4.1 Derive the transformation that rotates an object point $\theta°$ about the origin. Write matrix representation for this rotation.

Refer to Fig. 4.14. Definition of the trigonometric functions sin and cos yields

$$x' = r\cos(\theta + \phi) \qquad y' = r\sin(\phi + \theta) \text{ and}$$
$$x = r\cos\theta \qquad y = r\sin\phi$$

Using trigonometric identities, we obtain

$$r\cos(\theta + \phi) = r(\cos\theta\cos\phi - \sin\phi\sin\theta) = x\cos\theta - y\sin\theta$$

and

$$r \sin(\theta + \phi) = r(\sin\theta \cos\phi + \cos\theta \sin\phi)$$
$$= x \sin\theta - y \cos\theta$$

or

$$x' = x\cos\theta - y\sin\theta \quad y' = x\sin\theta + y\cos\theta$$

Writing $P' = (x', y')$, $P = (x, y)$, and

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

We can now write $P' = P \cdot R_\theta$

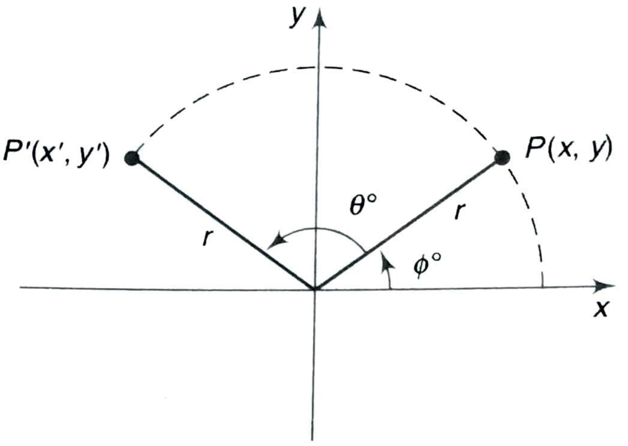

Fig. 4.14

4.2 **(a)** Find the matrix that represents rotation of an object by 30° about the origin.
(b) What are the new coordinates of the point $P(2, -4)$ after the rotation?

(a) From Problem 4.1:

$$R_{30^\circ} = \begin{pmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(b) The new coordinates can be found by multiplying:

$$(2 \quad -4)\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \left(\sqrt{3}+2 \quad 1-2\sqrt{3}\right)$$

4.3 Describe the transformation that rotates an object point, $Q(x, y)$, θ° about a fixed center of rotation $P(h, k)$ (Fig. 4.15).

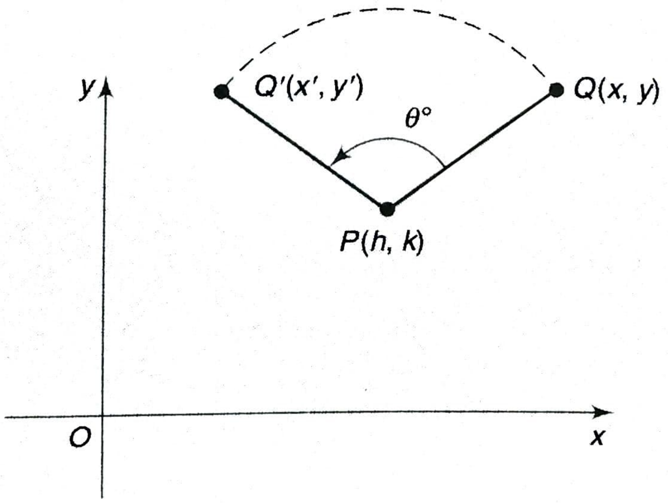

Fig. 4.15

We determine the transformation $R_{\theta,P}$ in three steps:

1. Translate so that the center of rotation P is at the origin,
2. Perform a rotation of θ degrees about the origin, and
3. Translate P back to (h, k).

Using $\mathbf{v} = h\mathbf{I} + k\mathbf{J}$ as the translation vector, we build $R_{\theta,P}$ by composition of transformations:

$$R_{\theta,0} = T_{-v} \cdot R_\theta \cdot T_v$$

4.4 Write the general form of the matrix for rotation about a point $P(h, k)$.

Following Solved Problem. 4.3, we write $R_{\theta,P} = T_{-v} \cdot R_\theta \cdot T_v$, where $\mathbf{V} = h\mathbf{I} + k\mathbf{J}$. Using the 3×3 homogeneous coordinate form for the rotation and translation matrices, we have

$$R_{\theta,P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -h & -k & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h & k & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -h\cos\theta + k\sin\theta + h & -h\sin\theta - k\cos\theta + k & 1 \end{pmatrix}$$

4.5 Perform a 45° rotation of triangle $A(0,0)$, $B(1, 1)$, $C(5, 2)$

(a) about the origin and
(b) about $P(-1, -1)$.

We represent the triangle by a matrix formed from the homogeneous coordinates of the vertices:

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}$$

(a) The matrix of rotation is

$$R_{45^\circ} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So the coordinates $A'B'C'$ of the rotated triangle ABC can be found as

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot R_{45^\circ} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{array}{c} A' \\ B' \\ C' \end{array} \begin{pmatrix} 0 & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ \frac{3\sqrt{2}}{2} & \frac{7\sqrt{2}}{2} & 1 \end{pmatrix}$$

Thus $A' = (0, 0)$, $B' = (0, \sqrt{2})$, and $C' = \left(\frac{3}{2}\sqrt{2}, \frac{7}{2}\sqrt{2}\right)$

(b) From Problem 4.4, the rotation matrix is given by $R_{45^\circ, P} = T_{-V} \cdot R_{45^\circ} \cdot T_V$, where $\mathbf{V} = -\mathbf{I} - \mathbf{J}$. So

$$R_{45^\circ, P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -1 & (\sqrt{2}-1) & 1 \end{pmatrix}$$

Now

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot R_{45^\circ, P} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -1 & (\sqrt{2}-1) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & (\sqrt{2}-1) & 1 \\ -1 & (2\sqrt{2}-1) & 1 \\ \left(\frac{3}{2}\sqrt{2}-1\right) & \left(\frac{9}{2}\sqrt{2}-1\right) & 1 \end{pmatrix}$$

So $A' = (-1, \sqrt{2} - 1)$, $B' = (-1, 2\sqrt{2} - 1)$, and $C' = \left(\frac{3}{2}\sqrt{2}-1, \frac{9}{2}\sqrt{2}-1\right)$

4.6 Find the transformation that scales (with respect to the origin) by

(a) a units in the X-direction,
(b) b units in the Y-direction, and
(c) simultaneously a units in the X-direction and b units in the Y-direction.

(a) The scaling transformation applied to a point $P(x, y)$ produces the point (ax, y). We can write this in matrix form as $P \cdot S_{a,1}$ or

$$(x \quad y)\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} = (ax \quad y)$$

(b) As in part (a), the required transformation can be written in matrix form as $P \cdot S_{1,b}$. So

$$(x \quad y)\begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix} = (x \quad by)$$

(c) Scaling in both directions is described by the transformation $x' = ax$ and $y' = by$. Writing this in matrix form as $P \cdot S_{a\ b}$, we have

$$(x \quad y)\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = (ax \quad by)$$

4.7 **Write the general form of a scaling matrix with respect to a fixed point $P(h, k)$.**

Following the same general procedure as in Problems 4.3 and 4.4, we write the required transformation with $\mathbf{v} = h\mathbf{I} + k\mathbf{J}$ as

$$S_{a,b,P} = T_{-v} \cdot S_{a,b} \cdot T_v$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -h & -k & 1 \end{pmatrix}\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h & k & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ -ah + h & -bk + k & 1 \end{pmatrix}$$

4.8 **Magnify the triangle with vertices A (0, 0), B (1, 1), and C (5, 2) to twice its size while keeping $C(5, 2)$ fixed.**

From Problem. 4.7, we can write the required transformation with $\mathbf{v} = 5\mathbf{I} + 2\mathbf{J}$ as

$$S_{2,2,C} = T_{-v} \cdot S_{2,2} \cdot T_v$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{pmatrix}$$

Representing a point P with coordinates (x, y) by the row vector $(x \quad y \quad 1)$, we have

$$A \cdot S_{2,2,C} = (0 \quad 0 \quad 1)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{pmatrix} = (-5 \quad -2 \quad 1)$$

$$B \cdot S_{2,2,C} = (1 \quad 1 \quad 1)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{pmatrix} = (-3 \quad 0 \quad 1)$$

$$C \cdot S_{2,2,C} = (5 \quad 2 \quad 1)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{pmatrix} = (5 \quad 2 \quad 1)$$

So $\bar{A}' = (-5, -2)$, $\bar{B}' = (-3, 0)$, and $\bar{C}' = (5, 2)$. Note that, since the triangle ABC is completely determined by its vertices, we could have saved much writing by representing the vertices using a 3×3 matrix

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}$$

and applying $S_{2,2,C}$ this. So

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} S_{2,2,C} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{pmatrix} = \begin{pmatrix} -5 & -2 & 1 \\ -3 & 0 & 1 \\ 5 & 2 & 1 \end{pmatrix} = \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}$$

4.9 Describe the transformation M_L which reflects an object about a line L.

Let line L in Fig. 4.16 have ay intercept $(0, b)$ and an angle of inclination $\theta°$ (with respect to the x axis). We reduce the description to known transformations:

1. Translate the intersection point B to the origin.
2. Rotate by $-\theta°$ so that line L aligns with the x axis.
3. Mirror-reflect about the x axis.
4. Rotate back by $\theta°$.
5. Translate B back to $(0, b)$.

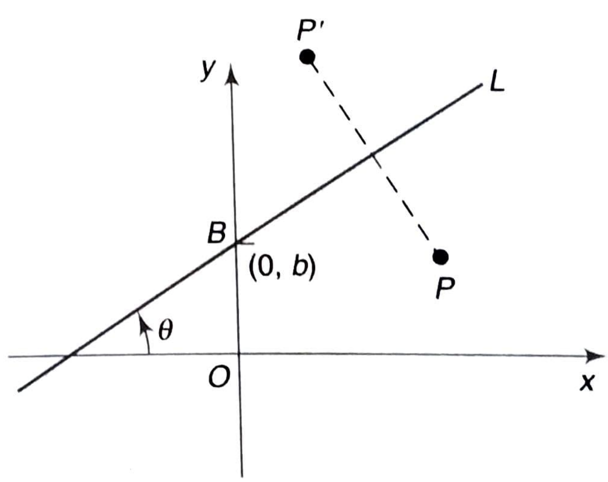

Fig. 4.16

In transformation notation, we have

$$M_L = T_{-v}\, R_{-\theta}\, M_x\, R_\theta\, T_v$$

where $\mathbf{v} = b\mathbf{J}$

4.10 Find the form of the matrix for reflection about a line L with slope m and y intercept $(0, b)$.

Following Solved Problem 4.9 and applying the fact that the angle of inclination of a line is related to its slope m by the equation $\tan\theta = m$, we have with $\mathbf{v} = b\mathbf{J}$,

$$M_L = T_{-v}\, R_{-\theta}\, M_x\, R_\theta\, T_v$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -b & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & b & 1 \end{pmatrix}$$

Now if $\tan\theta = m$, standard trigonometry yields $\sin\theta = m/\sqrt{m^2+1}$ and $\cos\theta = 1/\sqrt{m^2+1}$. Substituting these values for $\sin\theta$ and $\cos\theta$ after matrix multiplication, we have

$$M_L = \begin{pmatrix} \dfrac{1-m^2}{m^2+1} & \dfrac{2m}{m^2+1} & 0 \\ \dfrac{2m}{m^2+1} & \dfrac{m^2-1}{m^2+1} & 0 \\ \dfrac{-2bm}{m^2+1} & \dfrac{2b}{m^2+1} & 1 \end{pmatrix}$$

.11 Reflect the diamond-shaped polygon whose vertices are $A(-1, 0)$, $B(0, -2)$, $C(1, 0)$, and $D\ (0, 2)$ about (a) the horizontal line $y = 2$, (b) the vertical line $x = 2$, and (c) the line $y = x + 2$.

We represent the vertices of the polygon by the homogeneous coordinate matrix

$$V = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

From Solved Problem 4.9, the reflection matrix can be written as

$$M_L = T_{-v}\, R_{-\theta}\, M_x\, R_\theta\, T_V$$

(a) The line $y = 2$ has y intercept $(0, 2)$ and makes an angle of 0° with the x axis. So with $\theta = 0$ and $v = 2\mathbf{J}$, the transformation matrix is

$$M_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

This same matrix could have been obtained directly by using the results of Problem 4.10 with slope $m = 0$ and y intercept $b = 2$. To reflect the polygon, we set

$$V \cdot M_L = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 1 \\ 0 & 6 & 1 \\ 1 & 4 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Converting from homogeneous coordinates, $A' = (-1, 4)$, $B' = (0, 6)$, $C' = (1, 4)$, and $D' = (0, 2)$.

(b) The vertical line $x = 2$ has no y intercept and an infinite slope. We can use M_y, reflection about the y axis, to write the desired reflection by (1) translating the given line two units over to the y axis, (2) reflect about the y axis, and (3) translate back two units. So with $\mathbf{v} = 2\mathbf{I}$,

Finally

$$M_L = T_{-V}\, M_y\, T_v$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

Finally

$$V \cdot M_L = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 1 \\ 4 & -2 & 1 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

or $A' = (5, 0)$, $B' = (4, -2)$, $C' = (3, 0)$, and $D' = (4, 2)$.

(c) The line $y = x + 2$ has slope 1 and a y intercept $(0, 2)$. From Solved Problem 4.10, with $m = 1$ and $b = 2$, we find

$$M_L = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{pmatrix}$$

The required coordinates A', B', C', and D' can now be found.

$$V \cdot M_L = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 1 \\ -4 & 2 & 1 \\ -2 & 3 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

So $A' = (-2, 1)$, $B' = (-4, 2)$, $C = (-2, 3)$, and $D' = (0, 2)$.

4.12 The matrix $\begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}$ defines a transformation called a *simultaneous shearing* or *shearing* for short.

The special case when $b = 0$ is called *shearing in the x direction*. When $a = 0$, we have *shearing in the y direction*. Illustrate the effect of these shearing transformations on the square $A(0, 0)$, $B(1, 0)$, $C(1, 1)$, and $D(0, 1)$ when $a = 2$ and $b = 3$.

Figure 4.17(a) shows the original square, Fig. 4.17(b) shows shearing in the x direction, Fig. 4.17(c) shows shearing in the y direction, and Fig. 4.17(d) shows shearing in both directions.

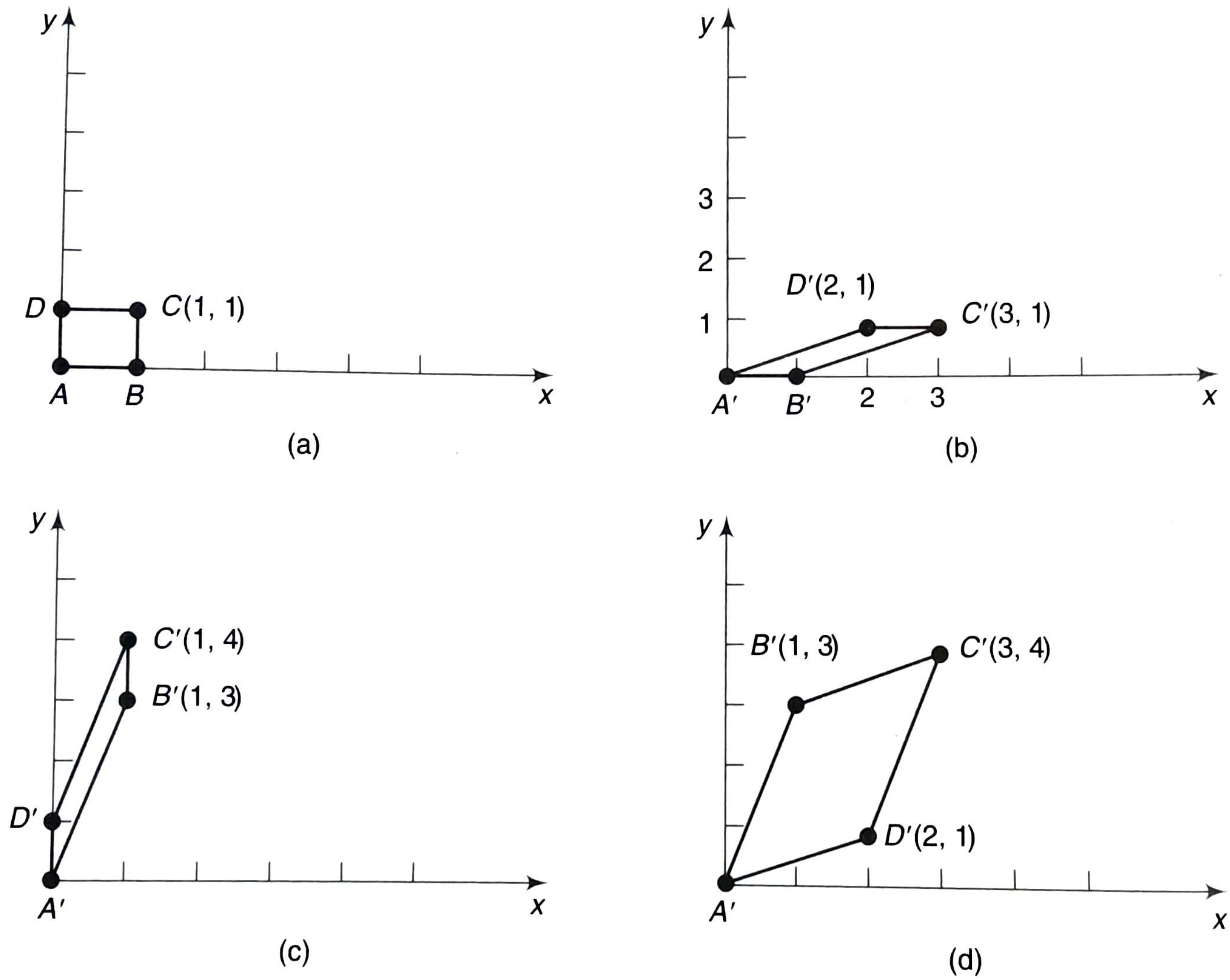

Fig. 4.17

4.13 An observer standing at the origin sees a point $P(1, 1)$. If the point is translated one unit in the direction $\mathbf{v} = \mathbf{I}$, its new coordinate position is $P'(2, 1)$. Suppose instead that the observer stepped back one unit along the x axis. What would be the apparent coordinates of P with respect to the observer?

The problem can be set up as a transformation of coordinate systems. If we translate the origin O in the direction $\mathbf{v} = -\mathbf{I}$ (to a new position at C) the coordinates of P in this system can be found by the translation $\bar{T}_v$

$$P \cdot T_V = (1 \quad 1 \quad 1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = (2 \quad 1 \quad 1)$$

So the new coordinates are $(2, 1)'$. This has the following interpretation: a displacement of one unit in a given direction can be achieved by either moving the object forward or stepping back from it.

4.14 **An object is defined with respect to a coordinate system whose units are measured in feet. If an observer's coordinate system uses inches as the basic unit, what is the coordinate transformation used to describe object coordinates in the observer's coordinate system?**

Since there are 12 inches to a foot, the required transformation can be described by a coordinate scaling transformation with $s = \frac{1}{12}$ or

$$\bar{S}_{1/12} = \begin{pmatrix} \frac{1}{12} & 0 \\ 0 & \frac{1}{12} \end{pmatrix}^{-1} = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$$

and so

$$(x \quad y) \cdot \bar{S}_{1/12} = (x \quad y)\begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix} = (12x \quad 12y)$$

4.15 **Find the equation of the circle $(x')^2 + (y')^2 = 1$ in terms of xy coordinates, assuming that the $x'y'$ coordinate system results from a scaling of a units in the x direction and b units in the y direction.**

From the equations for a coordinate scaling transformation, we find

$$x' = \frac{1}{a}x \qquad y' = \frac{1}{b}y$$

Substituting, we have

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Notice that as a result of scaling, the equation of the circle is transformed to the equation of an ellipse in the xy coordinate system.

4.16 **Find the equation of the line $y' = mx' + b$ in xy coordinates if the $x'y'$ coordinate system results from a $90°$ rotation of the xy coordinate system.**

The rotation coordinate transformation equations can be written as

$$x' = x \cos 90° + y \sin 90° = y \qquad y' = -x \sin 90° + y \cos 90° = -x$$

Substituting, we find $-x = my + b$. Solving for y, we have $y = (-1/m)x - b/m$.

4.17 Find the instance transformation which places a half-size copy of the square $A(0, 0)$, $B(1, 0)$, $C(1, 1)$, $D(0, 1)$ [Fig. 4.18(a)] into a master picture coordinate system so that the center of the square is at $(-1, -1)$ [Fig. 4.18(b)].

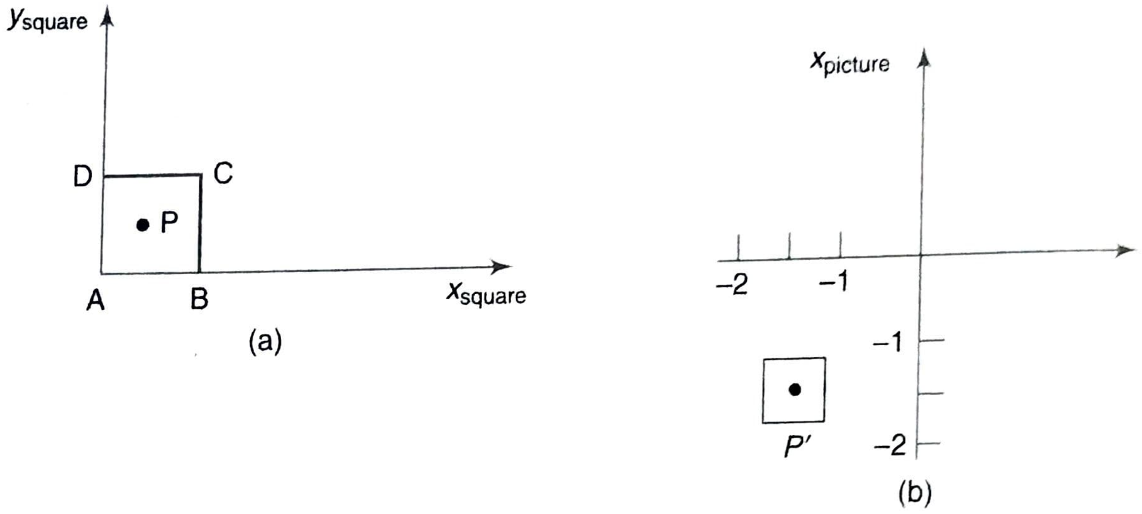

Fig. 4.18

The center of the square $ABCD$ is at $P\left(\frac{1}{2}, \frac{1}{2}\right)$. We shall first apply a scaling transformation while keeping P fixed (see Problem 4.7). Then we shall apply a translation that moves the center P to $P'(-1, -1)$. Taking $t_x = (-1) - \frac{1}{2} = -(3/2)$ and similarly $t_y = -(3/2)$ (so $\mathbf{v} = -3/2\ \mathbf{I} - 3/2\ \mathbf{J}$), we obtain

$$N_{\text{picture,object}} = S_{1/2,1/2,\,P} \cdot T_v = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-3}{2} & \frac{-3}{2} & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{-5}{4} & \frac{-5}{4} & 1 \end{pmatrix}$$

4.18 Write the composite transformation that creates the design in Fig. 4.20 from the symbols in Fig. 4.19.

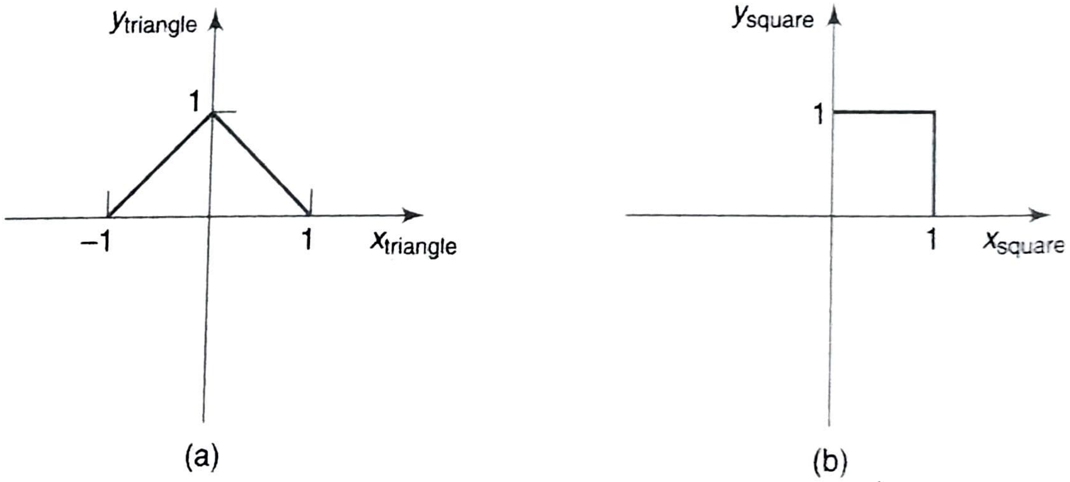

Fig. 4.19

First we create an instance of the triangle [Fig. 4.19(a)] in the square [Fig. 4.19(b)]. Since the base of the triangle must be halved while keeping the height fixed at one unit, the appropriate instance transformation is $N_{\text{square, triangle}} = T_{1/2,\mathbf{I}} \cdot S_{1/2,1}$.

The instance transformation needed to place the square at the desired position in the picture coordinate system (Fig. 4.19) is a translation in the direction $\mathbf{v} = \mathbf{I} + \mathbf{J}$:

$$N_{\text{picture, square}} = T_v$$

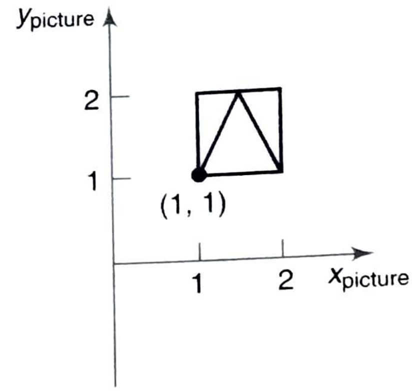

Fig. 4.20

Then the composite transformation for placing the triangle into the picture is

$$C_{\text{picture, triangle}} = N_{\text{square, triangle}} \cdot N_{\text{picture, square}}$$

and the composite transformation to place the square into the picture is

$$C_{\text{picture, square}} = N_{\text{picture, square}}$$

4.19 Write an algorithm for even odd method for polygon inside test.

Suppose the polygon vertices are $P_1(x_1, y_1)$, $P_2(x_1, y_1)$, ... $P_n(x_n, y_n)$ and the point in question is $P(x, y)$

1. Choose a point $Q\ (x_0\ , y_0)$ outside the polygon. This can be done by choosing $x_0 < \min(x_1, x_2, \ldots, x_n)$ or $x_0 > \max(x_1, x_2, \ldots x_n)$ or $y_0 < \min(y_1, y_2, \ldots y_n)$ or $y_0 > \max(y_1, y_2, \ldots y_n)$.
2. Draw a line joining P and Q.
3. Count the number of intersections as follows

   3.1 If the intersection point is a vertex point then check whether the points P_{i+1} and P_{i-1} are on the same side of the line PQ, if so count the number of intersections as two.
   else

   3.2 Count the number of intersections as one
4. Find the total number of intersections in step 3 above.
5. If the total number is even then the point P is outside the polygon otherwise it is inside.

4.20 Write an algorithm for drawing an ellipse using 2D transformations and any circle generating algorithm from chapter 3.

Step 1: Input a, b, x_0, y_0.

Step 2: If $a > b$ then

2.1 Draw the circle $(x - x_0)^2 + (y - y_0)^2 = a^2$

2.2 Scale the circle with scaling factor b/a along y-axis

else

Step 3: Draw the circle with $(x - x_0)^2 + (y - y_0)^2 = b^2$

3.1 Scale the circle with scaling factor b/a along x-axis