Robust Contract with Career Concerns[*]

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Abstract

An employer contracts with a worker to incentivize efforts whose productivity depends on ability; the worker then enters a market that pays him contingent on ability evaluation. With non-additive monitoring technology, the interdependence between market expectations and worker efforts can lead to multiple equilibria (contrasting Holmström (1982/1999); Gibbons and Murphy (1992)). We identify a sufficient and necessary criterion for the employer to face such strategic uncertainty—one linked to skill-effort complementarity, a pervasive feature of labor markets. To fully implement work, the employer optimally creates private wage discrimination to iteratively eliminate pessimistic market expectations and low worker efforts. Our result suggests that present contractual privacy, employers' coordination motives generate within-group pay inequality. The comparative statics further explain several stylized facts about residual wage dispersion.

JEL codes: D23, D62, D81, D86, J31

Keywords: contracting with career concerns, wage transparency, wage dispersion, adversarial equilibrium selection

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1 Introduction

How career concerns interact with contracting remains a significant question in agency problems with moral hazard. Career concerns arise frequently whenever labor markets evaluate workers' past performance to reassess their ability, subsequently determining future compensation based on these revised assessments. Therefore, employers who contract with workers to incentivize their efforts must take this reputation effect into account. Anticipating how labor markets interpret performance, workers work if the incentives provided by employers together with career concerns are able to cover their working costs; on the other hand, in equilibrium, the markets interpret worker performance by correctly anticipating their work decision.

The interdependence of market beliefs and worker behavior hence reveals an intrinsic coordination concern of the employer: if a wage scheme is merely enough for the worker to work when the market optimistically believes worker performance is informative, as a result of it believing that the worker works diligently, the wage scheme may be insufficient to induce the same effort when the market pessimistically expects the worker to shirk, which will indeed sustain shirking in equilibrium. Hence, *strategic uncertainty* (one contract induces multiple equilibria) may play a crucial role in contracting.

Following Holmström (1982/1999)'s response to Fama (1980) that career concerns and contracts are not necessarily perfect substitutes for resolving incentive problems, Gibbons and Murphy (1992) were the first who explicitly model contracting in the presence of career concerns. However, neither Holmström (1982/1999) nor Gibbons and Murphy (1992) captured the aforementioned issue of strategic uncertainty, as their settings always admit a unique equilibrium. The reason, as pointed out by Dewatripont et al. (1999a,b), is that they considered a special monitoring system where the worker's ability and effort enter additively in the determination of performance. Dewatripont et al. (1999b) also showed in a multiplicative example that multiple equilibria may arise sometimes. However, that paper did not study contracting or focus on addressing the coordination concern.

When will strategic uncertainty be an issue? Is this problem prevalent? How do employers optimally address strategic uncertainty in contracting? To provide answers, we study a contracting model under career concerns, where an employer privately contracts with a worker who later enters a labor market to receive a future income flow. Within the employer's firm, the worker privately chooses whether to work (and incur a cost) or shirk on a project, whose outcome is either success or failure. The success probability depends on both the worker's action and ability; and working always increases the likelihood of success. Our main setting follows the literature and assumes that all players initially do not observe the worker's ability.[1] In each contract, the employer specifies a bonus wage paid conditional on project success (failure is not rewarded). Crucially, we allow the employer to make private contract offers. That is, the employer can commit to a wage policy that is a distribution of such wage contracts. The realized contract is then privately offered to the worker, who makes his private work decision contingent on this

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[1]See the foundational papers on career concerns such as Holmström (1982/1999), Gibbons and Murphy (1992), Dewatripont et al. (1999a,b), etc.

realization. After leaving the employer, the worker's continuation payoff in the market is determined by the market's posterior belief about worker ability, which the market forms by observing the wage policy (only the distribution but not the realization of contract) and the project's outcome. To address strategic uncertainty, the employer designs a wage policy (that is, a contract distribution) that induces work as the unique equilibrium (PBE) outcome at the least expected wage cost.

Our model assumes *contractual privacy* or *inter-firm wage opaqueness*: labor markets cannot observe the past wages of workers.[2] A number of lawsuits and policies have put into practice the restrictions on firms' access to wage history, which also sparks heated policy debates about the effect of these practices on wage discrimination.[3] By modeling contractual privacy explicitly, we are able to join these discussions by addressing the following problems: Is wage opaqueness helpful in dealing with strategic uncertainty? What can we say about inequality, transparency, and career concerns?

To illustrate the forces at play, we first highlight three benchmark cases. First, if the employer can select her most preferred equilibrium, the optimal wage policy is degenerate at the partially optimal wage that makes the worker indifferent between working and shirking given that the market forms posterior under its expectation that the worker works. We say that strategic uncertainty becomes an issue if this partially optimal wage induces another undesirable equilibrium in which the worker shirks with positive probability. Second, if the employer aims to fully implement work but is restricted to use a single wage contract (rather than a distribution), she must offer the robustly optimal wage, which makes the worker indifferent under the market's maximal skepticism, i.e., its expectation that minimizes career concerns. This wage is high enough to overcome any pessimistic market expectations. Third, if the employer must keep contracting public (so the market also observes contract realization), her optimal wage policy is again degenerate at this robustly optimal wage. Notably, although the employer can randomize in both the first and third benchmarks, no wage dispersion arises in the optimal solution in either case.

Our first main result, Proposition 1, states a sufficient and necessary condition for strategic uncertainty: partial implementation suffers from multiple equilibria if and only if the on-path market expectation (that the worker works with probability one) does not generate the smallest career concerns. This condition holds when career concerns tend to be stronger when the market expects a higher effort. To illustrate the economic relevance of this result, we use a sufficient interpretation that alludes to the "complementarity" between the market's valuation of skills and the productivity of worker efforts. In particular, if we further assume that the market pays better when the worker is more likely to have "good ability", the condition holds if these "good ability" types are actually more skilled in the sense that their work increases the probability of success by a larger amount. This shows that strategic uncertainty arises

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[2]To see how our setup captures contractual privacy, note that an alternative interpretation of a wage policy is the wage structure (chosen by the employer) of a group of ex-ante identical workers who work on i.i.d. projects. Contractual privacy then requires that the market cannot tell which contract is offered to which worker, which corresponds to our market's inability to observe contract realization.

[3]For example, 22 states and 24 local districts in the US have enacted the policy of Salary History Bans since 2017, which forbids future employers to inquire about the past wages of workers; also, the supreme court case Rizo v. Yovino 2020 determined that contracting should not be based on discrimination from the past. See, for example, Bessen et al. (2024) for a recent survey on the debate about the impact of these privacy practices on pay inequality.

when the labor market demonstrates *skill-effort complementarity* in the reputation effect of career concerns. Evidence[4] suggests that very few, if any, markets manifest non-positive assortativeness between effort productivity and skill premiums, so strategic uncertainty could be rather ubiquitous. However, this is not captured by Gibbons and Murphy (1992) who follow Holmström (1982/1999) and assume that effort and ability are perfect substitutes in determining outcomes.[5]

Our second main result, Theorem 1, fully characterizes the optimal wage policy. It turns out that the employer resorts to randomization in contracting if and only if strategic uncertainty becomes an issue. Specifically, she randomizes over all wages between the partially optimal and the robustly optimal levels. She assigns probabilities in a greedy fashion: for each wage in this range, if the market expects the worker to work only at wages higher than it, the optimal policy generates just enough career concerns to make the worker indifferent between working and shirking at that wage. In the formal proof, we must deal with rich career concern structures as we do not impose restrictions on the market's valuation. We show that the employer introduces mass points in the wage distribution precisely when skill-effort substitution appears locally.

The employer ensures work at the minimal wage cost by ruling out low-effort equilibria that involve on-path shirking. She begins by offering the robustly optimal wage, which is high enough to induce work regardless of career concerns. This eliminates the market expectations that are pessimistic toward the worker's effort, making the market more responsive to observed performance and thereby eliminating low career concerns. Given the worker's work incentives strengthened in this way, the employer can ensure more effort with even lower wages. Therefore, in an iterative manner, she continues to raise career concerns and lower her wage offers by gradually shoring up market expectations. Eventually, the procedure ends once the worker is willing to work at the partially optimal wage. To reduce wage cost, the employer greedily assigns the smallest possible mass to high wages while making sure that every pessimistic market expectation is made non-credible: the worker finds it optimal to deviate to work *slightly more* given this expectation.

Our results offer a new perspective to the ongoing debate over the effectiveness of privacy policies in addressing wage discrimination. Can reducing transparency about worker wage history help close pay gaps? The prevailing view (e.g., Sinha (2019); Hansen and McNichols (2020); Bessen et al. (2024)) generally answers yes—arguing that such policies reduce pay disparities across gender and racial groups. In sharp contrast, our findings suggest the opposite, but from a different angle. We show that when firms face strategic uncertainty in contracting, the motives for incentive provision and coordination lead to within-group wage discrimination, if and only if wage history becomes private. In our model, this dispersion arises despite workers being ex-ante identical and uninformed about their own ability. As a result, policies such as Salary History Bans may inadvertently shift discrimination from across groups to within

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[4]We show in a linear-market-value example that the condition is *equivalent* to positive skill-wage assortativeness, which is extensively documented by, e.g., Abowd et al. (1999); Card et al. (2013); Song et al. (2019), among many others.

[5]In Appendix A.1, we revise our setup to accommodate their monitoring structure with continuous action, ability, and outcome. There, we show career concerns enter the condition that pins down equilibrium merely as a constant, so market expectations do not shift work incentives and equilibrium is hence unique regardless of contracts.

groups, potentially overstating their net effect.[6,7] Furthermore, we discuss what happens if contracting is not completely private by considering four cases where wage information can be partially transmitted.[8] Our prediction remains robust in these settings: as long as contracting is not fully transparent, the incentive-coordination channel continues to generate wage dispersion, underscoring the nuanced implications of wage privacy policies.

Moreover, we use our results to reveal a new mechanism that gives rise to residual wage inequality. As Mortensen (2003) opens his book: "... observably identical workers are found (to receive) both good and bad (wages) ..." and "(in many studies, o)bservable worker characteristics ... typically explain no more than 30 percent of the variation in compensation ..." Our approach finds that within-group pay dispersion can emerge from employers' coordination motives when designing incentive schemes in the presence of career concerns. To connect our theory with empirical patterns, we examine the comparative statics of optimal wage policies. First, we show that when labor markets match more productive workers to higher-pay positions more effectively, employers optimally choose more dispersed wage structures with a wider range and a larger variance. Card et al. (2013) document precisely this pattern in Germany: a positive correlation between skill-wage assortativeness and residual wage dispersion. Our model provides a simple underlying mechanism: when skills and wages are matched more associatively, worker efforts and career incentives become more complementary, which intensifies strategic uncertainty in contracting; to resolve this, employers deliberately introduce greater wage inequality, even among observably identical workers. Next, we show that increasing skill premiums and occupational mobility also leads to higher wage dispersion, as this amplifies career concerns. While these factors are commonly understood to drive wage inequality through observable worker heterogeneity, our result is surprising in showing that they also shape residual variation.

Finally, we discuss an extension where the worker is privately informed of his ability. We start with binary types and assume that the market pays better if the worker is more likely to have higher productivity (high type). We find a similar sufficient and necessary criterion for strategic uncertainty and so contract dispersion: the difference between full-working and full-shirking career incentives should exceed a threshold. This again highlights skill-effort complementarity. To guarantee work with informed worker, the employer first offers wages that, given skeptical market expectations and thus small career concerns, attract only the high type into work but discourage the low type from working; this guarantees larger career concerns because the market now attributes project success [failure] to high-type work [low-type shirk]; in this way, the employer can elicit higher effort with even lower wages. By iteratively

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[6]Which channel dominates remains an open empirical question. For instance, Sran et al. (2020) and Davis et al. (2022) find that cross-group wage convergence is often negligible, while others report only modest (e.g., 1% in Hansen and McNichols (2020)) or mixed effects (Sinha (2019)). Our result thus calls for a re-evaluation of these policies.

[7]The same evidence also documents that Salary History Bans reduce the average pay (e.g., Sran et al. (2020); Davis et al. (2022)). Our prediction is consistent with this because, compared to public contracting, the employer with robustness concern manages to reduce wage offers by shifting market expectations. In contrast, when the employer ignores strategic uncertainty and merely implements work partially, her contract choice is the same with or without contractual privacy.

[8]See Section 6 for more detailed discussions. The four cases we consider there are: (i) the market has some probability to observe wage; (ii) the worker has some probability to be able to disclose wage; (iii) the employer can release information about wage; (iv) some firms in the market can observe wage.

strengthening career concerns, maintaining that the high type works "more than" the low type, the employer reduces her wage cost. As a result, the optimal wage policy now takes a more specific form: it contains a lower-bound mass point and upward continuous dispersion. To implement this policy, the employer can publicly announce a minimum wage (as, e.g., a living wage policy) while privately giving personalized lucky wage raises to some workers. Furthermore, we extend this result to settings where the employer induces type-dependent working probabilities or the worker has more than two types.

Outline. Section 2 lays out the main model. Section 3 discusses the sufficient and necessary criterion for strategic uncertainty, and then illustrates the main idea with an example. Section 4 first introduces an auxiliary result we apply to show our main result. After presenting the main result, we make implications on wage dispersion. Section 5 analyzes the informed-worker extension. All proofs are relegated to the appendix.

Literature review This paper relates to three strands of literature. First, there is an expanding literature on contracting with externalities that focuses on adversarial equilibrium selection. One branch of this literature follows the seminal work of Segal (2003) and studies settings in which agents' decisions are bilaterally contractible.[9] Our paper belongs to another branch that investigates moral hazard problems with unobservable actions, which follows Winter (2004).[10] We depart from the second branch in that we consider a setting of contracting with career concerns in which externalities result from market expectations. Moreover, while the vast majority of this literature considers complementary actions, we allow for a flexible structure of externalities that nests supermodularity, submodularity, and their arbitrary mixture. In addition, our informed-worker extension characterizes the optimal contracting when the worker's private type is hidden from the employer.[11]

In the context of wage privacy and discrimination in incentive provision, the most closely related work is Halac et al. (2021), who study wage opaqueness among co-workers in the setting of Winter (2004). Their results are complementary to ours. While they examine intra-firm wage transparency—specifically, whether co-workers observe each other's contracts—we focus on inter-firm transparency, where future employers may or may not observe a worker's past wage. In their setting, the prevailing policy concern (as discussed in their paper) is that wage privacy may facilitate discrimination within firms. However, they find that privacy induces symmetric wages for identical workers. Our context contrasts sharply: the common policy view in our case holds that wage transparency can perpetuate past discrimination, while privacy is expected to mitigate it. In contrast, we show that privacy across firms can enable within-group

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[9]For the first branch, see also, e.g., Bernstein and Winter (2012); Sakovics and Steiner (2012); Halac et al. (2020); Ali et al. (2022); Gan and Li (2024); Halac et al. (2024a); Chan (forthcoming).

[10]For more examples, Halac et al. (2024b), Camboni and Porcellacchia (forthcoming), and Cusumano et al. (2023) examine an employer's endogenous choice of monitoring technology when contracting with a team; Moriya and Yamashita (2020), Halac et al. (2022), and Morris et al. (2024) study the optimal provision of information as a way to provide incentives; Eliaz and Spiegler (2015) and Halac et al. (2021) introduce contractual uncertainty into Winter (2004).

[11]Within the first branch, Ali et al. (2022) and Gan and Li (2024) also study the multiplicity concern arising from endogenous beliefs, but in the problems of a seller of information; although Bernstein and Winter (2012) mainly deal with supermodular payoffs in their analysis of coordinating multi-agent participation, they discuss mixed externalities in an extension; robust implementation with hidden types is also the focus of Halac et al. (2024a) and a few recent contributions cited there.

wage discrimination, even among observably identical workers.

Second, this paper also relates to the literature on career concerns, pioneered by Holmström (1982/1999). Our novelty lies in two aspects: we consider a flexible monitoring system such that efforts and skills are not simple complements or substitutes; and we explicitly address strategic uncertainty by studying full implementation and allowing contracts to be random. Holmström (1982/1999) and Gibbons and Murphy (1992) among many others[12] build their analysis in additive Gaussian environments, in which multiplicity is always absent. Dewatripont et al. (1999b) and Bonatti and Hörner (2017) assume complementary skills and efforts, but adopt equilibrium selection rules to bypass multiplicity. Dewatripont et al. (1999a) and Rodina (2017) compare flexible monitoring systems, but do not focus on employers' incentives provision problems. We are not aware of work in this literature that studies full implementation, and this paper is the first to show that random contracts are helpful in providing robust explicit incentives in the presence of implicit incentives from career concerns.[13]

Finally, an extensive literature focuses on discrimination in firms and, more broadly, wage differentials. We discuss the connections between our results and empirical studies in Section 4.3. On the debate about policies protecting inter-firm privacy, Bardhi et al. (2024) provides complementary answers, from a different angle, by identifying conditions for early-career discrimination to be persistent.

2   Model

A risk-neutral employer (she) offers a wage policy to hire a risk-neutral worker (he) to work on a project. The worker has an ability type, drawn from a finite space Θ that contains at least two types, according to a fully supported prior distribution $\mu^0 \in \Delta(\Theta)$. We use k to denote a typical element of Θ. If the type k worker chooses to shirk, then the project succeeds with probability $p_{0k} \geq 0$. On the other hand, if the type k worker works, this raises the success probability to $p_{0k} + p_k \leq 1$ with $p_k > 0$. To keep both success and failure on path, we assume there exist some type k' such that $p_{0k'} > 0$ and some type k'' such that $p_{0k''} + p_{k''} < 1$. If the type k worker works, he pays a cost $c_k > 0$. Following the standard framework of contracting with career concerns, Gibbons and Murphy (1992) (who also follow Holmström (1982/1999) on this), we assume both the employer and the worker do *not* observe the ability type and, initially, they share the prior. The employer faces moral hazard: the worker's work choice is private while the project outcome (success or failure) is public and contractible. As we will see later, the worker's decision hinges on the *effective cost* λ:

$$\lambda := \frac{\mathbb{E}_0[c_k]}{\mathbb{E}_0[p_k]} = \frac{\sum_{k \in \Theta} \mu_k^0 c_k}{\sum_{k \in \Theta} \mu_k^0 p_k}, \tag{1}$$

Where the expectation $\mathbb{E}_0$ is taken over the ability type distributed according to the prior.

Career concerns. After the project is closed, the worker leaves the employer and receives future wages in a market. The market cares about the worker's ability and holds the prior initially. In particular, the

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[12]For a few examples, see also Hakenes and Katolnik (2017); Cisternas (2018); Hörner and Lambert (2021).

[13]More broadly, another branch of the literature studies career concerns with hidden outcomes and observable actions. A seminal work is Prendergast and Stole (1996). See also, e.g., Halac and Kremer (2020); Rappoport (2022); Ke et al. (2023).

post-employment wage is captured by a general function $v : \Delta(\Theta) \to \mathbb{R}_+$, which specifies a transfer to the worker for every posterior belief the market may have about his ability.

For our main result, we only need $v(.)$ to be continuous (with respect to the Euclidean norm).

Random wage. We now introduce the employer's choice. To begin with, we assume that the worker is protected by limited liability. As a result, it is without loss to focus on success-contingent wages, that is, the employer pays the worker only upon project success, while failure is not rewarded. Hence, each such wage represents a contract, and the two terms (wage/contract) are interchangeable in the context. Given this, we allow the employer to commit to a contract distribution. Specifically, the employer picks a *wage policy* $F \in \Delta(\mathbb{R}_+)$ randomizing over nonnegative success-contingent wages. Each wage $w \geq 0$ represents the realized contract where the employer pays w contingent on success.

Here is an alternative interpretation of such random wages: imagine that the employer hires many ex-ante identical workers who are assigned to i.i.d. projects, and offers each of them a potentially different contract. Hence, choosing a wage policy is equivalent to choosing a wage structure.

Timing and payoffs. Holding the prior belief, the employer *publicly* offers a wage policy $F \in \Delta(\mathbb{R}_+)$. The ability type k is realized from μ^0 but unobserved by anyone. The worker *privately* learns the wage level, say w, which is a realization of the policy F. Given this information, the worker *privately* chooses whether to work ($a = 1$), shirk ($a = 0$), or randomize. Based on worker effort and ability, the project outcome x is *publicly* and stochastically realized, being either success ($x = 1$) or failure ($x = 0$). At this point, the market can only observe the wage policy and the project outcome, but not the ability type, the realized wage contract, or the worker's effort. Subsequently, the market forms its posterior belief μ via the Bayes' rule and pays the worker the post-employment wage $v(\mu)$.

The worker's ex-post payoff includes the working cost, the realized wage, and the post-employment wage: $-a \times c_k + x \times w + v(\mu)$. The employer's objective will be specified shortly below.

Unable to price ability, the employer offers contracts to different types according to the same policy. Therefore, our setup suppresses the presence of *cross-group* wage dispersion, and whenever the employer chooses a non-degenerate wage policy, it directly represents *within-group* discrimination.

Equilibrium. Each wage policy F induces a dynamic Bayesian game played by the worker and the market. We consider the set of all Perfect Bayesian Equilibria of the game. In particular, an equilibrium specifies a tuple $g = (\sigma, \overline{\mu}, \underline{\mu})$ where $\sigma : \mathbb{R}_+ \to [0, 1]$ denotes the worker's probability of working upon receiving a wage realization $w \in \mathbb{R}_+$, and $\overline{\mu}$ and $\underline{\mu}$ refer to the market's posterior beliefs when it observes project success and failure, respectively.

A perfect Bayesian equilibrium satisfies: (i) Given market's beliefs, for every wage realization w, the worker maximizes his expected payoff by deciding whether to work:

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\begin{aligned}
&\sigma(w) \in \operatorname{argmax}_{\widetilde{\sigma} \in [0,1]} \widetilde{\sigma}(V^1 - V^0), \text{ where} \\
&V^1 = \mathbb{E}_0[-c_k + (p_{0k} + p_k)(w + v(\overline{\mu})) + (1 - p_{0k} - p_k)v(\underline{\mu})], \text{ and} \\
&V^0 = \mathbb{E}_0[p_{0k}(w + v(\overline{\mu})) + (1 - p_{0k})v(\underline{\mu})];
\end{aligned}
\tag{2}
$$

And (ii) given the worker's total working probability $q := \mathbb{E}_F[\sigma(w)] = \int_{\mathbb{R}_+} \sigma(w)dF(w)$ pinned down by his strategy, the market's beliefs satisfy the Bayes' rule, that is, for every type k, $\overline{\mu}$ and $\underline{\mu}$ assign respective probabilities to it as follows:

$$\overline{\mu}_k = \frac{\mu_k^0[q(p_{0k} + p_k) + (1-q)p_{0k}]}{\sum_{j \in \Theta} \mu_j^0[q(p_{0j} + p_j) + (1-q)p_{0j}]}, \text{ and}$$

$$\underline{\mu}_k = \frac{\mu_k^0 - \mu_k^0[q(p_{0k} + p_k) + (1-q)p_{0k}]}{1 - \sum_{j \in \Theta} \mu_j^0[q(p_{0j} + p_j) + (1-q)p_{0j}]}. \tag{3}$$

The Bayes' rule can always be applied because we have assumed that success and failure are always on path. If without any confusion, we refer to a tuple $g = (\sigma, \overline{\mu}, \underline{\mu})$ defined above simply as an *equilibrium*. In addition, let $\mathcal{E}(F)$ collect all the equilibria induced by F. If F is degenerate at some wage level w, we also denote the equilibrium set simply as $\mathcal{E}(w)$.

Career value. For notational convenience, we define the *career value* $D : [0, 1]^2 \to \mathbb{R}$ as the difference between the post-employment wage for project success and that for project failure. In (3), the worker's total working probability q uniquely pins down the belief system through the Bayes' rule, so we rewrite the beliefs as two functions: $\overline{\mu}(q)$ and $\underline{\mu}(q)$. The career value is thus defined by:

$$D(q) := v(\overline{\mu}(q)) - v(\underline{\mu}(q)). \tag{4}$$

This intermediate concept admits a simpler definition of our equilibrium: by rewriting (2), we notice that the worker's strategy σ forms an equilibrium if, given $q = \mathbb{E}_F[\sigma(w)]$, for all wage $w \geq 0$:

$$\sigma(w) \in \text{argmax}_{\widetilde{\sigma} \in [0,1]} \, \widetilde{\sigma}[w + D(q) - \lambda], \tag{5}$$

Where λ is defined in (1). In other words, the worker decides by comparing his effective cost λ and his effective gain $w + D(q)$, which includes both the explicit wage incentives and the implicit incentives of career concerns. Moreover, the career value $D(q)$ is produced by the market's correct expectation about the worker's total effort q.

Since we have assumed $v(.)$ is continuous, the fact that $\overline{\mu}_k$ and $\underline{\mu}_k$ defined in (3) are both continuous in q implies that career value $D(.)$ is continuous in q. Note that even when $v(.)$ may manifest an intuitive pattern such as increasing with higher types' probability, $D(.)$ is in general nonmonotonic, so the market's optimism about effort can both enhance and reduce work incentives. This requires us to deal with a potentially mixed structure of externalities that may be an arbitrary mixture of supermodularity and submodularity, which complicates our robust focus as will be specified immediately.

Robustness. We investigate the robustly optimal wage policy that minimizes the employer's expected wage cost while inducing the worker to work with probability *one* in *all* equilibria. Section 4.2 shows (in a footnote) that the result can be readily extended to the full implementation of any working probability. The current full-working case can be viewed as the employer's optimal choice if she faces a sufficiently

high valuation of project success over project failure.

More formally, a wage policy F fully implements full working if, in every equilibrium $g \in \mathcal{E}(F)$, the worker must work with probability $q = \mathbb{E}_F[\sigma(w)] = 1$. We collect all such wage policies in $\mathcal{F}^{FI}$. Notice that $\mathcal{F}^{FI}$ is nonempty because the employer can always guarantee full working by offering a sufficiently high wage. Thus, we can define the following objective of the employer:

Definition 1. The employer's *minimal wage guarantee* for fully implementing full working is:

$$W^* = \inf_{F \in \mathcal{F}^{FI}} \mathbb{E}_F[w].^{14} \tag{6}$$

A wage policy F^* is *robustly optimal* if there is a sequence of wage policies $(F_n)_{n=1}^{\infty}$ such that:

1. The sequence $(F_n)_{n=1}^{\infty}$ is contained in $\mathcal{F}^{FI}$ and weakly converges to F^*;
2. The wage guarantee, $\mathbb{E}_{F_n}[w]$, converges to W^*.

Finally, we assume $\lambda > \max_q D(q)$ to ensure limited liability will not play a restrictive role when the employer designs the wage policy. This allows us to ignore some unnecessary discussion.

3 Strategic Uncertainty and Main Idea

In this section, we investigate two questions: When will strategic uncertainty become a problem if the employer designs the contract while ignoring the underlying issue of coordination? How can the employer tackle strategic uncertainty by using private random contracts? Section 3.1 begins by illustrating what would happen if (i) the employer partially implements work, or (ii) she can only use a deterministic wage. The result identifies the sufficient and necessary condition for strategic uncertainty to be present. We interpret this condition with two examples and discuss why strategic uncertainty is absent in classic career concern models. Section 3.2, using a simple example, demonstrates how randomization decreases the employer's wage costs while fully implementing work.

3.1 When will Strategic Uncertainty be a Problem?

Our main setting considers an employer who aims to fully implement work by offering random wages. This section studies two benchmark problems, FD and PR, each relaxing one of the two model elements. In the FD problem, the employer fully implements full working but can only use a deterministic wage instead of a distribution.[15] Her optimal choice is called the *FD wage level*. Moreover, in the PR problem,

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[14]Given full working, the expected wage is actually $\sum_{k \in \Theta} \mu_k^0 (p_{0k} + p_k) \, \mathbb{E}_F[w]$, where $\sum_{k \in \Theta} \mu_k^0 (p_{0k} + p_k)$ is the probability of paying each wage realization. Here, this term enters the objective as a constant, so we omit it for conciseness.

[15]In fact, we can further consider another benchmark where the employer fully implements work under public contracting, namely, when the market also observes the realization of wage contract. However, this case coincides with the FD problem because randomization is not useful for the employer. To see this, note that now each contract realization (rather than the distribution) induces a worker-market game where the employer minimizes wages independently. Hence, it is without loss to offer a single wage that fully implements work at the least cost.

the employer selects a wage policy to partially implement work, that is, she ignores strategic uncertainty and can choose her most preferred equilibrium. The wage-minimizing design is called the *PR wage policy*. We define the two problems formally:

$$\text{[FD]: } \inf_{w \geq 0} w, \text{ s.t. } \forall g \in \mathcal{E}(w), \sigma(w) = 1;$$

$$\text{[PR]: } \min_{F \in \Delta(\mathbb{R}_+)} \mathbb{E}_F[w], \text{ s.t. } \exists g \in \mathcal{E}(F), \mathbb{E}_F[\sigma(w)] = 1.$$

To derive solutions, we first highlight the following two critical wage levels:

$$\underline{w} = \lambda - D(1); \qquad \overline{w} = \lambda - \min_{q \in [0,1]} D(q).$$

Recall that when the market expects the worker to work with total probability $q \in [0, 1]$, the worker faces career value $D(q)$. The equilibrium condition (5) then implies that the wage $w_q := \lambda - D(q)$ makes the worker indifferent between working and shirking given the market expectation q. Moreover, if the wage policy offers wages no less than w_q with exactly the probability q, then such a market expectation is self-fulfilling in equilibrium.

The following result shows that wage dispersion is not useful for partial implementation, but partial implementation suffers from multiple equilibria whenever the two problems diverge in solutions:

Proposition 1. *The FD wage level is $\overline{w}$ whereas the PR wage policy is degenerate at $\underline{w}$. Furthermore, $\mathcal{E}(\underline{w})$ contains multiple equilibria if and only if there is some $q < 1$ such that $D(q) \leq D(1)$.*

To understand Proposition 1, we explain how multiple equilibria emerge in our setting. The key lies in the interdependence between the worker's work decisions and the market's beliefs. On the one hand, as long as the offered wages are no less than $\underline{w}$ (equal to w_q with $q = 1$), the full-working equilibrium exists. As a result, the PR wage policy assigns all probability to the lower bound $\underline{w}$. On the other hand, this wage does not guarantee full implementation because other market expectations may also be self-fulfilling. For example, if there is some $q < 1$ with $D(q) = D(1)$, we have $\underline{w} = w_q$, so the career value $D(q)$ makes it optimal for the worker to randomize at the PR wage (with working probability q). An undesirable mixed-strategy equilibrium thus emerges. The second possibility is that if $D(1) > D(0)$, $\underline{w}$ is lower than w_q with $q = 0$, so the career value $D(0)$ drives the worker to shirk at the PR wage. Therefore, full shirking forms an undesirable equilibrium in this case. The only scenario where strategic uncertainty disappears under the PR wage policy is when $D(1) < D(q)$ for all $q < 1$. Furthermore, for the full implementation of $q = 1$, the FD wage must avoid the previous two cases, and thus is larger than w_q for all $q \in [0, 1]$. As a result, the infimum of these wages is given by $\overline{w}$.

Two messages are delivered by Proposition 1. First, in spite of the ability to offer a wage distribution, the PR employer who ignores the issue of strategic uncertainty still uses a deterministic wage, creating no wage dispersion. Moreover, the employer's goal may be jeopardized by the existence of an undesirable

equilibrium that occurs whenever $\underline{w} < \overline{w}$, equivalent to the following criterion:

$$D(1) > \min_{q \in [0,1]} D(q), \tag{7}$$

Which requires the career value to be somewhat increasing in market expectation. This, roughly speaking, asserts that if the monitoring system is such that the worker's effort tends to make project success [failure] "better" ["worse"] news for the market, the employer will face strategic uncertainty and might have to account for a robust objective in designing wage structure. As is straightforward, we say *strategic uncertainty becomes a problem* (of partial implementation) if criterion (7) holds. We further interpret our result, Proposition 1, with the following three examples.

Example: Skill-Effort Complementarity   To illustrate the insight behind the criterion (7) for strategic uncertainty, we impose a weak assumption on the post-employment wage $v(.)$ that the market pays the worker. We label the types as integers $\Theta = \{1, 2, ..., K\}$ with $K \geq 2$, and suppose that the ability types are ordered by the market's "willingness-to-pay" for each of them. More specifically, let $v(.)$ be *strictly increasing in the first-order stochastic dominance*: for all $\mu^1, \mu^2 \in \Delta(\Theta)$,

$$\sum_{j=1}^{k} \mu_j^1 \leq \sum_{j=1}^{k} \mu_j^2 \text{ for all } k, \text{ and some inequality is strict} \Rightarrow v(\mu^1) > v(\mu^2).$$

In other words, when the market believes that the worker has a better chance of being higher types, the worker expects to receive a greater continuation value in the labor market.

Given such market environment, we identify a useful property of the monitoring system. To do so, we define each type k's effort effect on project outcome as P_k, and his shirking effect on success probability and failure probability as Q_k^S and Q_k^F, respectively, as follows:

$$P_k := \frac{\mu_k^0 p_k}{\sum_{j=1}^{K} \mu_j^0 p_j}; \qquad Q_k^S := \frac{\mu_k^0 p_{0k}}{\sum_{j=1}^{K} \mu_j^0 p_{0j}}; \qquad Q_k^F := \frac{\mu_k^0 (1 - p_{0k})}{\sum_{j=1}^{K} \mu_j^0 (1 - p_{0j})}.$$

In other words, P_k equals p_k (type k's ability to enhance success probability via work) averaged relative to other types while taking into account the prior weights. Likewise, Q_k^S and Q_k^F measure the relative chances of success and failure, respectively, when type k shirks. Let P, Q^S, and Q^F be the respective vectors of all types' effort effects, shirking effects on success, and shirking effects on failure. Note that $P, Q^S, Q^F \in \Delta(\Theta)$. With $v(.)$ rewarding higher types, we say the contracting environment demonstrates *skill-effort complementarity* if P first-order stochastically dominates both Q^S and Q^F:

$$\sum_{j=1}^{k} P_j \leq \sum_{j=1}^{k} Q_j^S \text{ and } \sum_{j=1}^{k} P_j \leq \sum_{j=1}^{k} Q_j^F \text{ for all } k, \text{ and some inequality is strict.}$$

This property represents complementarity because it requires the effort effect P to increase (in types)

faster than Q^S and Q^F. It is natural to interpret p_k as the productivity of type k's effort, so the weighted term P_k captures a similar concept. Thus, skill-effort complementarity demands that both the productivity of worker effort P_k and the market valuation of skills $v(.)$ be co-increasing and thus complementary. In particular, if the shirking rate p_{0k} is type-independent so $Q^S = Q^F = \mu^0$, the property holds when p_k increases strictly in k.

By embedding into the monitoring system this form of complementarity between skills and efforts, we show that strategic uncertainty must be a problem:

Claim 1. *If the post-employment wage is strictly increasing in the first-order stochastic dominance, strategic uncertainty becomes a problem if the contracting environment demonstrates skill-effort complementarity.*

Skill-effort complementarity is sufficient for criterion (7) because it makes sure that market beliefs are more reactive if it holds a more optimistic expectation about effort. Specifically, the market's posterior belief contingent on success [failure] is determined by P and Q^S [Q^F], and when P dominates Q^S [Q^F], the worker's effort makes success [failure] "better" ["worse"] news for the market.

On the other hand, strategic uncertainty will not form a problem if P is weakly first-order stochastically dominated by both Q^S and Q^F. A special case is when the base productivity p_{0k} is type-independent while the effort productivity p_k weakly decreases in k.[16]

Example: Linear Value A natural question arising next is whether skill-effort complementarity can be a ubiquitous market feature, so strategic uncertainty is prevalent in practice. In the example above, one can get the sense that strategic uncertainty occurs whenever the wages offered by the market are positively correlated with the worker's skills. To better establish this point and build connection with evidence, we further restrict to a setting that is nested by the previous example: let the post-employment wage be linear, namely, $v(\mu) = \Sigma_{k \in \Theta} u_k \mu_k$ where each type's market value u_k is nonnegative; moreover, let the shirking rate p_{0k} be type-independent. If we relabel types so that u_k weakly increases in k, this linear environment becomes a special case of the previous example. We will later refer to this setup as the *linear environment* to study comparative statics. In this example, the career value function depends on market values $u = (u_k)_{k \in \Theta}$ and effort productivity $p = (p_k)_{k \in \Theta}$ as follows:

$$D(q) = \text{Cov}(u, p) T(q; \mathbb{E}_0[p_k]), \tag{8}$$

Where $\text{Cov}(u, p)$ is the covariance between market values and productivity:

$$\text{Cov}(u, p) = \mathbb{E}_0[u_k p_k] - \mathbb{E}_0[u_k] \, \mathbb{E}_0[p_k].$$

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[16]The classic career concern setups (e.g., Holmström (1982/1999); Gibbons and Murphy (1992)) consider additive monitoring systems whose informativeness about ability is independent of effort. This corresponds to $P_k = Q_k^S = Q_k^F$ in our case, which generates a constant career value in this knife-edge specification and will not cause strategic uncertainty.

Above, T is strictly positive and increases strictly in the worker's total working probability q when $q > 0$, and p enters T only through its expectation.[17] Note that $\mathrm{Cov}(u, p) = \mathrm{Corr}(u, p)\,\mathrm{Var}(u)\,\mathrm{Var}(p)$ where the correlation term $\mathrm{Corr}(u, p)$ determines the sign of $\mathrm{Cov}(u, p)$. Therefore, by assuming nontrivial variance of market value u and productivity p, we push Claim 1 further by identifying a sufficient and necessary condition for strategic uncertainty:

Claim 2. *In the linear environment, strategic uncertainty becomes a problem if and only if* $\mathrm{Corr}(u, p) > 0$.

As a result, strategic uncertainty becomes an issue if and only if the worker's skills (p_k) are positively correlated with his expected payoffs in the labor market (u_k). This criterion is likely satisfied, as extensive evidence documents systematic sorting of high-skill workers into high-wage positions (e.g., Abowd et al. (1999); Card et al. (2013); Song et al. (2019), among many others). Therefore, our result highlights that strategic uncertainty may be a prevalent concern in contract design.

Example: Classic Monitoring   The classic career concern setups (e.g., Holmström (1982/1999); Gibbons and Murphy (1992)) usually do not feature a multiplicity issue because they assume a specific monitoring system that fails to incorporate the effort's effect on the outcome-generating process. In particular, they consider a project whose outcome y is linear in worker's action a and his ability k: $y = a + k + \epsilon$, where ϵ is a random noise drawn from a distribution independent of a and k. In this case, whenever the worker increases his action, all the outcome realizations simply increase by the same amount uniformly. As a result, the joint distribution of types and market posteriors in equilibrium does not vary with worker effort, and career concerns are pinned down solely by the post-employment wage.

To demonstrate this more formally, in Appendix A.1, we focus on the setting of Gibbons and Murphy (1992) who also study contracting with career concerns as we do. We show that, even in a more general framework that does not necessarily depend on some other assumptions they need, such as firm competition or linear contracts, strategic uncertainty is absent solely due to the special monitoring system they consider. The reason is that career concerns enter the condition that pins down equilibrium simply as a constant, opposite to what criterion (7) requires.

To see an example in which multiple equilibria may arise, one must look at Dewatripont et al. (1999b) who instead considered a multiplicative monitoring system. Such a multiplicative form leads to a force that is consistent with our skill-effort complementarity (in their terminology, "explicit and implicit incentives may become complements"). However, it would be difficult to study contracting in Dewatripont et al. (1999a,b) for general monitoring systems (both non-robust and robust cases are hardly tractable). One contribution of our work is thus to provide a flexible framework in which we are able to pin down a sharp criterion for multiplicity and derive the optimal contracting under distinct objectives.

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[17]In fact, $T(q; \mathbb{E}_0[p_k]) = q/[(p_0 + \mathbb{E}_0[p_k]q)(1 - p_0 - \mathbb{E}_0[p_k]q)]$ where $p_0 = p_{0k}$ is the type-independent shirking rate.

3.2 How to Tackle Strategic Uncertainty?

Proposition 1 implies that when criterion (7) fails, namely $D(1) = \min_{q \in [0,1]} D(q)$, offering the PR wage $\underline{w}$ with probability one is robustly optimal. This shows that accounting for a robust objective alone does not necessarily lead to dispersed contracts. However, if criterion (7) holds, this is no longer true.

In this section, we assume criterion (7) holds. We show although $\overline{w}$ is the robustly optimal deterministic wage, it is very expensive for the employer. We construct a new wage policy that fully implements full working and involves wage dispersion while paying the worker a lower expected wage than $\overline{w}$. This improving example delivers the main insight into why within-group discrimination helps the employer reduce wage costs when strategic uncertainty arises.

Now that we have assumed $\overline{w} > \underline{w}$, we can find some $\widetilde{w} \in (\underline{w}, \overline{w})$. The following lemma demonstrates that the employer can guarantee a career value by guaranteeing the market's expectation about effort:

Lemma 1. *For all $\widetilde{w} \in (\underline{w}, \overline{w})$ and $\widetilde{q} := \max\{q' : \widetilde{w} = \lambda - D(q')\}$, we have $D(q) > D(\widetilde{q})$ for all $q > \widetilde{q}$.*[18]

In other words, by ensuring that the market expects the worker to work with at least the total probability $\widetilde{q}$, the worker's career concerns are bounded below by $D(\widetilde{q})$. This $\widetilde{q}$ is chosen as the largest market expectation that sustains $\widetilde{w}$ in equilibrium. We then consider the following binary-wage policy $\widetilde{F}$ (with a small $\epsilon > 0$) as an improvement upon the FD wage $\overline{w}$:

$$\widetilde{F} \text{ offers } \overline{w} + \epsilon \text{ with probability } \widetilde{q} \text{ and offers } \widetilde{w} + \epsilon \text{ with probability } 1 - \widetilde{q}. \tag{9}$$

Because $\widetilde{w} < \overline{w}$, by choosing a small ϵ, $\widetilde{F}$ demands a lower expected wage than the FD wage. Therefore, it suffices to show that $\widetilde{F}$ also fully implements full working. First, since both $\widetilde{w}$ and $\overline{w}$ are no lower than $\underline{w}$, the desired full-working equilibrium exists under $\widetilde{F}$. The remaining job is to break all the undesirable equilibria, which fall into two categories: (i) the worker works with some probability $q < 1$ at the high wage while fully shirking at the low wage; (ii) the worker fully works at the high wage and works with some probability $q < 1$ at the low wage. Recall that the high wage $\overline{w} + \epsilon$ is even higher than the FD wage, which is by construction so high that the worker always works regardless of career concerns. As a result, those equilibria in case (i) are eliminated because working is dominant when this high wage is realized. Anticipating this, the market must now expect that the worker's total working probability is at least the probability of the high wage, $\widetilde{q}$, which brings the career value to at least $D(\widetilde{q})$ (Lemma 1). Consequently, the low wage greater than $\widetilde{w} = \lambda - D(\widetilde{q})$ is also high enough to make working optimal given the career-value guarantee $D(\widetilde{q})$. The equilibria in case (ii) are thus ruled out.

Intuitively, $\widetilde{F}$ reduces wage costs compared to the FD wage by first offering a high wage to preclude those equilibria in which career values are too low. Given a guaranteed high career value, the worker's work incentives then increase, and he is willing to work even at a lower wage.

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[18]To see this result, we suppose there is some $q' > \widetilde{q}$ such that $D(q') \leq D(\widetilde{q})$. Note that $\widetilde{w} > \underline{w}$ implies $D(\widetilde{q}) < D(1)$. As a result, we have $D(q') \leq D(\widetilde{q}) < D(1)$, and the continuity of $D(.)$ guarantees the existence of some $q \in [q', 1)$ that satisfies $\widetilde{w} = \lambda - D(q)$, a contradiction to $\widetilde{q}$ being the largest such probability.

As one can imagine, there is room for the employer to continue to improve: she finds another wage $\widehat{w} \in (\underline{w}, \widetilde{w})$ and the largest probability $\widehat{q}$ that satisfies $\widehat{w} = \lambda - D(\widehat{q})$. Then, the wage policy that offers $\overline{w}$ with probability $\widetilde{q}$, $\widetilde{w}$ with probability $\widehat{q} - \widetilde{q}$, and $\widehat{w}$ with probability $1 - \widehat{q}$ further improves upon $\widetilde{F}$. If we let the process continue until the wage policy is fully supported on the entire interval $[\underline{w}, \overline{w}]$, this construction in fact leads to the robustly optimal wage policy, the optimality of which will be stated and explained in Section 4.

4   Optimal Wage Policy

Before formally characterizing the robustly optimal wage policy, Section 4.1 reduces the main problem (6) to an auxiliary problem that helps identify the key constraint that our robust focus introduces. Section 4.2 presents and interprets the main result, Theorem 1. Finally, Section 4.3 studies comparative statics and builds connections with several stylized empirical facts on (residual) wage dispersion.

4.1   An Auxiliary Problem

In this section, we build the equivalence between the employer's problem (6) and an auxiliary problem. The auxiliary problem identifies two constraints that fulfill the two main goals of the employer, respectively: first, the employer must make sure the desired full-working equilibrium exists; second, the wage policy should destabilize all the undesirable equilibria for full implementation.

The first constraint follows directly from our arguments in Section 3.1 that, to keep the full-working equilibrium, the offered wages must be no less than the partially optimal wage $\underline{w}$. Formally, take a wage policy F, and it must satisfy the following *equilibrium-keeping* constraint:

$$\text{supp}(F) \geq \underline{w}. \tag{EK}$$

Moreover, the second constraint introduces a series of conditions for breaking all the bad equilibria. In every bad equilibrium, the worker's total working probability q is strictly lower than one. According to the equilibrium condition (5), the worker in equilibrium must play a threshold strategy. That is, there exists some wage level w such that he works [shirks] if the wage realization is higher [lower] than w, and he is indifferent at w. Hence, every such equilibrium specifies a q and a w, and these parameters are consistent with the same worker strategy if and only if:

$$q \in [1 - F(w^+), 1 - F(w^-)], \text{ also denoted } w \in F^{-1}(1 - q). \tag{10}$$

On the one hand, when the market expects the worker to work with a total probability q, the career value is $D(q)$. On the other hand, to rationalize the threshold wage w, (5) requires that the career value

be $\lambda - w$. Thus, this equilibrium does not exist if and only if the two career values differ:

$$D(q) \neq \lambda - w. \tag{11}$$

In particular, if $D(q) > \lambda - w$, the market's expectation is too "aggressive", creating such a high career value that the worker wants to deviate to work more than expected. Likewise, when $D(q) < \lambda - w$, the "conservative" market expectation will give rise to deviation to more shirking.

However, despite the two possible manners of breaking equilibrium above, we will show that a wage policy that fully implements full working must break those bad equilibria by inducing "aggressive" market expectation alone. Formally, it must satisfy the *equilibrium-breaking* constraint:

$$\text{For all } q \in [0, 1) \text{ and } w \in F^{-1}(1 - q), w + D(q) \geq \lambda. \tag{EB}$$

To understand why equilibrium elimination must be directional, we first examine a candidate equilibrium with a very high threshold wage w_H and thus zero working probability $q_H = 0$, (EB) holds because $D(0)$ is bounded. Therefore, had there been some q_L and w_L with $w_L \in F^{-1}(1-q_L)$ such that $w_L + D(q_L) < \lambda$, we would be able to construct $w' \in [w_L, w_H)$ and some associated $q' < 1$ that form a bad equilibrium by applying the continuity of $D(.)$. This, however, violates full implementation.

$$\left.\begin{array}{l} w_H + D(q_H) > \lambda \\ w_L + D(q_L) < \lambda \end{array}\right\} \Rightarrow \text{ there exists } w' + D(q') = \lambda. \tag{12}$$

Crucially, the two constraints are also sufficient for finding the robustly optimal wage policy:

Proposition 2. *F^* is robustly optimal if and only if it solves the following problem:*

$$\min_{F \in \Delta(\mathbb{R}_+)} \int_{\underline{w}}^{\infty} (1 - F(w))dw \qquad \text{s.t. (EK) and (EB).} \tag{13}$$

Recall that our setting incorporates arbitrarily mixed externalities because career value may be non-monotonic in the market's expectation. Proposition 2 translates the employer's robustness problem to a standard cost-minimization problem with the (EB) constraint restricting the direction in which the employer breaks all bad outcomes. This makes the general characterization possible. Analogous results will be derived for related problems where we later consider partial working and informed worker.

4.2 Main Result

This section presents and discusses our main result, Theorem 1. The main result shows that the robustly optimal wage policy is fully supported in $[\underline{w}, \overline{w}]$. As a result, within-group wage dispersion occurs if and only if strategic uncertainty becomes a problem, abiding by the same criterion (7). From the examples discussed in Section 3.1, we also know that in markets where workers face skill-effort complementarity

(and positive skill-wage assortativeness), such private discrimination can be ubiquitous.

Theorem 1. *To induce full working, the unique robustly optimal wage policy F^* has* $\text{supp}(F^*) = [\underline{w}, \overline{w}]$:

$$\text{For all } w \in [\underline{w}, \overline{w}), \, F^*(w) = 1 - \overline{q}(w^+), \text{ where } \overline{q}(w) = \max\{q : w + D(q) = \lambda\}.^{19,20} \tag{14}$$

Moreover, F^* is continuous if and only if $D(.)$ is strictly increasing.

The employer guarantees work at a low wage cost by precluding low-effort equilibria with on-path shirking. She begins by offering high wages down from the FD wage $\overline{w}$, which is high enough to guarantee work regardless of career concerns. This eliminates the market expectations that are pessimistic toward the worker's effort, thereby rendering the market more reactive to work outcomes and thus shutting down low career concerns. By robustly strengthening the worker's work incentives in this way, the employer is able to elicit more effort with even lower wages. Therefore, in an iterative manner, she continues to raise career concerns and lower her wage offers by gradually shoring up market expectations. Eventually, the procedure ends once the worker is guaranteed to work at the PR wage $\underline{w}$, as is the case in the desired equilibrium. As a result, F^* is *fully supported* from $\underline{w}$ to $\overline{w}$. Moreover, a defining feature of the optimal contracting is that the employer finds it optimal to *greedily* place as little mass on high wages as is needed such that every pessimistic market expectation is made non-credible because the worker is lured to deviate to work but only *slightly* more than what the market expects.

In particular, the employer balances two goals. On the one hand, she must assign a sufficiently large probability to wages no less than each w to ensure that the worker's strategy with this threshold w will generate market expectations no lower than $\overline{q}(w)$. Based on our construction of $\overline{q}(w)$ in (14), Lemma 1 implies that this will guarantee a career value $D(\overline{q}(w))$ facing the worker, and this guarantee is equal to $\lambda - w$. Therefore, the career concerns facing the worker increase strictly as the employer offers a lower and lower wage w in the iterative coordination procedure. The employer must abide by the above rule to keep enhancing work incentives while breaking bad outcomes. On the other hand, to reduce wage costs, the employer must place as little mass on high wages as is needed. As a result, she breaks the bad equilibria in a greedy manner by making (EB) bind pointwise:

$$\text{For all } w \in [\underline{w}, \overline{w}], \, w + D(1 - F^*(w)) = \lambda. \tag{15}$$

In other words, whenever the market expects the worker to play a strategy with threshold w, its beliefs will generate the career value $D(1 - F^*(w))$ that almost sustains its expectation in equilibrium, yet the worker deviates to work slightly more at the wages just below w. In this way, the employer breaks every undesirable outcome on the verge.

Figure 1 illustrates our result with two different examples. The left panel demonstrates the geometric construction of the robustly optimal wage policy. The graph of G (dashed line) represents all pairs (w, F) where $F = 1 - q$ and $w = D(q) - \lambda$ for some $q \in [0, 1]$. Here, G exhibits a zigzag pattern because the induced career value function is nonmonotonic. Theorem 1 states that the robustly optimal wage policy

F^* is given by the highest nondecreasing curve bounded below G. In addition, this example illustrates that mass points arise precisely when the career value is not strictly increasing, or in other words, when skills and efforts become "local substitutes". The right panel examines a linear value parameterization that we discussed in Section 3.1. In the linear environment, career values are always monotonic, so F^* is continuous and free of mass points.
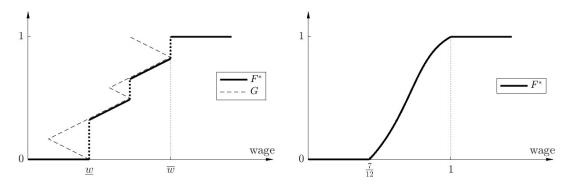

Figure 1: Robustly optimal wage policy F^*. In the left panel, we have $G = \{(w, F) : F = 1 - q$ and $w = D(q) - \lambda$ for all $q \in [0, 1]\}$. The right-panel example considers linear market valuation, parameterized by $\Theta = \{H, L\}$ with prior $\mu_H^0 = \mu_L^0 = 0.5$, market value $u_H = 1$ and $u_L = 0$, shirking success rates $p_0 = p_{0H} = p_{0L} = 0.1$, additional success rates due to working $p_H = 0.7$ and $p_L = 0.3$, and effective cost $\lambda = 1$. This gives $D(q) = \frac{10q}{(1+5q)(9-5q)}$, $\overline{w} = 1 - D(0) = 1$, and $\underline{w} = 1 - D(1) = \frac{7}{12}$.

Theorem 1 establishes a sharp prediction that characterizes the sufficient and necessary condition (7) for wage dispersion to be optimal given the employer's robust objective. While this condition specifies a reduced-form relationship between career concerns and worker effort, its broader economic implications deserve further analysis. The next section examines comparative statics to connect our theoretical results with well-known empirical patterns of (residual) wage dispersion.

4.3   Implications on Wage Dispersion

In this section, comparative analysis establishes that greater wage dispersion is associated with greater skill-wage assortativeness, larger skill premiums, and higher occupational mobility. Crucially, our results appear surprising because they explain how these variables can shape the *residual* wage dispersion that cannot be explained by worker heterogeneity. This contribution is possible due to our consideration of ex-ante identical workers who are even uninformed of their potential characteristics (ability types), so a non-degenerate F^* indicates the endogenous use of within-group wage differentials.

To compare the robustly optimal wage policies under different parameters, we define that one wage policy is *more dispersed* than another if it exhibits both a wider range and a larger variance. This definition aligns our theoretical analysis with empirical evidence, where wage dispersion is typically measured by either the wage range or standard deviation. Formally, we define:

Definition 2. Take two wage policies F^1 and F^2 with $\text{supp}(F^1) = [\underline{w}^1, \overline{w}^1]$ and $\text{supp}(F^2) = [\underline{w}^2, \overline{w}^2]$. F^2 is *more dispersed* than F^1 if $\overline{w}^2 - \underline{w}^2 > \overline{w}^1 - \underline{w}^1$ and $\text{Var}(F^2) > \text{Var}(F^1)$.

Throughout our analysis, we use superscripts to differentiate parameter sets and their corresponding robustly optimal wage policies. For instance, we let v^1 and v^2 denote two post-employment wages, while F^{*1} and F^{*2} are the associated robustly optimal wage policies, respectively. Unless otherwise specified, all other parameters not mentioned in each result are assumed to be identical across the two settings.

We illustrate our first set of implications on wage dispersion by focusing on the linear value example discussed in Section 3.1 where the post-employment wage is linear $v(\mu) = \sum_{k \in \Theta} u_k \mu_k$, and the shirking success rates are type-independent $p_{0k} = p_0$. We refer to it as the linear environment below. Denote by $u = (u_k)_{k \in \Theta}$ the profile of market values and by $p = (p_k)_{k \in \Theta}$ the profile of productivity.

Proposition 3. *Consider the linear environment. Take two market values u^1 and u^2, and two productivities p^1 and p^2. Suppose F^{*2} contains wage dispersion, and maintain $\mathbb{E}_0[p_k^1] = \mathbb{E}_0[p_k^2]$. Then, F^{*2} is more dispersed than F^{*1} if $\mathrm{Cov}(u^2, p^2) > \mathrm{Cov}(u^1, p^1)$. Moreover, this condition holds if $p := p^1 = p^2$, $\mathbb{E}_0[u_k^1] = \mathbb{E}_0[u_k^2]$, and u^2 single-crosses u^1 from below, that is, there is $k^* \in \Theta$ such that $u_k^2 > [<] u_k^1$ for all $p_k > [<] p_{k^*}$.*

Proposition 3 identifies two determinants of the residual wage inequality. The first part of the result predicts that greater wage differentials emerge when more productive workers (whose efforts increase success probability more effectively) are more likely to match with higher-paying positions. To illustrate with an example, we consider $\Theta = \{1, 2, ..., K\}$ with a uniform prior. We let the second market be "efficient" with both u_k^2 and p_k^2 strictly increasing in k, control the productivity $p^1 = p^2$, and make the first market less "efficient" by setting $u_k^1 = u_{t(k)}^2$ where t is a non-identity permutation of Θ that represents a less assortative matching of skills to positions. As a result, we have $\mathrm{Cov}(u^2, p^2) > \mathrm{Cov}(u^1, p^1)$. Moreover, the uniform prior controls the variances up to a permutation, so the condition is also equivalent to $\mathrm{Corr}(u, p) > \mathrm{Corr}(u, p)$ with correlation being a more commonly used measure of assortativeness in empirical estimates. This prediction is in line with the famous work by Card et al. (2013) who show that the increasing assortativeness in the assignment of workers to plants can partially explain the increasing dispersion of West German wages. Our result provides a novel mechanism underlying their findings: assortative skills and wages give rise to complementary worker efforts and career concerns, which poses a concern for strategic uncertainty in contracting; moreover, to optimally address it, employers deliberately create wage inequality despite identical workers.

The second part of Proposition 3 maintains the skills $p^1 = p^2$ while varying market value in a single-crossing direction, with rising skill premiums as a special case. In particular, consider $\Theta = \{1, 2, ..., K\}$ with p_k strictly increasing in k. Construct u^2 to be steeper than u^1, namely $u_k^2 - u_{k-1}^2 > u_k^1 - u_{k-1}^1$ for all $k > 1$. By normalizing $\mathbb{E}_0[u_k^1] = \mathbb{E}_0[u_k^2]$, we ensure that u^2 single-crosses u^1 from below. The persistent parallel increase in skill premiums and wage inequality has been well documented, for instance, by many studies examining skill-biased technical change and wage dispersion (see Krusell et al. (2000), Acemoglu (2002), and the citations there for a few examples). While the standard explanation attributes this relationship to wage differentiation across skill levels, we identify a novel channel: present greater skill premiums in the labor market, career concerns become more important vis-a-vis the wage incentives, and thus strategic uncertainty grows to a larger scale. What is surprising is that skill premiums can shape wage inequality even when we shut down discrimination.

By decomposing covariance as $\mathrm{Cov}(u,p) = \mathrm{Corr}(u,p)\,\mathrm{Var}(u) \times \mathrm{Var}(p)$, one can see from above that wage dispersion increases when we strengthen the relationship between market value u and skill p in two distinct ways: we can either increase their correlation $\mathrm{Corr}(u,p)$ (assortativeness), or we can expand the spread of market value $\mathrm{Var}(u)$ (skill premiums).

In addition, the following result returns to the general model, and considers discounting the payoffs that a worker expects in the labor market:

Proposition 4. *Take two post-employment wages v^1 and v^2. Suppose F^{*2} contains wage dispersion. Then, F^{*2} is more dispersed than F^{*1} if there is a discount factor $\delta \in [0,1)$ such that $v^1 = \delta v^2$.*

Proposition 4 establishes that when workers place less weight on future income, the employer tends to lay out narrower wage dispersion. This discounting effect may naturally pertain to occupational mobility, which represents both job separation rates and job search duration. Formally, workers discount post-employment wages by e^{-rt}, where r is the discount rate and t captures the combined duration of current employment and subsequent job search. Higher occupational mobility reduces t and increases the discount factor, thereby amplifying the importance of career concerns within the industry. This positive association between occupational mobility and wage dispersion is documented in several empirical studies, including Bloom and Michel (2002) and Kambourov and Manovskii (2009), for example. Interestingly, we provide an argument for how occupational mobility leads to wage differentiation, whereas others mostly content the other direction and often leverage the variations in the wage or skill structure to explain the change in occupational mobility.

5 Informed Worker

The main model follows Gibbons and Murphy (1992) (who also follow Holmström (1982/1999) on this) and assumes the worker does not know his ability. This section relaxes this assumption. In particular, after the worker observes his ability type $k \in \Theta$, he has type-dependent working cost c_k, shirking success rate p_{0k}, and additional success rate due to working p_k. We again ask: what is the robustly optimal wage policy? The model timing is unchanged except the worker observes ability before making work decision: given wage policy F, upon receiving wage w, each worker type k chooses a working probability $\sigma_k(w)$. The market, only observing the wage policy and the project outcome, forms a posterior belief about the worker's ability and pays him the post-employment wage.

Setup and Assumption　We begin by defining the concept of career value for the new specification, and then describe equilibrium and the assumption we use. Each worker type's work decision σ_k induces a type-contingent total working probability $q_k := \mathbb{E}_F[\sigma_k(w)] = \int_{\mathbb{R}_+} \sigma_k(w)dF(w)$. The market's beliefs

$\overline{\mu}$ and $\underline{\mu}$ when seeing success and failure, respectively, are given by: for every type k,

$$\overline{\mu}_k = \frac{\mu_k^0[q_k(p_{0k} + p_k) + (1 - q_k)p_{0k}]}{\sum_{j \in \Theta} \mu_j^0[q_j(p_{0j} + p_j) + (1 - q_j)p_{0j}]}, \text{ and}$$

$$\underline{\mu}_k = \frac{\mu_k^0 - \mu_k^0[q_k(p_{0k} + p_k) + (1 - q_k)p_{0k}]}{1 - \sum_{j \in \Theta} \mu_j^0[q_j(p_{0j} + p_j) + (1 - q_j)p_{0j}]}.$$

One can see the beliefs are continuous functions of the total working probabilities $(q_k)_{k \in \Theta}$, so we write them as $\overline{\mu}((q_k)_{k \in \Theta})$ and $\underline{\mu}((q_k)_{k \in \Theta})$. The career value is then defined accordingly:

$$D((q_k)_{k \in \Theta}) = v(\overline{\mu}((q_k)_{k \in \Theta})) - v(\underline{\mu}((q_k)_{k \in \Theta})).$$

As a result, a worker strategy profile $(\sigma_k)_{k \in \Theta}$ forms a Perfect Bayesian Equilibrium if, for all type k, for all realization of wage contract w:

$$\sigma_k(w) \in \operatorname{argmax}_{\widetilde{\sigma} \in [0,1]} \widetilde{\sigma}[w + D((q_k)_{k \in \Theta}) - \lambda_k], \text{ where } q_k = \mathbb{E}_F[\sigma_k(w)], \tag{16}$$

Where $\lambda_k := \frac{c_k}{p_k}$ is type k's effective cost. Different types of worker face heterogeneous work incentives, captured by $w + D - \lambda_k$ in (16), because the effective cost varies with worker ability. More specifically, in an equilibrium with career value equal to D, type k worker's threshold wage is $w_k := \lambda_k - D$, which pins down the worker's strategy (only except for his randomization decision at w_k).

To keep the section compact, we focus on binary types $\Theta = \{H, L\}$ with the high [low] type having a low [high] effective cost, namely $\lambda_H < \lambda_L$. In addition, we first characterize the robustly optimal wage policy that minimizes the wage costs while fully implementing full working. In Appendix A.2, we extend the result to cases with either partial working or multiple types.

Our result here depends on the following assumption: the post-employment wage $v(.)$ is continuous and strictly increasing in the high type's probability. In other words, we let worker ability and market value be assortative since $\lambda_H = \frac{c_H}{p_H} < \frac{c_L}{p_L} = \lambda_L$, which means the high type is more skilled in the sense that he must be able to either work with a lower cost or raise the success rate more effectively. Therefore, an increasing $v(.)$ captures a market that values skill. Under this assumption, the career value function $D(q_L, q_H)$ is strictly increasing [decreasing] in q_H [q_L]. This is because if the high type works more, success is more [less] likely to occur, so it becomes a stronger signal that the ability is high [low]. Therefore, we consider an environment where the two types have asymmetric externalities, that is, the high [low] type's work increases [decreases] career value, and thus increases [decreases] work incentives.

Preliminary Analysis   Again, our analysis begins with two critical wages:

$$\underset{\sim}{w} = \lambda_L - D(1,1); \qquad \widetilde{w} = \max\{\lambda_H - D(0,0), \underset{\sim}{w}\}. \tag{17}$$

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Notice that w is partially optimal (the PR wage) because in a full-working equilibrium, the career value is $D(1,1)$, and the wage must be able to induce work from the worker with lower work incentives. Another wage $\lambda_H - D(0,0)$ (in the definition of $\widetilde{w}$) is the lowest wage needed for breaking the bad equilibrium where no working happens as the career value there is $D(0,0)$, and the wage must be able to lure work from the worker with higher work incentives. In fact, we show (see Proposition 9 in Appendix A.2) that $\widetilde{w}$ is the minimal wage for fully implementing full working (i.e., the FD wage); and that the PR wage, $\underset{\sim}{w}$, suffers from multiplicity if and only if the following happens:

$$\widetilde{w} > \underset{\sim}{w}, \text{ namely } D(1,1) - D(0,0) > \lambda_L - \lambda_H. \tag{18}$$

This extends criterion (7). The difference is that, with the worker informed, his effort must segregate the market's respective opinions on success and failure to a sufficiently large extent, as $\lambda_L - \lambda_H > 0$.

Following Section 4.1, we derive the equilibrium-keeping and equilibrium-breaking constraints, and hence write an auxiliary problem. For notational convenience, we associate each wage policy F with its *tail wage policy* $R(w) := 1 - F(w^-)$. Intuitively, $R(w^*) = \mathrm{Pr}_F(w \geq w^*)$ is the total working probability if the worker works with probability one at all wages no less than the threshold w^*. Let $\Delta_T(\mathbb{R}_+)$ denote the space of tail wage policies. The first constraint, equilibrium-keeping, asserts that the employer should assign all the probability to wages no less than the partially optimal level $\underset{\sim}{w}$:

$$R(\underset{\sim}{w}) = 1. \tag{EK'}$$

By definition, (EK') is equivalent to $F(\underset{\sim}{w}^-) = 0$. The second constraint, equilibrium-breaking, applies the following property of each candidate equilibrium: both types' threshold wages differ by a constant, that is, $w_H = \lambda_H - D$ and $w_L = \lambda_L - D$ where D is the equilibrium level career value, which implies $w_L - w_H = \lambda_L - \lambda_H$. Let $\lambda_0 := \lambda_L - \lambda_H$ denote the *cost gap*. One can see the worker's strategy profile is pinned down (except for the decision at thresholds) by a one-dimensional scalar, for example, the high type's threshold wage. Therefore, the equilibrium-breaking constraint comprises a set of conditions for breaking a one-dimensional continuum of bad equilibria:

$$\text{For all } w \geq \underset{\sim}{w} - \lambda_0, \, w + D(R(w + \lambda_0), R(w)) \geq \lambda_H. \tag{EB'}$$

For every high-type threshold w, the low type's threshold wage is $w + \lambda_0$. Thus, (EB') demands that in a candidate equilibrium indexed by w, the market's beliefs formed by anticipating the high type's total working probability $R(w)$ and the low type's total working probability $R(w + \lambda_0)$ must generate a career value $D(R(w+\lambda_0), R(w))$ that is too high to rationalize the anticipated worker behavior. In other words, given the induced career value, both type would like to work more than expected. This constraint follows the spirit of (EB) by requiring the wage policy induce excessively optimistic expectations. Notice that in a desired full-working equilibrium, the high type's threshold wage is $\lambda_H - D(1,1) = \underset{\sim}{w} - \lambda_0$, so the bad equilibria are indexed by $w > \underset{\sim}{w} - \lambda_0$ in (EB').

We show (EK') and (EB') are also sufficient for finding the robustly optimal wage policy:

Proposition 5. $F^\dagger$ *is robustly optimal if and only if its associated tail wage policy* $R^\dagger$ *solves the following problem:*

$$\min_{R \in \Delta_T(\mathbb{R}_+)} \int_{\underset{\sim}{w}}^\infty R(w)dw \qquad \text{s.t. (EK') and (EB').} \tag{19}$$

The key to solving (19) again relies on a greediness result: the employer finds it optimal to place as little mass on high wages as is needed. Formally, (EB') binds. We give a name, w-EB', to the equilibrium-breaking constraint indexed by w. To understand why (EB') optimally binds, take a $w > \underset{\sim}{w}$ (so that (EK') is not restrictive at w) with $R(w) > 0$, and suppose w-EB' is slack. However, the employer can be strictly better off by lowering R around w because: (i) this reduces wage costs $\int R(w)dw$; (ii) the resulting tail wage policy still satisfies (EB') and thus remains feasible in problem (19). To see (ii), we notice that the value $R(w)$ enters two EB' constraints, w-EB' and $(w - \lambda_0)$-EB':

$$w + D(R(w + \lambda_0), R(w)) \geq \lambda_H;$$
$$(w - \lambda_0) + D(R(w), R(w - \lambda_0)) \geq \lambda_H.$$

Because w-EB' is slack, a small perturbation of $R(w)$ will not violate this constraint. On the other hand, $R(w)$ enters $(w - \lambda_0)$-EB' as the total working probability of the low type, so lowering this value will make project success a stronger signal that the worker has high ability. This increases the career value, $D(R(w), R(w - \lambda_0))$, on the left-hand side, and thus the constraint cannot become violated.

Result and Generalizations We present below the extension result, Proposition 6, which states that the robustly optimal wage policy $F^\dagger$ is fully supported in $[\underset{\sim}{w}, \widetilde{w}]$, containing a mass point at the minimal wage while being otherwise continuous. Hence, within-skill wage inequality occurs following the same criterion (18) for strategic uncertainty to be an issue. Figure 2 depicts a generic example. The solution form admits an intriguing interpretation of the optimal wage structure: the employer publicly announces a minimum wage $\underset{\sim}{w}$ and offers personalized lucky wage raises to some workers.
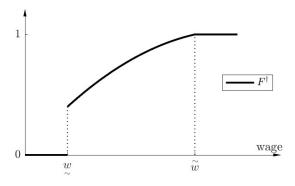

Figure 2: Robustly optimal wage policy $F^\dagger$ with an informed worker.

Proposition 6. *To induce full working from a binary-type informed worker, the unique robustly optimal wage policy $F^\dagger$ is given by:*

$$\text{supp}(F^\dagger) = [\underset{\sim}{w}, \widetilde{w}], \text{ and } w\text{-EB' binds for all } w \in (\underset{\sim}{w}, \widetilde{w}]. \tag{20}$$

Moreover, $F^\dagger$ is continuous on $(\underset{\sim}{w}, \widetilde{w}]$ and has a mass point at $\underset{\sim}{w}$.

To understand Proposition 6, one can imagine the following procedure that pins down the robustly optimal wage policy. First of all, the employer offers the wage $\widetilde{w}$ that, given skeptical market expectations and thus small career concerns, attract only the high type into work but discourage the low type from working; this guarantees larger career concerns since the market now attributes project success [failure] to high-type work [low-type shirk]; as a result, the employer can ensure greater effort with lower wages. What follows is that the employer sequentially assigns probabilities to different wages down from $\widetilde{w}$ to keep strengthening career concerns by maintaining that the high type works "more than" the low type. In particular, conditional on the probabilities assigned to wages higher than each threshold w, the employer places as little mass on w as is needed, so w-EB' binds. Eventually, the procedure ends once it reaches the equilibrium-keeping wage $\underset{\sim}{w}$, leaving a mass point at the bottom.

To see how the binding (EB') yields employer behavior, note that the right-most part of the associated tail policy $R^\dagger$ must have $R^\dagger(w) = 0$ for all $w > \widetilde{w}$ because it is suboptimal to offer these extremely high wages. Given this, for every $w \in (\underset{\sim}{w}, \widetilde{w}]$, once the part of $R^\dagger$ on w's right-hand side is already determined (so $R^\dagger(w + \lambda_0)$ is given), the binding w-EB' pins down $R^\dagger(w)$. We show that (see Lemma 2 in Appendix A.2) $R^\dagger$ is unique, continuous, and strictly decreasing on $(\underset{\sim}{w}, \widetilde{w}]$.

The other solution characteristic is the mass point at minimum. The insight underlying this feature is closely related to the employer's greedy strategy for designing the continuous part. In the "last" binding constraint $\underset{\sim}{w}$-EB', the career value is $\lambda_H - \widetilde{w} = D(1,1) - \lambda_0$, lower than the desired level $D(1,1)$. Hence, the employer uses a mass point to ensure the high type works fully, forcing the career value to jump to a sufficiently high tier to guarantee the low type's full working while abiding by (EK').

To extend the result beyond the binary-type full-working case, in Appendix A.2, we further consider partial working and multiple types. For robustly inducing a certain profile of working probabilities, the employer may introduce multiple mass points at each type's threshold wage in the desired equilibrium (see Proposition 7). With multiple types, we identify assumptions that ensure that the simple wage structure in Proposition 6 is still optimal (see Proposition 10). The proof relies on generalizing the auxiliary problem (19), which is intrinsically challenging to solve because these problems involve a continuum of non-linear constraints that are interdependent with each other. Therefore, the optimality of greediness is crucial in providing necessary conditions (binding EBs) that uniquely pin down solutions. Even though more general cases could be less tractable, we extend criterion (7) in Proposition 9 which shows that our main message is still preserved: discrimination occurs precisely when strategic uncertainty becomes a problem, usually following complementary reputation effects in labor markets.

6 Discussions

General market value   If one interprets the post-employment wage $v(.)$ as the worker's continuation value in a "larger equilibrium" played by the entire labor market, then typically it is also a function of the worker's posterior belief. Able to observe his effort privately, the worker's belief can be different from that of the market in off-path play, which matters since his continuation payoffs depend on his optimal decision in the market that may vary with his belief.

However, this will not change our result at all since we eventually work with the function of career value and the only technical assumption we need is the continuity of career value. Specifically, denote the worker's posterior beliefs contingent on success by $\overline{\nu}_a$, where $a = 1$ represents that the worker has decided to work while $a = 0$ indicates his shirking. Likewise, the beliefs contingent on failure are $\underline{\nu}_a$. Note that these worker beliefs are four constants exogenously given by the Bayes' rule. To incorporate the above discussion, we set the worker's post-employment wage to be $v(\mu, \nu)$ where μ and ν represent the posterior beliefs about worker ability held by the market and the worker, respectively. Next, we can see the worker's optimal choice in equilibrium is still determined by comparing his effective cost λ, the contract realization w, and the career value $D(q)$ with q being market expectation. What changes is the definition of career value:

$$D(q) = \frac{1}{\mathbb{E}_0[p_k]} \Big\{ \mathbb{E}_0[p_{0k} + p_k] \big[v(\overline{\mu}(q), \overline{\nu}_1) - v(\underline{\mu}(q), \underline{\nu}_1) \big] - \mathbb{E}_0[p_{0k}] \big[v(\overline{\mu}(q), \overline{\nu}_0) - v(\underline{\mu}(q), \underline{\nu}_0) \big]$$
$$+ \big[v(\underline{\mu}(q), \underline{\nu}_1) - v(\underline{\mu}(q), \underline{\nu}_0) \big] \Big\}.$$

Although this new expression is complex, it is continuous once the post-employment wage is continuous in both beliefs. Our main analysis thus follows smoothly.

Partial privacy   Contractual privacy is the crucial element in our model, but it is only compared with public contracting. To check robustness, we discuss the intermediate case where contracting is neither completely private nor completely public in the following four scenarios: (i) the market has some probability to observe wage; (ii) the worker has some probability to be able to disclose wage; (iii) the employer can release information about wage; (iv) some firms in the market can observe wage.

The main message of the paper will not change in the above four cases because strategic uncertainty occurs subject to the same criterion (7). To see this, consider restricting the employer's decision to a single wage instead of a contract distribution, so communication about wage is unnecessary since the contract is deterministic. Hence, Section 3.1 has demonstrated that the partial- and robust-implementation solutions here are $\underline{w}$ and $\overline{w}$, respectively, and strategic uncertainty emerges if and only if they diverge.

To see why randomization is helpful, for case (i) and (ii), we assume the silent event where no wage information can be transmitted must happen with a positive probability. Hence, randomization can still shift market expectations by exploiting the market's ignorance in the silent event. The deviation from our main result is that, in iteratively constructing the optimal wage structure, the employer must guarantee

work at each wage while taking into account the equilibrium behavior in the non-silent even. For case (iii), randomization improves upon the FD wage when, for example, the employer commits to not release any information and uses the robustly optimal wage policy. To understand how randomization emerges in case (iv), we consider the following setup. Let the posterior belief of the firms that can observe wage be μ_O, and that of those firms that cannot see contract realization be μ_N. The worker's post-employment wage is defined as the highest wage offer from the market $v(\mu_O, \mu_N) := \max\{v_0(\mu_O), v_0(\mu_N)\}$ in which $v_0(\mu)$ represents a firm's offer when it holds the belief μ. Recall that the main goal in guaranteeing effort is to lure the worker to work at wages where he previously shirks. When these wages are realized, the firms that observe contract realization simply know that the worker shirks ($q = 0$). Therefore, the career value (that matters for this task of inducing deviation) is the following one:

$$D(q) = \max\left\{v_0(\overline{\mu}(q)), v_0(\overline{\mu}(0))\right\} - \max\left\{v_0(\underline{\mu}(q)), v_0(\underline{\mu}(0))\right\}.$$

To deliver the insight, suppose that we have skill-effort complementarity (Section 3.1). Therefore, the uninformed firms have more reactive beliefs, which implies $v_0(\overline{\mu}(q)) \geq v_0(\overline{\mu}(0))$ and $v_0(\underline{\mu}(q)) \leq v_0(\underline{\mu}(0))$. In other words, the worker joins the informed firms if failure is the outcome, whereas he joins the uninformed firms if he succeeds. As a result, the career value here $D(q) = v_0(\overline{\mu}(q)) - v_0(\underline{\mu}(0))$ will attenuate compared to the standard definition (4), so the optimal wage structure will contain narrower dispersion, which, nonetheless, will not degenerate to a deterministic contract.

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Appendix A

A.1 Classic Monitoring System

This section studies a one-period version of the classic contracting-with-career-concerns setup considered by, for example, Gibbons and Murphy (1992), and shows that with the additive monitoring system they assume, every contract induces a unique equilibrium. In addition, by using this one-period result, even in a multiple-period model, one obtains uniqueness by working backward from the last period to the first period (this relies on that the market's posterior belief pins down its equilibrium contract offer in each period, and that the market and the worker have symmetric information on the equilibrium path). The timing coincides with our main model. However, the worker now chooses a continuous action $a \geq 0$ and the project's outcome can be any real number $y \in \mathbb{R}$. The worker's ability type $k \in \mathbb{R}$ is normally distributed with mean m_0 and variance σ_0^2. The monitoring system is additive and takes the following linear form:

$$y = a + k + \epsilon, \tag{21}$$

Where ϵ is an independent noise normally distributed with mean 0 and variance σ_ϵ^2. We consider the equilibria (PBE) induced by any contract $w(y)$ that specifies the outcome-contingent transfer to the worker. Moreover, we also do not impose any restriction on the post-employment wage $v(.)$, which can be viewed as the continuation value in a multiple-period equilibrium with the play after the current period already pinned down. Since we allow $w(.)$ and $v(.)$ to be arbitrary, we are considering a setting that is potentially more general than Gibbons and Murphy (1992) who consider perfect firm competition and linear contracts. Because we discuss the strategic uncertainty facing a partially implementing contract, it suffices to consider a deterministic one. We assume the worker's working cost is $c(a)$.

We begin by characterizing the worker's career concerns when the market believes that he chooses some action $\hat{a}$. Given this belief, the well-known formulas from DeGroot (2005) suggest that conditional on observing outcome y, the market's posterior belief about type k is a normal distribution with:

$$\text{mean } \frac{\sigma_\epsilon^2 m_0 + \sigma_0^2(y - \hat{a})}{\sigma_\epsilon^2 + \sigma_0^2} \text{ and variance } \frac{\sigma_0^2 \sigma_\epsilon^2}{\sigma_0^2 + \sigma_\epsilon^2}. \tag{22}$$

This implies the post-employment wage, which is a function of posterior, can be rewritten as a reduced-form function of $y - \hat{a}$ alone, denoted simply as $v(y - \hat{a})$. Note that when the market believes the worker chooses $\hat{a}$ while he actually chooses a, $y - \hat{a}$ is normally distributed with mean $m_0 + a - \hat{a}$ and variance $\sigma^2 := \sigma_0^2 + \sigma_\epsilon^2$. Hence, the worker's future expected value in the market is:

$$U(a|\hat{a}) = \int_{\mathbb{R}} \frac{1}{\sigma^2 \sqrt{2\pi}} e^{-\frac{[s - (m_0 + a - \hat{a})]^2}{2\sigma^2}} v(s) ds. \tag{23}$$

Notice that $U(a|\hat{a}) = U(a - \hat{a}|0)$. We let $U_0(a) := U(a|0)$ and thus $U(a|\hat{a}) = U_0(a - \hat{a})$.

Moreover, the worker's payoff from the employer when choosing a is:

$$W(a) = \int_{\mathbb{R}} \frac{1}{\sigma^2 \sqrt{2\pi}} e^{-\frac{[s - (m_0 + a)]^2}{2\sigma^2}} w(s) ds. \tag{24}$$

Consequently, an equilibrium where the worker chooses $\hat{a}$ exists if and only if $\hat{a}$ is the worker's optimal choice given the market belief that it is also his actual decision:

$$\hat{a} \in \text{argmax}_{a \geq 0} W(a) - c(a) + U_0(a - \hat{a}). \tag{25}$$

The first-order condition of the worker's problem above is $W'(\hat{a}) - c'(\hat{a}) + U_0'(0) = 0$. Crucially, one can see career concerns enter the condition through a constant $U_0'(0)$ that does not vary with the action $\hat{a}$. When one compares the condition with (5), roughly speaking, $U_0'(0)$ is the "counterpart" of career value. As a result, the setup fails to incorporate efforts' influence on the outcome-generating process.

If we follow Gibbons and Murphy (1992) who assume that $w(.)$ is linear, and we also let $c(.)$ be strictly concave, then the equilibrium is unique. Therefore, any contract induces an outcome robustly.

A.2 Informed Worker: Partial Working and Multiple Types

This section defines the robust solution concept for fully implementing partial working when the worker has multiple types, and presents the general results. To begin with, we label the ability types as integers $\Theta = \{1, 2, ..., K\}$ and fix a profile $Q \in [0, 1]^K$ such that the kth coordinate, Q_k, refers to each type k's total working probability. We assume that the effective cost λ_k strictly decreases with the type label k, so higher types are more skilled in the sense that their efforts either take lower costs or produce success more effectively. Given the equilibrium concept specified in Section 5, we collect all wage policies that induce Q as the unique equilibrium outcome in $\mathcal{F}^{FI}(Q)$. Hence, for every $g \in \mathcal{E}(F)$ with $F \in \mathcal{F}^{FI}(Q)$, $\mathbb{E}_F[\sigma_k(w)] = Q_k$ for all k. At first, we take as granted that Q is implementable with $\mathcal{F}^{FI}(Q) \neq \emptyset$, and will provide the condition for implementability later. The following then defines the employer's objective conditional on her goal of inducing Q robustly.

Definition 3. The employer's *minimal wage guarantee* for fully implementing Q is:

$$W_Q^\dagger = \inf_{F \in \mathcal{F}^{FI}(Q)} \sup_{g \in \mathcal{E}(F)} \mathbb{E}_F \left[w \sum_{k \in \Theta} \mu_k^0 (p_{0k} + p_k \sigma_k(w)) \right]. \tag{26}$$

A wage policy $F_Q^\dagger$ is *robustly optimal* if there is a sequence of wage policies $(F_n)_{n=1}^\infty$ such that:
1. The sequence $(F_n)_{n=1}^\infty$ is contained in $\mathcal{F}^{FI}(Q)$ and weakly converges to $F_Q^\dagger$;
2. The wage guarantee, $\sup_{g \in \mathcal{E}(F_n)} \mathbb{E}_{F_n} \left[w \sum_{k \in \Theta} \mu_k^0 (p_{0k} + p_k \sigma_k(w)) \right]$, converges to $W_Q^\dagger$.

Note that the above contains a min-max problem where we assume the employer always expects the worst-case equilibrium in $\mathcal{E}(F)$ to occur. However, this selection rule is not essential because each type takes a threshold strategy in equilibrium, so Q and F suffice to pin down the expected wage cost.

Binary Types with Partial Working   First, we investigate the full implementation of partial working of a binary-type worker. The assumption needed is the same as the one made in Section 5, namely, $v(.)$ is continuous and strictly increasing in the high type's probability. We also borrow the notations there: $\Theta = \{H, L\}$, career value $D : [0, 1]^2 \to \mathbb{R}$, and tail wage policy R.

The analysis again starts with the following critical wages:

$$\underline{w}_L = \lambda_L - D(Q_L, Q_H); \qquad \underline{w}_H = \lambda_H - D(Q_L, Q_H); \qquad \widetilde{w} = \lambda_H - D(0, 0). \tag{27}$$

Note that $\underline{w}_k$ denotes type k's on-path threshold wage while $\widetilde{w}$ refers to the wage that guarantees high type's work. Similar to full working, the employer's optimal strategy here stems from greediness, where she iteratively breaks the bad outcomes by inducing small deviations. To formalize greediness, we first define a tail wage policy's support as its associated wage policy's support, namely $\text{supp}(R) := \text{supp}(F)$. We then pin down the greedy behavior of the employer, which yields binding (EB'):

Definition 4. The *greedy wage policy* $R^G \in \Delta_T(\mathbb{R}_+)$ is defined by the following:

1. w-EB' holds if $R^G(w) < 1$;
2. w-EB' binds if and only if $w \in \text{supp}(R^G)$.

It turns out that R^G can be uniquely pinned down. We first show that the upper bound of its support must be $\widetilde{w}$. Next, proceeding from this wage to lower ones, the binding EBs back out the entire function given that $R^G = 0$ at wages higher than $\widetilde{w}$. The construction stops when the function hits the boundary $R^G = 1$. We summarize a few properties of R^G below:

Lemma 2. R^G *is unique, continuous, and fully supported on* $[w_l, \widetilde{w}]$ *with some* $w_l < \underset{\sim}{w}$.

Next, we construct the Q-equilibrium-breaking policy R_Q^G and the Q-equilibrium-keeping policy R_Q^K, each of which translates the corresponding constraints (EB/EK) into the employer's behavior:

$$R_Q^G(w) = \min\{R^G(w), Q_H\}.$$

$$R_Q^K(w) = \begin{cases} 1 & \text{if } w = 0; \\ Q_H & \text{if } w \in (0, \underline{w}_H]; \\ Q_L & \text{if } w \in (\underline{w}_H, \underline{w}_L]; \\ 0 & \text{if } w > \underline{w}_L. \end{cases}$$

Notice that R_Q^G basically represents the greedy strategy R^G while being capped by Q_H because Q_H is the maximal working probability the employer induces for the high type. Moreover, R_Q^K captures the need to maintain the existence of the desired equilibrium where the career value is $D(Q_L, Q_H)$ and each type k's threshold wage is $\underline{w}_k$. Given these, R_Q^K ensures that type k's working probability is exactly Q_k.

Now, we are ready to formally present the extension result:

Proposition 7. *To induce* Q *from a binary-type informed worker, the unique robustly optimal wage policy* $F_Q^\dagger$ *is associated with the following tail wage policy, for all* $w \geq 0$:

$$R_Q^\dagger(w) = \max\{R_Q^K(w), R_Q^G(w)\}. \tag{28}$$

Moreover, $F_Q^\dagger$ *takes one of the three forms below. If* $Q_H > Q_L > 0$, $F_Q^\dagger$ *contains two mass points at* $\underline{w}_H$ *and* $\underline{w}_L$, *and there is* $\widehat{w} \in [\underline{w}_H, \underline{w}_L)$ *such that* $F_Q^\dagger$ *is fully supported and continuous on* $(\underline{w}_H, \widehat{w}]$ *and* $(\underline{w}_L, \widetilde{w}]$. *If* $Q_H = Q_L > 0$, $F_Q^\dagger$ *contains a mass point at* $\underline{w}_L$ *and is fully supported and continuous on* $(\underline{w}_L, \widetilde{w}]$. *If* $Q_H \geq Q_L = 0$, $F_Q^\dagger$ *contains no mass point and is fully supported and continuous on* $[\underline{w}_H, \widetilde{w}]$.

Proposition 7 extends the full-working result, Proposition 6, and highlights the optimality of greediness as (28) demonstrates that everywhere in the wage range, either EK or EB must bind. Figure 3 depicts how the optimal wage structure varies with Q. The interesting case is where $Q_H > Q_L > 0$. Recall that in the full-working case, the optimal design takes on a base-bonus wage structure, that is, the employer announces a minimum wage $\underset{\sim}{w}$ while privately offering dispersed wage raises to some workers. When
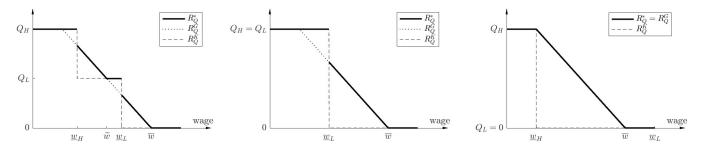
Figure 3: Partial working of a binary-type worker.

$Q_H > Q_L > 0$, she instead uses two base-bonus structures. In particular, she randomly divides workers into two groups and announces a minimum wage $\underline{w}_H$ [$\underline{w}_L$] in the first [second] group. Moreover, she privately offers dispersed wage raises to some workers in each group.

Finally, we provide the sufficient and necessary criterion for implementability of Q:

Proposition 8. *For $Q_H \geq Q_L$, Q is not implementable if and only if $Q_L \leq R^G(\underline{w}_L)$ and $\underline{w}_L \in \mathrm{supp}(R^G)$.*

In short, Proposition 8 shows that fixing every Q_H, there is a lower bound $\underline{Q}_L$ such that Q is implementable if and only if $Q_L \in (\underline{Q}_L, Q_H]$. On the one hand, we must have $Q_L \leq Q_H$ because the high type is willing to work at all wages where the low type works; on the other hand, Q_L cannot be lower than $\underline{Q}_L$ because the need to break bad equilibria forces the employer to assign probabilities to high wages (as in the greedy policy R^G), which guarantees a minimal working probability of the low type, $\underline{Q}_L$.

Multiple Types with Full Working   Next, we investigate the full implementation of full working of a finite-type worker. Apart from the continuity of $D(.)$, we assume that $D(.)$ is quasi-concave in q_k for each k. One can see that this extends the binary-type assumption that $D(.)$ increases [decreases] strictly in q_H [q_L], as a result of $v(.)$ strictly increasing in high-type probability. We are then able to derive the sufficient and necessary condition for strategic uncertainty and wage dispersion:

Proposition 9. *To induce full working, wage dispersion is robustly optimal if and only if:*

$$\underset{\sim}{w} := \lambda_1 - D(1, ..., 1) < \widetilde{w} := \max \left\{ \max_{k \in \Theta} \lambda_k - D(\underbrace{0, ..., 0}_{k \text{ zeros}}, 1, ..., 1), \underset{\sim}{w} \right\}. \tag{29}$$

Moreover, the FD wage level is $\widetilde{w}$ and the PR wage policy is degenerate at $\underset{\sim}{w}$.

Proposition 9 preserves the main message of this paper, which connects employers' motives of incentive provision and coordination with how contractual privacy impacts discrimination. However, fully characterizing the robustly optimal wage policy is difficult when the worker has multiple types. The reason is that greediness may no longer be optimal. Specifically, the EB constraints in this case are:

$$\text{For all } w \geq \underset{\sim}{w} - (\lambda_1 - \lambda_K), \ w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) \geq \lambda_K. \tag{EB''}$$

Two facts complicate solving the employer's problem: first, how career value depends on different types' working probabilities can be complex; second, each value $R(w)$ enters at most K constraints, so (EB") contains interdependent conditions. Fortunately, we find the following three assumptions that maintain the optimality of the greedy strategy.

Assumption 1. *$D(.)$ is continuously differentiable and concave.*

To find solutions, we first reduce the original problem of the employer to an auxiliary one, in analogy to (13) and (19). With interdependent constraints, we solve this problem by writing down KKT conditions. Thus, we need Assumption 1 to ensure that the KKT conditions are meaningful and sufficient.

To impose a structure on career value, we let $D_k(.)$ denote the partial derivative of career value with respect to q_k for each type k. Assumption 2 then assumes an increasing-in-type effect of efforts on career concerns. In other words, higher types' work is more effective in enhancing career concerns.

Assumption 2. *$D_K(.) > 0$, and for all $k_2 > k_1$ and $Q \in [0,1]^K$, $D_{k_2}(Q) > D_{k_1}(Q)$.*

Furthermore, the last assumption we need is Assumption 3. This assumption ensures the optimality of greediness. Roughly speaking, conditions (30) and (31) below together highlight the highest type's salient ability in increasing career concerns through working. Moreover, (31) requires that the wage dispersion should be relatively small to prevent the interdependence among EB constraints from introducing non-greedy motives.

Assumption 3. *For each $k < K$, for all $0 \leq q_1 \leq q_2 \leq ... \leq q_k \leq 1$, we have:*

$$D(1, ..., 1) - D(q_1, q_2, ..., q_k, 1, ..., 1) < \lambda_1 - \lambda_k. \tag{30}$$

Moreover, we have:

$$D(1, ..., 1) - D(0, ..., 0) < (\lambda_1 - \lambda_K) + \min_{k<K}\{\lambda_k - \lambda_{k+1}\}. \tag{31}$$

Given these conditions, we are able to construct Lagrangian multipliers explicitly and verify that the greedy solution below satisfies the KKT conditions. Proposition 10 finds an optimal wage structure that has the same base-bonus feature (minimum wage and continuous raises) as that in Proposition 6.

Proposition 10. *Suppose Assumption 1, 2, and 3 hold. To induce full working from a finite-type informed worker, a robustly optimal wage policy $F^\dagger$ is given by:*

$$\text{supp}(F^\dagger) = [\underset{\sim}{w}, \widetilde{w}], \text{ and } w\text{-EB" binds for all } w \in (\underset{\sim}{w}, \widetilde{w}]. \tag{32}$$

Moreover, $F^\dagger$ is continuous on $(\underset{\sim}{w}, \widetilde{w}]$ and has a mass point at $\underset{\sim}{w}$.

Appendix B

B.1 Proof of Claim 1 and Claim 2

First, we show Claim 1 is true. It suffices to show that with skill-effort complementarity, $D(q)$ is strictly increasing in q. In fact, we can show that given the working probability q, the market's posterior belief contingent on observing success [failure] $\overline{\mu}(q)$ [$\underline{\mu}(q)$], given by the Bayes' rule (3), is strictly increasing [decreasing] in the order of first-order stochastic dominance, so the fact that $v(.)$ increases strictly with respect to strict first-order stochastic dominance yields the result. To do this, we want to show that for all k, we have $\frac{d}{dq}\sum_{j=1}^{k}\overline{\mu}_j(q) \le 0$ and $\frac{d}{dq}\sum_{j=1}^{k}\underline{\mu}_j(q) \ge 0$, and at least one inequality is strict:

$$
\frac{d}{dq}\sum_{j=1}^{k}\overline{\mu}_j(q) \le 0 \Leftrightarrow \frac{d}{dq}\frac{\sum_{j=1}^{k}\mu_j^0(p_{0j}+p_j q)}{\sum_{j=1}^{K}\mu_j^0(p_{0j}+p_j q)} \le 0
$$

$$
\Leftrightarrow \left(\sum_{j=1}^{k}\mu_j^0 p_j\right)\left[\sum_{j=1}^{K}\mu_j^0(p_{0j}+p_j q)\right] \le \left(\sum_{j=1}^{K}\mu_j^0 p_j\right)\left[\sum_{j=1}^{k}\mu_j^0(p_{0j}+p_j q)\right]
$$

$$
\Leftrightarrow \left(\sum_{j=1}^{k}\mu_j^0 p_j\right)\left(\sum_{j=1}^{K}\mu_j^0 p_{0j}\right) \le \left(\sum_{j=1}^{K}\mu_j^0 p_j\right)\left(\sum_{j=1}^{k}\mu_j^0 p_{0j}\right)
$$

$$
\Leftrightarrow \sum_{j=1}^{k}P_j \le \sum_{j=1}^{k}Q_j^S; \text{ and}
$$

$$
\frac{d}{dq}\sum_{j=1}^{k}\underline{\mu}_j(q) \ge 0 \Leftrightarrow \text{ with similar steps } \Leftrightarrow \sum_{j=1}^{k}P_j \le \sum_{j=1}^{k}Q_j^F.
$$

By skill-effort complementarity, both above hold and for at least one k, one inequality above is strict.

Next, we show (8), the career value function in the linear environment. By plugging in (3), we have:

$$
\begin{aligned}
D(q) &= \sum_{k\in\Theta}u_k\overline{\mu}_k - \sum_{k\in\Theta}u_k\underline{\mu}_k = \frac{\sum_{k\in\Theta}u_k\mu_k^0(p_0+p_k q)}{\sum_{k\in\Theta}\mu_k^0(p_0+p_k q)} - \frac{\sum_{k\in\Theta}u_k[\mu_k^0-\mu_k^0(p_0+p_k q)]}{1-\sum_{k\in\Theta}\mu_k^0(p_0+p_k q)} \\
&= \frac{\mathbb{E}_0[u_k]p_0+\mathbb{E}_0[u_k p_k]q}{p_0+\mathbb{E}_0[p_k]q} - \frac{\mathbb{E}_0[u_k]-(\mathbb{E}_0[u_k]p_0+\mathbb{E}_0[u_k p_k]q)}{1-(p_0+\mathbb{E}_0[p_k]q)} \\
&= \frac{\mathbb{E}_0[u_k]p_0+\mathbb{E}_0[u_k p_k]q-\mathbb{E}_0[u_k](p_0+\mathbb{E}_0[p_k]q)}{(p_0+\mathbb{E}_0[p_k]q)[1-(p_0+\mathbb{E}_0[p_k]q)]} \\
&= \frac{(\mathbb{E}_0[u_k p_k]-\mathbb{E}_0[u_k]\mathbb{E}_0[p_k])q}{(p_0+\mathbb{E}_0[p_k]q)(1-p_0-\mathbb{E}_0[p_k]q)} = \text{Cov}(u,p)T(q;\mathbb{E}_0[p_k]),
\end{aligned}
\tag{33}
$$

Where for all $q > 0$, T can be rewritten as follows:

$$
T(q;\mathbb{E}_0[p_k]) = \frac{1}{p_0(1-p_0)/q-(\mathbb{E}_0[p_k])^2 q+(1-2p_0)\mathbb{E}_0[p_k]}.
\tag{34}
$$

One can see that $T(0;\mathbb{E}_0[p_k]) = 0$ and T is strictly increasing in q for all $q > 0$.

B.2 Proof of Proposition 1

First, we show the PR wage policy, denoted as F^P, assigns all the probability to $\underline{w}$. This is true because, in the full-working equilibrium, the worker faces a career value $D(1)$. Thus, this equilibrium exists if and only if a wage policy does not assign any positive probability to wages lower than $\lambda - D(1)$, equivalent to $\text{supp}(F^P) \subset [\underline{w}, \infty)$. Moreover, any such wage policy can be improved by shifting all the probability to $\underline{w}$, which lowers the expected wage while ensuring the existence of the desired equilibrium. Therefore, the wage policy degenerate at $\underline{w}$ is the (almost surely) unique PR wage policy.

Second, to determine the FD wage, denoted as w^F, we characterize the equilibrium set induced by each deterministic wage, say w. There are three cases: (i) if $w \geq \lambda - D(1) = \underline{w}$, the worker works with probability one; (ii) if $w \leq \lambda - D(0)$, the worker shirks; (iii) if $w = \lambda - D(q)$, the worker randomizes and works with probability q. Therefore, to ensure that case (i) exists while case (ii) does not, we must have $w^F \geq \underline{w}$ and $w^F > \lambda - D(0)$. To make sure that case (iii) vanishes in the equilibrium set, we need $w^F \neq \lambda - D(q)$ for all $q < 1$, which means $w^F \notin \{\lambda - D(q) : q < 1\}$. The continuity of $D(.)$ further implies that $\{\lambda - D(q) : q \in [0,1]\} = [\lambda - \max_{q \in [0,1]} D(q), \lambda - \min_{q \in [0,1]} D(q)]$. Combining these three conditions, we find that w^F fully implements full working if and only if $w^F > \overline{w} = \lambda - \min_{q \in [0,1]} D(q)$ if $D(1) \neq \min_{q \in [0,1]} D(q)$, or $w^F \geq \overline{w}$ if $D(1) = \min_{q \in [0,1]} D(q)$. In either case, the infimum is $\overline{w}$.

Lastly, we show $\mathcal{E}(\underline{w})$ contains multiple equilibria if and only if there is a $q' \neq 1$ such that $D(q') \leq D(1)$. When such a q' does not exist, $D(1) = \min_{q \in [0,1]} D(q)$ and 1 is the unique minimizer. As a result, $\underline{w} > \{\lambda - D(q) : q < 1\}$ and thus $\underline{w} > \lambda - D(0)$, so $\mathcal{E}(\underline{w})$ is a singleton. On the other hand, if there exists a $q' \neq 1$ such that $D(q') \leq D(1)$, $\underline{w}$ must induce an equilibrium in either case (ii) or case (iii). To see this, we first assume $D(1) < \max_{q \in [0,1]} D(q)$, so $D(1) \in [D(q'), \max_{q \in [0,1]} D(q))$. By the continuity of $D(.)$, there must exist a $\widetilde{q} \neq 1$ such that $D(\widetilde{q}) = D(1)$. Hence, $\underline{w} = \lambda - D(\widetilde{q})$, forming an equilibrium in case (iii). Otherwise, we have $D(1) = \max_{q \in [0,1]} D(q)$. This implies that $D(1) \geq D(0)$ and we thus have $\underline{w} \leq \lambda - D(0)$, forming an equilibrium in case (ii). At this point, we complete the proof.

B.3 Proof of Proposition 2 and Theorem 1

Here is the proof sketch. We first show that every robustly optimal wage policy must be feasible in the auxiliary problem (13). Next, we show that problem (13) has a unique solution given by Theorem 1. To conclude, we show that the solution can be approximated by a sequence of wage policies in the sense of Definition 1, and that it is the unique robustly optimal wage policy. In this proof, we assume $\overline{w} > \underline{w}$ since the other case is trivial (see our arguments at the beginning of Section 3.2). Also, notice:

$$F^{-1}(1 - q) := \{w : q \in [1 - F(w^+), 1 - F(w^-)]\}. \tag{35}$$

Step 1. We take any robustly optimal wage policy $\widetilde{F}^*$ and a sequence of wage policies $(F_n)_{n=1}^{\infty}$ that approximates it in the sense of Definition 1. Since every F_n fully implements full working, it must satisfy (EK) to ensure that there is a full-working equilibrium. Suppose there is $\widehat{w} < \underline{w}$ such that $\widetilde{F}^*(\widehat{w}) > 0$.

Then, the right-continuous nondecreasing function $\widetilde{F}^*$ must be continuous at some $w' \in (\widehat{w}, \underline{w})$ with $\widetilde{F}^*(w') > 0$. However, this is impossible because $F_n(w') = 0$ for all $n \geq 1$ and weak convergence implies $F_n(w') \to \widetilde{F}^*(w')$. Hence, $\widetilde{F}^*$ also satisfies (EK).

Moreover, for all $q \in [0, 1)$ and $w \in F_n^{-1}(1-q)$, the analysis in Section 4.1 shows that the candidate bad equilibrium parameterized by q and w vanishes if and only if $D(q) \neq \lambda - w$. Now, take a small $\epsilon > 0$, and suppose $\widetilde{F}^*$ assigns positive probability to wages larger than $\overline{w} + \epsilon$. Then, we define:

$$\overline{F}_n(w) = \begin{cases} F_n(w) & \text{if } w < \overline{w} + \epsilon; \\ 1 & \text{otherwise.} \end{cases} \qquad \overline{F}^* = \begin{cases} \widetilde{F}^*(w) & \text{if } w < \overline{w} + \epsilon; \\ 1 & \text{otherwise.} \end{cases} \tag{36}$$

One can see such truncation preserves the weak convergence of $(\overline{F}_n)_{n=1}^\infty$ to $\overline{F}^*$, and $\mathbb{E}_{\overline{F}_n}[w] \to \mathbb{E}_{\overline{F}^*}[w]$. Furthermore, each $\overline{F}_n$ still fully implements full working because: (i) $\text{supp}(\overline{F}_n) \geq \underline{w}$ so a full-working equilibrium exists; (ii) the candidate bad equilibria under $\overline{F}_n$ are the same as under F_n except for those with a threshold wage $w \geq \overline{w} + \epsilon$, but these cases satisfy $D(q) > \lambda - w$ regardless of q and thus these equilibria still cannot exist. As a result, by Definition 1, the fact that $\overline{F}^*$ has a strictly lower expectation than $\widetilde{F}^*$ forms a contradiction to $\widetilde{F}^*$ being robustly optimal. In summary, it is without loss to assume $\text{supp}(\widetilde{F}^*)$ and $\text{supp}(F_n) \leq \overline{w} + \epsilon$. Therefore, let $q_H = 0$ and take some $w_H > \overline{w} + \epsilon$, which satisfies $w_H \in F_n^{-1}(1 - q_H)$ and $w_H \in \widetilde{F}^{*-1}(1 - q_H)$. In this case, $w_H + D(q_H) > \lambda$. Now, suppose there is $\widehat{q} \in [0, 1)$ and $\widehat{w} \in \widetilde{F}^{*-1}(1 - q_L)$ such that $\widehat{w} + D(\widehat{q}) < \lambda$. Since the graph $\{(w, q) : w \in F_n^{-1}(1 - q)\}$ converges to $\{(w, q) : w \in \widetilde{F}^{*-1}(1 - q)\}$ (with respect to the Hausdorff metric) due to weak convergence, and $D(.)$ is continuous, we can find a large n, and some (w_L, q_L) close to $(\widehat{w}, \widehat{q})$ such that $w_L \in F_n^{-1}(1 - q_L)$ and $w_L + D(q_L) < \lambda$. Therefore, we have the following two nonempty sets:

$$\begin{aligned} A_n &= \{q \in [q_H, q_L] : \exists w \in F_n^{-1}(1 - q) \text{ s.t. } w + D(q) \geq \lambda\}; \\ B_n &= \{q \in [q_H, q_L] : \exists w \in F_n^{-1}(1 - q) \text{ s.t. } w + D(q) \leq \lambda\}. \end{aligned} \tag{37}$$

The continuity of $D(.)$ and $F_n^{-1}(.)$ implies that A_n and B_n are both closed sets. Since $A_n \cup B_n = [q_H, q_L]$ by construction, $A_n \cap B_n$ is nonempty. Namely, there are $q' \leq q_L < 1$ and $w' \in F_n^{-1}(1 - q')$ such that $w' + D(q') = \lambda$. However, this contradicts F_n fully implementing full working as w' and q' parameterize a bad equilibrium. This shows $\widetilde{F}^*$ satisfies (EB) and is hence feasible in problem (13).

Before the second step, we show $F^*(.)$ is strictly increasing on $(\underline{w}, \overline{w})$. For every $\widetilde{w} \in (\underline{w}, \overline{w})$, let $\widetilde{q}$ be $\overline{q}(\widetilde{w})$. Lemma 1 shows that $D(q) \geq D(\widetilde{q})$ for all $q \geq \widetilde{q}$. Take any $\widehat{w} \in (\widetilde{w}, \overline{w})$. Hence, every $\widehat{q}$ such that $\widehat{w} + D(\widehat{q}) = \lambda$ must have $D(\widehat{q}) = \lambda - \widehat{w} < \lambda - \widetilde{w} = D(\widetilde{q})$, so $\widehat{q}$ has to be lower than $\widetilde{q}$. This implies $\overline{q}(\widehat{w}) < \overline{q}(\widetilde{w})$. Namely, $\overline{q}(.)$ is strictly decreasing and thus F^* is strictly increasing.

Also, we characterize F^* by showing it is continuous if and only if $D(.)$ is strictly increasing. When $D(.)$ is strictly increasing, for all $w \in [\underline{w}, \overline{w}]$, $\{q : w + D(q) = \lambda\}$ is a singleton that contains $D^{-1}(\lambda - w)$. Hence, $\overline{q}(w) = D^{-1}(\lambda - w)$, continuous in w. Moreover, $\overline{q}(\underline{w}) = D^{-1}(\lambda - \underline{w}) = D^{-1}(D(1)) = 1$ and $\overline{q}(\underline{w}) = D^{-1}(\lambda - \overline{w}) = D^{-1}(\min_{q \in [0,1]} D(q)) = D^{-1}(D(0)) = 0$. As a result, F^* is continuous. On the

other hand, let $F^*(.)$ be continuous. This is equivalent to $\overline{q}(.)$ is continuous, $\overline{q}(\underline{w}) = 1$, and $\overline{q}(\overline{w}) = 0$. We know $\overline{q}(.)$ is also strictly decreasing, so $\overline{q}^{-1}(.)$ is a well-defined strictly decreasing function. Therefore, $D(.)$ is strictly increasing because for all $q^2, q^1 \in [0, 1]$ with $q^2 > q^1$, we have $D(q^2) = D(\overline{q}(\overline{q}^{-1}(q^2))) = \lambda - \overline{q}^{-1}(q^2) > \lambda - \overline{q}^{-1}(q^1) = D(\overline{q}(\overline{q}^{-1}(q^1))) = D(q^1)$.

Step 2. We suppose there is a solution to problem (13), denoted as $\widetilde{F}^*$, that is different from F^* given in Theorem 1. Note that (EK) requires $\widetilde{F}^*(w) = 0$ for all $w < \underline{w}$, and that $F^*(w) = 1$ for all $w \geq \overline{w}$. As a result, the linear objective of problem (13) can obtain at $\widetilde{F}^*$ a value no less than at F^* only when:

$$\int_{\underline{w}}^{\overline{w}} \widetilde{F}(w)^* dw \geq \int_{\underline{w}}^{\overline{w}} F^*(w) dw. \tag{38}$$

Hence, $\widetilde{F}^*$ differs from F^* only when there is some open subset of $[\underline{w}, \overline{w})$ in which $\widetilde{F}^* > F^*$. We choose one $\widetilde{w}$ in this set, at which $\widetilde{F}^*$ is continuous. An implication is that $\widehat{w} := \min F^{*-1}(\widetilde{F}^*(\widetilde{w})) > \widetilde{w}$ since, otherwise, we have $\widetilde{F}^*(\widetilde{w}) \leq F^*(\widetilde{w})$. Moreover, $F^*(\widehat{w}^-) \leq \widetilde{F}^*(\widetilde{w})$, so if we take $w' \in \widetilde{F}^{*-1}(F^*(\widehat{w}^-))$, we have $w' \leq \widetilde{w} < \widehat{w}$. Let $q' = 1 - F^*(\widehat{w}^-)$, and since $F^*(.)$ is strictly increasing on $(\underline{w}, \overline{w})$ and $\widehat{w} \in (\underline{w}, \overline{w})$, we have $q' < 1$. Note that $F^*(\widehat{w}^-) = 1 - \overline{q}(\widehat{w})$, so $q' = \overline{q}(\widehat{w})$ and thus $\widehat{w} + D(q) = \lambda$. However, a contradiction to (EB) occurs as $q' < 1$ and $w' \in \widetilde{F}^{*-1}(1 - q')$ but $w' + D(q') < \widehat{w} + D(q) = \lambda$. Hence, such an $\widetilde{F}^*$ does not exist. Finally, it is easy to verify that F^* satisfies (EK) and (EB) by its construction. In this way, we have shown F^* is the unique solution to problem (13).

Step 3. We show F^* can be approximated by a sequence $(F_n)_{n=1}^\infty$ feasible in problem (6). Since $F^*(.)$ is strictly increasing on $(\underline{w}, \overline{w})$, fixing a small $\epsilon > 0$, we can construct the weakly approximating sequence so that every $F_n \in \Delta(\mathbb{R}_+)$ is continuous on $\mathbb{R}_+$ (that is, atomless), and is strictly below F^* on $(\underline{w}, \overline{w} + \epsilon)$ while coinciding with F^* outside this interval (except we fix $F_n(\underline{w}) = 0$). Such an F_n induces a full-working equilibrium since $F_n(w) = F^*(w) = 0$ for all $w < \underline{w}$, namely $\mathrm{supp}(F_n) \geq \underline{w}$. Moreover, for every candidate bad equilibrium parameterized by some $q < 1$ and $w \in F_n^{-1}(1 - q)$: (i) if $w \in (\underline{w}, \overline{w}]$, we have $q = 1 - F_n(w) > 1 - F^*(w^-) = \overline{q}(w)$, which along with Lemma 1 implies $D(q) > D(\overline{q}(w))$, so $w + D(q) > w + D(\overline{q}(w)) = \lambda$; (ii) if $w > \overline{w}$, we always have $w + D(q) > \lambda$. In either case, the bad equilibria cannot exist, so F_n fully implements full working. In addition, weak convergence here implies $\mathbb{E}_{F_n}[w] \to \mathbb{E}_{F^*}[w]$. As a result, $(F_n)_{n=1}^\infty$ approximates F^* in the sense of Definition 1.

With all three steps above, we have completed the proof. To see this, notice that steps 1 and 3 together show that problems (6) and (13) each have a solution that is feasible in the other problem, so their optimal values coincide. Next, steps 2 and 3 together show F^* achieves the optimal value in problem (13) while being feasible in problem (6), so it is a robustly optimal wage policy. Finally, suppose there is another robustly optimal wage policy $\widetilde{F}^*$, then step 1 implies it is feasible in problem (13) and thus optimal there. However, step 2 says that problem (13) has a unique solution, so we reach a contradiction. In summary, F^* is the unique robustly optimal wage policy (Theorem 1), and problems (6) and (13) coincide in their solution sets (Proposition 2).

B.4 Proof of Proposition 3

We begin by showing the first part of Proposition 3. This part is true because: (i) $\mathbb{E}_0[p_k^1] = \mathbb{E}_0[p_k^2]$ controls for the first term of (8), $T(1; \mathbb{E}_0[p_k^1]) = T(1; \mathbb{E}_0[p_k^2])$; (ii) the fact that F^{*2} contains wage dispersion implies $\mathrm{Cov}(u^2, p^2) > 0$, which along with $\mathrm{Cov}(u^2, p^2) > \mathrm{Cov}(u^1, p^1)$ further implies $\max\{0, \mathrm{Cov}(u^2, p^2)\} = \mathrm{Cov}(u^2, p^2) > \max\{0, \mathrm{Cov}(u^1, p^1)\}$. Hence, there is $\eta \in [0, 1)$ such that the career value functions have $D^1 = \eta D^2$ (in Proposition 4, the condition $v^1 = \delta v^2$ also implies this with η equal to δ, so the following proof applies there as well). Then, we have $\overline{w}^1 - \underline{w}^1 = D^1(1) - \min_q D^1(q) = \eta D^2(q) - \min_q \eta D^2(q) = \eta(\overline{w}^2 - \underline{w}^2) < \overline{w}^2 - \underline{w}^2$. Moreover, for all $\Delta w \in (0, \overline{w}^2 - \underline{w}^2)$, we have $\eta \Delta w \in (0, \overline{w}^1 - \underline{w}^1)$, and:

$$
\begin{aligned}
\overline{q}^1(\underline{w}^1 + \eta \Delta w) &= \max\{q : \underline{w}^1 + \eta \Delta w + D^1(q) = \lambda\} = \max\{q : \eta \Delta w + D^1(q) = D^1(1)\} \\
&= \max\{q : \eta \Delta w + \eta D^2(q) = \eta D^2(1)\} = \max\{q : \Delta w + D^2(q) = D^2(1)\} \\
&= \max\{q : \underline{w}^2 + \Delta w + D^2(q) = \lambda\} = \overline{q}(\underline{w}^2 + \Delta w).
\end{aligned}
\tag{39}
$$

Above, the second and fifth equalities apply $\underline{w}^i = \lambda - D^i(1)$ for both $i = 1, 2$. (39) implies, if we construct for both $i = 1, 2$, a random variable x^i that has the cumulative distribution $G^i(w) := F^i(\underline{w}^i + w)$, we have $x^1 = \eta x^2$. By construction, $\mathrm{Var}(F^{*1}) = \mathrm{Var}(G^1) = \mathrm{Var}(x^1) = \mathrm{Var}(\eta x^2) = \eta^2 \mathrm{Var}(x^2) = \eta^2 \mathrm{Var}(G^2) = \eta^2 \mathrm{Var}(F^{*2}) < \mathrm{Var}(F^{*2})$. Thus, according to Definition 2, F^{*2} is more dispersed than F^{*1}.

Next, we show the second part of Proposition 3. We assume $p = p^1 = p^2$, $\mathbb{E}_0[u_k^1] = \mathbb{E}_0[u_k^2]$, and that there is $k^* \in \Theta$ such that $u_k^2 > [<]u_k^1$ for all $p_k > [<]p_{k^*}$. Then, the covariances differ by the following:

$$
\begin{aligned}
\mathrm{Cov}(u^2, p) - \mathrm{Cov}(u^1, p) &= \mathbb{E}_0[u_k^2 p_k] - \mathbb{E}_0[u_k^1 p_k] = \sum_{k \in \Theta} \mu_k^0 p_k (u_k^2 - u_k^1) \\
&= \sum_{k \in \Theta: p_k > p_{k^*}} \mu_k^0 p_k (u_k^2 - u_k^1) + \sum_{k \in \Theta: p_k < p_{k^*}} \mu_k^0 p_k (u_k^2 - u_k^1) + \sum_{k \in \Theta: p_k = p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1) \\
&> \sum_{k \in \Theta: p_k > p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1) + \sum_{k \in \Theta: p_k < p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1) + \sum_{k \in \Theta: p_k = p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1) \\
&= \sum_{k \in \Theta} \mu_k^0 p_{k^*} (u_k^2 - u_k^1) = \left(\mathbb{E}_0[u_k^2] - \mathbb{E}_0[u_k^1] \right) p_{k^*} = 0.
\end{aligned}
\tag{40}
$$

Above, we have a strict inequality because we have assumed $\mathrm{Cov}(u^2, p) > 0$, which means p_k cannot be type-independent, so there is some $k \neq k^*$ such that $p_k \neq p_{k^*}$, and we have either $\sum_{k \in \Theta: p_k > p_{k^*}} \mu_k^0 p_k (u_k^2 - u_k^1) > \sum_{k \in \Theta: p_k > p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1)$ or $\sum_{k \in \Theta: p_k < p_{k^*}} \mu_k^0 p_k (u_k^2 - u_k^1) > \sum_{k \in \Theta: p_k < p_{k^*}} \mu_k^0 p_{k^*} (u_k^2 - u_k^1)$. At this point, we complete the proof.

Appendix C

C.1 Proof of Proposition 5

We show a generalized version of Proposition 5 for the case with multiple types $\Theta = \{1, 2, ..., K\}$ and an arbitrary implementable working probability profile $Q = (Q_k)_{k \in \Theta}$. To be consistent with the notations in Appendix A.2, let $\underline{w}_k := \lambda_k - D(Q)$, $\underline{w}_0 := \infty$, and $\underline{w}_{K+1} := 0$. Also, we define: for all $R \in \Delta_T(\mathbb{R}_+)$,

$$V(R) = \sum_{k=1}^{K+1} \left(\sum_{j=1}^{k-1} \mu_j^0 p_{0j} + \sum_{j=k}^{K} \mu_j^0 (p_{0j} + p_j) \right) \int_{\underline{w}_k}^{\underline{w}_{k-1}} R(s) ds. \tag{41}$$

Here, $\sum_{j=1}^{0} = 0$ and $\sum_{j=K+1}^{K} = 0$. We then claim an lower bound of the minimal wage guarantee $W_Q^\dagger$:

Proposition 11. *For an implementable Q, there exists a constant C_Q such that:*

$$W_Q^\dagger \geq C_Q + \min_R V(R)$$

$$\begin{aligned}
s.t. \quad &\text{[EK]} \textit{ For all } k \in \Theta, R(\underline{w}_k) \geq Q_k \text{ and } R(\underline{w}_k^+) \leq Q_k; \\
&\text{[EB-r]} \textit{ For all } w > \underline{w}_K, w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) \geq \lambda_K; \\
&\text{[EB-l]} \textit{ For all } w < \underline{w}_K, w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) \leq \lambda_K; \\
&\text{[FE]} \textit{ } R \textit{ is nonincreasing and left-continuous, and } R \in [0, 1].
\end{aligned} \tag{42}$$

Moreover, every robustly optimal tail wage policy $R_Q^\dagger$ is feasible in this problem and satisfies $W_Q^\dagger = C_Q + V(R_Q^\dagger)$. In turn, a solution to this problem is robustly optimal if it can be approximated by a sequence of wage policies in the sense of Definition 3.

Henceforth, we refer to the four constraints in Problem (42) as [EK], [EB-r], [EB-l], and [FE]. Problem (42) provides a lower bound for the minimal wage guarantee. While it is generally not equivalent to the original problem (26), Proposition 11 establishes the criterion for building such an equivalence. That is, we first obtain a solution to the relaxed problem and next verify it is also "feasible" in the original problem. At the end of the proof, we use this result to show Proposition 5.

To show Proposition 11, we start by proving a useful lemma:

Lemma 3. *For every robustly optimal wage policy $R_Q^\dagger$, there exists a sequence $(R_n)_{n=1}^\infty$ converging to $R_Q^\dagger$ in the sense of Definition 3 such that every R_n fully implements Q in the unique pure-strategy equilibrium where each type works with probability one at his threshold wage $\underline{w}_k$.*

To show Lemma 3 is true, we take any robustly optimal $R_Q^\dagger$ and its approximating sequence $(R_n)_{n=1}^\infty$. Our goal is to construct another sequence $(\widetilde{R}_n)_{n=1}^\infty$ that also approximates $R_Q^\dagger$, and each $\widetilde{R}_n$ fully implements Q in a unique pure-strategy equilibrium. In fact, whenever some R fully implements Q, it must induce a unique equilibrium because on the equilibrium path, each type k's threshold wage is fixed as $\underline{w}_k$ and the probability of him working at $\underline{w}_k$ is pinned down as $1 - \frac{R(\underline{w}_k) - Q_k}{R(\underline{w}_k) - R(\underline{w}_k^+)}$. Notice that some type k

may randomize only because $R(\underline{w}_k) > Q_k \geq R(\underline{w}_k^+)$. Therefore, we simply need to make the potentially mixed equilibrium induced by each R_n pure, by perturbing the values of R_n around $\underline{w}_k$ for some k such that $R_n(\underline{w}_k) > Q_k$. To do so, we consider a sequence of sufficiently small positive numbers $(\epsilon_n)_{n=1}^\infty$ that converges to 0. Then, we construct each $\widetilde{R}_n$ in the following way:

$$\widetilde{R}_n(w) = \begin{cases} Q_k - \frac{R_n(\underline{w}_k - \epsilon_n) - Q_k}{\epsilon_n}(w - \underline{w}_k) & \text{if } w \in [\underline{w}_k - \epsilon_n, \underline{w}_k], \text{ for all } k; \\ R_n(w) & \text{otherwise.} \end{cases} \tag{43}$$

In other words, instead of having a jump at $\underline{w}_k$, we linearize the function on the small interval $[\underline{w}_k - \epsilon_n, \underline{w}_k]$ to maintain $\widetilde{R}_n(\underline{w}_k) = Q_k$. As a result, $\widetilde{R}_n$ induces Q in one pure-strategy equilibrium where each type works with probability one at his threshold wage $\underline{w}_k$. In addition, whenever $R_n(w)$ converges to $R_Q^\dagger(w)$ pointwise, which covers all continuous points w of $R_Q^\dagger$, $\widetilde{R}_n(w)$ converges pointwise to the same value as $R_n(w)$ does. Thus, $\widetilde{R}_n$ also weakly converges to $R_Q^\dagger$.

Now, we show each $\widetilde{R}_n$ fully implements Q. Consider any undesirable candidate equilibrium where each type k's threshold wage is w_k'. Let D^0 be the career value generated by the worker's strategy in this candidate equilibrium. If $\widetilde{R}_n(w_k') = R_n(w_k')$ for all k, this candidate equilibrium cannot exist because it is broken under R_n. Thus, we consider a candidate equilibrium where some type k's threshold wage lies in some open linearization interval $(\underline{w}_k - \epsilon_n, \underline{w}_k)$. We collect all such types in $\widetilde{\Theta}$. In fact, by choosing a sufficiently small ϵ_n, we can make sure that, in every such equilibrium, each type's threshold wage is a continuous point of $\widetilde{R}_n$. This is because we have finite types, so these threshold wages can potentially come from finitely many intervals of the form, $(w' - \epsilon_n, w')$ where $w' = \underline{w}_{k'} + \lambda_{k'} - \lambda_{k''}$ for some k', k''. It is possible to find an ϵ_n such that $\widetilde{R}_n$ is continuous on all these intervals. Moreover, suppose two types' threshold wages, say $w_{k'}'$ and $w_{k''}'$, are in two linearization intervals $(w^1 - \epsilon_n, w^1)$ and $(w^2 - \epsilon_n, w^2)$, respectively. By choosing a small ϵ_n, we can make sure that this happens only if $w^1 - w^2 = \lambda_{k'} - \lambda_{k''}$. Hence, we take any threshold wage w_k' that lies in some linearization interval $(\underline{w}_{k'} - \epsilon_n, \underline{w}_{k'})$, denote by $e := w_k' - \underline{w}_{k'}$ its upward deviation from $\underline{w}_{k'}$, and let $b_{k'} := \frac{\widetilde{R}_n(w_k') - R_n(\underline{w}_{k'}^+)}{R_n(\underline{w}_{k'}) - R_n(\underline{w}_{k'}^+)}$. Now, consider an auxiliary equilibrium under R_n where each type k's threshold wage is $w_k' + e$, and if $k \in \widetilde{\Theta}$, type k works with probability b_k at threshold $w_k' + e$, and if $k \notin \widetilde{\Theta}$, type k works with probability 1 at threshold $w_k' + e$. Let D^1 be the career value generated by the worker's strategy in this auxiliary equilibrium (under R_n). From all the arguments above, and due to the left-continuity of $\widetilde{R}_n$ and the continuity of $D(.)$, we have that $D^0 - D^1$ and e both approximate zero as ϵ_n goes to zero. As a result, the fact that the auxiliary equilibrium is broken under R_n implies $w_k' + e + D^1 \neq \lambda_k$ for all k, which further implies $w_k' + D^0 \neq \lambda_k$ for all k whenever ϵ_n is small enough. Consequently, the candidate equilibrium cannot exist under $\widetilde{R}_n$. Finally, the last case we need to consider is one where some type k's threshold wage equals exactly some $\underline{w}_{k'}$. Likewise, by choosing a small ϵ_n, we make sure that for each k, either $w_k' = \underline{w}_{k'}$ with some type k', or $\widetilde{R}_n(w_k') = R_n(w_k')$. We again collect all the former category of types in $\widetilde{\Theta}$. For each $k \in \widetilde{\Theta}$, let $b_k := \sigma_k(w_k') \frac{Q_k - R_n(\underline{w}_{k'}^+)}{R_n(\underline{w}_{k'}) - R_n(\underline{w}_{k'}^+)}$. We again construct a similar auxiliary equilibrium under R_n where each type k's threshold wage is w_k', and if $k \in \widetilde{\Theta}$, type k works with probability b_k at threshold w_k', and if

$k \notin \widetilde{\Theta}$, type k works with probability 1 at threshold w'_k. Note that the career value generated by the worker's strategy in this auxiliary equilibrium (under R_n) is exactly D^0. Thus, the fact that the auxiliary equilibrium is broken under R_n implies $w'_k + D^0 \neq \lambda_k$ for all k. This condition further breaks the candidate equilibrium under $\widetilde{R}_n$.

To conclude our proof of Lemma 3, we notice that the expected wage payment under $\widetilde{R}_n$ only differs from that under R_n at those wages in the linearization intervals. However, once ϵ_n approximates zero as n goes to infinity, this difference vanishes and the wage guarantee of $\widetilde{R}_n$ converges to $W_Q^\dagger$ as well.

One caveat is that Lemma 3 does not guarantee that the robustly optimal wage policy has $R_Q^\dagger(\underline{w}_k) = Q_k$ for all k, because $R_Q^\dagger$ is approximated by a policy sequence in the sense of weak convergence, so at its discontinuous points, the sequence may not converge pointwise. Instead, we have $R_Q^\dagger(\underline{w}_k) \geq Q_k$ for all k, because every wage policy is nonincreasing and left-continuous.

Hereafter, we return to the proof of Proposition 11. We first derive the form of the objective function $V(.)$ in (42). To do so, we fix a wage policy R that fully implements Q. Lemma 3 suggests that it is without loss to consider a policy R that fully implements Q in the unique pure-strategy equilibrium where each type works with probability one at his threshold wage $\underline{w}_k$. On the equilibrium path, when those wages in the interval $[\underline{w}_k, \underline{w}_{k-1})$ are realized, the types no lower than k will work with probability one. Hence, the total probability of each such wage being paid is $\sum_{j=1}^{k-1} \mu_j^0 p_{0j} + \sum_{j=k}^K \mu_j^0 (p_{0j} + p_j)$, which we denote as r_k. Recall that $F(w^-) = 1 - R(w)$. Therefore, let $Q_{K+1} = R(0) = 1$, and the expected wage is:

$$
\begin{aligned}
\sum_{k=1}^{K+1} r_k \int_{\underline{w}_k}^{\underline{w}_{k-1}} w\, d(1 - R(w)) &= \sum_{k=1}^{K+1} r_k \left(w(1 - R(w)) \Big|_{\underline{w}_k}^{\underline{w}_{k-1}} + \int_{\underline{w}_k}^{\underline{w}_{k-1}} R(w)\, dw \right) \\
&= \sum_{k=1}^{K+1} r_k \int_{\underline{w}_k}^{\underline{w}_{k-1}} R(w)\, dw + \sum_{k=1}^{K+1} r_k [\underline{w}_{k-1}(1 - R(\underline{w}_{k-1})) - \underline{w}_k(1 - R(\underline{w}_k))] \\
&= \underbrace{\sum_{k=1}^{K+1} r_k \int_{\underline{w}_k}^{\underline{w}_{k-1}} R(w)\, dw}_{V(R)} + \underbrace{\sum_{k=1}^{K+1} r_k [\underline{w}_{k-1}(1 - Q_{k-1}) - \underline{w}_k(1 - Q_k)]}_{C_Q}.
\end{aligned}
\tag{44}
$$

The first equality above results from integration by parts. The last equality above applies $R(\underline{w}_k) = Q_k$ for all k, because each type k works with probability one at his threshold wage $\underline{w}_k$. Hence, the expected wage payment equals $V(R) + C_Q$, where C_Q's value does not depend on R. Finally, since the objective function is linear and thus continuous in R, it correctly measures the wage cost also at limits in the sense of Definition 3. In particular, $W_Q^\dagger = V(R_Q^\dagger) + C_Q$.

Now, to complete the proof of Proposition 11, it suffices to show the following claim: Every robustly optimal $R_Q^\dagger$ must be feasible in problem (42). This is sufficient because, along with $W_Q^\dagger = V(R_Q^\dagger) + C_Q$, this claim implies that the optimal value of (42) offers a lower bound for $W_Q^\dagger$. Moreover, the claim makes sure that, for any solution to problem (42), say $\overline{R}$, that can be approximated by a sequence $(R_n)_{n=1}^\infty$ in the sense of Definition 3, this solution $\overline{R}$ must be robustly optimal. This is because, even though we have

$W_Q^\dagger \geq V(\overline{R}) + C_Q$, the weak inequality must be equality since, otherwise, the employer could achieve a lower wage guarantee with $(R_n)_{n=1}^\infty$. Thus, $W_Q^\dagger = V(\overline{R}) + C_Q$ and $\overline{R}$ is robustly optimal.

To show the claim above, we take any robustly optimal $R_Q^\dagger$ and the sequence $(R_n)_{n=1}^\infty$ that approximates $R_Q^\dagger$ in the sense of Definition 3. Lemma 3 implies that it is without loss to assume $R_n(\underline{w}_k) = Q_k$ for all n, k. We know $R_Q^\dagger$ satisfies [FE]. Also, we have shown above that $R_Q^\dagger(\underline{w}_k) \geq Q_k$ for all k. Moreover, suppose $R_Q^\dagger(\underline{w}_k^+) > Q_k$. Then, there are a small $\epsilon > 0$ and a large n' such that for all $n > n'$, $R_n(\underline{w}_k + \epsilon) > Q_k$, a contradiction to $R_n(\underline{w}_k) = Q_k$ and $R_n(\underline{w}_k) \geq R_n(\underline{w}_k + \epsilon)$. Thus, $R_Q^\dagger$ satisfies [EK].

We show $R_Q^\dagger$ satisfies [EB-r] by contradiction. Suppose there is a $w' > \underline{w}_K$ such that [EB-r] is violated: $w' + D((R_Q^\dagger(w' + \lambda_k - \lambda_K))_{k=1}^K) < \lambda_K$. Then, by the continuity of $D(.)$ and the left-continuity of $R_Q^\dagger$, there exists some w^2 slightly lower than w' (or even w' itself) such that $w^2 + D((R_Q^\dagger(w^2 + \lambda_k - \lambda_K))_{k=1}^K) < \lambda_K$ and all the threshold wages $w^2 + \lambda_k - \lambda_K$ are continuous points of $R_Q^\dagger$. This implies that R_n converges to $R_Q^\dagger$ pointwise at these threshold wages. Thus, the continuity of $D(.)$ further admits a large n such that $w^2 + D((R_n(w^2 + \lambda_k - \lambda_K))_{k=1}^K) < \lambda_K$. By the left-continuity of $R_Q^\dagger$ and R_n, we can always choose w^2 such that the threshold wages $w^2 + \lambda_k - \lambda_K$ also are continuous points of R_n. On the other hand, for a large wage $w^1 > \max_{k,Q} \lambda_k - D(Q)$, we always have EB-r: $w^1 + D((R_n(w^1 + \lambda_k - \lambda_K))_{k=1}^K) > \lambda_K$. We also choose w^1 such that the threshold wages $w^2 + \lambda_k - \lambda_K$ are continuous points of R_n. Next, we define the following correspondence, for all $w \in [w^2, w^1]$:

$$\phi(w) = \{D(Q) : Q_k = b_k R_n(w + \lambda_k - \lambda_K) + (1 - b_k)R_n(w + \lambda_k - \lambda_K^+); b_k \in [0, 1]\}, \qquad (45)$$

Which, fixing w while varying all the b_k's, collects all the possible career values generated by the worker strategy that each type k works with probability b_k at his threshold wage $w - \lambda_k + \lambda_K$. One can check that the continuity of $D(.)$ implies continuity for $\phi(.)$. By construction, we have $\lambda_K - w^2 < \phi(w^2)$ and $\lambda_K - w^1 > \phi(w^1)$. Therefore, there must be some $w' \in (w^2, w^1)$ such that $\lambda_K - w' \in \phi(w')$, which means R_n induces an equilibrium where the highest type's threshold wage is $w' > w^2 > \underline{w}_K$. This contradicts the assumption that R_n fully implements Q. As a result, $R_Q^\dagger$ satisfies [EB-r] for all $w > \underline{w}_K$.

A similar argument shows that $R_Q^\dagger$ also satisfies [EB-l]. Notice that, for a small wage $w^1 < \min_{k,Q} \lambda_k - D(Q)$ (we assumed such $w^1 > 0$ exists in Section 2), w^1-[EB-l] always holds. Hence, by supposing [EB-l] is violated at some point and repeating the previous construction of an intermediate equilibrium, we obtain a contradiction. In summary, we have shown $R_Q^\dagger$ is feasible in (42), further giving Proposition 11.

In fact, our arguments above also show that the closure (in terms of weak convergence) of $\mathcal{F}^{FI}(Q)$ is feasible in problem (42). To see this, pick any $R^\dagger \in \mathrm{cl}(\mathcal{F}^{FI}(Q))$ and the sequence $(R_n)_{n=1}^\infty \subset \mathcal{F}^{FI}(Q)$ that weakly converges to $R^\dagger$. We also use (43) to adjust each R_n while making it remain in $\mathcal{F}^{FI}(Q)$ and weakly converge to $R^\dagger$. Then, everything above goes through.

The last step is to prove Proposition 5, which considers binary types $\Theta = \{H, L\}$ and full working $Q = (1, 1)$. We start by showing problem (42) is equivalent to problem (19) in this special case. First, the [EK] constraint for each k degenerates to $R(\underline{w}_k) = 1$ because $Q_k = 1$, and [FE] guarantees $R(\underline{w}_k^+) \leq$

$R(\underline{w}_k) = 1$. Second, the objective function $V(R)$ is thus equal to $\alpha + \beta \int_{\widetilde{w}}^{\infty} R(s)ds$ with some positive α and β since $R(w) = 1$ for all $w \le \widetilde{w} = \underline{w}_L$. Third, [EB-r] is also satisfied at $\underline{w}_H$. In fact, we have $\underline{w}_H + D(R(\underline{w}_H + \lambda_L - \lambda_H), R(\underline{w}_H)) = \underline{w}_H + D(1,1) = \lambda_H$. Finally, we show [EB-l] is implied by [EK] and [FE], so we can omit it from the problem. In particular, [EK] and [FE] require $R(w) = 1$, for all $w < \underline{w}_H$, which means $w + D(R(\underline{w}_H + \lambda_L - \lambda_H), R(\underline{w}_H)) = w + D(1,1) < \lambda_H$.

Recall that Proposition 5 also claims problem (19) produces the same set of solutions as the original problem (26) does. In the proof of Proposition 6 below, we show problem (19) yields a unique solution that is also robustly optimal. That proof does not depend on the argument in this paragraph. Thus, the robustly optimal wage policy must also be unique because, otherwise, there would be a different such wage policy, which, by Proposition 11, is feasible in problem (19) while producing the same minimal wage guarantee. Nonetheless, this means problem (19) would not have a unique solution, forming a contradiction. Hence, the two problems induce the same set of solution.

C.2 Proof of Lemma 2

Notice that R^G must have a bounded support because, for those wages greater than $\max_{k,Q} \lambda_k - D(Q)$, the associated (EB') constraints cannot be binding, so $\text{supp}(R^G)$ cannot contain these wages. Let the finite upper bound be $w^1 = \max \text{supp}(R^G)$. Thus, $R^G(w') = 0$ for all $w' > w^1$. Since w-EB' holds if $R^G(w) < 1$, for all $w' > w^1$, we have $w' + D(R^G(w' + \lambda_0), w') = w' + D(0,0) \ge \lambda_H$, implying $w' \ge \lambda_H - D(0,0) = \widetilde{w}$. This can be possible only if $w^1 \ge \widetilde{w}$. On the other hand, w^1-EB' binds because $w^1 \in \text{supp}(R^G)$, which means $\lambda_H = w^1 + D(R^G(w^1 + \lambda_0), R^G(w^1)) = w^1 + D(0, R^G(w^1)) \ge w^1 + D(0,0) \ge \widetilde{w} + D(0,0) = \lambda_H$ where the first inequality applies the assumption that D is strictly increasing in q_H, and the second inequality comes from $w^1 \ge \widetilde{w}$. Hence, the inequalities must be equalities, so $w^1 = \widetilde{w}$ and $R^G(w^1) = 0$.

Next, we show R^G must be fully supported on an interval $[w_l, \widetilde{w}]$. To do this, suppose this is not true. That is, there exists some $w' \in \text{supp}(R^G)$ such that $w' > w_l$ and, for a small $\epsilon > 0$, $w' - \epsilon \notin \text{supp}(R^G)$. Note that $w' > w_l$ implies $R^G(w') = R^G(w' - \epsilon) < 1$ (otherwise, $R^G(w) = 1$ for all $w \le w'$, and $w_l \notin \text{supp}(R^G)$). Hence, w'-EB' binds, while $(w' - \epsilon)$-EB' holds but does not bind. Formally:

$$w' + D(R^G(w' + \lambda_0), R^G(w')) = \lambda_H < w' - \epsilon + D(R^G(w' - \epsilon + \lambda_0), R^G(w' - \epsilon))$$
$$< w' + D(R^G(w' + \lambda_0), R^G(w' - \epsilon)) = w' + D(R^G(w' + \lambda_0), R^G(w')),$$

(46)

Which forms a contradiction. Here, the second inequality uses $R^G(w' + \lambda_0) \le R^G(w' - \epsilon + \lambda_0)$ and the assumption that D is strictly decreasing in q_L. Thus, R^G is fully supported on an interval. Note that the same argument shows $R^G(w_l) = 1$ because, otherwise, $R^G(w_l) = R^G(w_l - \epsilon) < 1$ and $w_l \in \text{supp}(R^G)$ while $w_l - \epsilon \notin \text{supp}(R^G)$.

From above, we obtain $R^G(w) = 0$ if $w \ge \widetilde{w}$, and $R^G(w) = 1$ if $w \le$ some w_l, and R^G is strictly decreasing on $[w_l, \widetilde{w}]$. Furthermore, we show R^G is also continuous on $[w_l, \widetilde{w}]$. Suppose this is not true, and we collect all the discontinuous points in W^m. For a small $\epsilon > 0$, there exists $w' \in W^m$ such that

$w' > (\sup W^m) - \epsilon$. In this case, R^G is continuous at $w' + \lambda_0$. However, this cannot happen because w'-EB' and $(w'+0)$-EB' cannot both bind if the career value in w'-EB' jumps due to the discontinuity at w' and the assumption that D is strictly increasing in q_H. Consequently, R^G is continuous. Finally, we claim R^G is unique. Again, suppose this is not true, and for two different greedy wage policies $\widetilde{R}^G$ and $\widehat{R}^G$, we collect all the points at which they differ in W^d. We have shown that $W^d \subset (w_l, \widetilde{w})$. For a small $\epsilon > 0$, there exists $w' \in W^d$ such that $w' > (\sum W^d) - \epsilon$. In this case, $q' := \widetilde{R}^G(w' + \lambda_0) = \widehat{R}^G(w' + \lambda_0)$ and w'-EB' binds for both policies. However, this means $D(q, \widetilde{R}^G(w')) = D(q, \widehat{R}^G(w'))$ while $\widetilde{R}^G(w') \neq \widehat{R}^G(w')$. The assumption that D is strictly increasing in q_H forbids such a situation. In summary, R^G is unique. The final thing to show is $w_l < \widetilde{w}$. One can see this by using the binding w_l-EB', which gives $\lambda_H = w_l + D(R^G(w_l + \lambda_0), R^G(w_l)) > w_l + D(1,1) = w_l + \lambda_L - \widetilde{w}$. The inequality here applies $R(w_l) = 1$ and the fact that R^G is strictly decreasing on an interval above w_l, so $R(w_l + \lambda_0) < R(w_l)$. This simply indicates that $w_l < \widetilde{w} - (\lambda_L - \lambda_H) < \widetilde{w}$.

C.3   Proof of Proposition 6

By the definition of the greedy wage policy R^G and Lemma 2, we observe that Proposition 6 states that the robustly optimal wage policy $F^\dagger$ is associated with the tail wage policy $R^\dagger$ given by:

$$R^\dagger(w) = \begin{cases} 1 & \text{if } w \leq \widetilde{w}; \\ R^G(w) & \text{otherwise.} \end{cases} \tag{47}$$

We first show that $R^\dagger$ is the unique solution to problem (42), which, in this case, coincides with problem (19) where (EK') and (EB') correspond to [EK] and [EB-r], respectively. Since [EB-l] is not important in the binary-type case, we also use [EB] as a shorthand for [EB-r]. Then, we apply Proposition 11 to verify that this solution is robustly optimal. In the proof of Proposition 5, we already showed this also implies the uniqueness of robustly optimal wage policy.

First, denote by $V(R) = \int_{\widetilde{w}}^\infty R(s)ds$ the objective function in problem (19), which differs from the objective in problem (42) only by a constant. Suppose there is another $\widetilde{R}$ feasible in problem (42) such that $V(\widetilde{R}) \leq V(R^\dagger)$ and $\widetilde{R} \neq R^\dagger$. The [EK] constraint in problem (42) then implies $\widetilde{R}(w) = R^\dagger(w) = 1$ for all $w \leq \widetilde{w}$, so their difference must lie in some wages greater than $\widetilde{w}$. Note that this can happen only if $\widetilde{w} > \widetilde{w}$ since, otherwise, $R^\dagger(w) = R^G(w) = 0$ for all $w > \widetilde{w}$ and one could not find a different policy that yields weakly lower expected wage. By assumption, there must be a wage w such that $\widetilde{R}(w) < R^\dagger(w)$, and we collect all such wages in W^d. We must have $W^d \subset (\widetilde{w}, \widetilde{w})$ since $R^\dagger(w) = R^G(w) = 0$ at a higher w. Therefore, we take a small $\epsilon > 0$ and find the $w' \in W^d$ such that $w' > (\sup W^d) - \epsilon$. This implies $\widetilde{R}(w') < R^\dagger(w')$ while $\widetilde{R}(w' + \lambda_0) \geq R^\dagger(w' + \lambda_0)$. However, w'-EB' binds at $R^\dagger$ by the definition of R^G, which gives $\lambda_H = w' + D(R^\dagger(w' + \lambda_0), R^\dagger(w')) > w' + D(\widetilde{R}(w' + \lambda_0), \widetilde{R}(w'))$. Hence, $\widetilde{R}$ violates w'-EB' and thus cannot be feasible in problem (19). As a result, $R^\dagger$ is the unique solution to problem (19).

To apply Proposition 11, we construct a sequence $(R_n)_{n=1}^\infty$ that approximates $R^\dagger$ in the sense of

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Definition 3 (in this case, Definition 1). We take a sequence of small numbers $(\epsilon_n)_{n=1}^{\infty}$ that converges to zero. Then, we construct each R_n by setting $R_n(w) = R^{\dagger}(w - \epsilon_n)$, which amounts to shifting $R^{\dagger}$ to the right by ϵ_n. Since R_n converges to $R^{\dagger}$ pointwise, it also weakly converges to $R^{\dagger}$. It suffices to show R_n fully implements full working. Since $R_n(\underline{w}_k) = 1$ for both k, full working is induced in one equilibrium. For all $w \geq \underline{w}_H + \epsilon_n$, the fact that $R^{\dagger}$ satisfies [EB] gives $w - \epsilon_n + D(R^{\dagger}(w + \lambda_0 - \epsilon_n), R^{\dagger}(w - \epsilon_n)) \geq \lambda_H$. Hence, under R_n, we have $w + D(R_n(w + \lambda_0), R_n(w)) = w + D(R^{\dagger}(w + \lambda_0 - \epsilon_n), R^{\dagger}(w - \epsilon_n)) = \lambda_H + \epsilon_n > \lambda_H$. For all $w \in (\underline{w}_H, \underline{w}_H + \epsilon_n)$, we have $w + D(R_n(w + \lambda_0), R_n(w)) = w + D(1, 1) > \underline{w}_H + D(1, 1) = \lambda_H$. In summary, every undesirable candidate equilibrium where types $k = H, L$ work with probability one at their respective threshold wages cannot exist. Next, we consider the mixed-strategy candidate equilibria. We start with those where the high type randomizes at his threshold wage $\underline{w}_H + \epsilon_n$, where R_n has a mass point. In this case, the assumption that D is strictly increasing in q_H guarantees that the career value generated by any worker strategy is weakly higher than that in the candidate equilibrium with a high-type threshold wage slightly higher than $\underline{w}_H + \epsilon_n$, so the fact that the latter equilibrium cannot exist also breaks the former. The remaining case includes those candidate equilibria where the low type randomizes at threshold $\underline{w}_L + \epsilon_n$. Likewise, the assumption that D is strictly decreasing in q_L guarantees that the career value generated by any worker strategy here is weakly higher than that in the candidate equilibrium with a low-type threshold wage slightly lower than $\underline{w}_L + \epsilon_n$. Thus, such an equilibrium cannot exist. Consequently, R_n fully implements full working, and Proposition 11 hence suggests that $R^{\dagger}$ is robustly optimal.

C.4   Proof of Proposition 7 and Proposition 8

We begin by showing one side of Proposition 8: For $Q_H \geq Q_L$, if $Q_L \leq R^G(\underline{w}_L)$ and $\underline{w}_L \in \text{supp}(R^G)$, then Q is not implementable. By Lemma 2, $\underline{w}_L \in \text{supp}(R^G) = [w_l, \widetilde{w}]$ implies $[\underline{w}_L, \widetilde{w}]$ is nonempty. Suppose Q is fully implemented by some R. The proof of Lemma 3 shows it is without loss to choose R such that $R(\underline{w}_L) = Q_L \leq R^G(\underline{w}_L)$. Therefore, if we collect all wages $w \in [\underline{w}_L, \widetilde{w}]$ such that $R(w) \leq R^G(w)$ in W^l, W^l is not empty. We take a small $\epsilon > 0$ and find some $w' \in W^l$ such that $w' > (\sup W^l) - \epsilon$. This means $R(w' + \lambda_0) \geq R^G(w' + \lambda_0)$ because either $w' + \lambda_0 \leq \widetilde{w}$ but $w' + \lambda_0 \notin W^l$, or $w' + \lambda_0 > \widetilde{w}$ so $R^G(w' + \lambda_0) = 0$. By the assumption that D is strictly increasing in q_H and strictly decreasing in q_L, and by the construction of R^G, we have $w' + D(R(w' + \lambda_0), R(w')) \leq w' + D(R^G(w' + \lambda_0), R^G(w')) = \lambda_H$. In addition, the proof of Proposition 11 implies that such $R \in \mathcal{F}^{FI}(Q)$ must be feasible in problem (42), so w'-[EB] gives: $w' + D(R(w' + \lambda_0), R(w')) \geq \lambda_H$. Combining both inequalities above yields $w' + D(R(w' + \lambda_0), R(w')) = \lambda_H$, which means there is an equilibrium where the high type's threshold wage is $w' \geq \underline{w}_L > \underline{w}_H$. This forms a contradiction and thus Q is not implementable.

Next, we show Proposition 7 by first demonstrating that the stated policy, $R_Q^{\dagger}$, given by (28) is the unique solution to problem (42) and then applying Proposition 11 to verify that it is robustly optimal. To repeat the argument at the end of Proposition 5's proof, we want to show $R_Q^{\dagger}$ is the unique robustly

optimal wage policy. Our analysis above implies that we must have either $\underline{w}_L > \tilde{w}$ or $Q_L > R^G(\underline{w}_L)$ for an implementable Q. In fact, $\underline{w}_L > \tilde{w}$ implies $Q_L \geq R^G(\underline{w}_L)$, so we always have the latter.

To start with, we show $R_Q^\dagger$ is feasible in problem (42). First, the construction of R^G and the definition of $\underline{w}_k$ give $\underline{w}_H + D(R^G(\underline{w}_L), R^G(\underline{w}_H)) = \lambda_H = \underline{w}_H + D(Q_L, Q_H)$. Hence, $Q_L \geq R^G(\underline{w}_L)$ and the assumption that D is strictly increasing in q_H and strictly decreasing in q_L together imply $Q_H \geq R^G(\underline{w}_H)$. (28) thus gives $R_Q^\dagger(\underline{w}_k) = Q_k$ for both k, which satisfies [EK]. Second, for all $w > \underline{w}_H$, $R_Q^\dagger$ satisfies [EB-r] because $w + \lambda_0 > \underline{w}_L$, meaning $R_Q^\dagger(w + \lambda_0) = R^G(w + \lambda_0)$. The maximum expression of (28) further gives $R_Q^\dagger(w) \geq R^G(w)$. Thus, $w + D(R_Q^\dagger(w + \lambda_0), R_Q^\dagger(w)) \geq w + D(R^G(w + \lambda_0), R^G(w)) = \lambda_H$, satisfying w-[EB-r]. Lastly, for all $w < \underline{w}_L$, $R_Q^\dagger$ satisfies [EB-l] since $w + \lambda_0 < \underline{w}_L$, implying $R_Q^\dagger(w + \lambda_0) \geq R_Q^\dagger(\underline{w}_L) = Q_L$. Hence, $R_Q^\dagger(w) = Q_H$ and the assumption that D is strictly decreasing in q_L together yield $w + D(R_Q^\dagger(w + \lambda_0), R_Q^\dagger(w)) \leq w + D(Q_L, Q_H) < \underline{w}_H + D(Q_L, Q_H) = \lambda_H$, satisfying w-[EB-l]. Of course, [FE] holds for $R_Q^\dagger$ (as R_Q^G and R_Q^P satisfy [FE]).

Knowing $R_Q^\dagger$ is feasible, we now show $R_Q^\dagger$ is the unique solution to problem (42). Suppose there is a different feasible $\widetilde{R}$ that produces weakly lower expected wage than $R_Q^\dagger$ does. In problem (42), the fact that the objective function is linear and the coefficients r_k are strictly positive implies $W^s := \{w \geq 0 : \widetilde{R}(w) < R_Q^\dagger(w)\}$ is nonempty. In addition, [EK] requires $\widetilde{R} \geq R_Q^P$, and hence the fact that $R_Q^\dagger(\underline{w}_k) = Q_k$ for both k means whenever $w \in W^s$, we have $R_Q^\dagger(w) = R^G(w)$. We take a small $\epsilon > 0$ and find $w' \in W^s$ such that $w' > (\sup W^s) - \epsilon$, so that $\widetilde{R}(w' + \lambda_0) \geq R_Q^\dagger(w' + \lambda_0) \geq R^G(w' + \lambda_0)$. Then, $\widetilde{R}(w') < R_Q^\dagger(w') = R^G(w')$ and the assumption that D is strictly increasing in q_H and strictly decreasing in q_L together imply $w' + D(\widetilde{R}(w' + \lambda_0), \widetilde{R}(w')) < w' + D(R^G(w' + \lambda_0), R^G(w')) = \lambda_H$, a contradiction to w'-[EB-r]. In summary, $R_Q^\dagger$ is the unique solution to problem (42).

To apply Proposition 11, we need to construct a sequence $(R_n)_{n=1}^\infty$ that approximates $R_Q^\dagger$ in the sense of Definition 3. First, we deal with the following two cases: either $Q_H = Q_L$ or $Q_L = 0$. From (28), $R_Q^\dagger$ in both cases only contains at most one mass point. In fact, the $Q_L = 0$ case does not produce any mass point as $\underline{w}_H + D(0, R_Q^\dagger(\underline{w}_H)) = \underline{w}_H + D(0, Q_H) = \lambda_H = \underline{w}_H + D(0, R^G(\underline{w}_H))$. Whereas, the $Q_H = Q_L > 0$ case contains one mass point at $\underline{w}_L$ because, at w' such that $R^G(w') = Q_H = Q_L(> R^G(w' + \lambda_0))$, we have $\lambda_H = w' + D(R^G(w' + \lambda_0), R^G(w')) > w' + D(Q_L, Q_H) = w' + \lambda_L - \underline{w}_L$, which gives $w' < \underline{w}_L - (\lambda_L - \lambda_H)$ and hence $Q_L > R^G(\underline{w}_L)$. Thus, the associated part of Proposition 6's proof where we essentially constructed $R_n(w) = R_Q^\dagger(w - \epsilon_n)$ for small $\epsilon_n > 0$ converging to 0 still works here. Note that, in the $Q_L = 0$ case, since $Q_L \leq R^G(\underline{w}_L)$, an implementable Q must have $\underline{w}_L > \tilde{w}$, which ensures that such construction maintains $R_n(\underline{w}_k) = Q_k$ for both k (namely [EK]) and thus also works in this case. Second, we consider the last case $Q_H > Q_L > 0$. Note that we must have $Q_L > R^G(\underline{w}_L)$ since, otherwise, $R^G(\underline{w}_L) \geq Q_L > 0$ implies $\underline{w}_L \leq \tilde{w}$, which, as we have shown, makes Q not implementable. We also have shown $D(R^G(\underline{w}_L), R^G(\underline{w}_H)) = D(Q_L, Q_H)$, which further implies $Q_H > R^G(\underline{w}_H)$. In summary, $R_Q^\dagger$ contains two mass points at $\underline{w}_k$ for both k. We therefore take a sequence of small positive

numbers $(\epsilon_n)_{n=1}^{\infty}$ that converges to 0, so as to construct each R_n in the following way:

$$R_n = \begin{cases} R_Q^{\dagger}(w - \epsilon_n) & \text{if } w \geq \underline{w}_L + \epsilon_n; \\ R_Q^{\dagger}(w - 2\epsilon_n) & \text{if } w < \underline{w}_L + \epsilon_n. \end{cases} \tag{48}$$

Due to the presence of two mass points, by choosing a small ϵ_n, we maintain $R_n(\underline{w}_k) = Q_k$ for both k, and R_n satisfies [FE] and induces Q in one equilibrium. We then show R_n fully implements Q. First, for all $w \geq \underline{w}_L + \epsilon_n$, we have $w + D(R_n(w + \lambda_0), R_n(w)) = \epsilon_n + (w - \epsilon_n) + D(R_Q^{\dagger}(w + \lambda_0 - \epsilon_n), R_Q^{\dagger}(w - \epsilon_n)) \geq \epsilon_n + \lambda_H > \lambda_H$. For all $w \in [\underline{w}_H + \epsilon_n, \underline{w}_L + \epsilon_n)$, we have $w + D(R_n(w + \lambda_0), R_n(w)) = w + D(R_Q^{\dagger}(w + \lambda_0 - \epsilon_n), R_Q^{\dagger}(w - 2\epsilon_n)) \geq w + D(R_Q^{\dagger}(w + \lambda_0 - \epsilon_n), R_Q^{\dagger}(w - \epsilon_n)) = \epsilon_n + (w - \epsilon_n) + D(R_Q^{\dagger}(w + \lambda_0 - \epsilon_n), R_Q^{\dagger}(w - \epsilon_n)) = \epsilon_n + \lambda_H > \lambda_H$. For all $w < \underline{w}_L + \epsilon_n$, we have $w + D(R_n(w + \lambda_0), R_n(w)) = 2\epsilon_n + (w - 2\epsilon_n) + D(R_Q^{\dagger}(w + \lambda_0 - 2\epsilon_n), R_Q^{\dagger}(w - 2\epsilon_n)) \geq \epsilon_n + \lambda_H > \lambda_H$. Therefore, as in the proof of Proposition 6, now we just need to break all the mixed equilibria. Two possibilities arise: The high type randomizes at threshold $\underline{w}_H + 2\epsilon_n$ (the low type's threshold $\underline{w}_L + 2\epsilon$ is a continuous point of R_n); Or, the low type randomizes at his threshold $\underline{w}_L + \epsilon_n$ (the high type's threshold $\underline{w}_H + \epsilon_n$ is a continous point of R_n). Similar to the proof of Proposition 6, the career value generated by any threshold behavior in the former equilibrium is no less than that in the candidate equilibrium with the a high-type threshold slightly higher than $\underline{w}_H + 2\epsilon_n$, while the latter case's career value cannot be less than in the candidate equilibrium with a low-type threshold slightly lower than $\underline{w}_H + \epsilon_n$. Since the two candidate equilibria are broken by R_n, so are the mixed ones. In summary, Proposition 11 suggests that $R_Q^{\dagger}$ is robustly optimal.

Notice that we have already shown that whenever $Q_L > R^G(\underline{w}_L)$ or $\underline{w}_L < \tilde{w}$, Q can be implemented by some R_n constructed above. As a result, we prove Proposition 8 at the same time.

C.5 Proof of Proposition 9

Similar to our discussion in Section 3.1, full working forms an equilibrium if the lowest type is willing to work conditional on the career value $D(1, ..., 1)$. Thus, the PR wage here is still $\underset{\sim}{w}$. Next, we consider the FD wage w^F. To keep the desired equilibrium, we must have $w^F \geq \underset{\sim}{w}$. To break all the undesirable equilibria, notice that given a deterministic wage w', such a candidate equilibrium must have some $k \in \Theta$ and some $q \in [0, 1]$ such that the total working probability profile is $Q_{k,q} = (0, ..., 0, q, 1, ..., 1)$ with $k - 1$ zeros. If $q \in (0, 1)$, the type k worker must be indifferent between working and shirking. To break an equilibrium with k and $q \in (0, 1)$, we must have $w' \neq \lambda_k - D(Q_{k,q})$; whereas, to break an equilibrium with k and $q = 0$ (or $k + 1$ and $q = 1$), we need $w' > \lambda_k - D(Q_{k,0})$ or $w' < \lambda_{k+1} - D(Q_{k+1,1})$. A subset of these conditions is $w' \neq \lambda_k - D(Q_{k,q})$ for all k and $q \in [0, 1]$. To check when this subset will be satisfied, we define $W^m = \{\lambda_k - D(Q_{k,q}) : k \in \Theta, q \in [0, 1]\}$. Due to the continuity of $D(.)$, for each k, the set $W_k^m := \{\lambda_k - D(Q_{k,q}) : q \in [0, 1]\}$ is an interval. Also, for all $k < K$, $Q_{k,0} = Q_{k+1,1}$, and thus $\lambda_k - D(Q_{k,0}) \in W_k^m \cap W_{k+1}^m$. As a result, W^m is also an interval. The fact that $w' \geq \underset{\sim}{w}$ and $\underset{\sim}{w} = \lambda_1 - D(Q_{1,1}) \in W^m$ implies $w' > \max W^m$. Notice that the conditions for breaking the two types of undesirable equilibria mentioned above are already implied by $w' > \max W^m \geq \lambda_k - D(Q_{k,q})$ for all

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k and $q \in [0,1]$, so the FD wage is simply the infimum wage satisfying this, yielding $w^F = \max W^m$. Finally, we show $\max W^m = \widetilde{w}$. This is true because, fixing each k, that fact that $D(.)$ is quasi-concave in q_k implies that $\lambda_k - D(Q_{k,q})$ is maximized (namely $D(Q_{k,q})$ is minimized) at either $q = 0$ or 1. However, if $k > 1$, $\lambda_{k-1} - D(Q_{k-1,0}) > \lambda_k - D(Q_{k,1})$, so $q = 1$ cannot form the maximizer in W^m. In summary, only two such cases can be the maximizer in W^m: some k and $q = 0$, or $q = 1$ with $k = 1$. The former corresponds to $\max_{k \in \Theta} \lambda_k - D(0, ..., 0, 1, ..., 1)$ with k zeros (in the definition of $\widetilde{w}$) while the latter corresponds to $\underset{\sim}{w}$. The analysis above further indicates that, if $\widetilde{w} = \underset{\sim}{w}$, the PR wage coincides with the FD wage, so it suffices to fully implement full working and thus it is a (unique) robustly optimal wage policy. As a result, dispersion is not robustly optimal.

In the opposite direction, we now consider the case $\widetilde{w} > \underset{\sim}{w}$. We construct a new wage policy with dispersion that improves upon the FD wage $\widetilde{w}$, which shows no dispersion cannot be robustly optimal and therefore concludes the proof. To do so, we take three positive numbers $e, \epsilon < 1$, and δ. The new wage policy, denoted as F', assigns probability ϵ to $\widetilde{w} + e$ and the rest of probability to $\widetilde{w} + e - \delta$. The expected wage of F' is $\widetilde{w} + e - \delta(1 - \epsilon)$, so by choosing $e < \delta(1 - \epsilon)$, we obtain an improvement. The remaining job is to construct a certain ϵ (that depends on δ but not e) to ensure that F' fully implements full working. By choosing $\delta < \min_{k<K} \lambda_k - \lambda_{k+1}$, we make sure that, in each candidate equilibrium, there is at most one type whose threshold lies between the two supporting wage levels, which means the total working probability profile must be $Q_{q,k}$ for some k and $q \in [0,1]$. Let $\widetilde{\Theta} = \text{argmax}_{k \in \Theta} \lambda_k - D(Q_{k,0})$ collect all the types k that attain $\widetilde{w}$ in (29). Let $E^d = \{(k,q) \in \Theta \times [0,1] : \lambda_k - D(Q_{k,q}) \geq \widetilde{w} - \delta\}$ collect all the candidate equilibria where the lower wage $\widetilde{w} + e - \delta$ is not high enough to induce more work from type k. All the other candidate equilibria (without full working) are broken because even the low wage $\widetilde{w} + e - \delta$ is sufficiently rewarding. In fact, we have $E^d \subset \cup_{k^* \in \widetilde{\Theta}} \{(k^*, q) : q \in [0, \epsilon_{k^*}]\}$ with some $(\epsilon_{k^*})_{k^* \in \widetilde{\Theta}}$. This is because $\lambda_{k+1} - D(Q_{k+1,1}) < \lambda_k - D(Q_{k,0})$ for all $k < K$, so we can always choose a small δ to ensure that these hard-to-break equilibria in E^d are only those close to and on the right-hand side of the hardest-to-break equilibria $(k^*, 0)$. In addition, for $k^* > 1$, $\lambda_{k^*-1} - D(Q_{k^*-1,0}) > \lambda_{k^*} - D(Q_{k^*,1})$, while if $k^* = 1$, we have $\lambda_{k^*} - D(Q_{k^*,1}) = \underset{\sim}{w} < \widetilde{w}$. Both cases suggest that $D(Q_{k^*,1}) < D(Q_{k^*,0})$. Hence, the continuity of $D(.)$ means, for a small δ, we can always have $\epsilon_{k^*} < 1$. We thus choose ϵ such that $1 > \epsilon > \epsilon_{k^*}$ for all $k^* \in \widetilde{\Theta}$. This construction makes sure that each candidate equilibrium (k,q) in E^d must be such that the type k worker's threshold wage is no less than the higher wage $\widetilde{w} + e$ (otherwise, $q > \epsilon$), which is even higher than the FD wage. As a result, all these candidate equilibria are broken, and hence F' fully implements full working.

C.6 Proof of Proposition 10

We start by characterizing the stated policy $F^\dagger$. The greedy policy R^G, in this case, is defined by Definition 4 but with each w-EB' replaced with the w-[EB-r] in problem (42). With similar argument at the beginning of the proof of Proposition 6, Proposition 10 states that the robustly optimal wage policy $F^\dagger$ is associated with the tail wage policy $R^\dagger$ that is, again, given by (47). Using the assumption that $D(.)$ is

continuous and $D_K(.) > 0$, the same arguments in the proof of Lemma 2 show that $\max \text{supp}(R^G) = \lambda_K - D(0, ..., 0)$ and R^G is continuous whenever $R^G < 1$. Notice that, if $\lambda_K - D(0, ..., 0) \leq \underset{\sim}{w}$, then this along with (30) in Assumption 3 implies $\widetilde{w} = \underset{\sim}{w}$. In this case, the stated $R^\dagger$ is degenerate at $\underset{\sim}{w}$, which is robustly optimal according to Proposition 9. Hereafter, we consider $\lambda_K - D(0, ..., 0) > \underset{\sim}{w}$, and thus $\widetilde{w} = \lambda_K - D(0, ..., 0)$. Next, we show R^G decreases strictly on $[\widetilde{w} - (\lambda_{K-1} - \lambda_K), \widetilde{w}]$. To do this, we first suppose R^G is constant at some value strictly lower than 1 on some $(w^l, w^u) \subset [\widetilde{w} - (\lambda_{K-1} - \lambda_K), \widetilde{w}]$. Due to continuity, for a small $\epsilon > 0$, $R^G(w^u) = R^G(w^u - \epsilon)$. Notice that for all $k < K$ and both $w' = w^u$ and $w^u - \epsilon$, $w' + \lambda_k - \lambda_K \geq w' + \lambda_{K-1} - \lambda_K \geq \widetilde{w}$, so $R^G(w' + \lambda_k - \lambda_K) = 0$. Hence, $w^u - \epsilon + D((R^G(w^u - \epsilon + \lambda_k - \lambda_K))_{k=1}^K) = w^u + D((R^G(w^u + \lambda_k - \lambda_K))_{k=1}^K) - \epsilon = \lambda_K - \epsilon < \lambda_K$, where the last equality applies the binding w^u-[EB-r]. This implies R^G fails $(w^u - \epsilon)$-[EB-r], a contradiction to $R^G(w^u - \epsilon) < 1$. As a result, R^G is strictly decreasing on $[\widetilde{w} - (\lambda_{K-1} - \lambda_K), \widetilde{w}]$ unless it hits 1 at some interior point w_l. This can be true only if $w_l + D(0, ..., 0, 1) = \lambda_K$. However, Assumption 3 implies that $w_l + D(0, ..., 0, 1) > w_l + D(1, ..., 1) - \lambda_1 + \lambda_{K-1} = \lambda_{K-1} - (\underset{\sim}{w} - w_l) > \lambda_{K-1} - (\widetilde{w} - w_l) > \lambda_{K-1} - (\lambda_{K-1} - \lambda_K) = \lambda_K$, where the first inequality comes from (30). This forms a contradiction. In summary, $R^G(w) = 0$ for all $w \geq \widetilde{w}$, $R^G(.)$ is strictly decreasing on $[\widetilde{w} - (\lambda_{K-1} - \lambda_K), \widetilde{w}]$, and $R^G(\widetilde{w} - (\lambda_{K-1} - \lambda_K)) \leq 1$. (31) in Assumption 3 says $\widetilde{w} - \underset{\sim}{w} < \min_{k<K}\{\lambda_k - \lambda_{k+1}\}$, which further indicates that $\underset{\sim}{w}$ is interior in $[\widetilde{w} - (\lambda_{K-1} - \lambda_K), \widetilde{w}]$, so $R^G(\underset{\sim}{w}) < 1$. Therefore, (47) requires that the stated $R^\dagger$ have a mass point at $\underset{\sim}{w}$ and is fully supported and continuous on $(\underset{\sim}{w}, \widetilde{w}]$.

From now on, in the interesting case $\lambda_K - D(0, ..., 0) = \widetilde{w} > \underset{\sim}{w}$, we show $R^\dagger$ is also robustly optimal. As usual, we first show $R^\dagger$ is a solution to problem (42) and then apply Proposition 11 to verify it is robustly optimal. To start with, we show problem (42) here is equivalent to the following:

$$
\begin{aligned}
&\min_R \int_{\underset{\sim}{w}}^{\infty} R(s)ds \\
&\text{s.t.} \quad \text{For all } w \geq \underset{\sim}{w}, \; w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) \geq \lambda_K, \text{ and } R(w) \in [0, 1].
\end{aligned}
\tag{49}
$$

Given our focus on full working, many arguments in the proof of Proposition 5 that show how problem (19) emerges can be directly applied here: the objective function above is the same as that in problem (19), and also we can omit [EB-l]. As is standard, we use [EB] as a shorthand for [EB-r]. In addition, Assumption 3 allows us to omit the w-[EB] for $w \leq \underset{\sim}{w}$. This is because, for each such w, $w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) = w + D((R(w + \lambda_k - \lambda_K))_{k=1}^{K-1}, 1) > w + D(1, ..., 1) - \lambda_1 + \lambda_{K-1} = \lambda_{K-1} - (\underset{\sim}{w} - w) \geq \lambda_{K-1} > \lambda_K$, where the first inequality comes from (30). As a result, problem (49) is essentially choosing an R defined on $[\underset{\sim}{w}, \infty)$. A solution to problem (42) is thus obtained by setting $R(w) = 1$ for all $w \leq \underset{\sim}{w}$ and adjusting R to be left-continuous. Thus, we also omit [EK] and left-continuity in [FE]. Finally, we relax the constraint that R must be nonincreasing. We will derive a nonincreasing solution to this relaxed problem, which hence is also optimal in the unrelaxed one.

Next, we prove that the Lagrangian method (KKT necessary conditions) applies for the solution to

problem (49). Since $D(.)$ is concave, problem (49) admits a convex feasible set. Therefore, the first-order conditions suffice to verify a solution. To show the existence of Lagrangian multipliers, we invoke the Robinson constraint qualification result (see, e.g., Steck (2018), Theorem 3.5; Bonnans and Shapiro (2013), Theorem 3.9). In particular, we reformulate (49) as follows:

$$\min_{R \in C} V(R) \tag{50}$$
$$\text{s.t.} \quad G(R) \geq 0,$$

Where X and Y are both real Banach spaces, $V : X \to \mathbb{R}$ and $G : X \to Y$ are continuously differentiable, and C is a nonempty closed convex subset of X. Take a large wage $w^1 > \max_{k,Q} \lambda_k - D(Q) + (\lambda_1 - \lambda_K)$ (so that it is not optimal to assign any probability beyond this wage). Then, we set $C = X = L^p([\underset{\sim}{w}, w^1])$ with any $1 < p < \infty$ and $V(R) = \int_{\underset{\sim}{w}}^{w^1} R(s)ds$. G (and Y accordingly) is given by:

$$G = (G_1, G_2, G_3), \text{ where}$$
$$G_1(R) = \left(w + D((R(w + \lambda_k - \lambda_K))_{k=1}^K) - \lambda_K \right)_{w \in [\underset{\sim}{w}, w^1 - (\lambda_1 - \lambda_K)]};$$
$$G_2(R) = (R(w))_{w \in [\underset{\sim}{w}, w^1]};$$
$$G_3(R) = (1 - R(w))_{w \in [\underset{\sim}{w}, w^1]}. \tag{51}$$

Here, each G_j corresponds to the three constraints of [EB], $R \geq 0$, and $R \leq 1$, respectively. Let G' be the Fréchet derivative of G. Then, the Lagrangian multipliers exist for solution $\widetilde{R}$ if the Robinson constraint qualification (RCQ) holds: There exists some $\epsilon > 0$ such that, for all perturbations $\delta_1 : [\underset{\sim}{w}, w^1 - (\lambda_1 - \lambda_K)] \to (0, \epsilon)$, $\delta_2 : [\underset{\sim}{w}, w^1] \to (0, \epsilon)$, and $\delta_3 : [\underset{\sim}{w}, w^1] \to (0, \epsilon)$, we can construct the $\widetilde{R}$'s perturbation $h \in C$ such that $G_j'(R)h(w) = \delta_j(w)$ if the j constraint at w binds at $\widetilde{R}$.

To check RCQ, we start by showing that [EB] cannot bind simultaneously with either $R \geq 0$ or $R \leq 1$. In fact, we have shown above that if $\widetilde{R}(w) = 1$, then $w + D((\widetilde{R}(w + \lambda_k - \lambda_K))_{k=1}^K) > \lambda_K$ as a result of (30) in Assumption 3. Thus, [EB] and $R \leq 1$ cannot bind at the same time. Moreover, let $w^u = \inf\{w : \widetilde{R} = 0\}$. Suppose, for some $w' > w^u$, w'-[EB] binds. Then, we pick $\epsilon' < w' - w^u$ and thus $w' - \epsilon' + D((\widetilde{R}(w' - \epsilon' + \lambda_k - \lambda_K))_{k=1}^K) = w' - \epsilon' + D(0, ..., 0) = w' + D((\widetilde{R}(w' + \lambda_k - \lambda_K))_{k=1}^K) - \epsilon' = \lambda_K - \epsilon' < \lambda_K$, violating $(w' - \epsilon')$-[EB], a contradiction to $\widetilde{R}$ being feasible. Therefore, there exists $w^l \leq w^u(< w^1)$ such that $W^1 := \{w : \widetilde{R}(w) = 1\} = [\underset{\sim}{w}, w^l)$, $W^{EB} := \{w : w\text{-[EB] binds at } \widetilde{R}\} \subset [w^l, w^u]$, and $W^0 := \{\widetilde{R}\} = (w^u, w^1]$. Hence, for a small ϵ, we construct h as the following:

$$h(w) = \begin{cases} \delta_2(w) & \text{if } w > w^u; \\ \frac{1}{D_K((\widetilde{R}(w + \lambda_k - \lambda_K))_{k=1}^K)} \left[\delta_1(w) - \sum_{k=1}^{K-1} D_k((\widetilde{R}(w + \lambda_k - \lambda_K))_{k=1}^K)h(w + \lambda_k - \lambda_K) \right] & \text{if } w \in [w^l, w^u]; \\ -\delta_3(w) & \text{if } w < w^l. \end{cases} \tag{52}$$

Note that such h exists because (i) Assumption 2 requires $D_K > 0$; (ii) and for each $w \in [w^l, w^u]$, the construction only relies on h's values at points greater than w, whereas h's value for all $w > w^u$ is already pinned down as $\delta_2(w)$. To see why this construction produces the desired h, notice that, if w-[EB] binds, we need $G_1'(\widetilde{R})h(w) = \sum_{k=1}^{K} D_k((\widetilde{R}(w + \lambda_k - \lambda_K))_{k=1}^{K})h(w + \lambda_k - \lambda_K) = \delta_1(w)$; and if $R(w) = 0$, $G_2'(\widetilde{R})h(w) = h(w) = \delta_2(w)$; whereas, if $R(w) = 1$, $G_3'(\widetilde{R})h(w) = -h(w) = \delta_3(w)$.

Consequently, every solution to problem (49) satisfies the KKT necessary conditions. Recall that the concavity of $D(.)$ makes problem (49)'s feasible set convex, so the KKT conditions are also sufficient. Now, we argue that the stated solution $R^\dagger$ satisfies the KKT conditions. Our analysis above indicates that [EB] binds at $R^\dagger$ for all $w \in [\underset{\sim}{w}, \widetilde{w}]$, $R^\dagger(w) = 0$ for all $w > \widetilde{w}$, and $R^\dagger(w) = 1$ never happens for $w \geq \underset{\sim}{w}$. Let η^{EB}, η^1, and η^0 be the multipliers associated with [EB], $R \leq 1$, and $R \geq 0$, respectively. We thus have the Lagrangian and the first-order conditions (FOC):

$$\mathcal{L}(R, \eta^{EB}, \eta^1, \eta^0) = \int_{\underset{\sim}{w}}^{w^1} ds \left[R^\dagger(s) - \eta^1(s)(1 - R^\dagger(s)) - \eta^0(s)R^\dagger(s) \right.$$

$$\left. - \eta^{EB}(s)\left(s + D((R^\dagger(s + \lambda_k - \lambda_K))_{k=1}^{K}) - \lambda_K\right)\right];$$

$$\text{FOC: for all } w \in [\underset{\sim}{w}, w^1], \, 0 = 1 + \eta^1(w) - \eta^0(w)$$

$$- \sum_{\ell=1}^{K} \eta^{EB}(w - \lambda_\ell + \lambda_K)D_\ell((R^\dagger(s + \lambda_k - \lambda_\ell))_{k=1}^{K}).$$

(53)

Here, we abuse the notation a little bit by setting $\eta^{EB}(w) = 0$ for any $w \notin [\underset{\sim}{w}, w^1]$. Since the condition (31) in Assumption 3 ensures that $\widetilde{w} - \underset{\sim}{w} < \lambda_{K-1} - \lambda_K$, the first-order condition for $w \in [\underset{\sim}{w}, \widetilde{w}]$ is, in fact, $1 - \eta^{EB}(w)D_K((R^\dagger(w + \lambda_k - \lambda_\ell))_{k=1}^{K}) = 0$. In addition, the first-order condition for $w > \widetilde{w}$ depends on whether there is $k(< K)$ such that $w - \lambda_k + \lambda_K \in [\underset{\sim}{w}, \widetilde{w}]$. (31) in Assumption 3 requires that there be at most one such k, denoted as k_w, if it exists. Hence, if k_w exists, the first-order condition is $1 - \eta^0(w) - \eta^0(w - \lambda_{k_w} + \lambda_K)D_{k_w}((R^\dagger(w + \lambda_k - \lambda_{k_w}))_{k=1}^{K}) = 0$; and if not, the condition is $1 - \eta^0(w) = 0$. Therefore, we construct the multipliers as the following:

$$\eta^1(w) = 0 \text{ for all } x \in [\underset{\sim}{w}, w^1];$$

$$\eta^{EB}(w) = \begin{cases} \frac{1}{D_K((R^\dagger(w + \lambda_k - \lambda_K))_{k=1}^{K})} & \text{if } w \in [\underset{\sim}{w}, \widetilde{w}]; \\ 0 & \text{if } w > \widetilde{w}; \end{cases}$$

$$\eta^0(w) = \begin{cases} 1 - \frac{D_{k_w}((R^\dagger(w + \lambda_k - \lambda_{k_w}))_{k=1}^{K})}{D_K((R^\dagger(w + \lambda_k - \lambda_{k_w}))_{k=1}^{K})} & \text{if } w > \widetilde{w} \text{ and } k_w \text{ exists}; \\ 1 & \text{if } w > \widetilde{w} \text{ and } k_w \text{ does not exists}; \\ 0 & \text{if } w \in [\underset{\sim}{w}, \widetilde{w}]. \end{cases}$$

(54)

From our analysis above, one can see that these multipliers satisfy the first-order conditions and equal zero whenever the associated constraints are slack. Moreover, Assumption 1 ($D_K > 0$ and $\frac{D_k}{D_K} < 1$)

guarantees that their nonzero part is strictly positive, corresponding exactly to the binding constraints. As a result, the KKT conditions are satisfied by $R^\dagger$, η^{EB}, η^1, and η^0. Following our logic from the start, we thus have shown $R^\dagger$ is a solution to problem (49), and also a solution to problem (42).

Finally, to apply Proposition 11, we construct a sequence $(R_n)_{n=1}^\infty$ that approximates $R^\dagger$ in the sense of Definition 3. In fact, just as in the proof of Proposition 6, we take a sequence of positive small numbers $(\epsilon_n)_{n=1}^\infty$ that approximates 0, and let $R_n(w) = R^\dagger(w - \epsilon_n)$ for all $w \geq 0$. The rest of the proof is nothing different from the proof there. Hence, $R^\dagger$ is robustly optimal.