PSY 769: HW1

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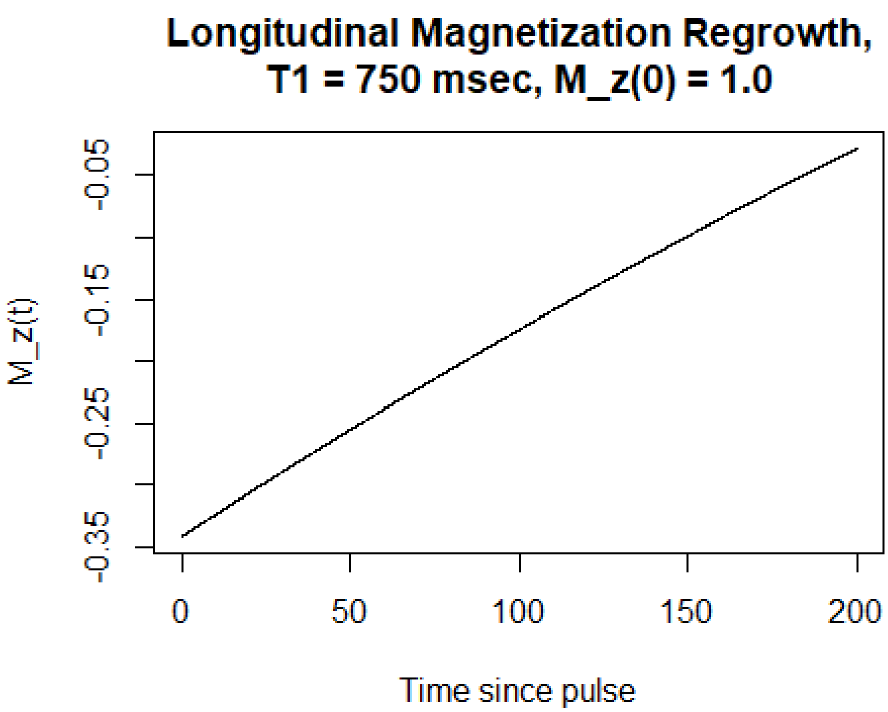
1.

Plot time course (t=0-2000 msec) of longitudinal magnetization regrowth, $M_z(t)$, following a single 100 degree RF pulse starting from equilibrium. Use equation below (the solution to the longitudinal relaxation part of the Bloch equation) assuming T1=750 msec, and equilibrium magnetization, $M_z^0 = 1.0$. (N.B.: $M_z(0_+)$ means longitudinal magnetization immediately after flip) Hint: don't confuse degrees/radians, or sin/cos!.

$$Mz(t) = M_z^0\left(1 - e^{-t/T1}\right) + M_z(0_+)e^{-t/T1}$$

Plotted here is the function $Mz(t) = (1)\left(1 - e^{-\frac{t}{750}}\right) + cos\left(\frac{110\pi}{180}\right)e^{-\frac{t}{750}}$.

```
t <- seq(0, 200, 0.05)
plot(t, (1 - exp(-t/750)) + (cos(110*pi/180)*exp(-t/750)), type = "l", xlab = "Time since pulse", ylab = "M
_z(t)", main = "Longitudinal Magnetization Regrowth,\nT1 = 750 msec, M_z(0) = 1.0")
```



2.

Apply the Bloch equation longitudinal magnetization solution (above) to the following simple spin echo sequence. At the very beginning of a spin-echo pulse sequence,

(a) the equilibrium longitudinal magnetization, M_z^0, is flipped by a 90 deg RF pulse,

(b) it recovers (from zero) for time t=TE/2,

(c) a 180 deg RF pulse is applied at time t=TE/2,

(d) an echo occurs at time t=TE,

(e) the longitudinal magnetization recovers for t=TR (measured from the first 90 deg RF pulse), and finally,

(f) the second 90 degree RF pulse occurs.

Derive the equation (this means show each step) for the amount of longitudinal magnetization present just before the second 90 degree RF pulse by giving equations for the longitudinal magnetization at each of the stages (a) to (e) above. No graphs needed.

Hint: remember M_z^0 is a constant while $M_z(0_+)$ is a variable!

$Let\ \ T^* = TR - \frac{TE}{2}.$

-> 180 degree pulse reverses longitudinal magnetization

$$M_{z\prime}^0 = -M_z^0$$

-> Recovery to end of first TI from longitudinal part of Bloch equation

$$M_{z\prime} = M_z^0\left(1 - 2e^{\frac{-T^*}{T1}}\right)$$

-> Longitudinal then regrows from zero

$$M_{z\prime} = M_z^0\left(1 - e^{\frac{-(TR-T^*)}{T1}}\right)$$

-> After second 180 degree pulse, just change sign again

$$M_{z\prime} = -M_z^0\left(1 - e^{\frac{-(TR-T^*)}{T1}}\right)$$

-> Apply relaxation equation again

$$M_{z\prime} = M_z^0\left(1 - e^{\frac{-T^*}{T1}}\right) - M_z^0\left(1 - e^{\frac{-(TR-T^*)}{T1}}\right)e^{\frac{-T^*}{T1}}$$

-> Continuing with the derivation

$$M_{z'} = M_z^0\left(1 - e^{\frac{-T^*}{T1}}\right) - M_z^0\left(e^{\frac{-T^*}{T1}} - e^{\frac{-(TR-T^*)}{T1} - \frac{T^*}{T1}}\right)$$

$$M_{z'} = M_z^0\left(1 - e^{\frac{-T^*}{T1}}\right) - M_z^0\left(e^{\frac{-T^*}{T1}} - e^{\frac{-TR}{T1}}\right)$$

$$M_{z'} = M_z^0 - M_z^0\left(e^{\frac{-T^*}{T1}}\right) - M_z^0\left(e^{\frac{-T^*}{T1}}\right) + M_z^0\left(e^{\frac{-TR}{T1}}\right)$$

$$M_{z'} = -2M_z^0\left(e^{\frac{-T^*}{T1}}\right) + M_z^0 + M_z^0\left(e^{\frac{-TR}{T1}}\right)$$

$$M_{z'} = M_z^0\left(1 - 2M_z^0 e^{\frac{-T^*}{T1}} + e^{\frac{-TR}{T1}}\right)$$

$$M_{z'} = M_z^0\left(1 - 2M_z^0 e^{\frac{-\left(TR-\frac{TE}{2}\right)}{T1}} + e^{\frac{-TR}{T1}}\right)$$

This is magnetization flipped to transverse, therefore made recordable.

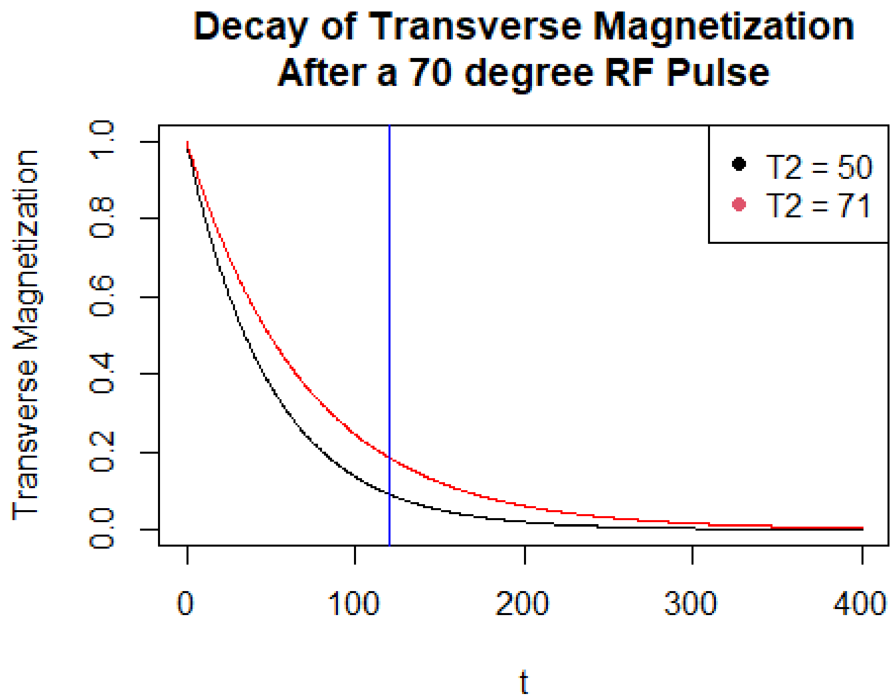
3.

Plot the time course of the decay of transverse magnetization after an α = 70 degree RF pulse for two different tissue types with T2=50 and T2=71 msec, and then plot the difference between these two curves to illustrate the time point where their transverse magnetizations are the most different. Use the equation below (assume longitudinal magnetization before the flip, M_z^0, is same for both tissue types, namely 1.0).

$$M_{x'y'}(t) = M_{x'y'}(0_+)e^{-t/T2}$$

Plotted here are the functions $M_{x'y'}(t) = (1)e^{-t/50}$ *and* $M_{x'y'}(t) = (1)e^{-t/71}$.

```r
# Is this calculation independent of flip angle?
t <- seq(0, 400, 0.5)

M_xpyp1 <- 1*exp(-t/50)
M_xpyp2 <- 1*exp(-t/71)
diff <- M_xpyp2 - M_xpyp1

plot(t, M_xpyp1, type = "l", col = "black", ylab = "Transverse Magnetization", main = "Decay of Transve
rse Magnetization\nAfter a 70 degree RF Pulse") # tissue with T2 = 50
lines(t, M_xpyp2, type = "l", col = "red") # tissue with T2 = 71
legend("topright", legend = c("T2 = 50", "T2 = 71"), col = 1:2, pch = 16)
abline(v = which.max(diff), col = "blue")
```



#which.max(diff) t = 120 is when the maximized difference is achieved

4.

Assume the following T1, T2, and proton-density (PD [$=M_z^0$]) values for gray matter (GM), white matter (WM), and cerebrospinal fluid (CSF): T1 (msec): GM=1150, WM=820, CSF=2200; T2 (msec): GM=86, WM=80, CSF=330; PD (water=1.0): GM=0.67, WM=0.58, CSF=0.98. Use this equation for spin-echo signal intensity:

$$M_{x\prime y\prime} = M_z^0\left(1 - e^{-TR/T1}\right)e^{-TE/T2}$$

(a) In T1-weighted images, WM > GM > CSF (signal intensity, brightness). For a fixed TE=6 msec, determine the TR that maximizes the contrast between GM and WM (TR that results in largest value of WM-minus-GM). Do this by plotting $M_{x\prime y\prime}(TR)$ curves for each tissue type (TR=0 to TR=2500). Briefly explain why T1-weighted images have intermediate TR and short TE.

Plotted here are the functions

$M_{x\prime y\prime}(TR)_{GM} = 0.67\left(1 - e^{-\frac{TR}{1150}}\right)e^{-\frac{6}{86}}$,

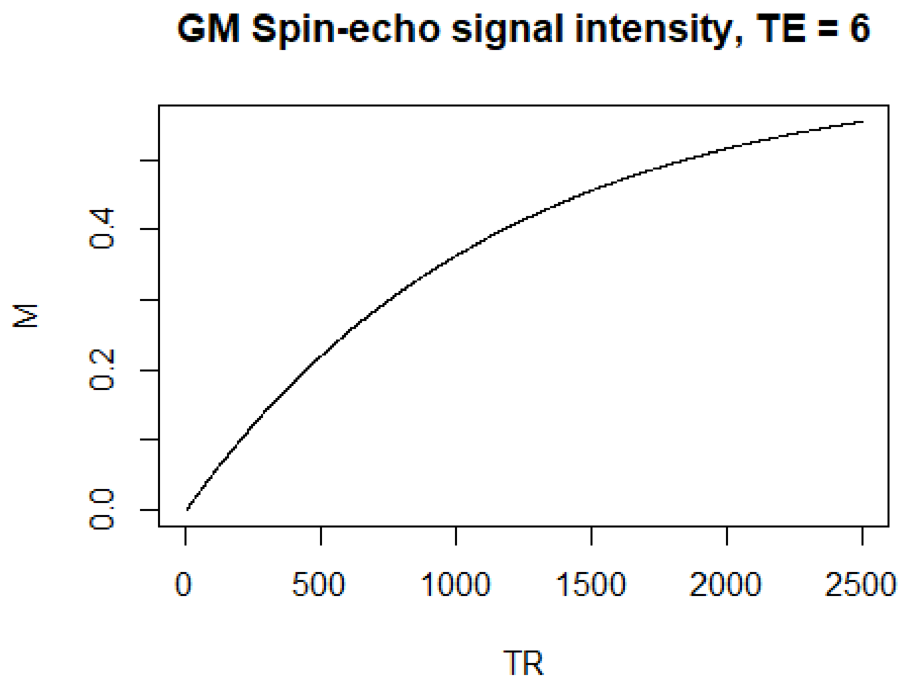
$M_{x\prime y\prime}(TR)_{WM} = 0.58\left(1 - e^{-\frac{TR}{820}}\right)e^{-\frac{6}{80}}$, *and*

$M_{x\prime y\prime}(TR)_{CSF} = 0.98\left(1 - e^{-\frac{TR}{2200}}\right)e^{-\frac{6}{330}}$.

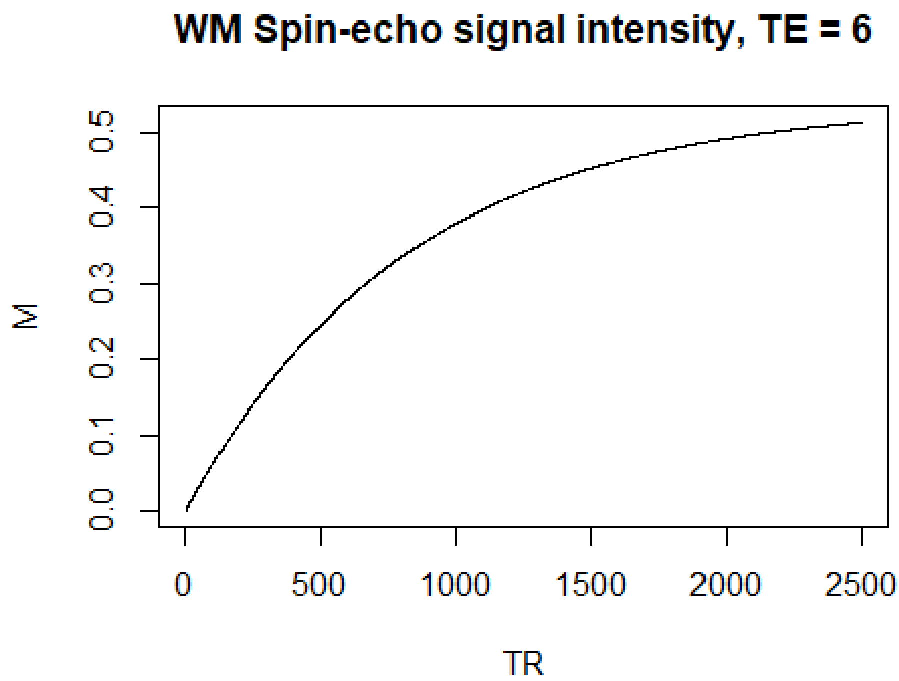
```
x <- seq(0:2500:2500)
```

```
## Warning in 0:2500:2500: numerical expression has 2501 elements: only the first
## used
```

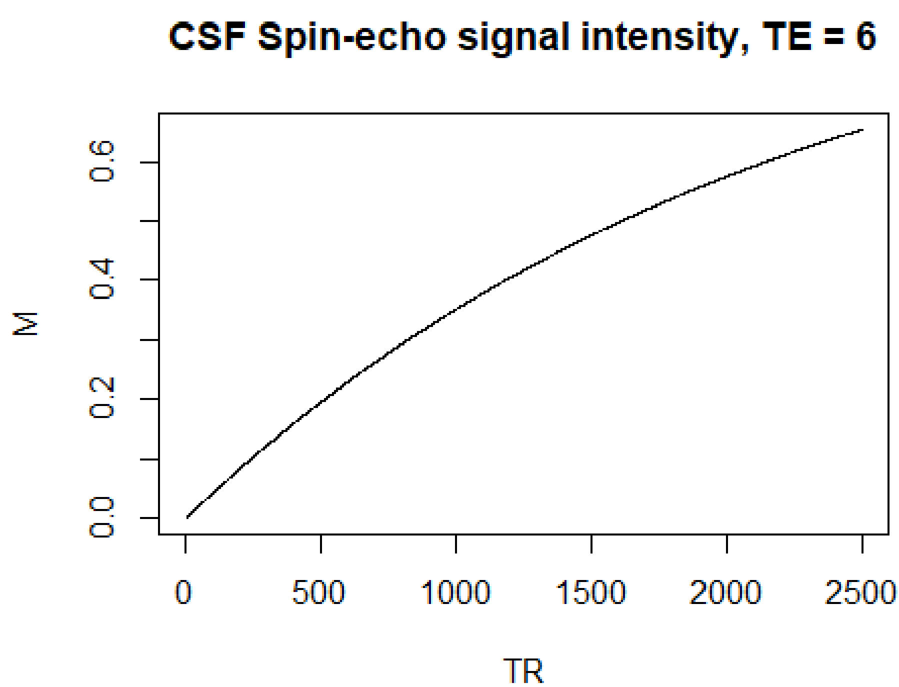
```
plot(x, 0.67*(1-exp(-x/1150))*exp(-6/86), type = "l", xlab = "TR", ylab = "M", main = "GM Spin-echo si
gnal intensity, TE = 6")
```




```
plot(x, 0.58*(1-exp(-x/820))*exp(-6/80), type = "l", xlab = "TR", ylab = "M", main = "WM Spin-echo sig
nal intensity, TE = 6")
```




```
plot(x, 0.98*(1-exp(-x/2200))*exp(-6/330), type = "l", xlab = "TR", ylab = "M", main = "CSF Spin-echo s
ignal intensity, TE = 6")
```



T1-weighted images have intermediate TR because at both extremities of TR, there is no clear difference between tissue types because longitudinal magnetization will be similar between tissue types; the right balance and clear differences in tissue contrast is achieved at intermediate values of TR. T1-weighted images have short TE values because short TE values lead to minimzed T2 contrast, thus yielding to exclusive T1 contrast.

(b) In typical T2-weighted images, CSF > GM > WM. For a fixed TR=3200 msec (vs. fixing the TE above), determine a TE that maximizes CSF-minus-WM contrast. Plot the curve of $M_{x'y'}(TE)$ for each tissue (from TE=1 to TE=200 msec). Briefly explain why T2-weighted images have long TR and intermediate TE.

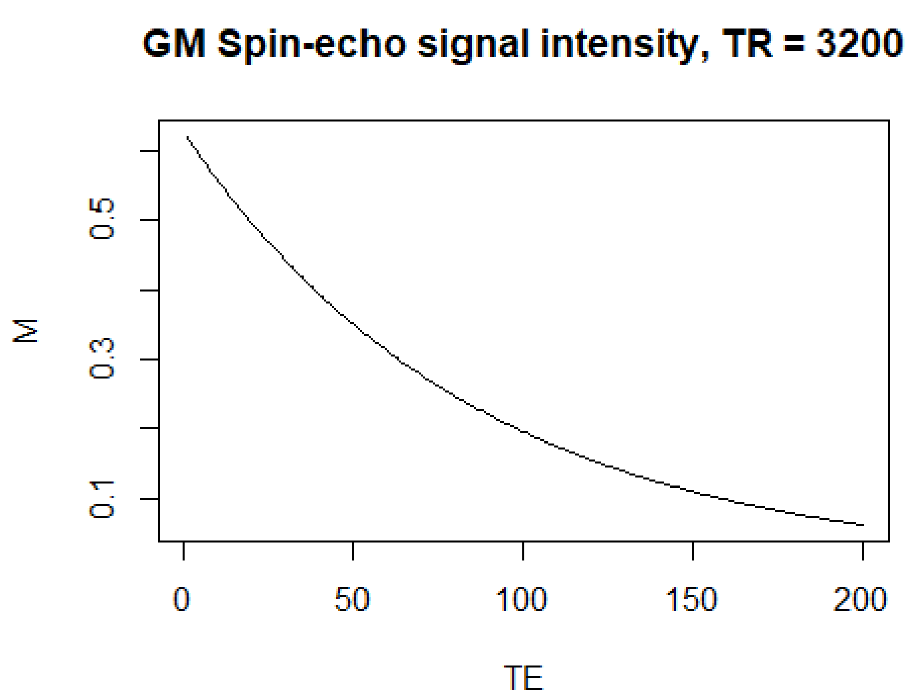
Plotted here are the functions

$$M_{x'y'}(TE)_{GM} = 0.67\left(1 - e^{-\frac{3200}{1150}}\right)e^{-\frac{TE}{86}},$$

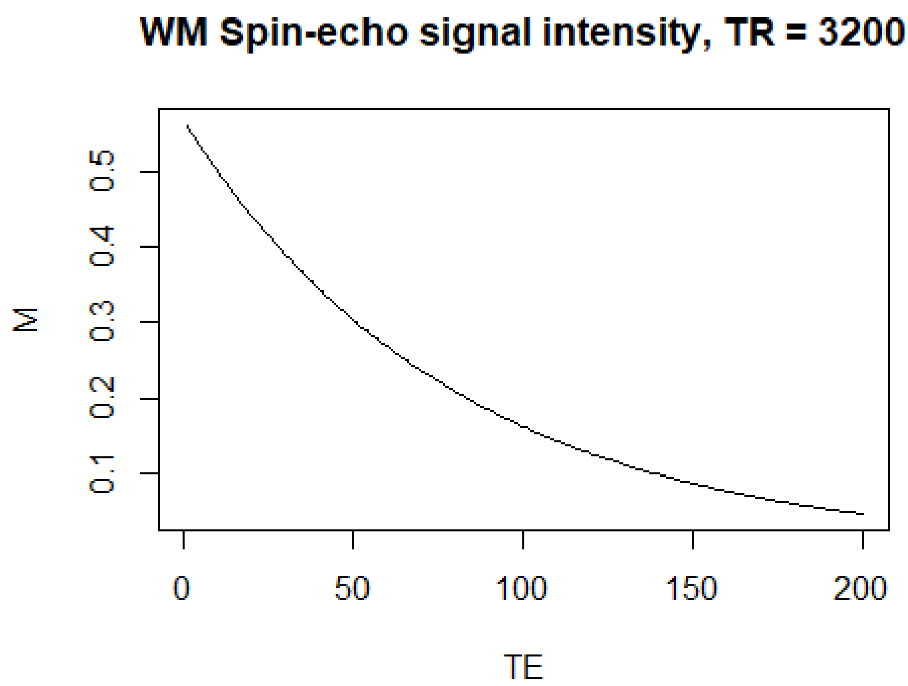
$$M_{x'y'}(TE)_{WM} = 0.58\left(1 - e^{-\frac{3200}{820}}\right)e^{-\frac{TE}{80}}, \text{ and}$$

$$M_{x'y'}(TE)_{CSF} = 0.98\left(1 - e^{-\frac{3200}{2200}}\right)e^{-\frac{TE}{330}}.$$

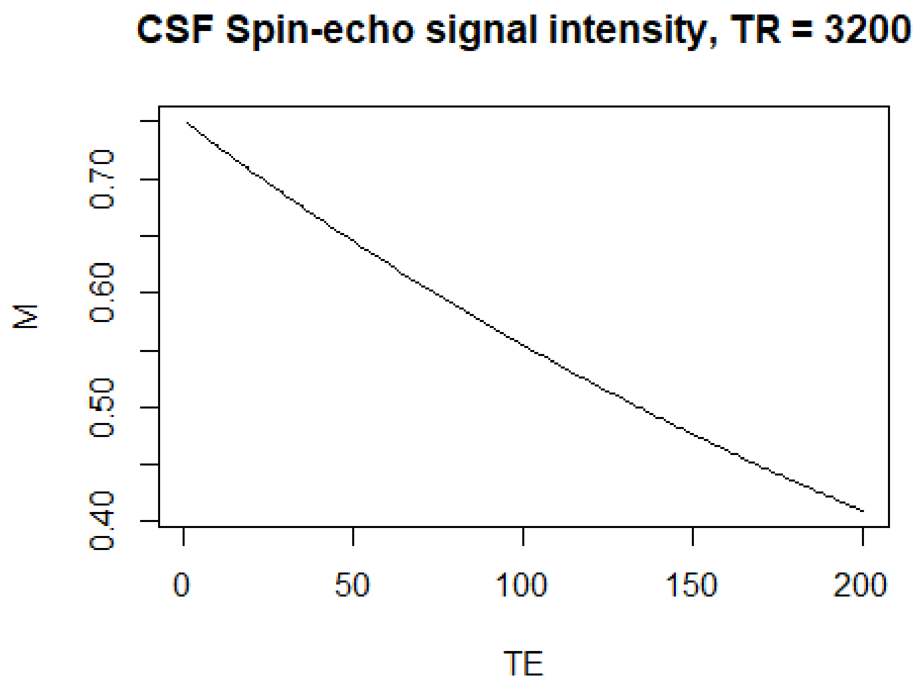
```
x <- seq(1:200:200)

## Warning in 1:200:200: numerical expression has 200 elements: only the first used

plot(x, 0.67*(1-exp(-3200/1150))*exp(-x/86), type = "l", xlab = "TE", ylab = "M", main = "GM Spin-ech
o signal intensity, TR = 3200")
```



```
plot(x, 0.58*(1-exp(-3200/820))*exp(-x/80), type = "l", xlab = "TE", ylab = "M", main = "WM Spin-echo signal intensity, TR = 3200")
```




```
plot(x, 0.98*(1-exp(-3200/2200))*exp(-x/330), type = "l", xlab = "TE", ylab = "M", main = "CSF Spin-echo signal intensity, TR = 3200")
```



T2-weighted images have long TR because long TR ensures almost complete longitudinal recovery and minimal T1 contrast. Intermediate TE is needed for T2-weighted images because extreme TE (whether long or short) leads to lost transverse magnetization (the recordable component of the signal) and to no T2 contrast; intermediate TE maximizes the difference in transverse magnetization.

5.

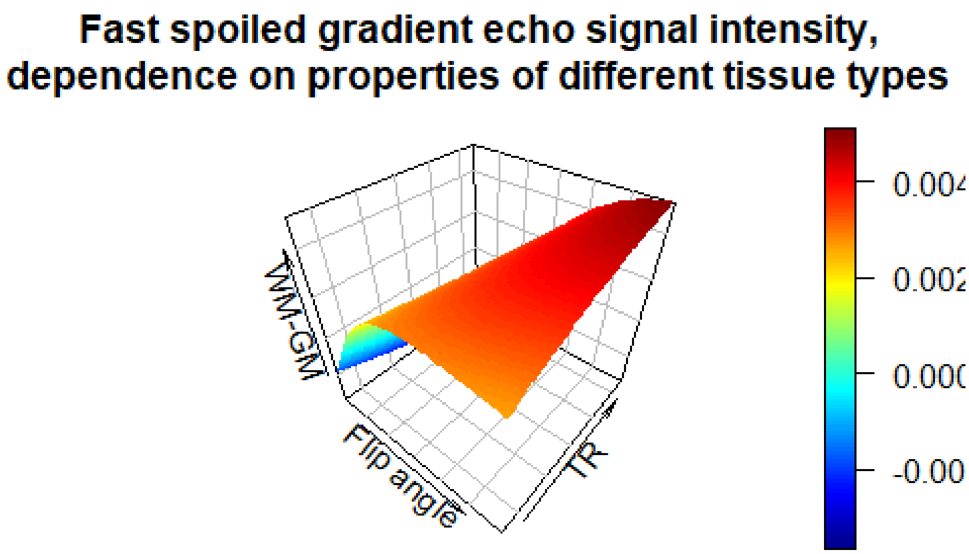
Use the following equation for fast spoiled gradient echo signal intensity (and T1, T2, and PD values from problem 4):

$$M_{x'y'} = \frac{M_z^0\left(1-e^{-\frac{TR}{T1}}\right)}{1-cos(\alpha)e^{-\frac{TR}{T1}}} sin(\alpha)e^{-\frac{TE}{T2}}$$

Make a 2D plot of the dependence of white matter $M_{x'y'}$ minus gray matter $M_{x'y'}$ (white-gray contrast) on both flip angle (from 3-20 deg) and TR (from 5-15 msec). Assume that the TE=4 msec. A '2D plot' means, show value of the difference in $M_{x'y'}$'s for regularly sampled combinations of flip angle and TR as a brightness map (contour map) or a height map (surface plot). Which adjustable scan parameter (TE, TR, flip angle) directly affects scan length?

Plotted here is the function $M_{x'y'_{WM}} - M_{x'y'_{GM}} = \frac{0.58\left(1-e^{-\frac{TR}{820}}\right)}{\left(1-cos(\alpha)e^{-\frac{TR}{820}}\right)} sin(\alpha)e^{-\frac{4}{80}} - \frac{0.67\left(1-e^{-\frac{TR}{1150}}\right)}{\left(1-cos(\alpha)e^{-\frac{TR}{1150}}\right)} sin(\alpha)e^{-\frac{4}{86}}$.

```
library(plot3D)
M <- mesh(seq(3*pi/180, 20*pi/180,length.out=100), seq(5, 15,length.out=100))

alpha <- M$x; TR <- M$y #flip angle (in degrees), TR

x <- alpha
y <- TR
zWM <- (0.58*(1-exp(-y/820))/(1-cos(x)*exp(-y/820)))*sin(x)*exp(-4/80)
zGM <- (0.67*(1-exp(-y/1150))/(1-cos(x)*exp(-y/1150)))*sin(x)*exp(-4/86)
z <- zWM - zGM


surf3D(x = x, y = y, z = z, colkey=TRUE, bty="b2", main="Fast spoiled gradient echo signal intensity,\ndependence on properties of different tissue types", xlab = "Flip angle", ylab = "TR", zlab = "WM-GM")
```



TR directly affects scan length.