Kinematics Analysis of 6 Degrees of Freedom Robotic Arm

SmartMethods

By Mojahed Nour– July 27, 2020

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01 Kinematics Analysis of 6 Degrees of Freedom

What Is Kinematics Analysis?

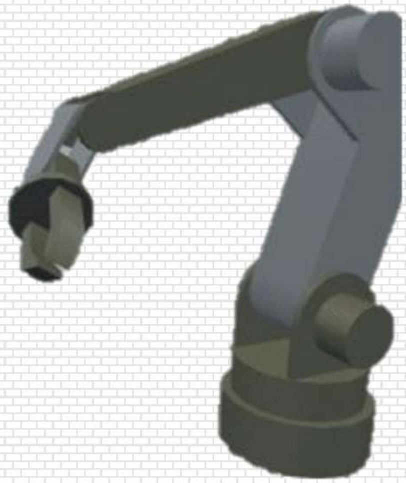
The kinematics problem is related to finding the transformation from the Cartesian space to the joint space and vice versa. The solutions of the kinematics problem of any robot manipulator have two types;.

I. The forward kinematic:

When all joints are known the forward kinematic will determine the Cartesian space, or where the manipulator arm will be.

II. The inverse kinematics:

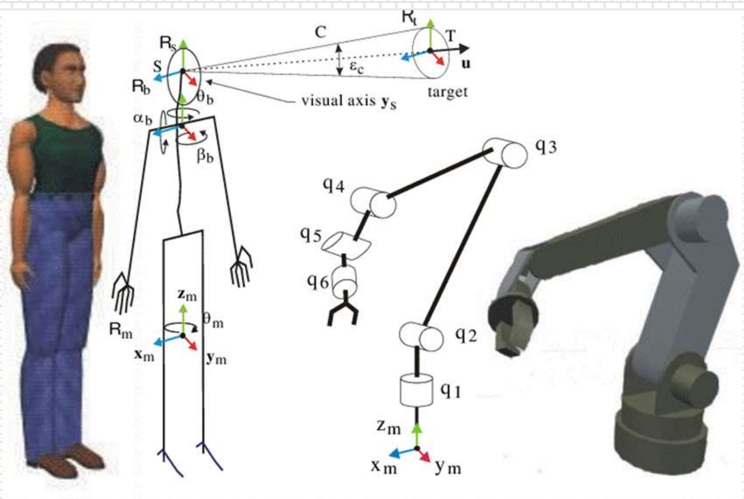
In the inverse kinematic the calculations of all joints is done if the desired position and orientation of the end- effectors is determined, that means by the inverse kinematic the robotic arm joint space angles will be calculated



02 THE AIM

What is the purpose of this task ?

The aim of this study is to analyze the robot arm kinematics which is very important for the movement of all robotic joints.



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FORWARD KINEMATICS

Assignments of joints and all parameters used to define the robot frames can be defined by using the DH parameters table explained by Tahseen (2013)

Res. J. Appl. Sci. Eng. Technol., 13(7): 569-575, 2016

Table 1: The DH parameters of the DFROBOT

Joint	Link	a_{i-1} min	α_{i-1} degree	d_i mm	θi degree
0-1	1	0	0	45	θ_1
1-2	2	0	90	0	θ_2
2-3	3	90	0	0	θ_3
3-4	4	90	0-90	0	θ_4
4-5	5	0	-90	30	θ_5
5-6	6	0	0	0	gripper

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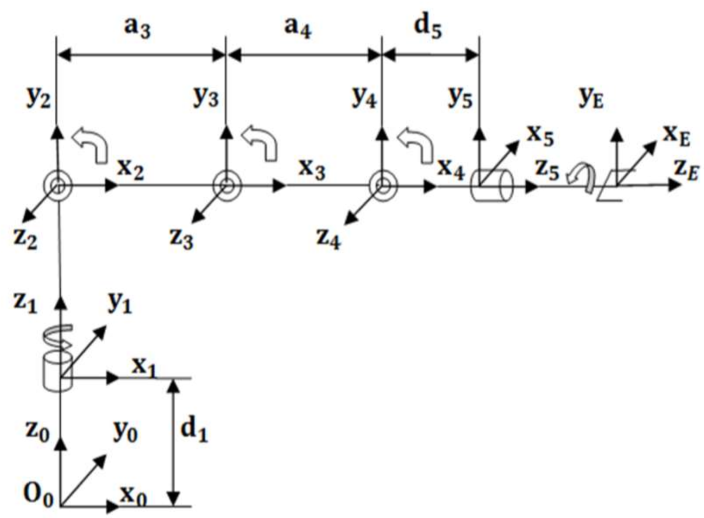
All joints as revolute. The main features of this kind: base rotation, single plane shoulder, elbow, wrist motion, functional gripper and optional wrist rotate. The kinematic modeling requires the solutions of the forward and inverse kinematics of the manipulator the link parameters are needed for the two solutions shown in the block diagram



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Forward kinematics analysis is the process of calculating the position and orientation of the end effector with given joints angles so by substituting these parameters in the homogenous transformation matrix from joint i to joint i+1 (Craig, 2005)

$$Ai = \begin{bmatrix} C\theta i & -S\theta iC\alpha i & S\theta iS\alpha i & aiC\theta i \\ S\theta i & C\theta iC\alpha i & -C\theta iS\alpha i & aiS\theta i \\ 0 & S\alpha i & C\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, the matrix A for example shows the transformation between frames 0 and 1, C = cosθ and S= sinθ:

Performing the composition from the n-th frame to the base frame we multiply the six matrices from :

$$A_n^0 = A_1^0 \,.\, A_2^1 \,\ldots..\, A_n^{n-1} = \prod_{i=1}^{n} A_i^{i-1} = \begin{bmatrix} R_n^0 & P_n^0 \\ 0 & 1 \end{bmatrix}$$

$$A_2^1 = A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & a_4C_4 \\ S_4 & C_4 & 0 & a_4S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = A_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^5 = A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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where, R is a 3×3 matrix for rotation and P is the position, so the total matrix of transformation:

$$A_6^0 = A_1^0 * A_2^1 * A_3^2 * A_4^3 * A_5^4 * A_6^5 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, p/,p2, p3 represent the position and {(n/, n2, n3), (o/, o2, o3), (a/, a2, a3}, represent the orientation of the end- effector, they can be calculated in terms of joint angles:

$$n_x = C_6 C_{12} C_{345} - S_6 S_{12}$$
$$n_y = C_6 S_{12} C_{345} + C_{12} S_6$$
$$n_z = C_6 S_{345}$$
$$o_x = -C_{12} S_6 C_{345} - S_{12} C_6$$
$$o_y = -S_6 S_{12} C_{345} + C_{12} C_6$$
$$o_z = -S_6 S_{345}$$
$$a_x = -C_{12} S_{345}$$
$$a_y = -S_{12} S_{345}$$
$$a_z = C_{345}$$

$$p_x = a_4 C_{12} C_3 C_4 - a_4 C_{12} S_3 S_4 + S_{12} d_5 + a_3 C_{12} C_3$$
$$p_y = a_4 S_{12} C_3 C_4 - a_4 S_{12} S_3 S_4 - C_{12} d_5 + a_3 S_{12} C_3$$
$$p_z = a_4 S_3 C_4 + a_4 C_3 S_4 + a_3 S_3 + d_1 \qquad (8)$$

where,

$C_{23} = \cos(\theta_2 + \theta_3), S_{23} = \sin(\theta_2 + \theta_3)$, $C_{234} = \cos(\theta_2 + \theta_3 + \theta_4)$ and $S_{234} = \sin(\theta_2 + \theta_3 + \theta_4)$

Making use of some trigonometric equations helps for easy solutions:

$$C_{12} = C_1 C_2 - S_1 S_2$$
$$S_{12} = C_1 S_2 + S_1 C_2$$
$$C_{234} = C_2(C_3 C_4 - S_3 S_4) - S_2(C_4 S_3 + C_3 S_4)$$
$$S_{234} = S_2(C_3 C_4 - S_3 S_4) + C_2(S_3 C_4 + C_3 S_4).$$

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The technique- Inverse kinematics

The solution of Inverse kinematics is more complex than forward kinematics and there is many solutions approach such as geometric and algebraic analysis used for finding the inverse kinematics considering the system structure of the robotic arm. In case of inverse kinematics the joint angles can be determined for any desired position and orientation in Cartesian space

$$A_1^{-1} * \left[\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\right] = A_1^{-1} * A_1^0 * A_2^1 * A_3^2 * A_4^3 * A_5^4 * A_6^5$$

The matrix manipulations has resulted the following matrix solutions:

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$$A_1^{-1} * \left[\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\right] = A_1^{-1} * A_1^0 * A_2^1 * A_3^2 * A_4^3 * A_5^4 * A_6^5$$

The matrix manipulations has resulted the following matrix solutions:

$$\begin{bmatrix} . & . & C_1a_x + S_1a_y & C_1p_x + S_1p_y \\ . & . & -S_1ax + C_1a_y & -S_1p_x + C_1p_y \\ . & . & a_z & p_z - d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} . & . & -C_2S_{345} & a_4C_2C_{34} + a_3C_2C_3 + S_2d_5 \\ . & . & -S_2S_{345} & a_4S_2C_{34} + a_3S_2C_3 - C_2d_5 \\ . & . & C_{345} & a_4S_{34} + a_3S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Both matrix elements in Eq. (17) are equated to each other and the resultant θ values are extracted. By taking (1, 4) (2, 4):

$$C_1p_x + S_1p_y = a_4C_2C_{34} + a_3\,C_2\,C_3 + S_2\,d_5$$

$$-S_1p_x + C_1p_y = a_4S_2C_{34} + a_3S_2C_3 - C_2d_5$$

Squaring and adding the two Eq. (18) and (19):

$$C_3 = \cos\theta_3 = \frac{\sqrt{p_x^2 + p_y^2 - d_5^2} - a_4C_{34}}{a_3} = n$$

$$\theta_3 = \cos^{-1}n = \mathrm{Atan2}\,(\mp\sqrt{1 - n^2}, n)$$

Eq. (3, 4):

$$p_z - d_1 = a_4S_{34} + a_3S_3$$

$$S_{34} = \frac{a_3S_3 - p_z + d_1}{a_4}$$

$$\theta_{34} = \mathrm{Atan2}\left[\frac{a_3S_3 - p_z + d_1}{a_4}, \mp\sqrt{1 - \left(\frac{a_3S_3 - p_z + d_1}{a_4}\right)^2}\right]$$

$$\theta_4 = \theta_{34} - \theta_3$$

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03 The technique- Inverse kinematics

$$S_2p_x - C_2p_y - d_1 = d_5$$
$$S_2p_x - C_2p_y = d_1 + d_5$$
$$\theta_2 = \mathrm{atan2}(p_x, -p_y) \mp \mathrm{atan2}\left[\sqrt{p_x^2 + p_y^2 - (d_1 + d_5)^2}, (d_1 + d_5)\right]$$

From Eq. (8) we can obtain:

$$a_x = -C_{12}S_{345}$$

$a_y = -S_{12}S_{345}$

Dividing the two equations:

$$\frac{S_{12}}{C_{12}} = \frac{a_y}{a_x} \theta_{12} = \mathrm{atan2}\,(a_y, a_x)$$

And then we find:

$$\theta_1 = \theta_{12} - \theta_2$$

Then also equating elements (3, 1) and (3, 2) of the two sides of the matrices in Eq. (24):

$$-S_6 = S_2n_x - C_2n_y \text{ Or} S_6 = C_2n_y - S_2n_x$$
$$-C_6 = S_2o_x - C_2o_y \text{Or } C_6 = C_2o_y - S_2o_x \theta_6 = \mathrm{Atan2}[(C_2n_y - S_2n_x), (C_2o_y - S_2o_x)]$$

$$A_1^{-1} * A_2^{-1} * A_3^{-1} * \left[\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\right] = A_4^3 * A_5^4 * A_6^5$$

$$\begin{bmatrix} C_1C_{23} & C_1S_{23} & S_1 & -a_3C_1C_2 \\ -S_1C_{23} & -S_1S_{23} & C_1 & a_3S_1C_2 \\ S_{23} & -C_{23} & 0 & -a_3S_2 - d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{RHS} \begin{bmatrix} C_6C_{45} & -S_6C_{45} & -S_{45} & a_4C_4 \\ C_6S_{45} & -S_6S_{45} & C_{45} & a_4S_4 \\ -S_6 & -C_6 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{LHS}$$

Equating elements (3, 4) from the two sides of Eq. (31):

$$S_{23}p_x - C_{23}p_y - a_3S_2 - d_1 = d_5$$
$$S_{23}p_x - C_{23}p_y = a_3S_2 + d_1 + d_5$$
$$\theta_{23} = \mathrm{atan2}\left(p_x, -p_y\right) \mp \mathrm{atan2}\left[\sqrt{p_x^2 + p_y^2 - (a_3S_2 + d_1 + d_5)^2}, (a_3S_2 + d_1 + d_5)\right]$$

$$\theta_3 = \theta_{23} - \theta_2$$

From the Eq. in (8) we can also obtain:

$$C_{345} = a_z\theta_{345} = \mathrm{atan2}\left(\mp\sqrt{1 - a_z^2}, a_z\right) \ldots$$

$$\theta_5 = \theta_{345} - \theta_3 - \theta_4$$

References

-https://www.youtube.com/watch?v=sJmkb8UEW34

-https://www.user.tu-berlin.de/mtoussai/teaching/14-Robotics/02-kinematics.pdf

-Research Article Kinematics Analysis and Modeling of 6 Degree of Freedom Robotic Arm from DFROBOT

THANK YOU

You are Welcome To Contact Me

Mojahed-nour

@MJD_noor

Mojahed nour

Mojahednour@email.com