Cristian Mojica

Prof. Bailey

STAT 673

6 December 2022

Report: *Spectral Analysis of EEG's by Autoregressive Decomposition of Time Series*

Abstract

Gersch's study seeks to improve on a traditional windowed periodogram model for EEG data analysis by means of taking EEG data and utilizing an autoregressive representation of it. From this representation, a researcher can extract such information as spectral densities of single and multiple EEG series, and transfer functions and coherences between EEG series pairs. The autoregressive representation spectral analysis generally gives spectral estimates that are smoother than traditional windowed periodogram spectral analysis, which has bumpy estimates. The autoregressive representation analysis also outperforms the windowed periodogram analysis in terms of asymptotic bias and variance and requires little to no subjective judgment compared to the windowed periodogram method.

Introduction

Electroencephalography (EEG) is a noninvasive method by which researchers in psychology or neuroimaging may understand electrical activity within the brain as an indication of where cerebral activity occurs. Specifically, this method takes excitatory postsynaptic potentials and inhibitory postsynaptic potentials from pyramidal cells near electrodes placed on the human scalp and adds up these potentials as a measure of electrical activity (Britton 81). Electrical activity is recorded as multivariate time series data by electrodes that are part of the EEG apparatus.

Windowed periodogram spectral analysis, the method improved upon by Gersch's study, was introduced by D.O. Walter in 1963, and that technique was thenceforth widely utilized and varied in its implementation in neurophysiology. It is "equivalent to the Fourier transform of the same record covariance function" (Gersch 211). In that same year, P. Whittle observed the stability of any fits of autoregressive representation models to stationary time series, and R.H. Jones in 1966 utilized autoregressive representations to make predictions in time series. This work laid the foundation for autoregressive representation spectral analysis research that would emerge in 1969 from the work of E. Parzen, H. Akaike, and R.E. Kromer.

Analysis of the statistical performance of the autoregressive representation was offered by the 1969 research. Parzen offered the conjecture that the spectral matrix scaled by n follows a complex Wishart distribution with degrees of freedom proportional to the reciprocal of p. He also suggested that autoregressive cross-spectral estimators have less bias than estimators from the windowed periodogram method, and ultimately that each spectral estimate has degrees of freedom proportional to the reciprocal of p; this was verified by Akaike and Kromer. Separately, "Kromer demonstrated that the [autoregressive representation] spectral estimates are asymptotically normal, consistent and efficient in decrease of bias and variance with sample size" (Gersch 215). This representation is superior to windowed periodogram analysis performed with any window choice, in terms of asymptotic bias and variance.

To improve upon windowed periodogram spectral analysis, Gersch's study proposes "an autoregressive representation method for computing the energy spectral density of single and multiple EEG test series and transfer function and coherence between pairs of EEG time series" (Gersch 205).

Methodology

An eight-second record of EEG data was taken from an epileptic patient, and the data was simultaneously recorded across six channels. (This data was provided by Dr. Barry Tharp of the Stanford University Medical Center.) These recordings were regularly sampled at intervals of 10 milliseconds and stored in memory. Windowed periodogram spectral analysis was performed with Akaike trigonometric window W2, bandwidth b = 1 Hz, and 16 degrees of freedom. Smoothing in windowed periodogram spectral analysis should improve the statistical properties of unsmoothed estimates of spectra, then "self- and cross-spectral densities are computed at each frequency" of the observed time series, and then transfer functions and coherence can be computed from these estimates (Gersch 211). In the same method, the bandwidth, degrees of freedom, and spectral estimate variance are related as indicated below. *T* represents record duration in seconds, and *S(f)* is the true spectral density at frequency *f*. In a spectral window, bandwidth concerns itself with distinguishable peak width in a spectrum.

$$b = \frac{n}{2T}\,;\, Var\,\hat{S}(f) = \frac{2}{n}S(f)^2;\, b\,Var\,\hat{S}(f) = \frac{S(f)^2}{T} = \text{constant}$$

There is a decision to make for spectral windows. Options include windows designed by Akaike, Daniell, Hamming, Parzen, and Tukey, but each distribute bias and variance errors differently. A spectral window with optimal distribution of bias and variance of spectral estimate errors cannot be designed, which points to the bias-variance tradeoff present in windowed periodogram spectral estimation. Such a window also cannot be designed since a spectral density for a given record is not known *a priori*, and so points to the highly empirical nature of this method.

The same EEG data was also used to compute these same results by the autoregressive representation technique, computed with 160 and 133 degrees of freedom. The spectral density calculated was parametric in *p*. These methods were then compared against each other by means of comparison against a known model. Autoregressive representation spectral analysis has the advantage of being more easily interpretable because it is smooth; windowed periodogram spectral analysis is generally bumpy and less readily interpretable. With regards to resolvability, the autoregressive and windowed periodogram methods are comparable in resolvability, but bandwidth does not translate readily to the autoregressive representation from the windowed periodogram representation.

Results

Various plots are provided in the original paper to display windowed periodogram spectral computations, as well as data from various EEG channels; spikes in a data record correspond to epileptic spikes. These plots are compiled into figures; these results are primarily pictorial and comparative in nature. Figure 2 of the study shows the energy spectral density and coherence that come from the data in channels 1-4 by means of the Akaike W2 window in windowed periodogram analysis. The W2 window and W1 windows were used and compared in analysis, and both returned similar results to what is shown in the study's Figure 2, suggesting stationary records and reasonable resolution and computational accuracy (Gersch 214). Comparison of Figures 2 and 3 suggest similarity in structural detail of the spectral estimates, and comparison of the same figures suggests that channels 1 and 2 have “no coherence” between them, and that there is distinct coherence between channels 3 and 4; this can be found immediately from the autoregressive representation (Gersch 214). Windowed periodogram analysis with Tukey and Parzen windows yields bumpy results under the conditions of low bias and $N = 400$, pointing to the possibility that the autoregressive representation is superior in statistical performance to windowed periodogram analysis regardless of window choice (which is an expertise-informed and subjective decision to make).

Part of the problem of fitting a spectral autoregressive representation to this data is deciding how many terms to include. By recursion, successive residual energies $v_1, v_2, \ldots, v_p$ are calculated, with p being selected such that no more than some explanatory proportion of observed energy k is attained. Here, k was set to be 0.05, and once p is found from this recursive procedure, 1 is added to it. It was found that k does not exceed 0.05 when p is at least 5, and that there is no great variation in spectral computations for values of p between 6 and 12. The value of $k = 0.05$ is thus adopted by Gersch for EEG data analysis, with the resulting p not exceeding 10.

Conclusions

Gersch’s study empirically demonstrates that it is possible to create an autoregressive model that fits observed data, where the model is specified with little to no subjectivity; the procedure is similar to determining the number of predictors to include in a regression model. The computation behind this has N/p degrees of freedom, “where N is the number of observed samples (in each record of a multidimensional time series), and p is the length or order of the autoregressive model fit to the data” (Gersch 217). Generally, the order for an autoregressive model does not exceed 10 for multichannel EEG record durations of 5 to 10 seconds sampled at 5 to 10-millisecond intervals, and this provides decent resolution and higher statistical performance compared to the traditional windowed periodogram method. Autoregressive representations are also smoother and more interpretable than the results provided by windowed periodogram analysis. This points to a trade-off between resolution fidelity and specious bumpiness.

Regarding future work, there is reference to extending the work on autoregressive representations “to the analysis of partial spectral coherences, multiple coherency spectra, their statistical performance, and other detailed applications” (Gersch 217). For future work on analyzing statistical performance, the autoregressive parameters calculate here come from empirical methods, so it would be good to see how estimates could come from bounds, maximum likelihood estimation, or even Bayesian estimation. In addition, computational power has advanced since 1970, and so with computers possessing vast memory and operating at higher speeds, it would be beneficial to replicate this study with higher sampling rate and longer recording times. This should improve spectral estimates and their properties in addition to the improvements expected from the autoregressive representation analytical design.

Bibliography

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