced by a single sentence " x is clever".

Note that " x is clever" is not a proposition. It will be a proposition if x is replaced by any name (noun). The sentence " x is clever" is symbolised as $P(x)$ where P stands for the predicate "is clever".

The variable x is called the propositional function.

In the propositional f_r $Q(x)$ denoting " x is fat" the predicate Q is "is fat". Thus an element say, "Humpty" of the set $\{x: Q(x)\}$ is an object for which $Q(\text{Humpty})$ i.e., "Humpty is fat" is true. This set $\{x: Q(x)\}$ is called the domain of ~~disc~~ discourse or universe of discourse or simply domain or universe of the predicate variable x of the propositional f_r $Q(x)$.

In the above discussion $P(x)$, $Q(x)$ are ex. of propositional f_rs of one variable & the predicates involve in these f_rs are called 1-place predicates.

Consider the following:

"Amitabh is taller than Aamir"

Here the predicate "is taller than" is a 2-place predicate because names of 2 objects are needed to complete a statement involving this predicate.

If S symbolizes "is taller than" then the statement can be expressed as a propositional fn $s(x, y)$ of two variables x, y where x represents 'Amitabh' & y represents 'Aamir'.

The domain of (x, y) is the set $\{(x, y) : s(x, y)\}$ of ordered pairs of names of objects. Similarly $T(x, y) : "x \text{ is to the South of } y"$. Statement instance of $T(x, y)$ may be "Sri Lanka is to the South of India".

The following ex. are 3-place & 4 place predicate respectively.

ex1. "The red ball is placed betⁿ the green ball & the black ball".

ex2. "Amit & Sumit is playing bridge against Souvik & Soumik".

The first one is symbolizes as $P(x, y, z)$ where P is the predicate "is placed betⁿ".
 $P(x, y, z) : "x \text{ is placed betⁿ } y \text{ and } z"$.

The domain of (x, y, z) is the set:
 $\{(x, y, z) : P(x, y, z)\}$ of order triplets
of names of 3 objects.

On the 2nd ex. the propositional f.
is $S(x, y, z, t)$ where S is predicate
"is playing bridge against" and
 $S(x, y, z, t)$: "x & y is playing bridge
against z & t".

In general if S is an n -place predical
& a_1, a_2, \dots, a_n are names of n objects,
then $S(a_1, a_2, \dots, a_n)$ is a statement.