Analisis numérico 24/abril/2020.

Ejemplo 1

K	X_K	$f(X_K)$	$f'(X_K)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

Polinomios de Lagrange y sus derivadas por posición $X_0 = 1.3$, $X_1 = 1.6$, $X_2 = 1.9$

$$L_{2,0}(X) = \frac{(X-X_1)(X-X_2)}{(X_0-X_1)(X_0-X_2)}$$

$$= \frac{(X-1.6)(X-1.9)}{(1.3-1.6)(1.3-1.9)}$$

$$= \frac{X^2-3.5X+3.04}{+0.18}$$

$$L_{2,0}(x) = \frac{50}{9}x^2 - \frac{175}{9}x + \frac{152}{9}$$

$$L'_{2,0}(x) = \frac{50}{9}(2x) - \frac{175}{9}(1)$$

$$= \frac{100}{9}x - \frac{175}{9}$$

$$X_0 = 1.3 \quad X_1 = 1.6 \quad X_2 = 1.9$$

$$L_{2,1}(x) = \frac{(x-X_0)(x-X_2)}{(X_1-X_0)(X_1-X_2)}$$

$$= \frac{(x-1.3)(x-1.9)}{(1.6-1.3)(1.6-1.9)}$$

$$= \frac{x^2 - 3.2x + 2.47}{-0.09}$$

$$= -\frac{100}{9}x^2 + \frac{320}{9}x - \frac{247}{9}$$

$$L'_{2,1}(x) = -\frac{100}{9}(2x) + \frac{320}{9}(1)$$

$$= -\frac{200}{9}x + \frac{320}{9}$$

$$L_{2,2}(x) = \frac{(x-X_0)(X-X_1)}{(X_2-X_0)(X_2-X_1)}$$

$$= \frac{(X-1.3)(X-1.6)}{(1.9-1.3)(1.9-1.6)}$$

$$= \frac{x^2 - 2.9X + 2.08}{0.18}$$

$$= \frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}$$

$X_0 = 1.3,\ X_1 = 1.6$
$X_2 = 1.9$

$$L'_{2,2}(x) = \frac{50}{9}(2x) - \frac{145}{9}(1)$$
$$= \frac{100}{9}x - \frac{145}{9}$$

$$H_{n,j}(x) = \left[1 - 2(x - x_j) L'_{n,j}(x_j)\right] L^2_{n,j}(x)$$

$$H_{2,0}(X_0) = \left[1 - 2(x - 1.3)\left[\frac{100}{9}(1.3) - \frac{175}{9}\right]\right]\left(\frac{50}{9}x^2 - \frac{175}{9}x + \frac{152}{9}\right)^2$$

$$\boxed{H_{2,0}(1.3) = (10x - 12)\left(\frac{50}{9}x^2 - \frac{175}{9}x + \frac{152}{9}\right)^2}$$

$$H_{2,1}(1.6) = \left[1 - 2(x - 1.6)\left(-\frac{200}{9}(1.6) + \frac{320}{9}\right)\right]\left[-\frac{100}{9}x^2 + \frac{320}{9}x - \frac{247}{9}\right]^2$$

$$H_{2,1}(1.6) = 1\left(-\frac{100}{9}x^2 + \frac{320}{9}x - \frac{247}{9}\right)^2$$

$$H_{2,2}(1.9) = \left[1 - 2(x - 1.9)\overbrace{\left(\frac{100}{9}(1.9) - \frac{145}{9}\right)}^{5}\right]\left(\frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}\right)^2$$

$$= (1 - (2x - 2(1.9))(5))\left(\frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}\right)^2$$

$$= (20 - 10x)\left(\frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}\right)^2$$

$$= 10(2 - x)\left(\frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}\right)^2$$

$$H_{2,2}(1.9) = (24-10x)\left(\frac{50}{9}x^2 - \frac{145}{9}x + \frac{104}{9}\right)^2$$

$$\hat{H}_{n,j}(x) = (x - x_j)\, L^2_{n,j}(x).$$

$X_0 = 1.3$
$X_1 = 1.6$
$X_2 = 1.9$

$$\hat{H}_{2,0}(X_0) = (X-1.3)\left(\frac{50}{9}X^2 - \frac{175}{9}X + \frac{152}{9}\right)^2$$

$$\hat{H}_{2,1}(X_1) = (x-1.6)\left(-\frac{100}{9}x^2 + \frac{320}{9}x - \frac{247}{9}\right)^2$$

$$\hat{H}_{2,2}(X_2) = (X-1.9)\left(\frac{50}{9}X^2 - \frac{145}{9}X + \frac{104}{9}\right)^2$$

Acá se expresa el polinomio de Hermite así

$$P_5(X) = f(X_0)H_{2,0}(X) + f(X_1)H_{2,1} + f(X_2)H_{2,2} + f'(X_0)\hat{H}_{2,0}(X) + f'(X_1)\hat{H}_{2,1} + f'(X_2)\hat{H}_{2,2}.$$

$$H_{2,0}(1{,}5) = (10(1{,}5) - 12)\left(\frac{50}{9}(1{,}5)^2 - \frac{175}{9}(1{,}5) + \frac{152}{9}\right)^2$$
$$= \left(\frac{4}{27}\right)$$

$$H_{2,1}(1{,}5) = \left(-\frac{100}{9}(1{,}5)^2 + \frac{320}{9}(1{,}5) - \frac{247}{9}\right)^2$$
$$= \left(\frac{64}{81}\right)$$

$$H_{2,2}(1{,}5) = 10(2 - 1{,}5)\left(\frac{50}{9}(1{,}5)^2 - \frac{145}{9}(1{,}5) + \frac{104}{9}\right)^2$$
$$= \frac{5}{81}$$

$$\hat{H}_{2,0}(1{,}5) = (1{,}5 - 1{,}3)\left(\frac{50}{9}(1{,}5)^2 - \frac{175}{9}(1{,}5) + \frac{152}{9}\right)^2$$
$$= \frac{4}{405}$$

$$\hat{H}_{2,1}(1{,}5) = (1{,}5 - 1{,}6)\left(-\frac{100}{9}(1{,}5)^2 + \frac{320}{9}(1{,}5) - \frac{247}{9}\right)^2$$
$$= -\frac{32}{405}$$

$$\hat{H}_{2,2}(1{,}5) = (1{,}5 - 1{,}9)\left(\frac{50}{9}(1{,}5)^2 - \frac{145}{9}(1{,}5) + \frac{104}{9}\right)^2$$
$$= -\frac{2}{405}$$

$x=1.5$

$$P_5(x)=0.62008860\,H_{2,0}(x)+0.4554022\,H_{2,1}(x)+0.2818186\,H_{2,2}(x)-0.5220232\,\hat{H}_{2,0}(x)-0.5698959\,\hat{H}_{2,1}(x)-0.5811571\,\hat{H}_{2,2}(x)$$

$x=1.5$

$$P_5(1.5)=0.6200860\left(\frac{4}{27}\right)+0.4554022\left(\frac{64}{81}\right)+0.2818186\left(\frac{5}{81}\right)-0.5220232\left(\frac{4}{405}\right)-0.5698959\left(\frac{-32}{405}\right)-0.5811571\left(-\frac{2}{405}\right)$$

$$\underline{P_5(1.5)=0.5118277}\ R//$$