Why you don't want controllers using derivatives of the same order as the equations themselves

Assume we have dynamics:

$$\dot{x} = Ax + Bu$$

Let $u = K_p x + K_v \dot{x}$. If B is invertible, we can pick K_v such that $K_v = B^{-1}$ and obtain:

$$\dot{x} = Ax + B(K_p x + B^{-1}\dot{x})$$

$$\dot{x} = Ax + BK_p x + \dot{x}$$

$$0 = (A + BK_p)x$$

Which means x is in the null space of $(A + BK_p)$. However, the initial problem did not specify this, therefore for any $x \notin \text{null}(A + BK_p)$, the proposed control should lead to an instantaneous change in x, which is not physically possible or meaningful.

Alternatively, pick $u = K_p x + B^{-1}\dot{x} - c$. Then we obtain:

$$\dot{x} = Ax + BK_p x + \dot{x} - Bc$$

$$(A + BK_p)x = Bc$$

$$x = (A + BK_p)^{+} Bc$$

Here it means that any x in column space of $(A + BK_p)^{+} B$ can be achieved instantaneously, which is not physical or meaningful.