Homework #1, EE556, Fall 2018

Due: 9/7/18; hand in #3,#4

Problem #1

Consider the case of 2 multivariate Gaussian classes with distinct means, equal class prior probabilities, and common covariance matrix $\Sigma = \sigma^2 I$. Derive the equation that specifies the locus of points representing the decision boundary between the two classes (the result for this case was given in lecture).

Problem #2

For the distributions given in problem 1, derive an expression for the probability of error, leaving the final result in a simple integral form. *Hint:* as mentioned in lecture, the decision rule amounts to applying a threshold to a scalar random variable Y that is a linear combination of Gaussian random variables. Hence, the probability of error expression can be written in a simple form, once the mean and variance of Y, conditioned on $c = 1, 2$ is known. There is a fair amount of calculation involved in this problem.

Problem #3

Consider the case of 2 multivariate Gaussian classes with distinct means, equal class prior probabilities, and common covariance matrix Σ that is *not* necessarily a diagonal matrix. Derive the equation that specifies the locus of points representing the decision boundary between the two classes (the result for this case was given in lecture). For $d = 2$, make a particular choice of Σ and accurately sketch the decision boundary in the plane.

Problem #4

Three categories with equal prior probabilities must be distinguished by observing a two-dimensional feature vector. The class-conditional pdfs are each Gaussian with uncorrelated components. The class 1 feature vector has mean (2, 0) and unit variances. The class 3 feature vector has mean (0, 2) and unit variances. The class 2 feature vector has mean (0,0) and variances (1, 2). Find the equations describing the decision boundaries between the classes and (roughly) sketch the decision regions in the plane.