

comprobar para nivel 3, $k+1=3 \Rightarrow k=2$.

$$D_{(3,i)} = \frac{4^2 D_{(2,i+1)} - D_{(2,i)}}{4^2 - 1}$$

$$= \frac{16 D_{(2,i+1)} - D_{(2,i)}}{15}$$

$$D_{(3,i)} = \frac{16}{15} D_{(2,i+1)} - \frac{1}{15} D_{(2,i)}$$

y así sucesivamente.

Obtener $f'(0.5)$ usando la extrapolación de Richardson y las diferencias finitas hacia adelante de segunda diferencia, hasta nivel 3.

✓ $f(x) = -0.10x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

con $h=0.1$

Fórmula
diferencias
finitas

$$f'(x_0) = \frac{-f(x_0+2h) + 4f(x_0+h) - 3f(x_0)}{2h}$$

Para nivel tres se necesita:

$$h_1 = 0.1, \quad h_2 = \frac{0.1}{2} = 0.05, \quad h_3 = \frac{0.05}{2} = 0.025$$

$$f'(0.5) = \frac{-f(0.5 + 2(0.1)) + 4f(0.5 + 0.1) - 3f(0.5)}{2(0.1)}$$

$$= \frac{-f(0.7) + 4f(0.6) - 3f(0.5)}{0.2}$$

$$= \frac{0.70454 + 3.29856 - 2.775}{0.2}$$

$$\boxed{f'(0.5) = 6.1405}$$

$$f'(0.5) = \frac{-f(0.5 + 2(0.05)) + 4f(0.5 + 0.05) - 3f(0.5)}{2(0.05)}$$

$$= \frac{-f(0.6) + 4f(0.55) - 3f(0.5)}{0.1}$$

$$= \frac{-0.82464 + 3.5085725 - 2.775}{0.1}$$

$$\boxed{f'(0.5) = -0.910675}$$

$$f'(0.5) = \frac{-f(0.5 + 2(0.025)) + 4f(0.5 + 0.025) - 3f(0.5)}{2(0.025)}$$

$$= \frac{-f(0.55) + 4f(0.525) - 3f(0.5)}{0.05}$$

$$= \frac{-0.877143125 + 3.606540469 - 2.775}{0.05}$$

$$f'(0.5) = -0.912053125$$

$$h_1 = 0.1 \rightarrow f'(0.5) = 6.1405$$

$$h_2 = 0.05 \rightarrow f'(0.5) = -0.910675$$

$$h_3 = 0.025 \rightarrow f'(0.5) = -0.912053125$$

h	Nivel 1	Nivel 2 (2,i)	Nivel 3 (3,i)
0.1	6.1405		
0.05	-0.910675	-3.261066667	
0.025	-0.912053125	-0.9125125	-0.755942222

Nivel 2

$$D(2,1) = \frac{4}{3}D(1,2) - \frac{1}{3}D(1,1)$$

$$= \frac{4}{3}(-0.910675) - \frac{1}{3}(6.1405)$$

$$\underline{D(2,1) = -3.261066667/}$$

$$D(2,2) = \frac{4}{3}D(1,3) - \frac{1}{3}D(1,2)$$

$$= \frac{4}{3}(-0.912053125) - \frac{1}{3}(-0.910675)$$

$$\underline{D(2,2) = -0.9125125/}$$

Nivel 3

$$D(3,1) = \frac{16}{15}D(2,2) - \frac{1}{15}D(2,1)$$

$$D(3,1) = \frac{16}{15}(-0.9125125) - \frac{1}{15}(-3.261066667)$$

$$\underline{D(3,1) = -0.7559422222}$$

Cálculo de errores.

$$f'(0.5) \approx -0.7559422222 \text{ Aproximado.}$$

Valor verdadero

$$f'(x) = -0.40x^3 - 0.45x^2 - 1x - 0.25$$

$$\begin{aligned} f'(0.5) &= -0.40(0.5)^3 - 0.45(0.5)^2 - (0.5) - 0.25 \\ &= -0.9125 \end{aligned}$$

Cálculo de errores.

$$E\% = \left| \frac{-0.9175 - (-0.7559422222)}{-0.9175} \right| \times 100.$$

$$E\% = 17.06351802\%$$

$$f'(x_0) = 6.1405$$

$$E\% = \left| \frac{-0.9175 - 6.1405}{-0.9175} \right| \times 100 = 769.2643052\%$$

Comparación de errores

Diferencia finita 769.2643052%

Richardson 17.06351802%

Ejemplo 2 Determine $f'(0.8)$ de e^x con $h=0.1$, por Richardson de nivel 4 y por diferencia finita centrada de primera diferencia.

Fórmula de la diferencia finita.

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$