We are in a position to solve some problems now.

Example 1: Use cohen-Sutherland algorithm to clip the line P_1(70, 20) and P_2 (100, 10) against a window lower left hand corner (50, 10) and upper right hand corner (80, 40).

[UPTU, B. Tech (CSE)– 5th sem., 2005-06]

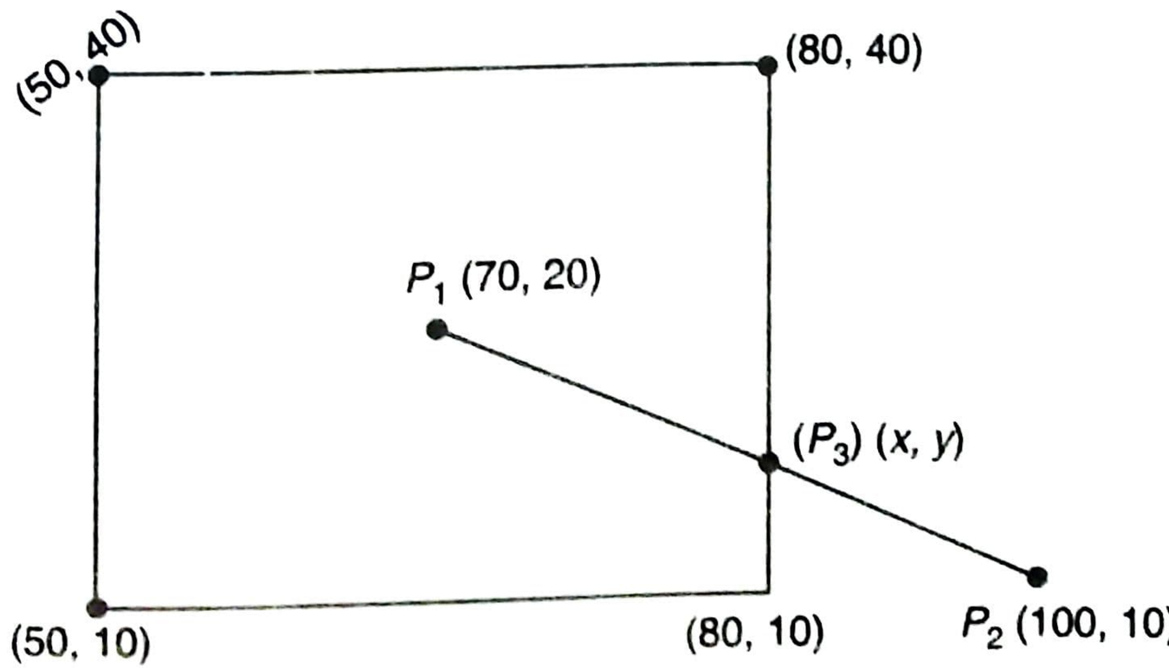
Solution 1. Given: P_1 (70, 20)

P_2 (100, 10)

Window's lower left corner is (50, 10)

Window's upper right corner is (80, 40)

∴ The window must be–



Here, we assign a 4 bit outcode,

$\because$ Point P_1 is inside window, so outcode $(P_1) = 0000$

and as point P_2 is outside window, so its outcode is 0010 (if our 4 bits are TBRL then R = 1, point P_2 is to the right of window).

$$\therefore \quad P_1 \text{ AND } P_2 = \begin{array}{r} 0000 \\ = 0010 \\ \hline 0000 \\ \hline \end{array} = \text{Zero i.e, line is partially visible.}$$

$$\text{Slope } (m) \text{ of line } P_1 P_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 20}{100 - 70} = \frac{-10}{30}$$

$$= \frac{-1}{3}$$

Now, we have to find point of intersection, P_3.

Say, $P_3 (x, y)$

But from figure, $x = 80$

$y =$? (to find)

If we use point $P_2 (x_2, y_2) = P_2 (100, 10)$

then $m = \text{slope of line} = \dfrac{y - y_2}{x - x_2}$

or $$\frac{-1}{3} = \frac{y - 10}{80 - 100}$$

or $$\frac{-1}{3} = \frac{y - 10}{-20}$$

$$\frac{20}{3} = y - 10$$

or $$y = 10 + \frac{20}{3}$$

$$= 10 + 6.667$$

$$y = 16.667$$

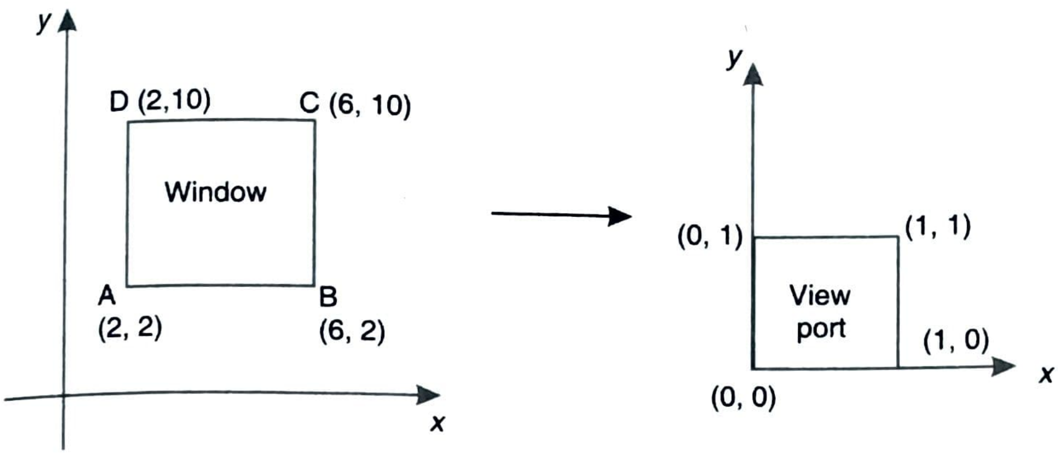
Now, the part $P_2 P_3$ of line $P_1 P_2$ is clipped as it is outside the window.

$$\therefore \quad P_3 (x, y) = P_3 \; (80, 16.667)$$

Example 2. Find the normalization transformation for window to viewport which uses the rectangle whose lower left corner at (2, 2) and upper right corner at (6, 10) as a window and the viewport that has lower left corner at (0, 0) and upper right corner at (1, 1).

[UPTU, B.Tech (CSE 5th sem., 2005-06)]

Solution.



Now,
$$S_x = \frac{\text{Viewport } x\text{-extent}}{\text{Window's } x\text{-extent}} = \frac{1}{6-2} = \frac{1}{4}$$

&
$$S_y = \frac{\text{Viewport } y\text{-extent}}{\text{Window's } y\text{-extent}} = \frac{1}{10-2} = \frac{1}{8}$$

∴ Overall transformation will be

(*a*) **Translate** the window to origin.

(*b*) **Scale** w.r.t. Origin to the required scaling factors (S_x and S_y).

(*c*) Return it to viewport position.

∴ In matrix form we can write–

$$[T_s]_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -xw_{\min} & -yw_{\min} & 1 \end{bmatrix} . \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ xV_{\min} & yV_{\min} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -S_x x w_{\min} + xV_{\min} & -S_y yw_{\min} + yV_{\min} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ -\frac{1}{4}.2+0 & -\frac{1}{8}.2+0 & 1 \end{bmatrix}$$

$$\left(\because S_x = \frac{1}{4}, S_y = \frac{1}{8}, xw_{\min} = 2, xV_{\min} = 0, yw_{\min} = 2, yV_{\min} = 1\right)$$

∴
$$[T] = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & 1 \end{bmatrix}$$

Example 3. Use any line-clipping algorithm to obtain the visible portion of the following line segments –

(*a*) $P_1 = (0.5, 0.4)$ and P_2 (1.6, 0.7)

(*b*) $P_1 = (0.4, -1.6)$ and P_2 (0.4, 2.7)

For window $(x_{umin}, y_{umin}) = (0, 0)$ & $(x_{wmax}, y_{wmax}) = (1, 1)$. **In each case determine the end points of clipped line.**

Solution. The region code for point (*x*, *y*) are as follows–

$$\left.\begin{aligned} \text{Bit } 1 &= \text{sign } (y - y_{wmax}) = \text{Sign } (y - 1) \\ \text{Bit } 2 &= \text{sign } (y_{wmin} - y) = \text{Sign } (-y) \\ \text{Bit } 3 &= \text{sign } (x - x_{wmax}) = \text{Sign } (x - 1) \\ \text{Bit } 4 &= \text{sign } (x_{wmin} - x) = \text{Sign } (-x) \end{aligned}\right] \quad \dots (A)$$

Here, $\text{sign } (a) = \begin{cases} 1 & \text{If } a \text{ is positive} \\ 0 & \text{otherwise} \end{cases}$

We use Cohen – Sutherland's approach here –

(a) $P_1(\overset{x}{0.5}, \overset{y}{0.4})$ and $P_2(\overset{x}{1.6}, \overset{y}{0.7})$

∴ Outcode for P_1 = 0000

Outcode for P_2 = 0010

[Putting P_1 & P_2 coordinates in equation A]

Now, parametric form of the line P_1P_2 is

$$x = x_1 + t\,(x_2 - x_1)$$
$$y = y_1 + t\,(y_2 - y_1) \qquad [0 \le t \le 1]$$

or

$$x = 0.5 + t\,(1.1)$$
$$y = 0.4 + t\,(0.3)$$

Because line P_1P_2 crosses the line $x = 1$

$$\therefore \quad y = y_1 + \left(\frac{a - x_1}{x_2 - x_1}\right)(y_2 - y_1)$$

$$= 0.4 + \left(\frac{1 - 0.5}{1.1}\right)(0.3)$$

$$\therefore \quad y = 0.57$$

Intersection Point, $I = (1, 0.57)$

Now, Region code for point I is 0000

∵ Both points P_1 and I have outcode of 0000, so line is visible and having end points P_1 (0.5, 0.4) and I (1, 0.57).

(b) Again for P_1 (0.4, – 1.6) outcode is 0100 and for P_2 (1.4, 2.7) outcode is 1000.

∵ P_1 AND P_2 = 0100

1000

$\boxed{0000}$

∴ This line is the candidate for clipping. This line intersects two lines $x = 0$ and $y = 1$.

∴ Parametric form of the line P_1P_2 is

$$x = 0.4 + t\,(0) = 0.4$$
$$y = -1.6 + t\,(4.3)$$

Please note that any window is formed with four line segments. Also note that if equation of the candidate line is solvable with equation of any line segment and this intersection point lies between the end points of the line segment then the candidate line crosses the window.

Now, for $y = 0$, $y_I = 0$

$$\therefore \quad y = 0.4 + \left(\frac{0-1.6}{2.7+1.6}\right)(0.4-0.4)$$

$$= 0.4$$

$\therefore$ Intersection point I is (0.4, 0)

Region code for I is 0000 and Region code for P_2 is 1000

$$\therefore \quad \text{I AND } P_2 = \begin{array}{c} 0000 \\ 1000 \\ \hline \boxed{0000} \end{array}$$

As P_2 is outside the window, so we clip P_1I.

Also, line segement, IP_2 intersects the $y = y_{max} = 1$.

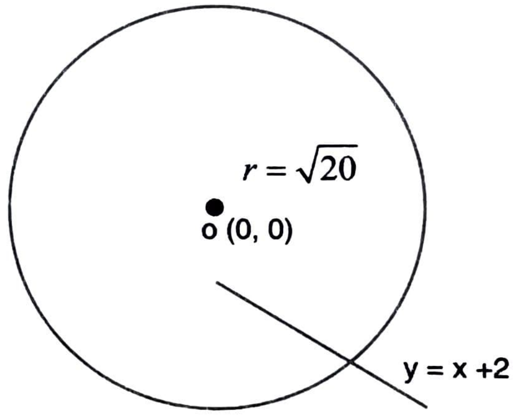
$$\therefore \quad y I_1 = 1$$

$$\text{and} \quad xI_1 = 0.4 + \left(\frac{1-0}{2.7-0}\right)(0.3)$$

$$= 0.4$$

$\therefore$ Region code for intersection point I_1 (0.4, 1) is 0000. So, we clip the line segment I_1P_2. Since both the end points of II_1 have region code of 0000, so, it is completely inside the window and has the end points as (0.4,0) and (0.4, 1).

Example 4. Find the coordinate of the line segment when a line $y = x + 2$ is clipped against a circular window of radius $\sqrt{20}$ and centre at (0, 0).



Solution. The equations of circle, at (x_c, y_c) is –

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

$$(x - 0)^2 + (y - 0)^2 = r^2$$

$$\text{or} \quad x^2 + y^2 = \left(\sqrt{20}\right)^2$$

$$= 20$$

$$\therefore \quad x^2 + y^2 = 20 \qquad \text{... (1)}$$

Now, Equations of line is $y = x + 2$ ... (2)

Put (2) in (1) and we get–

$$x^2 + (x + 2)^2 = 20$$

$$\text{or} \quad x^2 + x^2 + 4x + 4 = 20$$

$$\text{or} \quad 2x^2 + 4x = 20 - 4 = 16$$

$$\text{or} \quad x^2 + 2x = 8$$

$$\text{or} \quad x^2 + 2x - 8 = 0$$

$\therefore$ It is a quadratic equation with roots $x = 2$ or $x = -4$.

$\therefore$ corresponding values of y are –

$$y = x + 2 = 2 + 2 = 4$$

$$\text{and} \quad y = x + 2 = -4 + 2 = -2$$

$\therefore$ Line segment bounded by (2, 4) and (–4, –2) will be visible in the circular window.

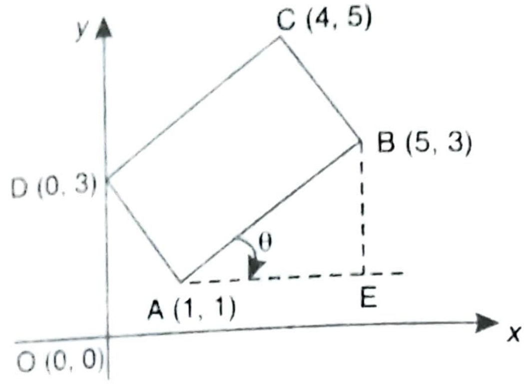
Example 5. Find the normalization transformation N, which uses the rectangle A B (5, 3), C (4, 5) and D (0, 3) as a window onto a normalized device screen as a view where x-extent is from 0 to 1 and y-extent from 0 to 1.

[UPTU, B.Tech (CSE) – 5th sem.,

or

Find the normalization transformation that maps a window whose corners are (10, 6), (8, 10) and (0, 6) onto a viewport which is the entire normalized device screen lower left corners at A.

Solution.



Firstly, we rotate the window (rectangle) clock wise about A so that it is aligned with

$$\sin\theta = \frac{BE}{AB} = \frac{3-1}{\sqrt{(5-1)^2+(3-1)^2}}$$

$$= \frac{2}{\sqrt{4^2+2^2}} = \frac{2}{\sqrt{16+4}} = \frac{2}{\sqrt{20}}$$

$$= \frac{2}{\sqrt{2\times2\times5}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\text{Similarly, } \cos\theta = \frac{AE}{AB} = \frac{5-1}{\sqrt{(5-1)^2+(3-1)^2}}$$

$$= \frac{4}{\sqrt{4^2+2^2}}$$

$$= \frac{4}{\sqrt{16+4}}$$

$$= \frac{4}{\sqrt{20}}$$

$$= \frac{4}{\sqrt{2\times2\times5}}$$

$$= \frac{4}{2\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

But θ is –ve

$$\therefore \quad \sin(-\theta) = -\frac{1}{\sqrt{5}}$$

and $$\cos(-\theta) = \cos\theta = \frac{2}{\sqrt{5}}$$

∴ Rotation matrix about A (1, 1) is given by a matrix–

$$[R_{-\theta}]_A = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \left(1-\frac{3}{\sqrt{5}}\right) \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \left(1-\frac{1}{\sqrt{5}}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

The x-extent of the rotated window is the length of AB = $\sqrt{2^2+4^2} = 2\sqrt{5}$ & the y-extent of rotated window is the length of AD = $\sqrt{1^2+2^2} = \sqrt{5}$

∵ x and y extent of the normalized device screen are 1 then

$$\frac{V_{x\max}-V_{x\min}}{W_{x\max}-W_{x\min}} = \frac{1-0}{2\sqrt{5}-0} = \frac{1}{2\sqrt{5}} \text{ and } \frac{V_{y\max}-V_{y\min}}{W_{y\max}-W_{y\min}} = \frac{1-0}{\sqrt{5}-0} = \frac{1}{\sqrt{5}}$$

∴ $$N = T_{\bar{V}} \cdot S_{sx,sy} \cdot T_{-\bar{V}}$$

$$= \begin{bmatrix} \frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ 0 & 0 & 1 \end{bmatrix}$$

∴ Normalization transformation is –

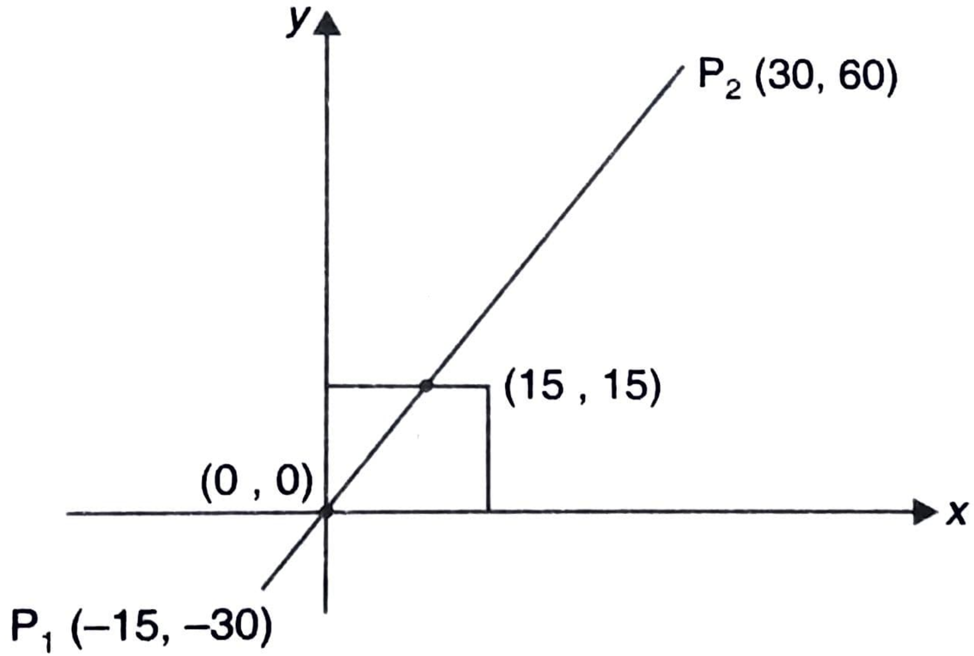
$$[N_R] = [N].[R_{-\theta}]_A$$

$$= \begin{bmatrix} \frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \left(1-\frac{3}{\sqrt{5}}\right) \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \left(1-\frac{1}{\sqrt{5}}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

or $$NR = \begin{bmatrix} \frac{1}{5} & \frac{1}{10} & -\frac{3}{10} \\ -\frac{1}{5} & \frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

We are in a position to solve some problems now.

Example 1. Use Liang-Barsky line clipping algorithm to clip the line P_1 (– 15 , – 30) to P_2 (30, 60) against the window having diagonally opposite corners as (0, 0) and (15, 15).



Solution. Given :

$$P_1 = (-15, -30) = P_1(x_1, y_1)$$
$$P_2 = (30, 60) = P_2(x_2, y_2)$$

Window coordinates are $\left.\begin{aligned} x_{min} &= 0 \\ x_{max} &= 15 \end{aligned}\right\}$

$$\left.\begin{aligned} y_{min} &= 0 \\ y_{max} &= 15 \end{aligned}\right\}$$

$\therefore$
$$dx = 30 - (-15) = 45 \quad \text{(or } \Delta x\text{)}$$
$$dy = 60 - (-30) = 90 \quad \text{(or } \Delta y\text{)}$$

$\therefore$
$$\left.\begin{aligned} p_1 &= -\Delta x = -45 \\ p_2 &= \Delta x = 45 \\ p_3 &= -\Delta y = -90 \\ p_4 &= \Delta y = 90 \end{aligned}\right\}$$

Also,
$$\left.\begin{aligned} q_1 &= x_1 - x_{min} = -15 - 0 = -15 \\ q_2 &= x_{max} - x_1 = 15 - (-15) = 30 \\ q_3 &= y_1 - y_{min} = -30 - 0 = -30 \\ q_4 &= y_{max} - y_1 = 15 - (-30) = 45 \end{aligned}\right\}$$

$\therefore$
$$u_1 = \frac{q_1}{p_1} = \frac{-15}{-45} = \frac{1}{3}$$

$$u_2 = \frac{q_2}{p_2} = \frac{30}{+45} = \frac{2}{3}$$

$$u_3 = \frac{q_3}{p_3} = \frac{-30}{-90} = \frac{1}{3}$$

and
$$u_4 = \frac{q_4}{p_4} = \frac{45}{90} = \frac{1}{2}$$

$\therefore$
$$u_1 = \left(\max\left(\frac{1}{3}, \frac{1}{3}, 0\right)\right) = \frac{1}{3} \qquad \text{(for } p_i < 0\text{)}$$

and
$$u_2 = \left(\min\left(\frac{2}{3}, \frac{1}{2}, 1\right)\right) = \frac{1}{2} \qquad \text{(for } p_i > 0\text{)}$$

$\because$ $u_1 < u_2$ **So there is a visible section.**

$\therefore$ **New endpoints are–**

$$x_1' = x_1 + (\Delta x \times u_1) = -15 + \left(45 \times \frac{1}{3}\right) = -15 + 15 = 0$$

$$y_1' = y_1 + (\Delta y \times u_1) = -30 + \left(90 \times \frac{1}{3}\right) = 0$$

$$x_2' = x_1 + (\Delta x \times u_2) = -15 + \left(45 \times \frac{1}{2}\right) = 7.5$$

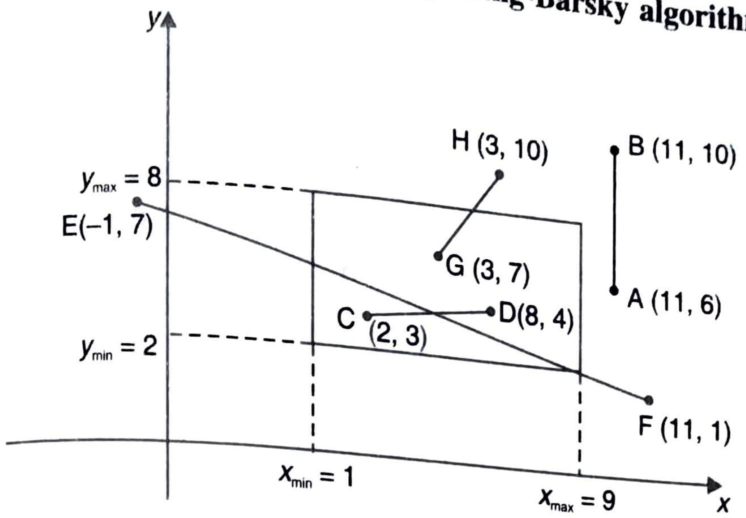
$$y_2' = y_1 + (\Delta y \times u_2) = -30 + \left[90 \times \frac{1}{2}\right] = 15$$

$\therefore$ **Visible line will be P_1' (0, 0) to P_2' (7.5, 15).**

Example 2. Consider the following clip window with

$$x_{min} = 1, \quad x_{max} = 9$$
$$y_{min} = 2, \quad y_{max} = 8$$

Now, clip the lines as shown in Figure using Liang-Barsky algorithm–



Solution. For line AB

$$\left.\begin{aligned} p_1 &= -\Delta x = 11 - 11 = 0 \\ p_2 &= \Delta x = 0 \\ p_3 &= -\Delta y = -(10-6) = -4 \\ p_4 &= \Delta_y = 4 \end{aligned}\right\}$$

and

$$\left.\begin{aligned} q_1 &= x_1 - x_{min} = 11 - 1 = 10 \\ q_2 &= x_{max} - x_1 = 9 - 11 = -2 \\ q_3 &= y_1 - y_{min} = 6 - 2 = 4 \\ q_4 &= y_{max} - y_1 = 8 - 6 = 2 \end{aligned}\right\}$$

$\because p_2 = 0$ and $q_2 < 0$; so the line AB is completely outside the boundary and thus can be eliminated.

For line CB

$$p_1 = -6 \qquad q_1 = 1 \qquad \therefore \quad r_1 = \frac{q_1}{p_1} = \frac{-1}{6}$$

$$p_2 = 6 \qquad q_2 = 7 \qquad r_2 = \frac{q_2}{p_2} = \frac{7}{6}$$

$$p_3 = -1 \qquad q_3 = 1 \qquad r_3 = \frac{q_3}{p_3} = -1$$

$$p_4 = 1 \qquad q_4 = 5 \qquad r_4 = \frac{q_4}{p_4} = 5$$

$\therefore$

$$u_1 = \max\left[0, -\frac{1}{6}, -1\right] = 0$$

and

$$u_2 = \min\left[1, \frac{7}{6}, 5\right] = 1$$

$$\therefore \quad u_1 = 0 \text{ and } u_2 = 1$$

$\therefore$ Line EF is completely inside the clipping window.

For line GH

$$p_1 = 0 \quad q_1 = 2$$
$$p_2 = 0 \quad q_2 = 6$$
$$p_3 = -3 \quad q_3 = 5 \qquad \because p_3 < 0,\ r_3 = \frac{q_3}{p_3}$$
$$p_4 = 3 \quad q_4 = 1 \qquad \because p_4 > 0,\ r_4 = \frac{q_4}{p_4}$$

$$\therefore \quad u_1 = \max\left(0, \frac{-5}{3}\right) = 0 \text{ and } u_2 = \min\left(1, \frac{1}{3}\right) = \frac{1}{3}$$

$\because$ $u_1 < u_2$, so the two endpoints of the clipped line are (3, 7) and $\left(3, 7 + 3 \times \frac{1}{3}\right) = (3,$

$$\left[\because \begin{array}{l} x = x_1 + \Delta x \\ y = y_1 + \Delta y \end{array}\right.$$

For line EF

$$p_1 = -12 \qquad q_1 = -2 \qquad \therefore \quad r_1 = \frac{1}{6}$$
$$p_2 = 12 \qquad q_2 = 10 \qquad r_2 = \frac{5}{6}$$
$$p_3 = 6 \qquad q_3 = 5 \qquad r_3 = \frac{5}{6}$$
$$p_4 = -6 \qquad q_4 = 1 \qquad r_4 = \frac{-1}{6}$$

$$\therefore \quad u_1 = \max\left(0, \frac{1}{6}, \frac{-1}{6}\right) = \frac{1}{6}$$

and

$$u_2 = \min\left(1, \frac{5}{6}, \frac{-5}{6}\right) = \frac{5}{6}$$

$\because$ $u_1 < u_2$, the endpoints of the clipped line are–

$$\left(-1 + 12 \times \frac{1}{6}, 7 + (-6) \times \frac{1}{6}\right) = (1, 6)$$

and

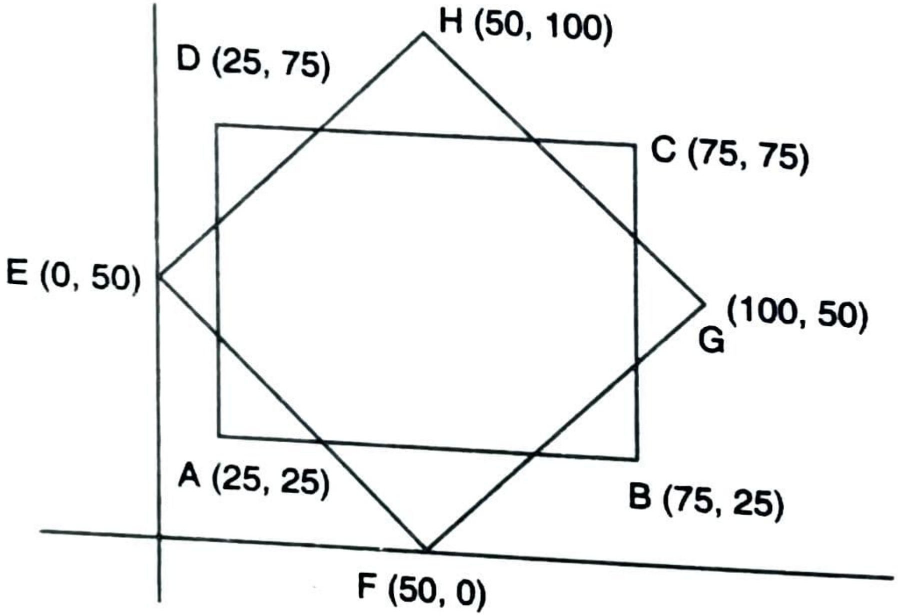
$$\left(-1 + 12 \times \frac{5}{6}, 7 + (-6) \times \frac{5}{6}\right) = (9, 2)$$

$\therefore$ Endpoints of clipped line are (1, 6) and (9, 2).

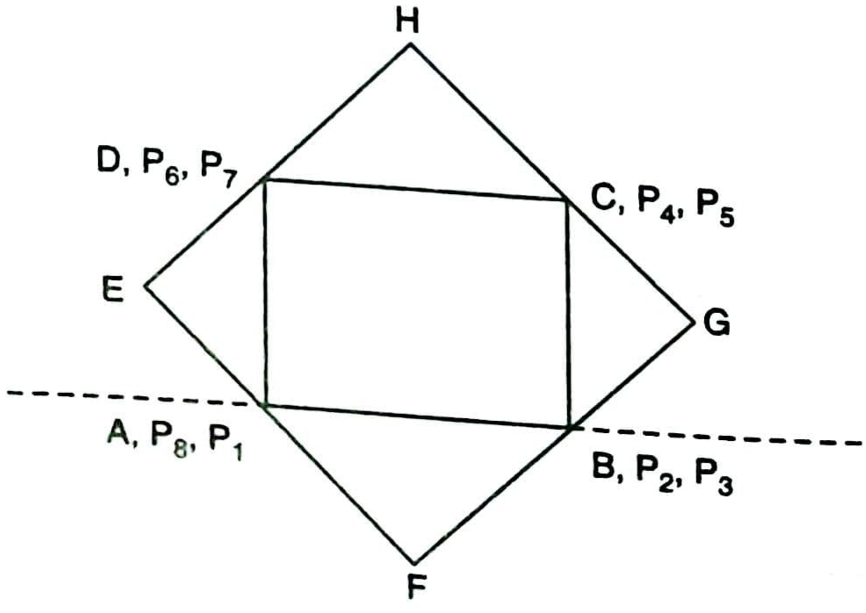
Line PQ is completely outside the window.

Q.3. Using Sutherland-Hodgeman algorithm, clip the following polygons against the rectangle –

[UPTU, B.Tech (IE) – 5th sem., 2004]



Ans. We need to clip polygon EFGHE against rectangle ABCD *i.e.*,



(*a*) Clip the polygon against the line AB

∴ Equation of line, AB is $y = 25$ ... (1)

Equation of line, EF is $y - 50 = -1\ (x - 0)$

or $\quad x + y = 50$ ... (2)

Equation of line, FG is $x - y = 50$ ... (3)

(*b*) Find intersection of the line segment AB with the line segments EF and FG, we get vertices P_1 and P_2 as –

$$\begin{aligned} P_1 &= (25, 25) \\ P_2 &= (75, 25) = P_3 \\ P_4 &= (75, 75) = P_5 \\ P_6 &= (25, 75) = P_7 \\ P_8 &= P_1 = (25, 25) \end{aligned}$$

Please note here that the intersection points of polygon and rectangle show that the polygon is totally outside the clipping boundaries i.e., not a single vertex is inside the clipping window. Therefore, the output will be zero.

Q.4. A clipping window ABCD is specified as *A* (0, 0), *B* (40, 0), *C* (40, 40), D (0, 40). Use midpoint subdivision algorithm to find the visible portion, if any, of the line segment joining the points P (– 10, 20) and Q (50, 10).

Ans. The outcodes of P is 0001 and Q is 0010. Both endpoint codes are not zero and their logical AND is zero, hence we can conclude that line cannot be rejected as invisible.

Now midpoint is

$$x_m = \frac{x_1 + x_2}{2} = \frac{-10+50}{2} = 20$$

$$y_m = \frac{y_1 + y_2}{2} = \frac{20+10}{2} = 15$$

Outcode of midpoint P_m (x_m, y_m) is 0000.

Neither segment PP_m nor P_mQ is either totally visible or trivially invisible. Lets keep segment PP_m for later processing, and we continue with P_mQ. This subdivision process continues until we find an intersection point with window edge i.e. (40, y). Table shows how the subdivision works.

P	Q	P_m	Comment
(– 10, 20)	(50, 10)	(20, 15)	Save PP_m and continue with P_mQ
(20, 15)	(50, 10)	(35, 12)	Continue with P_mQ
(35, 12)	(50, 10)	(42, 11)	Continue with PP_m
(35, 12)	(42, 11)	(38, 11)	Continue with P_mQ
(38, 11)	(42, 11)	(40, 11)	This is the intersection pint of line with right window edge.
(–10, 20)	(20, 15)	(5, 17)	Recall PP_m and continue with PP_m
(–10, 20)	(5, 17)	(–3, 18)	Continue with P_mQ
(–3, 18)	(5, 17)	(1, 17)	Continue with PP_m
(–3, 18)	(1, 17)	(–1, 17)	Continue with P_mQ
(–1, 17)	(1, 17)	(0, 17)	This is the intersection point of line with left window edge

Thus visible portion of line segment PQ is from (0, 17) to (40, 11).