

## Clases de análisis numérico

Ejemplo 3: Determinar por extrapolación de Richardson de nivel 5, la  $f'(0.7)$  de la función  $f(x) = \ln x$  por el método de diferencias centrales de orden 4. Con  $h = 0.1$ .

$$f'(x_0) = \frac{-f(x_0 + 2h) + 8f(x_0 + h) - 8f(x_0 - h) + f(x_0 - 2h)}{12h}.$$

Como es de nivel 5, se necesitan 5 subintervalos.  
los:

$$h_1 = 0.1, \quad h_2 = \frac{h_1}{2} = \frac{0.1}{2} = 0.05 \quad h_3 = \frac{h_2}{2} = \frac{0.05}{2} = 0.025$$

$$h_4 = \frac{0.025}{2} = 0.0125 \quad h_5 = \frac{h_4}{2} = \frac{0.0125}{2} = 6.25 \times 10^{-3}$$

Luego encontramos las diferencias centrales.

$$f'(0.7) = \frac{-f(0.7 + 2(0.1)) + 8f(0.7 + 0.1) - 8f(0.7 - 0.1) + f(0.7 - 2(0.1))}{12(0.1)}$$

$\Rightarrow$

$$f'(0.7) = \frac{-f(0.9) + 8f(0.8) - 8f(0.6) + f(0.5)}{1.2} \quad \text{Pg 2.}$$

$$\underline{\underline{f'(0.7) = 1.428058262}}$$

$$h = 0.05$$

$$f'(0.7) = \frac{-f(0.7+2(0.05)) + 8f(0.7+0.05) - 8f(0.7-0.05) + f(0.7-2(0.05))}{12(0.05)}$$

$$= \frac{-f(0.8) + 8f(0.75) - 8f(0.65) + f(0.6)}{0.6}$$

$$\underline{\underline{= 1.428541128}}$$

$$h = 0.025$$

$$f'(0.7) = \frac{-f(0.7+2(0.025)) + 8f(0.7+0.025) - 8f(0.7-0.025) + f(0.7-2(0.025))}{12(0.025)}$$

$$= \frac{-f(0.75) + 8f(0.725) - 8f(0.675) + f(0.65)}{0.3}$$

$$\underline{\underline{= 1.428569561}}$$



$$r \quad h = 0.0125$$

$$f'(0.7) = \frac{-f(0.7+2(0.0125)) + 8f(0.7+0.0125) - 8f(0.7-0.0125) + f(0.7-2(0.0125))}{12(0.0125)}$$

$$= \frac{-f(0.725) + 8f(0.7125) - 8f(0.6875) + f(0.675)}{0.15}$$

$$= \underline{\underline{1.428571312}} /$$

$$r \quad h = 6.25 \times 10^{-3}$$

$$f'(0.7) = \frac{-f(0.7+2(6.25 \times 10^{-3})) + 8f(0.7+6.25 \times 10^{-3}) - 8f(0.7-6.25 \times 10^{-3}) + f(0.7-2h)}{12(6.25 \times 10^{-3})}$$

$$= \frac{-f(0.7125) + 8f(0.70625) - 8f(0.69375) + f(0.6875)}{0.075}$$

$$= 1.428571421$$

$$\text{Nível 2} \rightarrow D(2, \bar{x}) = \frac{4}{3} D(1, \bar{x}+1) - \frac{1}{3} D(1, \bar{x})$$

$$\text{Nível 3} \rightarrow D(3, \bar{x}) = \frac{16}{15} D(2, \bar{x}+1) - \frac{1}{15} D(2, \bar{x})$$

$$\text{Nível 4} \rightarrow D(4, \bar{x}) = \frac{64}{63} D(3, \bar{x}+1) - \frac{1}{63} D(3, \bar{x})$$

$$\text{Nível 5} \rightarrow D_{(k+1, \bar{x})} = \frac{4^k D_{(k-1, \bar{x}+1)} - D_{(k-1, \bar{x})}}{4^k - 1}$$

$$K+1=5$$

$$K=4$$

$$D(5, \bar{x}) = \frac{4^4 D(4, \bar{x}+1) - D(4, \bar{x})}{4^4 - 1}$$

$$= \frac{256}{255} D(4, \bar{x}+1) - \frac{1}{255} D(4, \bar{x})$$

Nivel 2

$$\boxed{D(2, \bar{x}) = \frac{4}{3} D(1, \bar{x}+1) - \frac{1}{3} D(1, \bar{x})}$$

$$\begin{aligned} D(2, 1) &= \frac{4}{3} (1.428541128) - \frac{1}{3} (1.428058262) \\ &= \underline{1.428702083} / \end{aligned}$$

$$\begin{aligned} D(2, 2) &= \frac{4}{3} (1.428569561) - \frac{1}{3} (1.428541128) \\ &= \underline{1.428579039} / \end{aligned}$$

$$\begin{aligned} D(2, 3) &= \frac{4}{3} (1.428571312) - \frac{1}{3} (1.428569561) \\ &= \underline{1.428553229} / \end{aligned}$$

$$\begin{aligned} D(2, 4) &= \frac{4}{3} (1.428571421) - \frac{1}{3} (1.428571312) \\ &= \underline{1.428571457} / \end{aligned}$$



Nivel 3

$$D(3, \bar{x}) = \frac{16}{15} D(2, \bar{x}+1) - \frac{1}{15} D(2, \bar{x})$$

$$\begin{aligned} D(3, 1) &= \frac{16}{15} (1.428579039) - \frac{1}{15} (1.428702083) \\ &= \underline{1.428570836, /} \end{aligned}$$

$$\begin{aligned} D(3, 2) &= \frac{16}{15} (1.428553229) - \frac{1}{15} (1.428579039) \\ &= \underline{1.428551508, /} \end{aligned}$$

$$\begin{aligned} D(3, 3) &= \frac{16}{15} (1.428571421) - \frac{1}{15} (1.428553229) \\ &= 1.428572634 \end{aligned}$$

Nivel 4

$$D(4, \bar{x}) = \frac{64}{63} D(3, \bar{x}+1) - \frac{1}{63} D(3, \bar{x})$$

$$\begin{aligned} D(4, 1) &= \frac{64}{63} (1.428551508) - \frac{1}{63} (1.428570836) \\ &= \underline{1.4285512011} \end{aligned}$$

$$\begin{aligned} D(4, 2) &= \frac{64}{63} (1.428572634) - \frac{1}{63} (1.428551508) \\ &= \underline{1.428572969} \end{aligned}$$

$$D(5, \bar{x}) = \frac{256}{255} D(4, \bar{x}+1) - \frac{1}{255} D(4, \bar{x})$$

$$\begin{aligned} D(5, 1) &= \frac{256}{255} (1.428572969) - \frac{1}{255} (1.428551201) \\ &= \underline{1.4285730551} \end{aligned}$$

Nivel 1	Nivel 2	Nivel 3	Nivel 4	Nivel 5
1.428658262	1.428702083	1.428570836	1.428551261	1.428573055
1.428541128	1.428579039	1.428551568	1.428572969	
1.428569561	1.428553229	1.428572634		
1.428571312	1.428571457			
1.428571421				

Valor verdadero

$$f(x) = \ln x \rightarrow f'(x) = \frac{1}{x}$$

$$f'(0.7) = \frac{1}{0.7} = 1.428571429$$

Diferencias finitas

$$\epsilon\% = \left| \frac{\frac{1}{0.7} - 1.42858262}{\frac{1}{0.7}} \right| \times 100 = 0.03592166\%$$

Richardson

$$\epsilon\% = \left| \frac{\frac{1}{0.7} - 1.428573055}{\frac{1}{0.7}} \right| \times 100 = 0.0001385\%$$