Example 7.1 Find the normalization transformation N which uses the rectangle A (1, 1), B (5, 3), C (4, 5) and D (0, 3) as a window and the normalized device screen as the viewport.

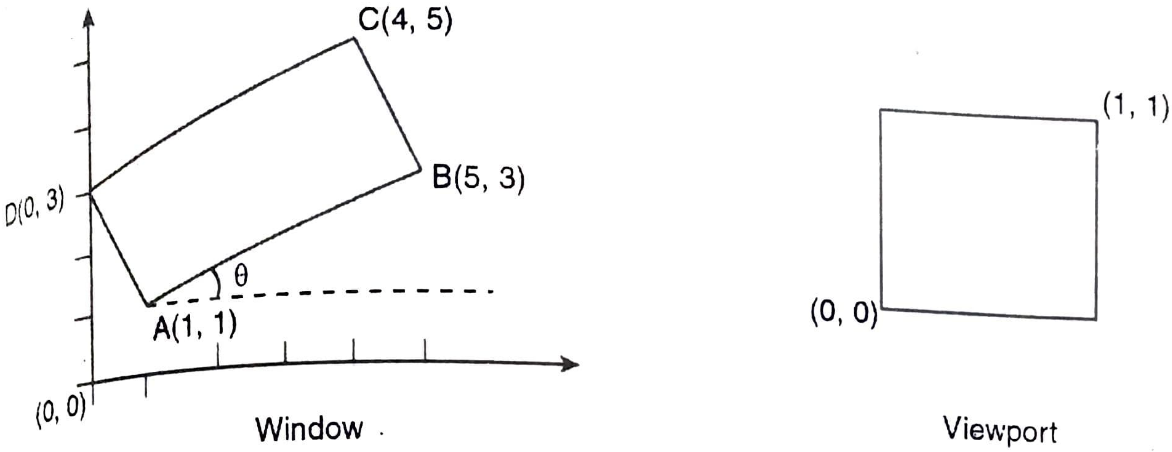

Fig. 7.7

Here we see that the window edges are not parallel to the coordinate axes. So we will first rotate the window about A so that it is aligned with the axes.

$$\text{Now,} \quad \tan\theta = \frac{3-1}{5-1} = \frac{1}{2}$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{5}}, \quad \cos\theta = \frac{2}{\sqrt{5}}$$

Here we are rotating the rectangle in clockwise direction. So θ is (–)ve i.e. $-\theta$.

The rotation matrix about $A(1, 1)$ is,

$$[T_{R,\,\theta}]_A = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \left(\frac{1-3}{\sqrt{5}}\right) \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \left(\frac{1-1}{\sqrt{5}}\right) \\ 0 & 0 & 1 \end{pmatrix}$$

The x extent of the rotated window is the length of $\overline{AB}$ which is $\sqrt{(4^2+2^2)} = 2\sqrt{5}$

Similarly, the y extent is length of $\overline{AD}$ which is $\sqrt{(1^2+2^2)} = \sqrt{5}$

For scaling the rotated window to the normalized viewport we calculate s_x and s_y as,

$$s_x = \frac{\text{viewport } x \text{ extent}}{\text{window } x \text{ extent}} = \frac{1}{2\sqrt{5}}$$

$$s_y = \frac{\text{viewport } y \text{ extent}}{\text{window } y \text{ extent}} = \frac{1}{\sqrt{5}}$$

As in expression (1), the general form of transformation matrix representing mapping of a window to a viewport is,

$$[T] = \begin{pmatrix} s_x & 0 & -s_x xw_{\min} + xv_{\min} \\ 0 & s_y & -s_y yw_{\min} + yv_{\min} \\ 0 & 0 & 1 \end{pmatrix}$$

In this problem [T] may be termed as N as this is a case of normalization transformation with,

$$xw_{\min} = 1 \qquad xv_{\min} = 0$$

$$yw_{\min} = 1 \qquad yv_{\min} = 0$$

$$s_x = \frac{1}{2\sqrt{5}} \qquad s_y = \frac{1}{\sqrt{5}}$$

By substituting the above values in [T] i.e. N,

$$N = \begin{pmatrix} \frac{1}{2\sqrt{5}} & 0 & \left(\frac{-1}{2}\right)\frac{1}{\sqrt{5}} + 0 \\ 0 & \frac{1}{\sqrt{5}} & \left(\frac{-1}{\sqrt{5}}\right)1 + 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now we compose the rotation and transformation N to find the required viewing transformation N_R

$$N_R = N\ [T_{R,\theta}]_A = \begin{pmatrix} \frac{1}{2\sqrt{5}} & 0 & \frac{-1}{2\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{5} & 1 - \frac{3}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 1 - \frac{1}{\sqrt{5}} \\ 0 & 0 & 1 \end{pmatrix}$$

Draw visible portion $P'P'_i$

Example 7.2 Given a window A (20, 20), B (60, 20), C (60, 40), D (20, 40) use any clipping algorithm to find the visible portion of the line P (30, 50) to Q (70, 30) inside the window.

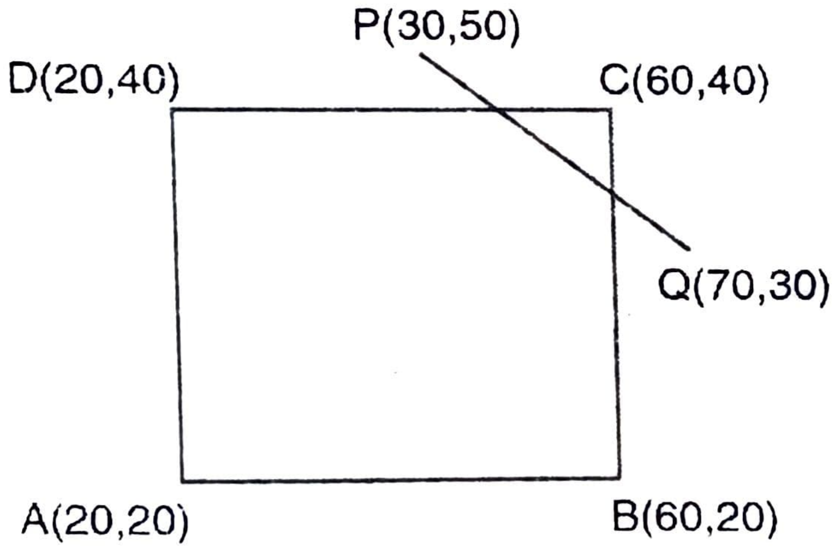

Fig. 7.11

For the line PQ the slope is

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \left(\frac{50 - 30}{30 - 70}\right) = -\left(\frac{20}{40}\right) = -\frac{1}{2}$$

and the intersections with the window edges are left : $x = 20$, $y = m\,(x_L - x_1) + y_1$

or, $y = -1/2\,(20 - 30) + 50 = -\,1/2\,(-10) + 50 = 5 + 50 = 55$

which is greater than y_T and is rejected

right : $x = 60$, $y = m\,(x_R - x_1) + y_1$

or, $y = -\,1/2\,(60 - 30) + 50 = -15 + 50 = 35$

$\therefore$ The intersection with the right edge is at point (60, 35).

top : $y = 40$, $x = x_i + 1/m\,(y_T - y_1)$

or, $x = 30 - 2\,(40 - 50) = 30 + 20 = 50$

$\therefore$ The intersection with the top edge is at point (50, 40)

bottom : $y = 20$, $x = x_1 + 1/m\,(y_B - y_1)$

or, $x = 30 - 2\,(20 - 50)$

$= 30 + 60 = 90$

which is greater than x_R and thus rejected.

So the visible part of the line PQ is from P (50,40) to Q (60,35).

Example 7.3 A Clipping window ABCD is located as follows:

A (100, 10), B (160, 10), C (160, 40), D (100, 40). Using Sutherland-Cohen clipping algorithm find the visible portion of the line segments EF, GH and $P_1 P_2$. E (50,0), F (70, 80), G (120, 20), H (140, 80), P_1 (120, 5), P_2 (180, 30)

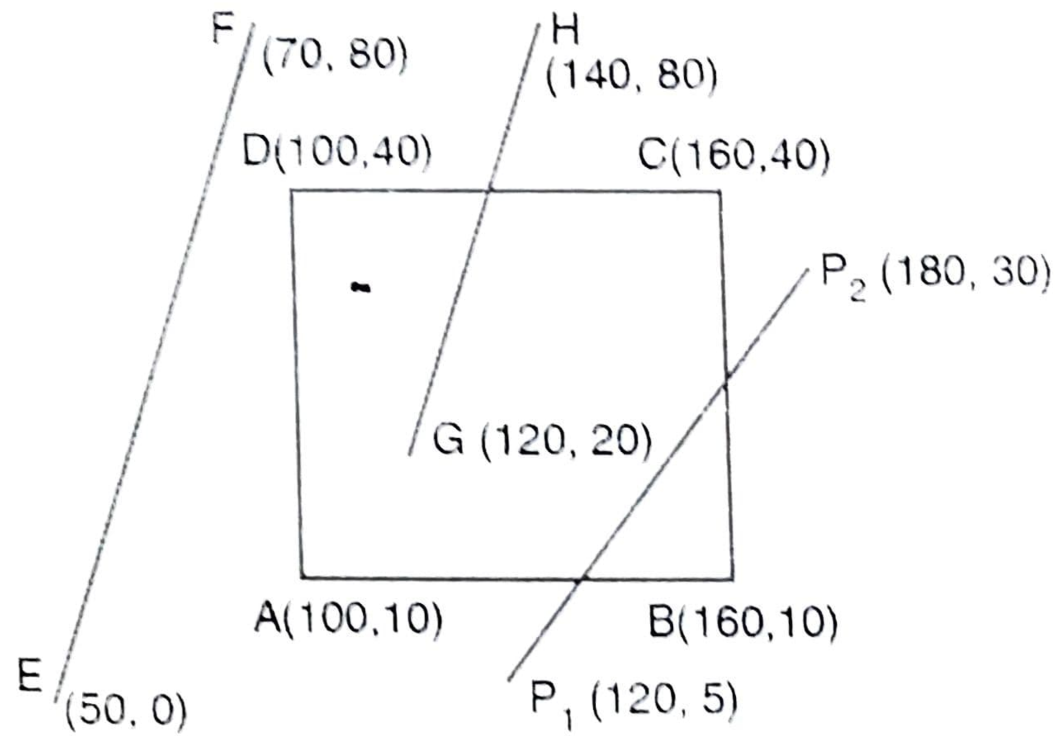

Fig. 7.13

At first considering the line $P_1\ P_2$

INPUT: P_1 (120, 5), P_2 (180, 30)
$x_L = 100$, $x_R = 160$, $y_B = 10$, $y_T = 40$

$x_1 > x_L$ then bit 1 of code–$P_1 = 0$ $C_{1\ \text{left}} = 0$
$x_1 < x_R$ then bit 2 of code–$P_1 = 0$ $C_{1\ \text{right}} = 0$
$y_1 < y_B$ then bit 3 of code–$P_1 = 1$ $C_{1\ \text{bottom}} = 1$
$y_1 < y_T$ then bit 4 of code–$P_1 = 0$ $C_{1\ \text{top}} = 0$

code–P_1 = 0100,

$x_2 > x_L$ then bit 1 of code–$P_1 = 0$ $C_{2\ \text{left}} = 0$
$x_2 > x_R$ then bit 2 of code–$P_1 = 1$ $C_{2\ \text{right}} = 1$
$y_2 > y_B$ then bit 3 of code–$P_1 = 0$ $C_{2\ \text{bottom}} = 0$
$y_2 < y_T$ then bit 4 of code–$P_1 = 0$ $C_{2\ \text{top}} = 0$

code–P_2 = 0010.

Both code–P_1 <> 0 and code–P_2 <> 0
then P_1P_2 not totally visible

code–P_1 AND code–P_2 = 0000
hence (code–P_1 AND code–P_2 = 0)
then line is not totally invisible.

As code–P_1 <> 0

for $i = 1$
{
$C_{1\ \text{left}}$ (= 0) <> 1 then nothing is done.
$i = i + 1 = 2$
}
code–P_1 <> 0 and code–P_2 <> 0
then P_1P_2 not totally visible.

code–P_1 AND code–P_2 = 0000
hence (code–P_1 AND code–P_2 = 0)
then line is not totally invisible.

for $i = 2$
{
$C_{1\ \text{right}}$ (=0) <> 1 then nothing is done.

$i = i + 1 = 2 + 1 = 3$
}
code–P_1 <> 0 code–P_2 <> 0

then P_1P_2 not totally visible.

code–P_1 AND code–P_2 = 0000
hence (code–P_1 AND code–P_2 = 0)
then line is not totally invisible.

for $i = 3$
{

$C_{1\text{ bottom}} = 1$ then find intersection of P_1 P_2 with bottom edge
$y_B = 10$
$x_B = (180 - 120)(10 - 5)/(30 - 5) + 120$
$= 132$

then $P_1 = (132, 10)$

$x_1 > x_L$	then bit 1 of code–P_1 =0	$C_{1\text{ left}} = 0$
$x_1 < x_R$	then bit 2 of code–P_1 =0	$C_{1\text{ right}} = 0$
$y_1 = y_B$	then bit 3 of code–P_1 =0	$C_{1\text{ bottom}} = 0$
$y_1 < y_T$	then bit 4 of code–P_1 =0	$C_{1\text{ top}} = 0$

code–P_1 = 0000
$i = i + 1 = 3 + 1 = 4$
}

code–P_1 = 0 but code–P_2 <> 0
then P_1P_2 not totally visible

code–P_1 AND code–P_2 = 0000
hence (code–P_1 AND code–P_2 = 0)
then line is not totally invisible

As code–P_1 =0
swap P_1 and P_2 along with the respective flags

$P_1 = (180,30)$
$P_2 = (132,10)$
code–P_1 = 0010
code–P_2 = 0000

$C_{1\text{ left}} = 0$	$C_{2\text{ left}} = 0$
$C_{1\text{ right}} = 1$	$C_{2\text{ right}} = 0$
$C_{1\text{ bottom}} = 0$	$C_{2\text{ bottom}} = 0$
$C_{1\text{ top}} = 0$	$C_{2\text{ top}} = 0$

Reset $i = 1$
for $i = 1$
{

$C_{1\text{ left}}$ (= 0) <> 1 then nothing is done

$i = i + 1 = 1 + 1 = 2$

}

code–P_1 <> 0, and code–P_2 <> 0
then P_1P_2 not totally visible.

code–P_1 AND code–P_2 = 0000
hence (code–P_1 AND code–P_2 = 0)
then line is not totally invisible

for $i = 2$

{

$C_{1\ right} = 1$ then find intersection of P_1 P_2 with right edge
$x_R = 160$
$y_R = (30 - 5)(160 - 120)/(180 - 120) + 5$
$= 21.6667$
$= 22$

then $P_1 = (160, 22)$
$x_1 > x_L$ then bit 1 of code–$P_1 = 0$ $C_{1\ left} = 0$
$x_1 = x_R$ then bit 2 of code–$P_1 = 0$ $C_{1\ right} = 0$
$y_1 > y_B$ then bit 3 of code–$P_1 = 0$ $C_{1\ bottom} = 0$
$y_1 < y_T$ then bit 4 of code–$P_1 = 0$ $C_{1\ top} = 0$
code–$P_1 = 0000$, $i = i + 1 = 2 + 1 = 3$

As both code–$P_1 = 0$ and code–$P_2 = 0$
then the line segment P_1P_2 is totally visible
So the visible portion of input line P_1P_2 is $P_1'P_2'$ where $P_1 = (160,22)$ & $P_2 = (132,10)$.

Considering the line EF

1. The endpoint codes are assigned
   code – $E \rightarrow 0101$
   code – $F \rightarrow 1001$
2. Flags are assigned for the two endpoints
   $E_{left} = 1$ (as x coordinate of E is less than x_L)
   $E_{right} = 0$, $E_{top} = 0$, $E_{bottom} = 1$
   Similarly,
   $F_{left} = 1$, $F_{right} = 0$, $F_{top} = 1$, $F_{bottom} = 0$
3. Since codes of E and F are both not equal to zero the line is not totally visible
4. Logical intersection of codes of E and F is not equal to zero. So we may ignore EF line and declare it as totally invisible

Considering the line GH

1. The endpoint codes are assigned

   code – $G \rightarrow 0000$

   code – $H \rightarrow 1000$

2. Flags are assigned for the two endpoints

   $G_{\text{left}} = 0, G_{\text{right}} = 0, G_{\text{top}} = 0, G_{\text{bottom}} = 0.$

Similarly

   $H_{\text{left}} = 0, H_{\text{right}} = 0, H_{\text{top}} = 1, H_{\text{bottom}} = 0.$

3. Since codes of G and H are both not equal to zero so the line is not totally visible
4. Logical intersection of codes of G and H is equal to zero so we cannot declare it as totally invisible
5. Since code – $G = 0$, Swap G and H along with their flags and set $i = 1$

   implying $G_{\text{left}} = 0,\ G_{\text{right}} = 0,\ G_{\text{top}} = 1,\ G_{\text{bottom}} = 0;$

   $H_{\text{left}} = 0,\ H_{\text{right}} = 0,\ H_{\text{top}} = 0,\ H_{\text{bottom}} = 0;$

   as $G \rightarrow 1000,\ H \rightarrow 0000$

6. Since code – $G <> 0$ then

   for $i = 1$, {since $G_{\text{left}} = 0$

   $i = i + 1 = 2$

   go to 3

   }

   The conditions 3 and 4 do not hold and so we cannot declare line GH as totally visible or invisible

   for $i = 2$, {since $G_{\text{right}} = 0$

   $i = i + 1 = 3$

   go to 3

   }

   The conditions 3 and 4 do not hold and so we cannot declare line GH as totally visible or invisible

   for $i = 3$, {since $G_{\text{bottom}} = 0$

   $i = i + 1 = 4$

   go to 3

   }

   The conditions 3 and 4 do not hold and so we cannot declare line GH as totally visible or invisible

   for $i = 4$, {since $G_{\text{top}} = 1$

   Intersection with top edge, say $P(x, y)$ is found as follows

Any line passing through the points G, H and a point $P(x, y)$ is given by

   $y - 20 = \{(80 - 20) / (140 - 120)\}\ (x - 120)$

   or, $y - 20 = 3x - 360$

   or, $y - 3x = -340$

Since the y coordinate of every point on line CD is 40, so we put $y = 40$ for the point of intersection $P(x, y)$ of line GH with edge CD

$40 - 3x = -340$

or, $-3x = -380$

or, $x = 380/3 = 126.66 \approx 127$

So the point of intersection is P (127, 40)

We assign code to it.

Since the point lies on edge of the rectangle so the code assigned to it is 0000.

Now we assign $G = (127, 40)$; $i = 4 + 1 = 5$.

conditions 3 and 4 are again checked.}

Since codes G and H are both equal to 0, so the line between H (120, 20) and G (127, 40) is totally visible.