Fractal Geometry

- To define natural objects having irregular and fragmented features.
- Euclidean methods do not realistically model these objects.
- Procedures rather than equations are used to model objects

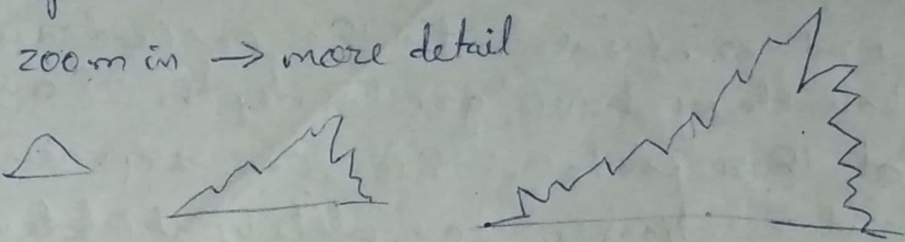
Two basic characteristics:

a) infinite detail at every point
b) Certain self-similarity between the object parts and the overall features of the object

The self similarity properties of an object can take different forms, depending on the choice of fractal representation.

zoom in → more detail

Jagged shape



Fractal dimension / Fractional dimension
- real number
- describe the amount of variation in the object detail

<u>Fractal Generation Procedures</u>

A fractal object is generated by repeatedly applying a specified transformation function to points within a region of space.

If $P_0 = (x_0, y_0, z_0)$ is a selected initial point, each iteration of a transformation function F generates successive levels of detail with the calculations

$P_1 = F(P_0)$, $P_2 = F(P_1)$, $P_3 = F(P_2)$ ...

The transformation function can be applied to
- a) a specified point set
- b) initial set of primitives : st. lines, curves, color areas, surfaces, solid objects.

At each iteration — we can use deterministic or random generation procedures.

- Apply transformation function a <u>finite number</u> of times.
  depends on the detailing and resolution of the display system

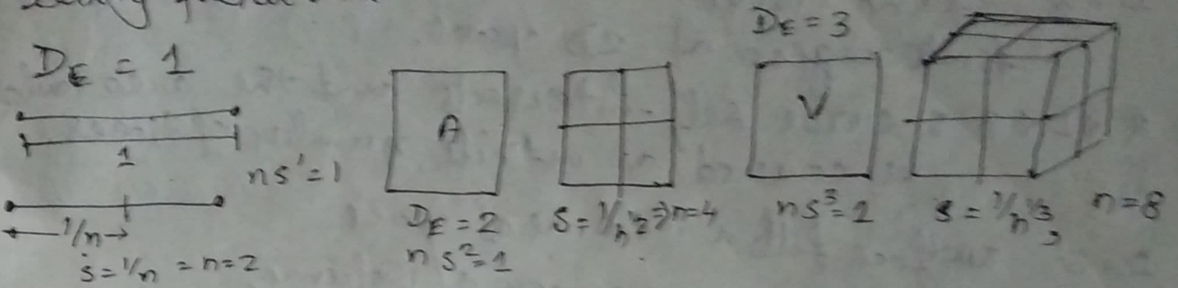
we cannot display detail variations that are smaller than the size of a pixel.

<u>Fractal Dimension</u>

is a measure of the roughness, or fragmentation of the object.
- More jagged-looking objects have larger fractal dimensions.
- Set up iterative procedures to generate fractal objects using a given value for the fractal dimension D.

With other procedures, we may be able to determine the fractal dimension from the properties of the constructed object. ⇒ fractal dimension is difficult to calculate.

scaling factor: $1/2$



Relation between the number of subparts and the scaling factor is $n \cdot s^{D_E} = 1$

Fractal dimension D for self-similar objects can be obtained from $n s^D = 1$

Solving this exp for D, the fractal similarity dimension, we have, $D = \dfrac{\ln n}{\ln(1/s)}$

A self-similar fractal constructed with different scaling factors for the different parts, the fractal similarity dimension is obtained from the implicit relationship

$\sum_{k=1}^{n} s_k^{D} = 1$ $\quad s_k$ is the scaling factor for subpart number k.

For general object shapes, we can use topological covering methods that approximate object subparts with simple shapes.

Ex: a subdivided curve can be approximated with straight-line sections.

Extension of above method → Hausdorff-Besicovitch dimension or fractional dimension.

↳ difficult to evaluate

↳ estimated with box covering methods using rectangles or parallelepiped.

Fractal dimension is estimated by box covering method

– subdivide the object into a number of small boxes using the scaling factors.

boxes n that it takes to cover an object is called the box dimension. n is related to fractal dimension

Fractal dimension > Euclidean dimension (or topological dimension)

parameters needed to specify the object.

D = 1 smooth fractal curve.

D = 2 Peano Curve : the curve completely fills a finite region of 2D space.

2 < D < 3 : curve self-intersects and the area could be covered an infinite number of times

D = 3 surface fills a volume of space.

D > 3 overlapping coverage of volume. Terrain, clouds, water

fractal surface : $2 < D \leq 3$

Fractal solid : $3 < D \leq 4$

D > 4 : self-overlapping object. : cloud properties water vapor density temperature within a region of space.

Classification of fractals

1) **Self-similar fractals** : have parts that are scaled-down versions of the entire object.

- Starting with an initial shape, we construct the object subparts by apply a scaling parameter s to the overall shape.
- same scaling factor s for all subparts or different scaling factors for different scaled-down parts of the object.
- Apply random variations to the scaled-down subparts, the fractal is said to be statistically self-similar.

→ used to model trees, shrubs, other plants.

ii) **Self-Affine Fractals** have parts that are formed with different scaling parameters, s_x, s_y, s_z in different coordinate directions. we can also include random variations to obtain statistically self-affine fractals.

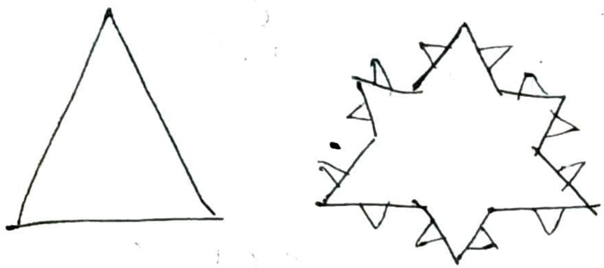
iii) **Invariant Fractal Sets** formed with nonlinear transformations.

- self-squaring fractals : Mandelbrot set (squaring functions ~~with nonlinear transformations~~ in complex space). Self inverse fractals formed with inversion procedures.

Self-inverse fractals formed with inversion procedures.

Geometric Construction of Deterministic Self-Similar

Fractals



Koch curves
(snowflake pattern)

$1, \frac{4}{3}, \frac{16}{9}$

initiator

Generator
→ four equal length segments

To construct a deterministic (non random) self-similar fractal, start with a given geometric shape, initiator. Subparts of the initiator are then replaced with a pattern called the generator.

scaling factor $1/3$ $D = \ln 4 / \ln 3 = 1.2619.$

Also, the length of each line segment in the initiator increases by a factor of $4/3$ at each step so that the length of the fractal curve tends to infinity as more detail is added to the curve.