Calcular spline lineal, cuadrático, cúbico de

X	-1	0	1	2	3	4
Y	-2	0	2	3	2	4

lineal: $f(x) = ax + b$

Intervalos $[-1, 0], [0, 1], [1, 2],$
$[2, 3], [3, 4]$

$$S = \begin{cases} a_0 x + b_0, & x \in [-1, 0] \\ a_1 x + b_1, & x \in [0, 1] \\ a_2 x + b_2, & x \in [1, 2] \\ a_3 x + b_3, & x \in [2, 3] \\ a_4 x + b_4, & x \in [3, 4] \end{cases}$$

$S(-1) = -2, \quad S(0) = 0 \quad S(1) = 2 \quad S(2) = 3$

$S(3) = 2, \quad S(4) = -4$

$$S(-1) = -2 \rightarrow -a_0 + b_0 = -2 \rightarrow \boxed{a_0 = 2}$$

$$S(0) = 0 \rightarrow \boxed{b_0 = 0}, \quad \boxed{b_1 = 0}$$

$$S(1) = 2 \rightarrow a_1 + b_1 = 2 \rightarrow \boxed{a_1 = 2}$$

$$a_2 + b_2 = 2$$

reducción

$$S(2) = 3 \rightarrow 2a_2 + b_2 = 3$$

$$2a_3 + b_3 = 3$$

$$S(3)=2 \; \begin{cases} 3a_3+b_3=2 \\ 3a_4+b_4=2 \end{cases}$$

$$\left.\begin{array}{l} 3a_4+b_4=2 \\ 4a_4+b_4=4 \end{array}\right\} \text{reducción}$$

$$S(4)=4 \rightarrow 4a_4+b_4=4$$

$$(a_2+b_2=2)(-1)$$

$$2a_2+b_2=3$$

$$-a_2-b_2=-2$$

$$2a_2+b_2=3$$

$$\boxed{a_2=1}$$

$$1+b_2=2$$

$$\boxed{b_2=1}$$

$$2a_3 + b_3 = 3$$

$$(3a_3 + b_3 = 2)(-1)$$

$$2a_3 + b_3 = 3$$

$$-3a_3 - b_3 = -2$$

$$(a_3 = 1)(-1)$$

$$\boxed{a_3 = -1}$$

$$2(-1) + b_3 = 3$$

$$-2 + b_3 = 3$$

$$\boxed{b_3 = 5}$$

$(3a_4 + b_4 = 2)(-1)$

$4a_4 + b_4 = 4$

$-3a_4 - b_4 = -2$

$4a_4 + b_4 = 4$

$\boxed{a_4 = 2}$

$3(2) + b_4 = 2$

$6 + b_4 = 2$

$\boxed{b_4 = -4}$

R/1

$$S = \begin{cases} \left.\begin{array}{ll} \underline{2x}, & x \in [-1, 0] \\ \underline{2x}, & x \in [0, 1] \end{array}\right\} x \in [-1, 1] \\ x+1, & x \in [1, 2] \\ -x+5, & x \in [1, 3] \\ 2x-4; & x \in [3, 4] \end{cases}$$

Cuadrático: $ax^2+bx+c=f(x)$

$$S=\begin{cases} a_0x^2+b_0x+c_0, & x\in[-1,0] \\ a_1x^2+b_1x+c_1; & x\in[0,1] \\ a_2x^2+b_2x+c_2; & x\in[1,2] \\ a_3x^2+b_3x+c_3; & x\in[2,3] \\ a_4x^2+b_4x+c_4; & x\in[3,4] \end{cases}$$

$S(-1) = -2 \rightarrow a_0 - b_0 + c_0 = -2 \quad E_1$

$S(0) = 0 \quad \begin{cases} \boxed{c_0 = 0} \\ \boxed{c_1 = 0} \end{cases}$

$S(1) = 2 \quad \begin{cases} a_1 + b_1 + c_1 = 2 \quad E_2 \\ a_2 + b_2 + c_2 = 2 \quad E_3 \end{cases}$

$S(2) = 3 \quad \begin{cases} 4a_2 + 2b_2 + c_2 = 3 \quad E_4 \\ 4a_3 + 2b_3 + c_3 = 3 \quad E_5 \end{cases}$

$S(3) = 2 \quad \begin{cases} 9a_3 + 3b_3 + c_3 = 2 \quad E_6 \\ 9a_4 + 3b_4 + c_4 = 2 \quad E_7 \end{cases}$

$S(4)=4 \Rightarrow 16a^4 + 4b_4 + c_4 = 4$ Eq 8

$$S'(x) = \begin{cases} 2a_0x + b_0, & x \in [-1, 0] \\ 2a_1x + b_1, & x \in [0, 1] \\ 2a_2x + b_2, & x \in [1, 2] \\ 2a_3x + b_3, & x \in [2, 3] \\ 2a_4x + b_4; & x \in [3, 4] \end{cases}$$

$2a_0x + b_0$; $2a_1x + b_1$;

$x = 0$

$2a_0(0) + b_0 = 2a_1(0) + b_1$

$b_0 = b_1$

$b_0 - b_1 = 0$ Eq

$2a_1x + b_1$, $2a_2x + b_2$

$x = 1$

$2a_1 + b_1 = 2a_2 + b_2$

$2a_1 - 2a_2 + b_1 - b_2 = 0$ E10

$2a_2x+b_2, \quad 2a_3x+b_3$

$x=2$

$4a_2+b_2=4a_3+b_3$

$4a_2-4a_3+b_2-b_3=0 \quad E11$

$2a_3x+b_3 \; ; \; 2a_4x+b_4$

$x=3$

$6a_3+b_3=6a_4+b_4$

$6a_3-6a_4+b_3-b_4=0 \quad E12$

Sistema de ecuaciones

$$a_0 - b_0 = -2$$
$$a_1 + b_1 = 2$$
$$a_2 + b_2 + c_2 = 2$$
$$4a_2 + 2b_2 + c_2 = 3$$
$$4a_3 + 2b_3 + c_3 = 3$$
$$9a_3 + 3b_3 + c_3 = 1$$
$$9a_4 + 3b_4 + c_4 = 2$$
$$16a_4 + 4b_4 + c_4 = 4$$
$$b_0 - b_1 = 0$$

$$2a_1 - 2a_2 + b_1 - b_2 = 0$$
$$4a_2 - 4a_3 + b_2 - b_3 = 0$$
$$6a_3 - 6a_4 + b_3 - b_4 = 0$$

grado de libertad

$$a_0 = 0$$
$$b_0 = 2$$
$$\boxed{b_1 = 2}$$

$$a_1 + 2 = 2$$
$$\boxed{a_1 = 0}$$

Sistema $(a_2, a_3, a_4, b_2, b_3, b_4, c_2, c_3, c_4) = k$

$$a_2 + 0 + 0 + b_2 + b_3 + 0 + c_2 + 0 + 0 = 2$$

$$4a_2 + 0 + 0 + 2b_2 + 0 + 0 + c_2 + 0 + 0 = 3$$

$$0 + 4a_3 + 0 + 0 + 2b_3 + 0 + 0 + c_3 + 0 = 3$$

$$0 + 9a_3 + 0 + 0 + 3b_3 + 0 + 0 + c_3 + 0 = 2$$

$$0 + 0 + 9a_4 + 0 + 0 + 3b_4 + 0 + 0 + c_4 = 2$$

$$0 + 0 + 16a_4 + 0 + 0 + 4b_4 + 0 + 0 + c_4 = 4$$

$$-2a_2 + 0 + 0 - b_2 + 0 + 0 + 0 + 0 + 0 = -2$$

$$4a_2 - 4a_3 + 0 + b_2 - b_3 + 0 + 0 + 0 + 0 = 0$$

$$0 + 6a_3 - 6a_4 + 0 + b_3 - b_4 + 0 + 0 + 0 = 0$$

$a_0 = 0 \quad b_0 = 2 \quad c_0 = 0$

$a_1 = 0 \quad b_1 = 2 \quad c_1 = 0$

$a_2 = \frac{1}{3} \quad b_2 = -\frac{8}{3} \quad c_2 = 7$

$a_3 = \frac{1}{3} \quad b_3 = -\frac{8}{3} \quad c_3 = 7$

$a_4 = \frac{2}{3} \quad b_4 = -\frac{14}{3} \quad 4 = 10$

$ax^2 + bx + c$

$\mathbb{R}$

$$S = \begin{cases} 2x, & x \in [-1, 1] \\ \frac{1}{3}x^2 - \frac{8}{3}x + 7; & x \in [1, 3] \\ \frac{2}{3}x^2 - \frac{14}{3}x + 10; & x \in [3, 4] \end{cases}$$

cúbica $\longrightarrow ax^3+bx^2+cx+d=f(x)$

$S=\left\{ \begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array} \right\}$ 5 trozos.

$S'(x)$ . . . .

$S''(x)$ . . . . . .

grados de libertad