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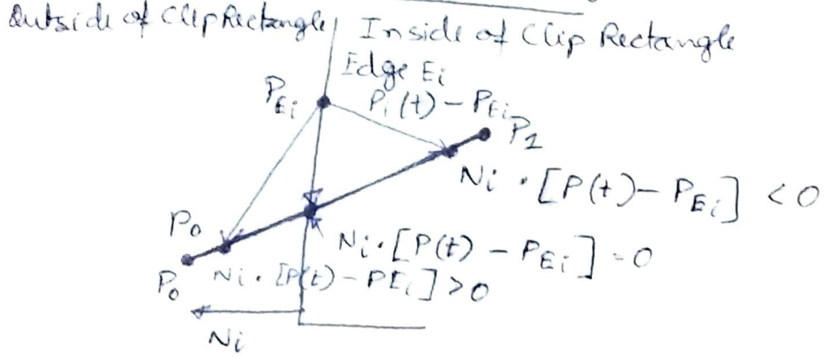
Cyrus-Beck Line Clipping



Based on the formulation of intersection between two lines.

Single edge E_i of clip rectangle and that edge's outward normal is N_i

The line segment from P_0 to P_1 that must be clipped to the edge

Either the edge or the line segment may have to be extended to find the intersection point.

The line is represented parametrically as

$$P(t) = P_0 + (P_1 - P_0)t;$$

where $t = 0$ at P_0 and $t = 1$ at P_1

Pick an arbitrary point P_{E_i} on edge E_i and consider the three vectors ~~from~~ $P(t) - P_{E_i}$ from P_{E_i} to three designated points on the line from P_0 to P_1: the intersection point to be determined, an endpoint of the line on the inside halfplane of the edge, and an endpoint on the line in the outside halfplane of the edge.

Which regions the point lie?

check value of $N_i \cdot [P(t) - P_{E_i}]$

< 0 for a point inside the halfplane

$= 0$ on the line containing the edge

> 0 for a point outside halfplane.

Solve for the value of t at the intersection of $P_0 P_1$ with the edge.

$$N_i \cdot [P(t) - P_{E_i}] = 0$$

$$N_i \cdot [P_0 + (P_1 - P_0)t - P_{E_i}] = 0$$

$$N_i \cdot [P_0 - P_{E_i}] + N_i \cdot [P_1 - P_0]t = 0$$

Let $D = (P_1 - P_0)$ be the vector from P_0 to P_1

$$\therefore t = \frac{N_i [P_0 - P_{E_i}]}{-N_i \cdot D}$$

denominator is nonzero

t has a valid value.

$N_i \neq 0$

$D \neq 0$ (as $P_1 \neq P_0$)

$\therefore N_i \cdot D \neq 0$ (as E_i & $P_0 P_1$ are not parallel)

If parallel → next case

Normal is determined

arbitrary P_{E_i} – say, an endpoint of the edge.

– for each clip edge.

– using this values for all segments

Given the four values of t for a line segment, the next step is to determine which of the values correspond to internal intersections of the line segment with edges of the clip rectangle.

Any value of t outside the interval [0, 1] can be discarded, since it lies outside $P_0 P_1$.

Next, determine whether the intersection lies on the clip boundary.

Potentially entering or potentially leaving the clip rectangle.

If moving from P_0 to P_1 causes us to cross a particular edge to enter edge's inside halfplane, the intersection is calculated, if it causes us to leave the edge's inside halfplane, the intersection is calculated.

$N_i \cdot D < 0 \Rightarrow PE$ (angle > 90) largest $t \Rightarrow t_E$

$N_i \cdot D > 0 \Rightarrow PL$ (angle < 90) smallest $t \Rightarrow t_L$

If $t_E > t_L \Rightarrow$ no portion of P_0P_1 is within the clip rectangle. Entire line is rejected.

if $dx > 0$ line moves from L to R
PE for left edge
PL for right edge

signed distances carry information.

Clip edge	Normal N_i	P_{Ei}	$P_0 - P_{Ei}$	$t = \frac{N_i \cdot (P_0 - P_{Ei})}{-N_i \cdot D}$
left : $x = x_{min}$	$(-1, 0)$	(x_{min}, y)	$(x_0 - x_{min}, y_0 - y)$	$\frac{-(x_0 - x_{min})}{(x_1 - x_0)}$
right : $x = x_{max}$	$(1, 0)$	(x_{max}, y)	$(x_0 - x_{max}, y_0 - y)$	$\frac{(x_0 - x_{max})}{-(x_1 - x_0)}$
bottom : $y = y_{min}$	$(0, -1)$	(x, y_{min})	$(x_0 - x, y_0 - y_{min})$	$\frac{-(y_0 - y_{min})}{(y_1 - y_0)}$
top : $y = y_{max}$	$(0, 1)$	(x, y_{max})	$(x_0 - x, y_0 - y_{max})$	$\frac{(y_0 - y_{max})}{-(y_1 - y_0)}$