Funciones trazadores de grado 3

Esta formado por un polinomio cúbico de la forma $f(x) = ax^3 + bx^2 + cx + d$ que se encarga de unir cada uno de los pares ordenados de la cúbica. - Esté método es el más empleado.

$$S(x) = \begin{cases} a_0x^3 + b_0x^2 + c_0x + d_0 \longrightarrow [x_0, x_1] \\ a_1x^3 + b_1x^2 + c_1x + d_1 \longrightarrow [x_1, x_2] \\ a_2x^3 + b_2x^2 + c_2x + d_2 \longrightarrow [x_2, x_3] \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ a_nx^3 + b_nx^2 + c_nx + d_n \longrightarrow [x_{n-1}, x_n] \end{cases}$$

Determinar la función spline de grado tres para tabla.

X	2	3	5
Y	-1	2	7

① Formar los intervalo $[2,3]$, $[3,5]$

②
$$S(x) = \begin{cases} a_0x^3 + b_0x^2 + c_0x + d_0, & x \in [2,3] \\ a_1x^3 + b_1x^2 + c_1x + d_1, & x \in [3,5] \end{cases}$$

3) $S(2) = -1 \quad S(3) = 2 \quad S(5) = 7$

4) Formar las ecuaciones

$S(2) = -1 \Rightarrow a_0(2)^3 + b_0(2)^2 + c_0(2) + d_0 = -1$

$\boxed{8a_0 + 4b_0 + 2c_0 + d_0 = -1}$ E1

$S(3) = 2 \rightarrow a_0(3)^3 + b_0(3)^2 + c_0(3) + d_0 = 2$

$\boxed{27a_0 + 9b_0 + 3c_0 + d_0 = 2}$ E2

$a_1(3)^3 + b_1(3)^2 + c_1(3) + d_1 = 2$

$\boxed{27a_1 + 9b_1 + 3c_1 + d_1 = 2}$ E3

$S(5) = 7 \quad a_1(5)^3 + b_1(5)^2 + c_1(5) + d_1 = 7.$

$\boxed{125a_1 + 25b_1 + 5c_1 + d_1 = 7}$ E4

Son 8 variables debe existir, vamos a buscar las 4 ecuaciones.

$$S'(x) = \begin{cases} 3a_0x^2 + 2b_0x + c_0 \rightarrow x \in [2,3] \\ 3a_1x^2 + 2b_1x + c_1 \rightarrow x \in [3,5] \end{cases}$$

Para el limite interno, es decir $x=3$.

$3a_0(3)^2 + 2b_0(3) + c_0 = 3a_1(3)^2 + 2b_1(3) + c_1$

$\boxed{27a_0 + 6b_0 + c_0 = 27a_1 + 6b_1 + c_1}$ E5

$$S''(x) = \begin{cases} 6a_0 x + 2b_0 \rightarrow x \in [2,3] \\ 6a_1 x + 2b_1 \rightarrow x \in [3,5] \end{cases}$$

$x = 3$

$$6a_0(3) + 2b_0 = 6a_1(3) + 2b_1$$

$$\boxed{18a_0 + 2b_0 = 18a_1 + 2b_1} \quad E6$$

Faltan 2 grados de libertad.

$S''(x_0) = 0 \qquad S''(x_n) = 0.$

$$S''(2) = 6a_0(2) + 2b_0 = 0$$

$$\boxed{12a_0 + 2b_0 = 0} \quad E7$$

$$S''(5) = 6a_1(5) + 2b_1 = 0$$

$$\boxed{30a_1 + 2b_1 = 0} \quad E8$$

$$\begin{cases} 8a_0 + 4b_0 + 2c_0 + d_0 = -1 \\ 27a_0 + 9b_0 + 3c_0 + d_0 = 2. \\ 27a_1 + 9b_1 + 3c_1 + d_1 = 2. \\ 125a_1 + 25b_1 + 5c_1 + d_1 = 7 \\ 27a_0 + 6b_0 + c_0 = 27a_1 + 6b_1 + c_1 \\ 18a_0 + 2b_0 = 18a_1 + 2b_1 \\ 12a_0 + 2b_0 = 0 \\ 30a_1 + 2b_1 = 0 \end{cases}$$

Se resuelve la matriz

$a_0=-1.25$, $b_0=7.5$, $c_0=-10.75$, $d_0=0.5$

$a_1=0.625$, $b_1=-9.375$, $c_1=39.875$, $d_1=-50.125$

$$\boxed{S(x)=\begin{cases}-1.25x^3+7.5x^2-10.75x+0.5, & x\in[2,3]\\ 0.625x^3-9.375x^2+39.875x-50.125, & x\in[3,5]\end{cases}}$$ R//

Ejemplo 2.

Determinar la función spline de:

X	-1	1	2	4
Y	-1	1	5	-2

① Se forman los intervalos $[-1,1]$, $[1,2]$ $[2,4]$

② $$S(x)=\begin{cases}a_1x^3+b_1x^2+c_1x+d_1, & x\in[-1,1]\\ a_2x^3+b_2x^2+c_2x+d_2, & x\in[1,2]\\ a_3x^3+b_3x^2+c_3x+d_3, & x\in[2,4]\end{cases}$$

③ $S(-1)=-1$, $S(1)=1$, $S(2)=5$, $S(4)=-2$

$S(-1)=-1 \rightarrow -a_1+b_1-c_1+d_1=-1$ (E_1)

$S(1)=1 \rightarrow a_1+b_1+c_1+d_1=1$ (E_2)

$a_2+b_2+c_2+d_2=1$ (E_3)

$$S(2)=5 \rightarrow 8a_2+4b_2+2c_2+d_2=5 \quad E4$$
$$8a_3+4b_3+2c_3+d_3=5 \quad E5$$

$$S(4)=2 \rightarrow 64a_3+16b_3+4c_3+d_3=2 \quad E6$$

$$S'(x)=\begin{cases} 3a_1x^2+2b_1x+c_1, & x\in[-1,1] \\ 3a_2x^2+2b_2x+c_2, & x\in[1,2] \\ 3a_3x^2+2b_3x+c_3, & x\in[2,4]. \end{cases}$$

$x=1.$

$$3a_1+2b_1+c_1=3a_2+2b_2+c_2 \quad E7$$

$x=2.$

$$3a_2(2)^2+2b_2(2)+c_2=3a_3(2)^2+2b_3(2)+c_3.$$

$$\boxed{12a_2+4b_2+c_2=12a_3+4b_3+c_3} \quad E8$$

Son 12 variables, se determinará $S''(x)$

$$S''(x)=\begin{cases} 6a_1x+2b_1, & x\in[-1,1] \\ 6a_2x+2b_2, & x\in[1,2] \\ 6a_3x+2b_3 & x\in[2,4] \end{cases}$$

$x=1$

$$6a_1+2b_1=6a_2+2b_2 \quad E9$$

$x=2.$

$$12a_2+2b_2=12a_3+2b_3 \quad E10$$

$S''(-1)=0 \longrightarrow -6a_1+2b_1=0$ E11

$S''(4)=0 \rightarrow 24a_3+2b_3=0$ E12

Se resuelve la matriz de las 12 ecuaciones

$a_1=\frac{51}{140}$ $b_1=\frac{153}{140}$ $c_1=\frac{89}{140}$ $d_1=-\frac{153}{40}$

$a_2=-\frac{21}{10}$ $b_2=\frac{297}{35}$ $c_2=-\frac{473}{70}$ $d_2=\frac{48}{35}$

$a_3=\frac{24}{35}$ $b_3=-\frac{288}{35}$ $c_3=\frac{1867}{70}$ $d_3=-\frac{732}{35}$

$$S(x)=\begin{cases} \frac{51}{140}x^3+\frac{153}{140}x^2+\frac{89}{140}x-\frac{153}{40}, & x\in[-1,1] \\ -\frac{21}{10}x^3+\frac{297}{35}x^2-\frac{473}{70}x+\frac{48}{35}, & x\in[1,2] \\ \frac{24}{35}x^3-\frac{288}{35}x^2+\frac{1867}{70}x-\frac{732}{35}, & x\in[2,4] \end{cases}$$