UNIVERSIDAD LINDA VISTA

EX-FINCA STA CRUZ #1 PUEBLO NUEVO SOLISTAHUACÁN, CHIAPAS

INGENIERÍA EN DESARROLLO DE SOFTWARE

CALCULO DIFERENCIAL

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ACTIVIDAD DE APRENDIZAJE:

EJERCICIOS DE LABORATORIO

FECHA DE ENTREGA:

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Contenido

Introducción

En el siguiente laboratorio se estudiará el concepto de derivadas, siguiendo reglas que nos facilitan la resolución de estas. Se hará por medio de ejercicios manuales y su comprobación con la herramienta de Scilab, un software matemático.

El objetivo es identificar la forma indicada para resolver los planteamientos y ejercitar las habilidades por medio de la práctica.

Definición

La derivada de *f* en *x* está dada por

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Siempre que exista ese límite. Para todos los *x* para los que exista este límite, *f'* es una función de *x* (Larson & Edwards, 2015).

Ejemplo:

$$f(x) = 3x^4 + 2x - 5$$

$$f'(x) = \frac{d}{dx}3x^4 + \frac{d}{dx}2x - \frac{d}{dx}5$$

$$f'(x) = 3 \cdot 4x^{4-1} + 2 - 0$$

$$f'(x) = 12x^3 + 2$$

Ejercicios

Manual, ejercicios 5.4

a) $y = \sqrt[3]{x} + \sqrt{x}\ ;\ x = 64 \qquad f'(x) = \frac{dy}{dx}(u) + \frac{dy}{dx}(v)$

$$\frac{dy}{dx}\left(x^{\frac{1}{3}}\right) = \frac{1}{3}\,x^{-\frac{2}{3}} \qquad \frac{dy}{dx}\left(\sqrt{x}\right) = \frac{1}{2}\,x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \left(\frac{1}{3\sqrt[3]{64^2}}\right) + \left(\frac{1}{2\sqrt{64}}\right) \rightarrow \frac{dy}{dx} = \frac{1}{3(16)} + \frac{1}{2(8)} = \frac{1}{48} + \frac{1}{16}$$

$$\frac{dy}{dx} = \frac{64}{768} = \underline{\frac{1}{12}}$$

-->deff('y=f(x)','y=(x.^(1/3))+(x.^(1/2))')

-->dy=numderivative(f,64)

dy =

0.0833333

b) $y = \frac{\sqrt{16+3x}}{x}\ ;\ x = 3 \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

$$u = \sqrt{16+3x} \qquad \frac{dy}{dx}\left(16+3x^{\frac{1}{2}}\right) = \left(\frac{1}{2}\,(16+3x)^{-\frac{1}{2}}\,(3)\right) \qquad \frac{dy}{dx}(x) = 1$$
$$v = x$$

$$\frac{dy}{dx} = \frac{x\left[\frac{1}{2}(16+3x)^{-\frac{1}{2}}(3)\right] - \sqrt{16+3x}}{x^2} \rightarrow \frac{dy}{dx} = \frac{x\left[\frac{3}{2\sqrt{16+3x}}\right] - \sqrt{16+3x}}{x^2}$$

$$\frac{dy}{dx} = \frac{\frac{3x}{2\sqrt{16+3x}} - \frac{\sqrt{16+3x}}{1}}{\frac{x^2}{1}} \rightarrow \frac{dy}{dx} = \frac{\frac{3x - 2(16+3x)}{2\sqrt{16+3x}}}{\frac{x^2}{1}} \qquad \frac{dy}{dx} = \frac{3x - 2(16+3x)}{(x^2)\,2\sqrt{16+3x}}$$

$$\frac{dy}{dx} = \frac{3x - (32+6x)}{(x^2)\,2\sqrt{16+3x}} = \frac{9-50}{9(2\sqrt{25})} = -\underline{\frac{41}{90}}$$

--> deff('y=f(x)','y=(sqrt(16+3*x))./(x)')

--> dy=numderivative(f,3)

dy =   -0.4555556

c) $y = \frac{x^2+2}{2-x^2}$; $x = 2$ $\qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{}$

$u = x^2+2 \qquad \frac{dy}{dx}(x^2+2) = 2x \qquad \frac{dy}{dx}(2-x^2) = -2x$

$v = 2-x^2$

$$\frac{dy}{dx} = \frac{2-x^2\,(2x) - x^2+2\,(-2x)}{(2-x^2)^2} \rightarrow \frac{dy}{dx} = \frac{(4x-2x^3)-(-2x^3-4x)}{(2-x^2)^2}$$

$$\frac{dy}{dx} = \frac{4x \cancel{-2x^3} \cancel{+2x^3} + 4x}{(2-x^2)^2} = \frac{4x+4x}{(2-x^2)^2} = \frac{dy}{dx} = \frac{8(2)}{4} = \frac{16}{4} = 4$$

```
--> deff('y=f(x)','y=((x.^2)+2)./(2-(x.^2))')

--> dy=numderivative(f,2)

 dy =    4.0000000
```

d) $y = \frac{\sqrt{5-2x}}{2x+1}$; $x = \frac{1}{2}$ $\qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

$u = \sqrt{5-2x} \qquad \frac{d}{dx}(u) = \frac{1}{2}\sqrt{5-2x}\,(-2) = \frac{1}{2\sqrt{5-2x}}(-2) \qquad \frac{d}{dx}(v) = 2$

$v = 2x+1$

$$\frac{dy}{dx} = \frac{2x+1\left[-\frac{1}{\sqrt{5-2x}}\right] - \frac{2\sqrt{5-2x}}{1}}{(2x+1)^2} \rightarrow \frac{dy}{dx} = \frac{\frac{-2x-1-(2(5-2x))}{\sqrt{5-2x}}}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x-11}{(2x+1)^2\sqrt{5-2x}} \rightarrow \frac{dy}{dx} = \frac{1-11}{4\sqrt{4}} = -\frac{10}{8} = -1.25$$

```
--> deff('y=f(x)','y=(sqrt(5-2*x))./(2*x+1)')

--> dy=numderivative(f,(1./2))

 dy =   -1.2500000
```

c) $y = x\sqrt{3+2x}\ ;\ x = 3 \qquad \frac{d}{dx} = UV = UV' + VU'$

$U = x \quad V = \sqrt{3+2x} \qquad \frac{dy}{dx}(U) = x = 1 \qquad \frac{d}{dx}(V) = 3+2x^{\frac{1}{2}} = \frac{1}{2}(3+2x)^{-\frac{1}{2}}(2)$

$$\frac{dy}{dx} = x\left((3+2x)^{-\frac{1}{2}}\right) + \sqrt{3+2x} = \frac{x}{\sqrt{3+2x}} + \frac{\sqrt{3+2x}}{1} = \frac{x+(3+2x)}{\sqrt{3+2x}}$$

$$\frac{dy}{dx} = \frac{3+3(3)}{\sqrt{3+2(3)}} = \frac{12}{3} = 4$$

--> deff('y=f(x)','y=(x*sqrt(3+2*x))')

--> dy=numderivative(f,3)

dy =   4.0000000

e) $y = \sqrt{\frac{4x+1}{5x-1}}\ ;\ x = 2 \qquad U = (4x+1)^{\frac{1}{2}} \quad V = (5x+1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{5x-1\,(4) - 4x+1\,(5)}{(5x-1)^2} = \frac{dy}{dx} = \frac{(20x-4)-(20x+5)}{(5x-1)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{20x}-4-\cancel{20x}-5}{(5x-1)^2} = \frac{-9}{(5x-1)^2} \cdot \left[\frac{1}{2}\left(\frac{(4x+1)}{(5x-1)}\right)^{\frac{1}{2}}\right]$$

$$\frac{dy}{dx} = \frac{-9}{2(5x-1)^2\sqrt{\frac{4x+1}{5x-1}}} = \frac{-9}{2(81)\sqrt{\frac{9}{9}}} = \frac{-9}{162} = -\frac{1}{18} = -0{,}0555556$$

--> deff('y=f(x)','y=(sqrt((4*x+1)./(5*x-1)))')1

--> dy=numderivative(f,2)

dy =   -0.0555556

g) $y=\sqrt{\frac{x^2-5}{10-x^2}}$; $x=3$ $\qquad \frac{d}{dx}\frac{U}{V}=\frac{V\cdot U'-U V'}{V^2}$

$U=(x^2-5)^{\frac{1}{2}}$ $\quad \frac{d}{dx}(x^2-5)=\underline{2x}$ $\quad \frac{d}{dx}(10-x^2)=\underline{-2x}$

$V=(10-x^2)^{\frac{1}{2}}$

$$\frac{dy}{dx}=\frac{10-x^2\,(2x)\;-\;x^2-5\,(-2x)}{(10-x^2)^2}\Rightarrow\frac{dy}{dx}=\frac{(20x-2x^3)-(-2x^3+10x)}{(10-x^2)^2}$$

$$\frac{dy}{dx}=\frac{20x\cancel{-2x^3}\cancel{+2x^3}-10x}{(10-x^2)^2}=\frac{10x}{(10-x^2)^2}\cdot\frac{1}{2}\left(\frac{x^2-5}{10-x^2}\right)^{-\frac{1}{2}}=\frac{10x}{2(10-x^2)^2\sqrt{\frac{x^2-5}{10-x^2}}}$$

$$\frac{dy}{dx}=\frac{30}{2(1)\sqrt{\frac{4}{1}}}=\frac{30}{4}=\underline{7.5}$$

deff('y=f(x)','y=(sqrt((x.^2-5)./(10-x.^2)))')

--> dy=numderivative(f,3)

dy = 7.5000001

h) $y=\ln(x^2+2)$; $x=2$ $\qquad \frac{d}{dx}(\ln v)=\frac{v'}{v}$

$\frac{d}{dx}(x^2+2)=\underline{2x}$

$$\frac{dy}{dx}=\frac{2x}{x^2+2}=\frac{2(2)}{(2)^2+2}=\frac{4}{6}=\underline{0.6666667}$$

--> deff('y=f(x)','y=(log(x.^2+2))')

--> dy=numderivative(f,2)

dy = 0.6666667

i) $y = xe^{-2x}$; $x = \frac{1}{2}$ $\quad \frac{d}{dx} uv = u'v + uv'$

$u = x$ $\quad \frac{d}{dx}(x) = 1$ $\quad \frac{d}{dx}(e^{-2x}) = e^{-2x} \cdot (-2) = -2e^{-2x}$
$v = e^{-2x}$

$$\frac{dy}{dx} = 1\left(e^{-2x}\right) + x\left(-2e^{-2x}\right) \Rightarrow \frac{dy}{dx} = 1\left(e^{-2\frac{1}{2}}\right) + \frac{1}{2}\left(-2e^{-2\frac{1}{2}}\right)$$

$$\frac{dy}{dx} = \left(e^{-1}\right) - e^{-1} = \underline{0}$$

```
--> deff('y=f(x)','y=x*exp(-2*x)')
--> dy=numderivative(f,0.5)
 dy =    4.584D-12
```

j) $y = \frac{\ln x^2}{x}$; $x = 4$ $\quad \frac{d}{dx}\frac{u}{v} = \frac{v \cdot u' - u \cdot v'}{v^2}$

$u = \ln x^2$ $\quad \frac{du}{dx}(\ln x^2) = \frac{2x}{x^2} = \frac{2}{x}$ $\quad \frac{dv}{dx}(x) = 1$
$v = x$

$$\frac{dy}{dx} = \frac{x\left(\frac{2}{x}\right) - \ln x^2 (1)}{x^2} = \frac{\frac{2x}{x} - 2\ln x (1)}{x^2} = \frac{2 - 2\ln(4)}{16}$$

$$\frac{dy}{dx} = \underline{-0.048286}$$

```
--> deff('y=f(x)', 'y=(log(x^2))./(x)')
--> dy = numderivative(f, 4)
 dy =   -0.0482868
```

k) $y = x \operatorname{Sen}\frac{x}{2}$; $x = 2$ $\qquad \frac{d}{dx} uv = u' \cdot v + u \cdot v'$

$u = x$ $\qquad \frac{du}{dx}(x) = 1$ $\qquad \frac{dv}{dx}\left(\operatorname{Sen}\frac{x}{2}\right) = \cos\frac{x}{2} \frac{d}{dx}\frac{x}{2} = \cos\frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2}\cos\frac{x}{2}$

$v = \operatorname{Sen} x/2$

$$\frac{dy}{dx} = x\left(\frac{1}{2}\cos\frac{x}{2}\right) + \operatorname{Sen}\frac{x}{2} \Rightarrow \frac{dy}{dx} \quad \frac{x}{2}\left(\cos\frac{x}{x}\right) + \operatorname{Sen}\frac{x}{2} =$$

$$\frac{dy}{dx} = \frac{2}{2}\cos\frac{2}{2} + \operatorname{Sen}\frac{2}{2} = \cos(1) + \operatorname{sen}(1) = \underline{1.381773}$$

--> deff('y = f(x)', 'y = x .* sin(x / 2)')

--> dy = numderivative(f, 2)

dy = 1.3817733

l) $y = \ln \cos x$; $x = 0.5$

$$\frac{d}{dx}(\cos x) = -\operatorname{Sen} x \frac{d}{dx} x = -\operatorname{sen} x \cdot 1 = \underline{-\operatorname{sen} x}$$

$$\frac{dy}{dx} = -\frac{\operatorname{Sen} x}{\cos x} = -\tan y = -\tan(0.5) = \underline{-0.54630}$$

--> deff('y = f(x)', 'y = log(cos(x))')

--> dy = numderivative(f,0.5)

dy = -0.5463025

m) $y = \operatorname{Sen} x \cos 2x; \quad x = 1 \qquad \frac{d}{dx} uv = v \cdot u' + u v'$

$U = \operatorname{Sen} x \qquad \frac{du}{dx}(\operatorname{Sen} x) = \cos x \frac{d}{dx} x = \underline{\cos x}$

$V = \cos 2x \qquad \frac{dv}{dx}(\cos 2x) = -\operatorname{sen} 2x \frac{d}{dx} 2x = -2 \operatorname{sen} 2x$

$\frac{dy}{dx} = \cos 2x (\cos x) + \operatorname{sen} x(-2 \operatorname{sen} 2x)$

$\frac{dy}{dx} = -0.2248 - 1.5302 = \underline{-1.7551}$

--> deff('y = f(x)', 'y=(sin(x)).*(cos(2*x))')

--> dy = numderivative(f,1)

dy =   -1.7551399

n) $y = 5 e^{\frac{x}{2}} \operatorname{sen} \frac{\pi x}{2}; \quad x = 2$

$U = 5 e^{\frac{x}{2}} \qquad \frac{du}{dx}(5 e^{\frac{x}{2}}) = 5 \frac{1}{2} e^{\frac{x}{2}} = \underline{\frac{5}{2} e^{\frac{x}{2}}}$

$V = \operatorname{Sen} \frac{\pi x}{2} \qquad \frac{dv}{dx}\left(\operatorname{Sen} \frac{\pi x}{2}\right) = \underline{\frac{\pi}{2} \cos \frac{\pi x}{2}}$

$\frac{dy}{dx} = \operatorname{sen} \frac{\pi x}{2} \left(\frac{5}{2} e^{\frac{x}{2}}\right) + 5 e^{\frac{x}{2}} \left(\frac{\pi}{2} \cos \frac{\pi x}{2}\right)$

$\frac{dy}{dx} = \frac{5}{2} e^{\frac{x}{2}} \operatorname{sen} \frac{\pi x}{2} + \frac{5\pi}{2} e^{\frac{x}{2}} \cos \frac{\pi x}{2}$

$= \frac{5\pi}{2} e(-1) + \frac{5}{2} e(0) = \underline{-21.3493}$

--> deff('y = f(x)', 'y = 5*exp(x./2)*sin(%pi*x./2)')

--> dy = numderivative(f,2)

dy =   -21.349336

o) $y = \ln\sqrt{tg\, x}$; $x = \frac{1}{4}\pi$ $\frac{d}{dx}\ln v = \frac{v'}{v}$

$y = \frac{1}{2}\ln(\tan x)$ $\frac{d}{dv}\ln(\tan x) = \frac{1}{\tan x} \cdot \sec^2 x$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\left(\frac{1}{\tan x}\cdot \sec^2 x\right) \rightarrow \frac{dy}{dx} = \frac{1}{2}\cdot\frac{\sec^2 x}{\tan x} = \frac{1}{2}\frac{\frac{1}{\cos^2 x}}{\frac{\operatorname{Sen} x}{\cos x}}$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{\cos x}{\cos^2 x \operatorname{Sen} x} \quad \frac{dy}{dx} = \frac{1}{2}\frac{1}{\cos x \operatorname{Sen} x} = \frac{1}{2(\cos x \operatorname{Sen} x)} = \frac{1}{2(\cos(\frac{\pi}{4})\operatorname{Sen}(\frac{\pi}{4}))}$$

$$\frac{dy}{dx} = \frac{1}{2(0.5)} = \frac{1}{1} = \underline{1}$$

--> deff('y = f(x)', 'y=log(sqrt(tan(x)))')

--> dy = numderivative(f,%pi./4)

dy =   1.0000000

p) $y = x\ln\sqrt{x+3} \rightarrow y = \frac{x}{2}\ln(x+3)$ $\frac{d}{dx} vu = vu' + uv'$

$x = 6$

$U = x/2$ $\frac{du}{dx}(x/2) = \frac{1}{2}$ $\frac{dv}{dx}\ln(x+3) = \frac{1}{x+3}$

$V = \ln(x+3)$

$$\frac{dx}{dx} = \ln(x+3)\left(\frac{1}{2}\right) + \frac{x}{2}\left(\frac{1}{x+3}\right) \rightarrow \frac{dy}{dx} = \ln(9)\frac{1}{2} + \frac{1}{3}$$

$$\frac{dy}{dx} = \left(2\ln 3 \cdot \frac{1}{2} = \ln 3\right) + \frac{1}{3} = 1.0986 + 0.3333 = \underline{1.4319}$$

--> deff('y=f(x)','y=x*(log(sqrt(x+3)))')

--> dy = numderivative(f,6)

dy =   1.4319456

$$\frac{d}{dx} vu = vu' + uv'$$

q) $y = x \operatorname{arc sen} x \; ; \; x = 1/2$

$u = x$ $\quad \frac{du}{dx} f(x) = 1$ $\quad \frac{dv}{dx}(\operatorname{arcsen}(x)) = \frac{1}{\sqrt{1-x^2}}$

$v = \operatorname{arcSen}(x)$

$$\frac{dy}{dx} = (\operatorname{arcsen}(x))(1) + x\left(\frac{1}{\sqrt{1-x^2}}\right) \div \frac{dy}{dx} = \operatorname{arcSen}\left(\frac{1}{2}\right) + \left(\frac{1/2}{\sqrt{1-\frac{1}{2}^2}}\right)$$

$$\frac{dy}{dx} = 0.52359 + \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = 0.52859 + 0.57735 = \underline{1.10094}$$

--> deff('y=f(x)', 'y=x*asin(x)')

--> dy = numderivative(f,0.5)

dy = 1.1009490

$$\frac{d}{dx} uv = uv' + u'v$$

r) $y = \frac{1}{x} \operatorname{arc tg} x \; ; \; x = 1$

$$\frac{du}{dx}\left(\frac{1}{x}\right) = \frac{x(0) - (1)1}{(x)^2} = \frac{-1}{x^2} \quad \frac{dv}{dx}(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right) + \left(-\frac{1}{x^2}\right)\operatorname{arctg} x = \frac{1}{2(1+x^2)} - \frac{\operatorname{arct} x}{x^2}$$

$$\frac{dy}{dx} = 0.5 - 0.78539 = \underline{-0.28539}$$

--> deff('y=f(x)', 'y=(1 ./ x) * atan(x)')

--> dy = numderivative(f,1)

dy = -0.2853982

5) $y = x^2 \operatorname{arccsc} \sqrt{x}$; $x = 2$ $\quad \frac{du}{dx}(x^2) = 2x$; $\frac{dv}{dx}(\operatorname{arccsc}\sqrt{x}) = \frac{1}{(\sqrt{x})\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x}}$

$$\frac{dy}{dx} = (x^2)\left(-\frac{1}{2x\sqrt{x-1}}\right) + \operatorname{arccsc}\sqrt{x}\,(2x)$$

$$\frac{dy}{dx} = -\frac{4}{4} + 4 \operatorname{arccsc}\sqrt{2} = 4\left(\operatorname{sen}^{-1}(1 \div \sqrt{2})\right) - 1 = 3.141592 - 1$$

$$\frac{dy}{dx} = \underline{2.141592}$$

--> deff('y=f(x)', 'y=x.^2*acsc(sqrt(x))')

--> dy = numderivative(f,2)

dy = 2.1415927

Larsson (2015), pág. 136

9) $g(x) = 3(4-9x)^4$ $\quad \frac{d}{dx} uv = uv' + vu'$

$$\frac{du}{dx} = (3) = \underline{0} ; \frac{dv}{dx} = (4-9x)^4 = 4(4-9x)^3 \cdot (-9)$$

$$\frac{dy}{dx} = 3\left[-36(4-9x)^3\right] + (4-9x)^4(0) = \frac{dy}{dx} = \left[-36(4-9x)^3\right] + 0 =$$

$$\frac{dy}{dx} = \underline{-108(4-9x)^3}$$

12) $g(x) = \sqrt{4-3x^2}$

$$\frac{dy}{dx}(4-3x^2)^{\frac{1}{2}} = \frac{1}{2}(4-3x^2)^{-\frac{1}{2}} = \frac{1}{2\sqrt{4-3x^2}} \cdot (6x) = -\frac{6x}{2\sqrt{4-3x^2}} = \underline{-\frac{3x}{\sqrt{4-3x^2}}}$$

15) $y = 2\sqrt[4]{9-x^2}$ $\quad U = 2$, $V = \sqrt[4]{9-x^2}$

$\frac{du}{dx}(2) = 0$ $\quad \frac{dv}{dx}(9-x^2)^{\frac{1}{4}} = \frac{1}{4}(9-x^2)^{-\frac{3}{4}} \cdot (-2x)$

$$\frac{dy}{dx} = 2\left[\frac{1}{4}(9-x^2)^{-\frac{3}{4}}(-2x)\right] = 2\left[-\frac{2x}{4}(9-x^2)^{-\frac{3}{4}}\right]$$

$$\frac{dy}{dx} = -\frac{4x}{4(9-x^2)^{-3/4}} = -\frac{x}{(9-x^2)^{-\frac{3}{4}}}$$

17) $y = \frac{1}{x-2}$ $\quad U = 1$, $V = x-2$ $\quad \frac{d'}{dx} = -\frac{c}{x^2} \cdot \frac{dv}{dx}$

$$\frac{dy}{dx} = -\frac{1}{(x-2)^2} \cdot \frac{1}{1} = -\frac{1}{(x-2)^2}$$

22) $g(t) = \frac{1}{\sqrt{t^2-2}} \Rightarrow g(t) = (t^2-2)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(t^2-2)^{-\frac{3}{2}} \cdot 2t = \frac{dy}{dx} = \frac{1}{2(t^2-2)^{+\frac{3}{2}}} \cdot 2x = \frac{2t}{2(t^2-2)^{+\frac{3}{2}}}$$

$$\frac{dx}{dx} = \frac{t}{(t^2-2)^{+\frac{3}{2}}}$$

23) $f(x) = x^2(x-2)^4$ $\quad U = x^2$, $V = (x-2)^4$

$\frac{du}{dx}(x^2) = 2x$ $\quad \frac{dv}{dx}(x-2)^4 = 4(x-2)^3 \cdot 1$

$$\frac{dy}{dx} = (2x)(x-2)^4 + x^2\left(4(x-2)^3\right)$$

$$\frac{dy}{dx} = 2x(x-2)(x-2)^3 + 4x^2(x-2)^3$$

$$\frac{dy}{dx} = 2x(x-2)^3[x-2+2x) = \frac{dy}{dx} = 2x(x-2)^3(3x-2)$$

24) $f(x) = x(2x-5)^3 \quad \frac{d}{dx} uv = vu' + uv'$

$\frac{du}{dx} f(x) = \underline{1} \quad \frac{dv}{dx} (2x-5)^3 = \underline{3(2x-5)^2 \cdot (2)}$

$\frac{dy}{dx} = (2x-5)^3 (1) + x(6(2x-5)^2) \quad \frac{dy}{dx} = (2x-5)^2 [2x-5+6x]$

$\frac{dy}{dx} = \underline{(2x-5)^2 (8x-5)}$

26) $y = \frac{1}{2} x^2 \sqrt{16-x^2}$

$\frac{du}{dx} f(x^2) = \underline{2x} \quad \frac{dv}{dx} (16-x^2)^{\frac{1}{2}} = \underline{\frac{1}{2\sqrt{16-x^2}} \cdot (-2x)} = -\frac{2x}{2\sqrt{16-x^2}} = -\frac{x}{\sqrt{16-x^2}}$

$\frac{dy}{dx} = x^2 \left(-\frac{2x}{2\sqrt{16-x^2}}\right) + \sqrt{16-x^2} \ (2x)$

$\frac{dy}{dx} = \frac{2x\sqrt{16-x^2}}{1} - \frac{x^3}{\sqrt{16-x^2}} \quad \frac{dy}{dx} = -\frac{2x \ (16-x^2) + x^3}{\sqrt{16-x^2}}$

$\frac{dy}{dx} = -\frac{32x - 3x^3}{\sqrt{16-x^2}} \cdot \frac{1}{2} \quad \frac{dy}{dx} = -\underline{\frac{3x^2-32}{2\sqrt{16-x^2}}}$

29) $g(x) = \left(\frac{x+5}{x^2+2}\right)^2 \quad \begin{matrix} u = x+5 = 1 \\ v = x^2+2 = 2x \end{matrix}$

$\frac{dy}{dx} = \frac{(x^2+2)(1) - (x+5)(2x)}{(x^2+2)^2} = \frac{-x^2+10x+2}{(x^2+2)^2}$

$\frac{dy}{dx} = 2\left(\frac{x+5}{x^2+2}\right) - \frac{x^2+10x+2}{(x^2+2)^2} = -\frac{2(x+5)(-x^2-10x+2)}{(x^2+2)^3}$

32) $g(x) = \left(\frac{3x^2-2}{2x+3}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{3x^2-2}{2x+3}\right)^2 \cdot \frac{d}{dx}\left(\frac{3x^2-2}{2x+3}\right)$$

$$\frac{d}{dx} = \frac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2} = \frac{12x^2+18x-6x^2+4}{(2x+3)^2}$$

$$\frac{dy}{dx} = 3\left(\frac{3x^2-2}{2x+3}\right)^2 \cdot \frac{6x^2+18x+4}{(2x+3)^2} = \frac{3(3x^2-2)^2(6x^2+18x+4}{(2x+3)^4}$$

45) $g(x) = 5 \tan 3x$

$$\frac{dy}{dx}(5\tan 3x) = 5\frac{d}{dx}(\tan g\, 3x) = 5(\sec^2 3x)(3) = \underline{15\sec^2 3x}$$

48) $y = \cos(1-2x)^2$

$$\frac{du}{dx} = 2(1-2x)\cdot -2 = -4(1-2x) \qquad \frac{du}{dx}\cos = -\operatorname{sen} x$$

$$\frac{dy}{dx} = -\operatorname{sen}(1-2x)^2 \cdot -4(1-2x) = \underline{4\operatorname{sen}(1-2x)^2 \cdot (1-2x)}$$

51) $f(x) = \frac{\cot x}{\operatorname{sen} x}$

$$\frac{du}{dx}\cot x = -\csc^2 x \cdot 1 = \underline{-\csc^2 x} \qquad \frac{dv}{dx}(\operatorname{sen} x) = \underline{\cos x}$$

$$\frac{dy}{dx} = \frac{(-\csc^2 x)(\operatorname{sen} x) - \cot x(\cos x)}{\operatorname{sen}^2 x} = \frac{-\frac{1}{\sin^2(x)}\operatorname{sen} x - \frac{\cos^2(x)}{\sin(x)}}{\sin^2(x)}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{\operatorname{sen} x} - \frac{\cos^2 x}{\sin(x)}}{\operatorname{sen}^2 x}$$

60) $y = 3x - 5\cos(\pi x)^2$

$\frac{du}{dx} 3x = 3$; $\frac{dv}{dx} \cos^2 = 2\cos(\pi x) \cdot (-\sin(\pi x)) \cdot \pi = -2\pi \cos(\pi x) \sin(\pi x)$

$= +10\pi \cos(\pi x) \sin(\pi x)$

$$\frac{dy}{dx} = \underline{3 + 10\pi \cos(\pi x) \sin(\pi x)}$$

64) $y = \cos\sqrt{\operatorname{sen}(\tan \pi x)}$

$$\frac{dy}{dx} = \left(-\operatorname{sen}\sqrt{\sin(\tan \pi x)}\right) \cdot \frac{d}{dx}\left(\sin(\tan \pi x)^{\frac{1}{2}}\right.$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\sin(\tan(\pi x)\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left(\sin(\tan \pi x)\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(-\operatorname{sen}(\tan(\pi x)\right)^{-\frac{1}{2}}\left(\cos(\tan \pi x) \cdot \frac{du}{dx}(\tan(\pi x))\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(-\operatorname{sen}(\tan(\pi x))\right)^{-\frac{1}{2}}\left[\cos(\tan(\pi x)) \cdot \sec^2(\pi x) \cdot \frac{d}{dx}(\pi x)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left(-\operatorname{sen}(\tan(\pi x))\right)^{-\frac{1}{2}}\left[\cos(\tan(\pi x)) \cdot \sec^2(\pi x) \cdot \pi\right]$$

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{\sin(\tan(\pi x)}} \cdot \left((\cos(\tan(\pi x))\right) \cdot \sec^2(\pi x) \cdot \pi\right)$$

$$\frac{dy}{dx} = -\underline{\frac{(\cos(\tan(\pi x))) \cdot (\sec^2(\pi x) \cdot \pi)}{2\sqrt{\sin(\tan(\pi x))}}}$$

Conclusión

Las derivadas muestran qué tan rápido cambian las funciones en un punto específico. En este trabajo, las resolví aplicando reglas y verifiqué mis resultados con el software Scilab.

Tuve algunas dificultades para identificar qué fórmulas usar, pero con la ayuda de mis compañeros y docente logré hacer las correcciones necesarias.

El propósito de este laboratorio se cumplió, ya que no solo se trató de resolver ejercicios, sino también de comprender los conceptos y fortalecer el razonamiento matemático.

Además de desarrollar habilidades cognitivas, este aprendizaje contribuye al crecimiento profesional, permitiéndonos ofrecer mejores soluciones y ayudar en distintos ámbitos.

Referencias

Larson, R., & Edwards, B. (2015). *Cálculo* (10.ª ed.). Cengage Learning.

Anexo

1. Selecciona las afirmaciones correctas sobre límites.

a) El límite de una función existe cuando existe una aproximación al mismo valor L tanto por izquierda y derecha en x.

c) No puede calcularse el valor de un límite cuando la función presenta un comportamiento no acotado.

2. Dado el siguiente límite

$$\lim_{x\to\infty} \frac{3x^3 + 2x^2 + x + 1}{4x^3 + 1}$$

¿Cuál es la afirmación correcta?

a) El límite es infinito

b) El límite es 0

c) El límite es indeterminado

d) <u>El límite es ¾</u>

$$= \frac{\frac{3x^3}{x^3} + \frac{2x^2}{x^3} + \frac{1}{x^3}}{\frac{4x^3}{x^3} + \frac{1}{x^3}} = \frac{3 + \cancel{\frac{2}{x^2}}^{\to 0} + \cancel{\frac{1}{x^3}}^{\to 0}}{4 + \cancel{\frac{1}{x^3}}^{\to 0}}$$

$=$ El límite es $\frac{3}{4}$

3. Sea

$$\lim_{x\to 2\sqrt{3}} \frac{\sqrt{x^2 - 8} - 2}{x^2 - 12}$$

¿Cuál es el valor del límite?

a) No existe límite

b) −1/4

c) <u>1/4</u>

d) 0

$$= \frac{\sqrt{x^2-8}-2}{x^2-12} \cdot \frac{\sqrt{x^2-8}+2}{\sqrt{x^2-8}+2} = \frac{x^2-8-4}{(x^2-12)(\sqrt{x^2-8}+2)}$$

$$= \frac{\cancel{x^2-12}}{\cancel{x^2-12}(\sqrt{x^2-8}+2)} = \frac{1}{\sqrt{x^2-8}+2} = \frac{1}{\sqrt{12-8}+2} = \frac{1}{2+2} = \frac{1}{4}$$

4. Considere la función definida a trozos

$$f(x) = \{\frac{x+2, x \leq 5}{-x+10, x > 5}$$

Determine el límite de f(x) cuando x → 5 por izquierda y por derecha. Justifique su respuesta.

R = No existe, pues para que haya un límite, los limites de ambos lados deben ser iguales.

5. Calcule a través del proceso de límite el valor de la pendiente de la recta tangente en (3, 1/9) para la función:

$$y = \frac{1}{x^2}$$

El valor encontrado es:

a) 1/9

b) 2/9

c) 2/27

d) <u>−2/27</u>

$$= \frac{\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}}{\Delta x} = \frac{\frac{x^2-(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{\Delta x} = \frac{\frac{\cancel{x^2}-\cancel{x^2}-2x\Delta x-\Delta x^2}{x^2(x+\Delta x)^2}}{\Delta x}$$

$$= \frac{\frac{-2x\Delta x-\Delta x^2}{x^2(x+\Delta x)^2}}{\frac{\Delta x}{1}} = \frac{-2x\Delta x-\Delta x^2}{(\Delta x)(x^2)(x+\Delta x)^2} = \frac{-2x-\Delta x}{(x^2)(x+\Delta x)^2}$$

$$\lim_{\Delta x \to 0} = \frac{-2x-\Delta x}{(x^2)(x^2+2x\Delta x+\Delta x^2)} = \frac{-2x}{x^2(x^2)} = \frac{-2x}{x^4} = \frac{-2}{x^3} = -\frac{2}{27}$$

6. Compruebe mediante límites si las asíntotas horizontales y verticales de la función dada corresponden a las mostradas en la gráfica.

$$f(x) = \frac{\sqrt{2x^2+1}}{3x-5}$$

Vertical

$3x - 5 = 0$

$3x = 5$

$$x = \frac{5}{3}$$

Horizontales

$$\frac{\frac{\sqrt{2x^2+1}}{x}}{\frac{3x-5}{x}} = \frac{\sqrt{2}}{3}$$