CSCI 665 ASSIGNMENT 2

1.a

Tf(0)=0

Tf(1)=0

Tf(n)=1+Tf(n-1)+Tf(n-2)

To Prove,

Tf(n)=F(n+1)-1

Base case,

Tf(2)= F(3)-1

=F(2)+F(1)-1

=((F(1))+F(0))+F(1)-1

=1+0+1-1

=1

Inductive step,

Tf(k)=F(k+1)-1 for 0<k<n

Tf(n)=1+Tf(n-1)+Tf(n-2)

=1+(F(n)-1)+(F(n-1)-1)

=F(n)+F(n-1)-1

=F(n+1)-1

b.F(n)=1/(√5)(Ψ^n-Ψ`^n) ,where Ψ=(1+√5)/2 & Ψ`=(1-√5)/2

lim Tf(n)=F(n+1)-1

n->∞ =1/√5 (Ψ^n-Ψ`^n) -1

=1/√5(Ψ^n+1-Ψ`^n+1)

=1/√5(Ψ^n+1-0)

=1/√5(Ψ^n+1)

$=1/\sqrt{5}(\Psi^n. \Psi)$
$=\Psi/\sqrt{5}$

2.Time complexity= O(n)
Tf(0)=0
Tf(1)=0
Tf(n)=Tf(n-1)+c. where c=constant
Iteration:
T(n)=T(n-1)+c
=[T(n-2)+c]+c
=T(n-2)+2c
=T(n-k)+kc
Let k=n
Tf(n)=Tf(n-n)+nc
=Tf(0)+nc
=0+nc
=nc

3.N^2->N^2:L

L(a,b)=(b,a+b)
So f(n;a,b)=(L^n(a,b))1

L^0(0,1)1=I(0,1)1=0
=(f(0;a,b),f(1;a,b))
=(0,1)

Inductive step,
For k,
L^k(a,b)=(f(k;a,b),f(k+1;a,b))

$L^{k+1}(a,b)=L(L^k(a,b))$

$=L(f(k;a,b),f(k+1;a,b))$

$=f(k+1;a,b),(f(k;a,b)+f(k+1;a,b))$

$=f(k+1;a,b),f(k+2;a,b)$

Hence,

$L^n(a,b)=(f(n;a,b),f(n+1;a,b))$

5.a. An algorithm whose worst case time complexity depends on numeric value of input (not number of inputs) is called Pseudo-polynomial algorithm.

b.Fib is a not Pseudo-polynomial time algorithm because its worst case complexity is bounded by the value of n and not the no. of elements. Hence if we give a large value to Fib we get delay as the n increases because the fib computes the value of n.

c.Fibit is not a Pseudo-polynomial time algorithm because while computing fibIt the complexity is bounded by n which is linear and not polynomial in time.

d.Fibpow is a Pseudo-polynomial time algorithm because complexity is bounded by O(log n) which is logarithmic.It is exponential in terms of length.

6.a.

$T(0)=0$

$T(n+1)=T(n)+5$

$T(x)=T(x-1)+5$

$Ti(n)=Ti(n-1)+5$

$=Ti(n-2)+5+5$

$=Ti(n-3)+5+5+5$

$Ti(n-k)+5k$

Let $k=n$

$Ti(n)=Ti(0)+5n$

$=5n$

b.$T(n+1)=n+T(n)$

$T(x)=n+T(x-1)$

$Ti(n)=n+T(n-1)$

$=n+n+T(n-2)$

$=nk+Ti(n-k)$

If $k=n$

$Ti(n)=n^2+T(0)$

$Ti(n)=n^2$

7.a. $T(n+1)=2T(n)$

$T(x)=2T(x-1)$

$Ti(n)=2T(n-1)$

$=2(2T(n-2))$

$=2^k T(n-k)$

Let $k=n$

$=2^n T(0)$

$T(n)=2^n$

b. $T(n+1)=2^n+1 +T(n)$

$T(x)=2^x+T(n-1)$

$T(n)=2^n +T(n-1)$

$=2^n+(2^{(n-1)}+T(n-2))$

$=2^n+(2^{(n-1)}+2^{(n-2)}+T(n-3))$

$=2^n+2^{(n-1)}+\ldots..+2^{(n-k)}+T(n-k)$

Let $k=n$

$=2^n+2^{(n-1)}+\ldots.+2^0+T(0)$

$=\Sigma\ 2^m \qquad 0<=m<=n$

$T(n)=2^{(n+1)}-1$

8.a

$T(n)=n+T(n/2) \qquad n=2^m$

$T(x)=x+T(x/2)$

$T(n)=n+T(n/2)$

$=n+n/2 +T(n/2^2)$

$=n+n/2+n/2^2+T(n/2^3)$

$=2^m+2^{(m-1)}+2^{(m-2)}+\ldots.2^{(m-(k-1))}+T(2^{(m-k)})$

Let $k=m$

$=n+n/2+n/2^2+\ldots..+2^{(m-1)}+2^{(m)}$

$=\Sigma\ 2^i$

$=2^{(m+1)}-1$

$=2n-1$

b.

$T(n)=1+T(n/3)$ $\qquad$ $n=3^m$

$=1+1+T(n^{3^{-2}})$

$=m+T(3^m.3^{-m})$

$T(n)=m+1$

9.a

$T(n) = aT(n\ n\ 1) + bn$

$= a[aT(n - 2) + b(n - 1)] + bn$

$= a[a[aT(n- 3) + b(n - 2)] + b(n - 1)] + bn$

$= a^3T(n-3)+a^2b(n-2)+ab(n-1)+bn$

$=a^kT(n-k)+\sum(a^i)b(n-i)$

$k=n-1$

$T(n)=a^n-1T(1)+ \sum a(i)b(n-i)$

$T(n)=a^n-1+bn+\ldots..+a^n-2+b.2$

If $a=1$

$T(n)=b(n(n+1)/2-1)$

$O(n)$

If $a>1$

$O(a^n)$

b.

$T(n) = aT(n - 1) + bn \log(n)$

$T(n) = aT(n - 1) + bn \log(n)$

$= a[aT(n - 2) + b(n - 1) \log(n - 1)] + bn \log(n)$

$= a[a[aT(n - 3) + b(n - 2) \log(n - 2)] + b(n - 1) \log(n - 1)] + bn \log(n)$

$= a3\ T(n\ n\ 3) + a2\ b(n - 2) \log(n - 2) + ab(n - 1) \log(n - 1) + bn \log(n)$

Identifying Pattern
$= (a^k) T(n - k) + \sum (a^i)b(n - i) \log (n - i)$
Let $k = n - 1$
So,
$T(n) = a^{(n- 1)} T(1) + \sum (a^i)b(n - i) \log (n - i)$
$= a^n - 1 + b \cdot n \cdot \log(n) + \cdots + a^n-2.b \cdot 2 \cdot \log(n)$
Hence, $O(a^n)$ if $a > 1$
$a=1$
$O(n\log n)$

c.
$T(n) = aT(n - 1) + bn^c$
$T(n) = aT(n - 1) + bn^c$
$= a[aT(n - 2) + b(n - 1)^c] + bn^c$
$= a[a[aT(n -3) + b(n - 2)^c] + b(n - 1)^c] + bn^c$
$= a^3 T(n - 3) + a^2 b(n - 2)^c + ab(n - 1)^c + bn^c$
,Identifying pattern
$= a^kT(n - k) + \sum (a^k- 1)b(n - i)^c$
Let $k = n - 1$
$T(n) = a^k- 1 T(1) + bn^c + ... + a^n- 2b \cdot 2c$
Hence, $O(a^n)$ if $a > 1$
and
for $a=1$
$O(n) = n^c$

d. $T(n) = aT(n/2) + bn^c$
$T(n) = aT(n/2) + bn^c$
$= a[aT(n/4) + b(n/2)^c] + bn^c$
$= a[a[aT(n/8) + b(n/4)^c] + b(n/2)^c] + bn^c$
$= a^3 T(n/8) + a^2 b(n/4)^c + ab(n/2)^c + bn^c$
$= (a^k)T(n/2^k) + \sum /(a^k)(n/2^k) + b(n - (k - 1))^c$

Let $k = \lg(n)$

$= (a^{\lg(n)}) T(n/2^{\lg(n)}) + \sum (a^{\lg(n)}) T(n/2^{\lg(n)}) + b(n - (\lg(n) - 1))^c$

So, $O(n) = n^c$