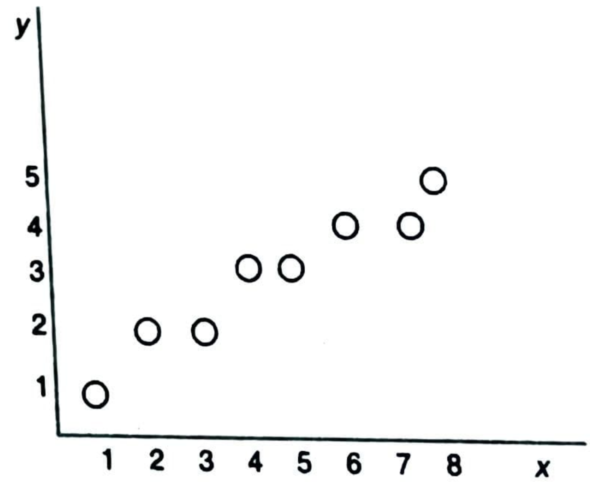
The following table shows the values–

d	x	y
1	1	1
$1 + inc_2 = -5$	2	2
$-5 + inc_2 = 3$	3	2
$3 + inc_2 = -3$	4	3
$-3 + inc_2 = 5$	5	3
$5 + inc_2 = -1$	6	4
$-1 + inc_1 = 7$	7	4
$7 + inc_2 = 1$	8	5



Point Plotting

Example 6. Consider a line from (0, 0) to (6, 7). Use Bresenham's algorithm to rasterize the line.

Solution. Given : $(x_1, y_1) = (0, 0)$

$(x_2, y_2) = (6, 7)$

$\therefore \Delta x = |x_2 - x_1| = 6$

$\Delta y = |y_2 - y_1| = 7$

$\therefore$ $\Delta y > \Delta x$, hence exchange Δx & Δy

Now $\Delta x = 7$ [After exchange]

$\Delta y = 6$

Plot (0, 0)

$$e = 2\,\Delta y - \Delta x$$
$$= 2\,(6) - 7$$
$$= 12 - 7$$
$$e = 5$$

if $e > 0$ then

$$x = x + S_1 \quad [S_1 = 1, S_2 = 1]$$
$$= 0 + 1 = 1$$

and

$$e = e - 2 * \Delta x$$
$$= 5 - 2 * 7$$
$$= 5 - 14$$
$$= -9$$

$\therefore$ $e < 0$, so exit while-loop.

Here exchange is made *i.e*, Δx and Δy are exchanged. So,

$$y = y + S_2$$
$$= 0 + 1 = 1$$
$$\therefore y = 1$$

and

$$e = e + 2\Delta y = -9 + 2\,(6) = -9 + 12 = 3$$
$$\therefore e = 3$$

Now plot (x, y) *i.e*, (1, 1)

$$\because e = 3 \geq 0$$
$$\therefore x = x + S_1 = 1 + 1 = 2$$
$$e = e - 2\,\Delta x = 3 - 2\,(7) = 3 - 14 = -11$$
$$y = y + S_2 = 1 + 1 = 2$$
$$e = e + 2 * \Delta y$$
$$= -11 + 2\,(6)$$
$$= -11 + 12 = 1$$
$$\therefore e = 1$$

Plot (x, y) *i.e*, (2, 2)

$$\text{Now } e = 1 \geq 0$$
$$\therefore x = 2 + 1 = 3$$
$$e = e - 2\,(\Delta x)$$
$$= 1 - 2\,(7)$$
$$= 1 - 14$$
$$= -13$$
$$y = 2 + 1 = 3$$
$$e = e + 2\,(\Delta y)$$
$$= -13 + 2(6)$$
$$= -13 + 12$$
$$= -1$$
$$\therefore e = -1$$

Plot (x, y) *i.e*, (3, 3)

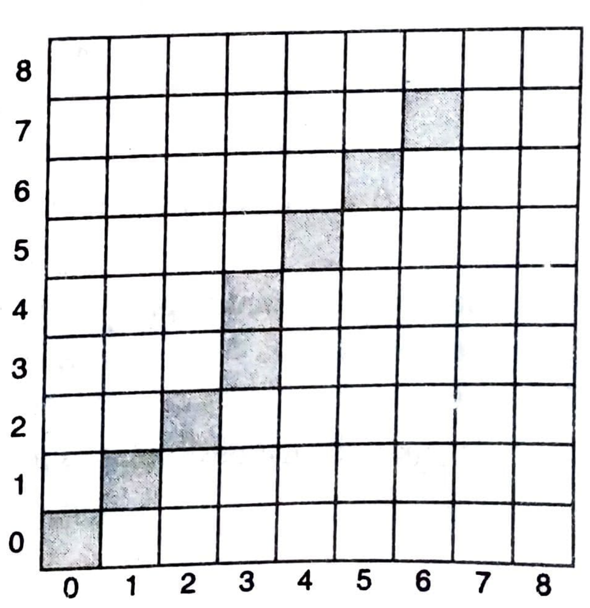
Now

$$e = -1 < 0$$
$$\therefore y = y + S_2$$
$$= 3 + 1 = 4$$
$$e = e + 2\,(\Delta y)$$
$$= -1 + 2\,(6)$$
$$= 11$$

Plot (x, y) *i.e*, (3, 4)

Now

$$e = 11 \text{ i.e., } e \geq 0$$
$$x = 3 + 1 = 4$$
$$e = e - 2 * \Delta x$$
$$= 11 - 2 * 7 = -3$$
$$y = 4 + 1 = 5$$
$$e = e + 2 * \Delta y$$
$$= -3 + 2*6 = 9$$



Plot (x, y) *i.e.*, (4,5)

Now
$$e = 9 \text{ i.e., } e \geq 0$$
$$x = 4 + 1 = 5$$
$$e = e - 2 * \Delta x = 9 - 2*7 = -5$$
$$y = 5 + 1 = 6$$
$$e = e + 2 * \Delta y = -5 + 2*6 = 7$$

Plot (x, y) *i.e.*, (5, 6)

Now
$$e = 7 \text{ i.e., } e \geq 0$$
$$x = 5 + 1 = 6$$
$$e = e - 2 * \Delta x = 7 - 2*7 = -7$$
$$y = 6 + 1 = 7$$
$$e = e + 2 * \Delta y = -7 + 2*6 = 5$$

Plot (6, 7)

Thus the pixels to be plotted are (0, 0) (1, 1) (2, 2) (3, 3) (3, 4) (4, 5) (5, 6) (6, 7).

Example 7. Consider a line from (5,5) to (13,9). Use Bresenham's algorithm to rasterize the line.

Solution. Given :
$$(x_1, y_1) = (5, 5)$$
$$(x_2, y_2) = (13, 9)$$
$$\therefore \Delta x = 13 - 5 = 8$$
$$\Delta y = 9 - 5 = 4$$
$$|m| = \frac{4}{8} = \frac{1}{2} = 0.5 < 1$$
$$p_0 = 2\Delta y - \Delta x = 2*4 - 8 = 0$$

The next point is (6, 6) and
$$p_1 = p_0 + 2\Delta y - 2\Delta x$$
$$= 0 + 2 * 4 - 2 * 8$$
$$= 8 - 16 = -8$$

$\because p_1 < 0$ so the next point to plot is (7, 6)

and
$$p_2 = p_1 + 2\Delta y = -8 + 2 \times 4 = 0$$

So, the next point to plot is (8, 7) and
$$p_3 = p_2 + 2\Delta y - 2\Delta x$$
$$= 0 + 2 \times 4 - 2 \times 8$$
$$= 8 - 16 = -8$$

$\because p_3 < 0$ so the next point to plot is (9, 7) and
$$p_4 = p_3 + 2\Delta y = -8 + 2 \times 4 = 0$$

and the next point to plot is (10, 8) and
$$p_5 = p_4 + 2\Delta y - 2\Delta x$$
$$= 0 + 2 \times 4 - 2 \times 8 = -8$$

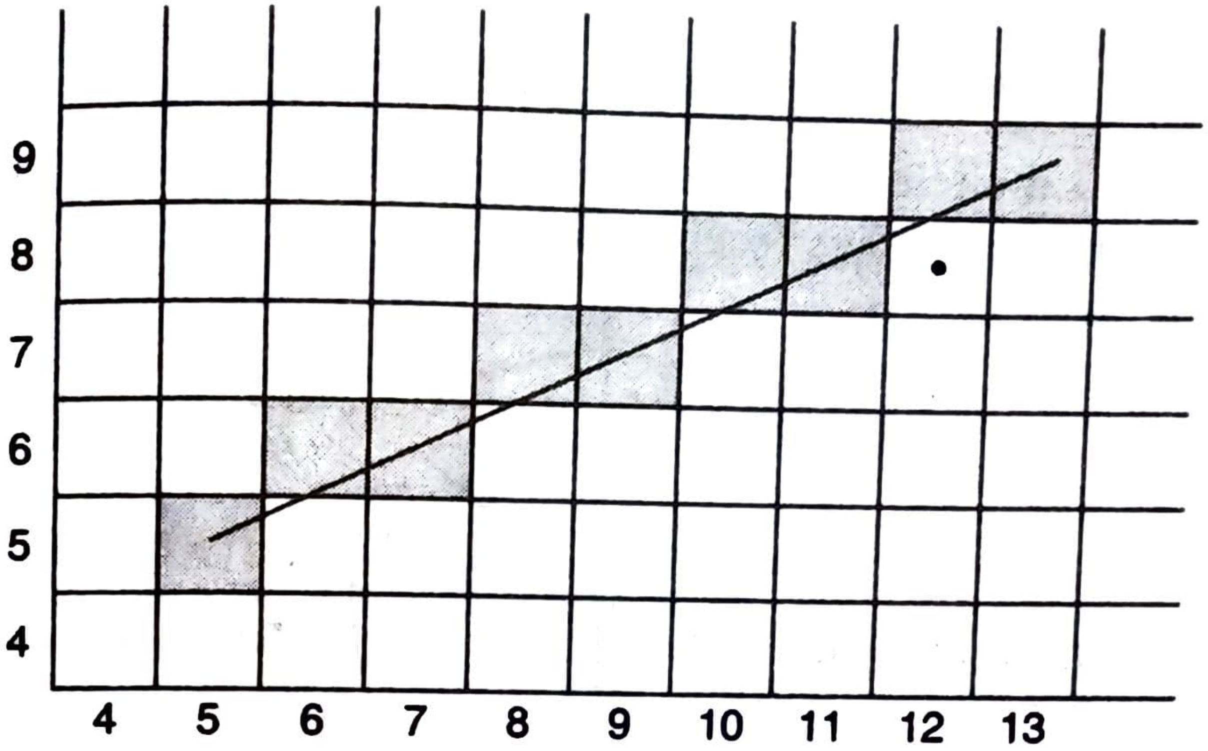
$\because p_5 < 0$, so the next point to plot is (11, 8) and
$$p_6 = p_5 + 2\Delta y$$
$$= -8 + 2 \times 4 = 0$$

and the next point to plot is (12, 9) and

$$p_7 = p_6 + 2\Delta y - 2\Delta x$$
$$= 0 + 2 \times 4 - 2 \times 8$$
$$= 8 - 16 = -8$$

$\because p_7 < 0$ so, the next point to plot is (13, 9)

The results are plotted as shown in figure below



Example 2. Plot a circle at origin having centre as (0,0) and radius = 8.

[GGSIPU, BCA-5th sem., Dec 2001]

Solution.

$$x = 0$$

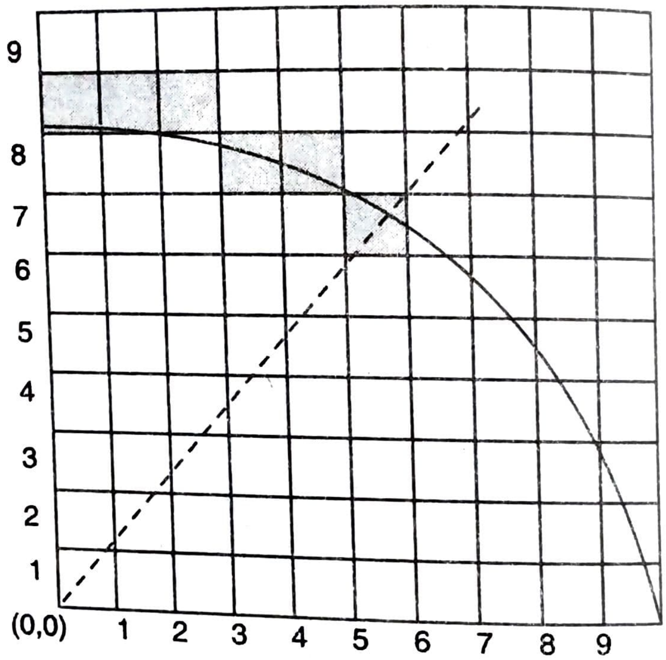
$$y = r = 8$$

$$d = 3 - 2 * r$$

$$\therefore \quad = 3 - 2 * 8 = -13$$

If we plot the points in 2nd octant, we will get–

Point	*x*	*y*	d_i+1
1	0	8	
2	1	8	$-13+4x+6 = -13+4(0)+6 = -7$
3	2	8	$-7+4x+6 = -7+4(1)+6 = 3$
4	3	7	$3+4(x-y)+10 = -11$
5	4	7	$-11+4x+6 = 7$
6	5	6	$-1+4(x-y)+10 = 5$
7	6	5	



Working:

Step 1. Let $(h, k) = (0, 0)$ = center of circle

Set $x = 0$ & $y = r = 8$

We check if $x > y$ *i.e,* $0 > 8$, which is false. So, do not stop and goto step 2.

Step 2. We will now point 8 points

$$i.e, (x+h, y+k) = (0, 8)$$
$$(y+h, x+k) = (8, 0)$$
$$(-y+h, x+k) = (-8, 0)$$
$$(-x+y, y+k) = (0, 8)$$
$$(-x+y, -y+k) = (0, -8)$$
$$(-y+h, -x+k) = (-8, 0)$$
$$(y+h, -x+k) = (8, 0)$$
$$(x+h, -y+k) = (0, -8)$$

Step 3.

$$d = 3 - 2r$$
$$= 3 - 2*8$$
$$= 3 - 16$$
$$= -13$$

$$d = -13 < 0$$

$\because$ $$d_{i+1} = d + 4n + 6$$

$\therefore$ $$= -13 + 4\,(0) + 6$$

$$= -7$$

and $$x = x + 1$$

$$= 0 + 1 = 1$$

Again check if $x > y$ or $1 > 8$, which is false. So, we again plot 8-points while taking $x = 1$

$\Rightarrow x = 1,\ h = 0,\ k = 0,\ y = 8$

$\therefore$ Points to be plotted are as follows –

(1, 8)

(8,1)

(–8, 1)

(–1, 8)

(–1, –8)

(–8, –1)

(8, –1)

(1, –8)

Now $d = -7 < 0$

$$\therefore\ d_{i+1} = d + 4x + 6$$

$$= -7 + 4\,(1) + 6$$

$$= -7 + 4 + 10$$

$$= 3$$

and $$x = x + 1 = 1 + 1 = 2$$

check if $x > y$ *i.e,* $2 > 8$, which is false.

So, again plot 8-points taking $x = 2,\ y = 8, h = 0,\ k = 0$ *i.e.*, these points are

(–8,2)

(–2,8)

(–2,–8)

(–8,–2)

(8,–2)

(2,–8)

Now, $d \geq 0$ *i.e.* $3 \geq 0$

$$\Rightarrow \quad d_{i+1} = d + 4\,(x - y) + 10$$

$$= 3 + 4\,(2-8) + 10$$

$$= 3 + 4\,(-6) + 10$$

$$= 3 - 24 + 10$$

$$= 13 - 24 = -11$$

$$x = x + 1$$

$$= 2 + 1 = 3$$

Now, $x > y$, No. $$y = y - 1 = 8 - 1 = 7$$

$\Rightarrow$ Again plotting 8 points, $x = 3$, $y = 7$, $h = 0$, $k = 0$

(3,7)

(7,3)

(–7,3)

(–3,7)

(–3,–7)

(–7,–3)

(7,–3)

(3,–7)

$$d = -11 < 0$$

$$\begin{aligned} d_{i+1} &= d + 4x + 6 \\ &= -11 + 4(3) + 6 \\ &= -11 + 12 + 6 = 7 \\ x &= x + 1 = 3 + 1 = 4 \end{aligned}$$

Check: If $x > y$ $\rightarrow$which is false

So, again computing 8 points. With $x = 4$, $y = 7$, $h = 0$, $k = 0$

$\Rightarrow$ (4,7)

(7,3)

(–7,4)

(–4,7)

(–4,–7)

(–7,–4)

(7,–4)

(4,–7)

Now, $d \geq 0$ *i.e*, $7 > 0$

$$\begin{aligned} \therefore \quad d_{i+1} &= d + 4\,(x-y) + 10 \\ &= 7 + 4\,(4-8) + 10 \\ &= 7 + 4\,(-4) + 10 \\ &= 7 - 16 + 10 = 1 \end{aligned}$$

$$\begin{aligned} \text{and} \quad x &= x + 1 \\ &= 4 + 1 \\ &= 5 \\ y &= y - 1 \\ &= 7 - 1 \\ &= 6 \end{aligned}$$

Check now if $x > y$ *i.e*, $5 > 6$, which is false

$\therefore$ again plot 8 points with $x = 5$, $y = 6$,

$h = 0$, $k = 0$ *i.e*,

(5,6)

(6,5)

(–6,5)

(–5,6)

(–5,–6)

(–6,–5)

(6,–5)

(5,–6)

Now $d \geq 0$ *i.e*, $d = 1$

$$\therefore d_{i+1} = d + 4(x - y) + 10$$
$$= 1 + 4(5 - 6) + 10$$
$$= 1 + 4(-1) + 10$$
$$= 7$$

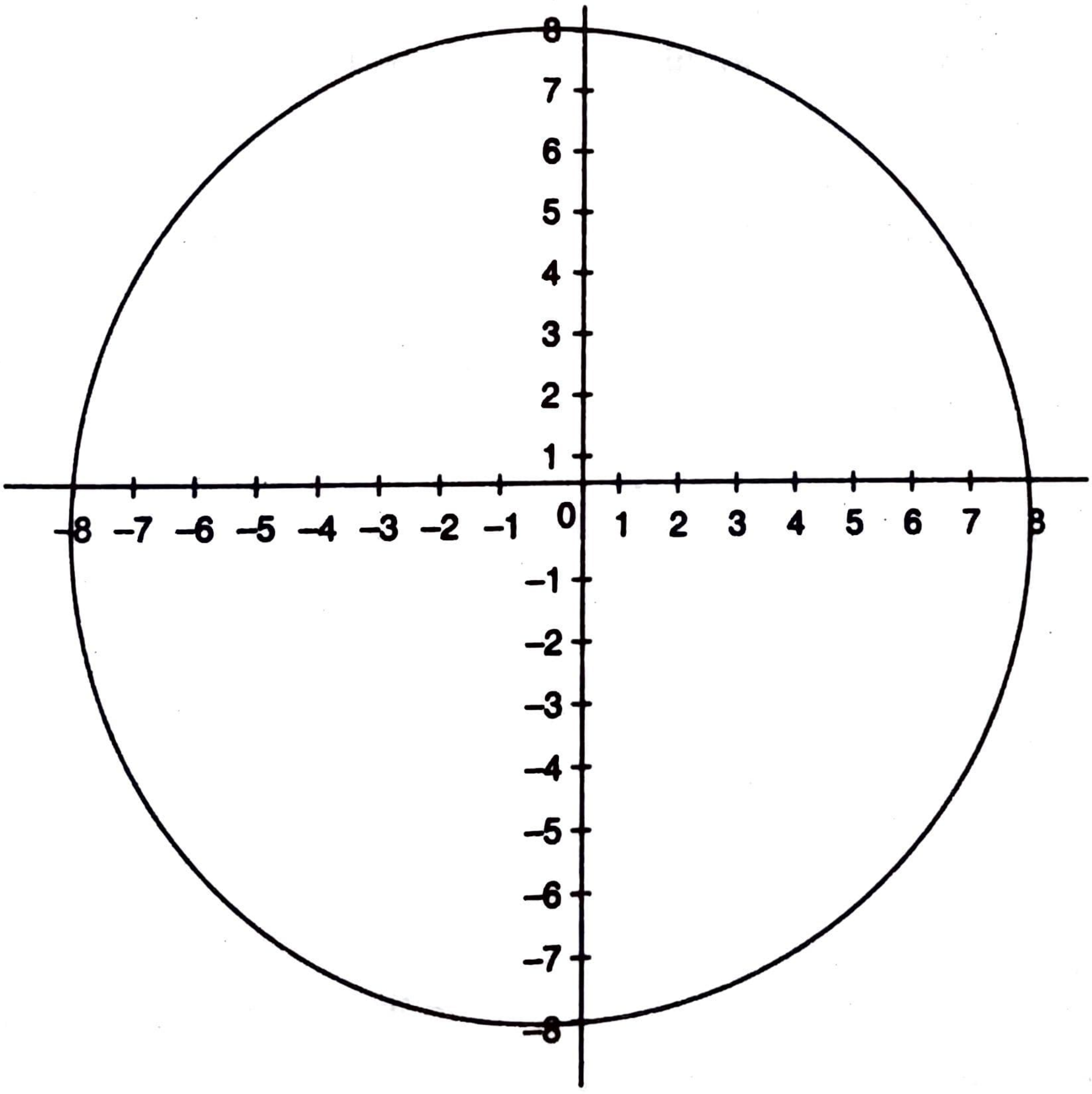
and

$$x = x + 1 = 5 + 1 = 6$$
$$y = y - 1 = 6 - 1 = 5$$

Now, check if $x > y$ *i.e*, $6 > 5$, which is true.

So, we stop now and the points to plot are –

(5, 6)	(–5, 6)	(6, –5)
(6, 5)	(–5, –6)	(5, – 6)
(–6, 5)	(–6, –5)	



Circle at (0, 0) with $r = 8$

Example 1. Plot a circle using mid point algorithm whose radius=3 and center **(0,0).**

Solution. Given :

$$r = 3$$

$$x = 0$$

$$y = 0$$

Plot first point as $(0,r)$ *i.e*, $y = r$ i.e, Plot (0,3).

Now, find out other points from (0,3) using symmetry property *i.e*,

Plot (3,0)

Plot (0,–3)

Plot (–3, 0)

Find

$$d = 1 - r$$

$$= 1 - 3$$

$$= -2$$

Till $x < y$, we perform the following –

if $d < 0$ *i.e.*, $-2 < 0$ then

$$x = x + 1 = 0 + 1 = 1$$

and

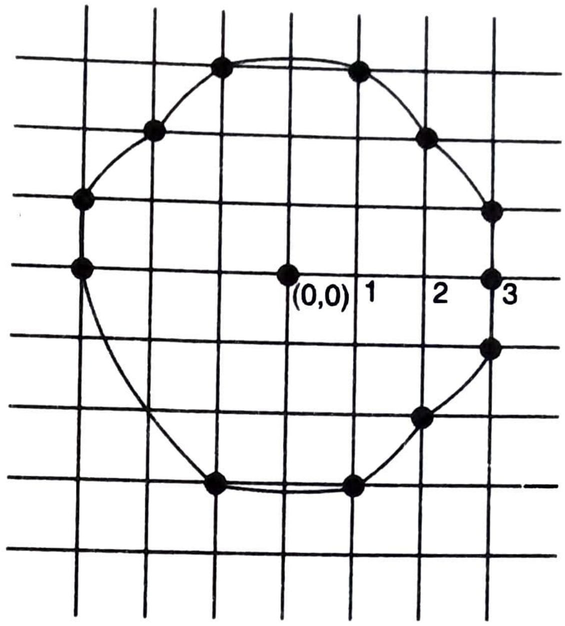
$$d = d + 2x + 1$$

$$= -2 + 2(1) + 1$$

$$= -2 + 2 + 1 = 1$$

Continuing in this way, we get these Coordinates to plot–

x	y
0	3
1	3
2	2
3	1



Please note that by plotting above table we get an arc only. So, to get a complete circle we use symmetry property as explained earlier to get all other points.

Example 2. Plot a circle using mid point circle method whose radius = 8 and center is at (0,0)

Solution. $x = 0$ (given)

$y = r = 8$ (given)

$\therefore \quad d = 1 - r = 1 - 8 = -7$

Now, the next decision parameter and the corresponding points are shown in tabular form–

Point	x	y	d_{i+1}
1	0	8	
2	1	8	$-7 + 2x + 3 = -4$
3	2	8	$-4+2x + 3 = 1$
4	3	7	$1 + 2(x-y) + 5 = -6$
5	4	7	$-6 + 2x + 3 = 3$
6	5	6	$3 + 2(x-y) + 5 = 2$
7	6	5	

Please plot these points yourself.

Example 3. Plot a circle using mid point circle algorithm where radius = 10 units.

Solution. Given:

$r = 10$

Initial point $(x, y) = (0, 10)$

$\therefore \quad d = p = 1 - r = 1 - 10 = -9$

or $\quad p = -9 < 0$

Firstly, we plot (0, 10)

$\because \quad p < 0$ so $x = 0 + 1 = 1$

and $\quad y = 10$

$p = -9 + 2 * 1 + 1$

$= -9 + 3 = -6$

$x < y$ i.e, $1 < 10$. So, condition is true and plot (1,10)

Now $$p = -6 < 0, \text{ so}$$
$$x = 1 + 1 = 2$$
$$y = 10$$
and $$p = -6 + 2*2+1$$
$$= -6+5$$
$$= -1$$

$x < y$ *i.e.* $2<10$, so plot (2, 10)

Now, $$p = -1 < 0, \text{ so}$$
$$x = 2 + 1 = 3$$
$$y = 10$$
$$p = -1 + 2 * 3 + 1 = 6$$

$x < y$ *i.e,* $3<10$, so plot (3, 10)

Now, $$p = 6 > 0, \text{ so}$$
$$x = x + 1 = 3 + 1 = 4$$
$$y = y - 1 = 10 - 1 = 9$$
$$p = p + 2x - 2y + 1$$
$$= 6 + 8 - 18 + 1$$
$$= -3$$

$x < y$, so the next point to plot is (4, 9)

Now $$p = -3 \ (< 0)$$
So $$x = x + 1 = 4 + 1 = 5$$
$$y = y = 9$$
$$p = p + 2x + 1$$
$$= -3 + 10 + 1$$
$$= 8$$

$x < y$, So next point to plot is (5, 9)

Now $$p = 8 \ (> 0)$$
So $$x = x + 1 = 5 + 1 = 6$$
$$y = y - 1 = 9 - 1 = 8$$
$$p = p + 2x - 2y + 1$$
$$= -8 + 12 - 16 + 1$$
$$= 5$$

$x < y$, So plot is (6, 8)

Now $$p = 5 \ (>0)$$
So $$x = x + 1 = 6 + 1 = 7$$
$$y = y - 1 = 8 - 1 = 7$$
$$p = p + 2x - 2y + 1$$
$$= 5 + 14 - 14 + 1$$
$$= 6$$

Plot (7, 7), now stop as x equal to y.

Note. We have to continue the iteration until $x \geq y$ *i.e.*, either $x > y$ or $x = y$.

Example 1. The input ellipse parameters are $r_x = 8$ and $r_y = 6$. Using midpoint ellipse method, rasterize this ellipse.

Solution. Given :

$$r_x = 8$$
$$r_y = 6$$

We will start with region - I

The initial point

for the ellipse

centered on the origin is $(x_0, y_0) = (0, r_y)$

$$i.e,\ (x_0, y_0) = (0, 6)$$

or

$$x_0 = 0$$
$$y_0 = 6$$

Calculating the initial decision parameter,

$$\begin{aligned} p1_0 &= r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 \\ &= (6)^2 - (8)^2.\,6 + \frac{1}{4}(8)^2 \\ &= 36 - 64.6 + \frac{1}{4}.64 \\ &= 36 - 384 + 16 \\ &= 52 - 384 \\ &= -332 \end{aligned}$$

Now check for $p1_k$ $i.e, p1_k < 0$

∴ Next point to plot is (x_k+1, y_k)

i.e. (1, 6)

and

$$\begin{aligned} p1_{k+1} &= p1_k + 2 r_y^2 .\, x_{k+1} + r_y^2 \\ &= -332 + 2(36).1 + 36 \\ &= -224 \end{aligned}$$

Now again we check, $-224 < 0$

∴ Next point to plot is (2, 6)

and

$$\begin{aligned} p1_{k+1} &= -224 + 2\,(36).\,2 + 36 \\ &= -224 + 144 + 36 \\ &= -44 \end{aligned}$$

Now, $-44 < 0$

$\therefore$ Next point is (3, 6)

and

$$pl_{k+1} = -44 + 2(36).3 + 36$$
$$= 208$$

Now, $208 > 0$

$\therefore$ Next point to plot is $(x_{k+1}, y_{k-1}) = (4, 5)$ $p_k + 2 r_y^2 \cdot x_k - 2 \cdot r_x^2 \cdot y_k + r_x^2$

$$\text{and } pl_{k+1} = 208 + 2\ (36).4 - 2\ (64).5 + 64$$
$$= 208 + 288 - 640 + 36$$
$$= -108$$

Again, $-108 < 0$

$\Rightarrow$ Next point is (5, 5)

and

$$pl_{k+1} = -108 + 2\ (36).\ 5 + 36$$
$$= -108 + 360 + 36 = 288$$

Again $288 > 0$

$\Rightarrow$ Next point to plot is (6, 4)

and

$$pl_{k+1} = 288 + 432 + 36 - 512$$
$$= 244 > 0$$

$\Rightarrow$ Next point to plot is (7, 3)

and

$$pl_{k+1} = 244 + 504 - 384 + 36$$
$$= 400$$

But here see that

$$\text{value of } 2\,r_y^2\,x_{k+1} = 504$$

and

$$2\,r_x^2\,y_{k+1} = 384$$

$$\Rightarrow \quad \boxed{2\,r_y^2\,x_{k+1} > 2\,r_x^2\,y_{k+1}}$$

Hence we move out of region 1,

Now calculating for region 2,

For region 2,

$$(x_0, y_0) = (7, 3)$$

$\Rightarrow x_0 = 7,\ y_0 = 3$

For region 2,

$$P2_0 = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2\,(y_0-1)^2 - r_x^2\,r_y^2$$

$$= 36\left(7+\frac{1}{2}\right)^2 + 64\ (3-1)^2 - 64.36$$

$$= 36\left(\frac{15}{2}\right)^2 + 64\ (4) - 64.36$$

$$= 36.\frac{225}{4} + 64.4 - 64.36$$
$$= 9.225 + 64.4 - 64.36$$
$$= 2025 + 256 - 2304$$
$$= -23$$

Now check $-23 < 0$

$\Rightarrow$ Next point to plot is (x_k+1, y_k-1)

i.e $(8, 2)$

and

$$P_{2k+1} = p_{2k} + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$
$$= -23 + 2(36).\ 8 - 2(64).2 + 64$$
$$= -23 + 16(36) - 4\ (64) + 64$$
$$= -23 + 576 - 256 + 64$$
$$= -279 + 640$$
$$= 361$$

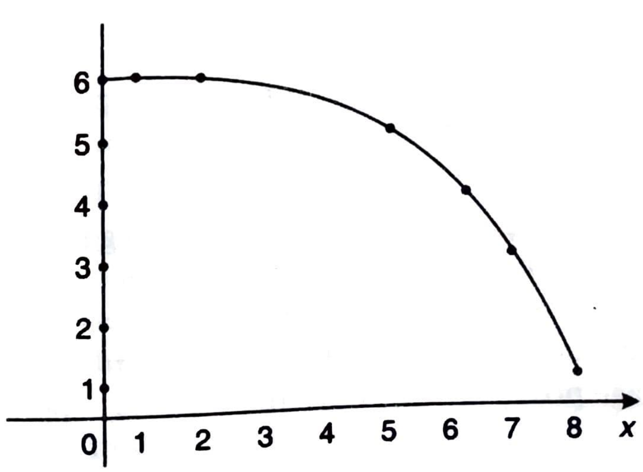
Now, $361 > 0$

$\therefore$ Next point is $(x_k, y_k - 1)$ *i.e.*, $(8, 1)$

$$\text{and } P_{2k+1} = P_{2k} - 2r_y^2 x_{k+1} + r_x^2$$
$$= 361 - 2\ (36).8 + 64$$
$$= 361 - 16\ (36) + 64$$
$$= 361 - 576 + 64$$
$$= -151$$

So, we get the following points to plot –

(0, 6)
(1, 6)
(3, 6)
(5, 5)
(6, 4)
(7, 3)
(8, 1)



The entire ellipse can be obtained by replicating this arc.

Step 4. Repeat step-3 until $x1 < x2$.

Q.2. Calculate the points to draw a circle having radius = 5 and center as (0,0).

Ans. We follow the following steps–

Step 1.

$$\begin{aligned} r &= 5 \\ X &= 0 \\ Y &= r = 5 \\ p &= 3 - 2 * r \\ &= 3 - 2 * 5 = -7 \end{aligned}$$

Step 2. $p < 0$ *i.e.*, $-7 < 0$

$$\begin{aligned} X &= x + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} Y &= Y \\ &= 5 \\ \text{Point is } (X, Y) &= (1, 5) \\ p &= p + 4 * x + 6 \\ &= -7 + 4 * 1 + 6 = 3 \end{aligned}$$

Step 3. $p > 0$ *i.e.*, $3 > 0$

$$\begin{aligned} X &= X + 1 \\ &= 1 + 1 = 2 \\ Y &= Y - 1 \\ &= 5 - 1 = 4 \\ \text{Point is } (X, Y) &= (2, 4) \\ p &= p + 4 * (X - Y) + 10 \\ &= 3 + 4 * (2 - 4) + 10 = 5 \end{aligned}$$

Step 4. $p > 0$ *i.e.*, $5 > 0$

$$\begin{aligned} X &= X + 1 \\ &= 2 + 1 = 3 \\ Y &= Y - 1 \\ &= 4 - 1 = 3 \\ \text{Point is } (X, Y) &= (3, 3) \end{aligned}$$

As X = Y Stop

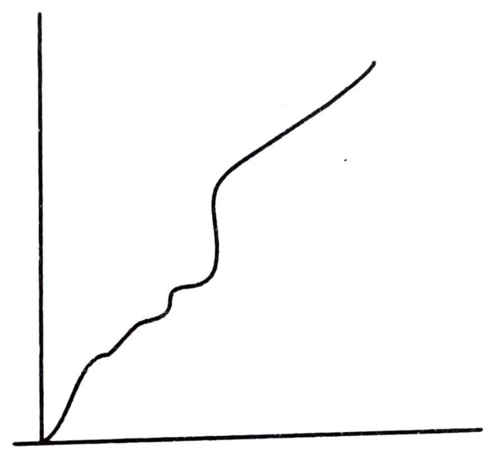
Now using 8-point symmetry we get all the points on the circle as below:

X, Y	0,5	1,5	2,4	3,3
X, – Y	0, –5	1, –5	2, –4	3, –3
– X, – Y	0, –5	–1, –5	–2, –4	–3, –3
– X, Y	0,5	–1,5	–2,4	–3,3
Y, X	5,0	5,1	4,2	3,3
Y, – X	5,0	5, –1	4, –2	3, –3
– Y, – X	–5,0	–5, –1	–4, –2	–3, –3
– Y, X	–5,0	–5,1	–4,2	–3,3

Q. 7. Execute Bresenham's straight line algorithm to produce a line from (0,0) to (17, 2) Use hand sketch]. **[UPTU, B.Tech (CSE/IT)-6th sem:, 2005-06]**

Ans. Bresenham's Straight line algo to produce a line from (0,0) to (17, 12)

D	X	Y
7	0	0
7 + inc 2 = –3	1	1
–3 + inc 1 = 21	2	1
21 + inc 2 = 11	3	2
11 + inc 2 = 1	4	3
1 + inc 2 = –9	5	4
–9 + inc 1 = 15	6	4
15 + inc 2 = 5	7	5
5 + inc 2 = –5	8	6
–5 + inc 1 = 19	9	6
19 + inc 2 = 9	10	7
9 + inc 2 = –1	11	8
–1 + inc 1 = 23	12	8
23 + inc 2 = 13	13	9
13 + inc 2 = 3	14	10
3 + inc 2 = –7	15	11
–7 + inc 1 = 17	16	11
17 + inc 2 = 7	17	12



Rough Sketch

$x1 = 0$ $x2 = 17$ $y1 = 0$ $y2 = 12$

$dx = 17$ $dy = 12$

decision factor $d = 2 * dy - dx = 7$

inc $1 = 2 * dy = 24$

inc $2 = 2 * dy - 2 * dx = -10$

Q.8 W ... specified ellipse into solid colour.