Implicatures of modified numerals

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Abstract

Krifka (1999) and Fox and Hackl (2006) note that *at least n* and *more than n* do not generate scalar implicatures to the effect that *not at least n+1* and *not more than n+1*, respectively. I argue that both Fox and Hackl's 2006 account for the absence of implicatures with comparative *more than n* and Krifka's 1999 and Nouwen's 2008 account for the parallel phenomenon with superlative *at least n* are empirically untenable. In particular, I show that an account is called for that treats comparative and superlative modifiers in the same way with respect to scalar implicatures. A new analysis is developed where not only the numeral but also the modifier introduces alternatives that have to be taken into account by the implicature-generating mechanism. More concretely, *at least*, on the one hand, forms a Horn-set with *at most* and *more than*, on the other hand, forms one with *fewer than*. The resulting alternatives are not ordered by informativeness. I therefore suggest that the grammatical view of implicatures should be adopted (Chierchia (2006), Chierchia et al. (2008), Fox (2007a)), whereby an exhaustivity operator which avoids contradictory strengthening of the basic meaning is put to use. This correctly predicts the empirical picture. Unless the modified numeral is embedded under an element of a certain class of operators, none of the alternatives can be used to strengthen the basic meaning. Otherwise a contradiction would arise.

1 Introduction

While bare numerals such as in the sentence in (1) usually receive an exact-interpretation, numerals modified by comparative *more than* or superlative *at least* such as in (2) do not give rise to such an interpretation.

(1) Three boys attended the party.

(2) a. More than three boys attended the party.

b. At least three boys attended the party.

Krifka (1999) and Fox and Hackl (2006) show that this state of affairs is surprising. If a scalar implicature were factored into the basic meaning of the sentences in (2), an exact-interpretation should result. For instance, for (2b) a scalar implicature negating the stronger alternative proposition *at least 4 boys attended the party* would have the effect that when conjoined with the basic interpretation of (2b) a proposition stating that exactly three boys attended the party should result. As will be seen such an implicature is even more expected given the fact that there are good arguments that the exact-interpretation of the bare numeral in (1) comes about by exactly such a process.

In the present paper, I argue that accounts for the facts in (2) are not completely satisfying empirically. The fully predictive theory by Fox and Hackl (2006), in particular, unfortunately only covers cases with comparative *more than*. I will show, however, that the cases in (2) should be treated on a par. But this has the consequence that the theory makes the wrong predictions for cases with superlative *at least*. Moreover, I will investigate in detail in which environments scalar implicatures surface for modified numerals. From this a descriptive generalization will emerge. I then present my own account of the facts in (2). Based on the assumption that *at least* has as a Horn-alternative *at most*, whereas *more than* has *fewer than* as an alternative, it can be shown that none of the derived propositional alternatives can be negated so to obtain a proposition stronger than the basic interpretation for either of the sentences in (2). This rather simple idea, however, has the consequence that the process deriving scalar implicatures cannot solely be based on a hearer's reasoning taking only into account considerations of informativeness. The reason is that the alternatives are not ordered by entailment, and therefore also not by informativeness. I will suggest that the grammatical view of scalar implicatures (cf. Chierchia (2006), Chierchia et al. (2008), Fox (2007a) a.o.) has no problem to explain the facts if Fox's 2007a exhaustivity operator is assumed. A Neo-Gricean theory would have to adapt Grice's maxim of quantity so that not only more informative propositions are considered when strengthening a the basic denotation of a sentence. This can, of course, be done but might not be the optimal solution given the nature of the Neo-Gricean program. I will also show that the present account has some empirical advantages over competing analyzes of the facts in (2).

The paper is structured as follows: in section 2 I review the Neo-Gricean theory of scalar implicatures and the problem presented by the data in (2). Section 3 discusses Fox and Hackl's 2006 account and presents evidence that the facts in (2) should be handled by one theory. Section 4 investigates in detail under which circumstances scalar implicatures reappear for modified numerals. Then I will present the new account in section 5. Section 6 discusses some consequences of the account and compares it to two competing analyses. Section 7 concludes the paper.

2 The problem

2.1 Gricean reasoning and bare numerals

It has long been noted that numerals should not receive an exact-interpretation but rather an at-least-interpretation as their basic non-strengthened denotation.[1] The exact-interpretation intuitively associated with sentences embedding a numeral should be treated as resulting from an inference, in particular a scalar implicature based on the hearer's reasoning employing Grice's 1975 maxim of quantity (cf. Gazdar (1979), Horn (1972), Levinson (1983) a.m.o.). The maxim of quantity essentially states that if two sentences ϕ and ψ are both relevant in the conversation, and ϕ is more informative than ψ – i.e., the denotation of ϕ asymmetrically entails the one of ψ – and the speaker believes both ϕ and ψ to be true, the speaker should choose ϕ over ψ.

Consider a current version of this theory, often called Neo-Gricean. (3) has an exact-inference. In particular it has the inference associated with it that Jack did not read (at least) four books (represented by $\rightsquigarrow$ throughout the paper) or any larger number of books. This taken together with the basic inference obtained from the assertion that Jack read at least three books derives the strengthened inference that Jack read exactly three books.

(3) Jack read three books.
$\rightsquigarrow$ Jack did not read four books or more

According to Horn (1972) scalar items are lexically collected in sets ordered by informativeness. The following are relevant Horn-sets $\{$or, and$\}$, $\{$some, all$\}$, $\{$1, 2, 3, 4, $\ldots\}$. From these sets of scalar alternatives one derives a set of alternative sentences $Alt(S)$ for a given sentence S by replacing the members of a given Horn-set with each other (Sauerland 2004). The Neo-Gricean version of the maxim of quantity can then be stated as follows:

(4) *Maxim of quantity*
In world w of evaluation if ϕ and ψ are both relevant in the conversation,

[1] When talking about the Neo-Gricean theory, I will refer to the meaning obtained before the scalar implicatures are factored in as the basic or the non-strengthened inference. The meaning where the implicatures are factored in is called the strengthened inference. When talking about the grammatical theory of scalar implicatures I will mostly replace 'inference' by 'meaning'.

$\phi \subset \psi$, $\phi \in Alt(\psi)$, and the speaker believes both $\phi(w) = 1$ and $\psi(w) = 1$, the speaker should choose ϕ over ψ.

For (3) this means that the hearer reasons as follows: the speaker believes that Jack read at least three books, which corresponds to the literal interpretation of (3). This is the basic inference in (5a), and it follows from Grice's maxim of quality, which says that a speaker should not present something as true that she believes to be false. Moreover, $Alt(3)$ includes the stronger propositions that Jack read four books, that Jack read five books and so on. Since these would also have been relevant to the topic of conversation, the speaker should have chosen one of these stronger propositions if she believed them to be true. But the speaker did not choose any of them. The hearer therefore concludes that the speaker does not believe any of these to be true, in particular (5b) follows. (Sauerland 2004:383) argues that if nothing in the context precludes it, the hearer will further strengthen (5b) to (5c), that is, the speaker does not believe that Jack read four books. In particular, this process of strengthening is allowed to apply when no contradiction results.[2] In the present example strengthening (5b) to (5c) does not lead to a contradiction. The stronger inference (5c) together with the speaker's believe that the basic meaning of (3) is true, leads the hearer to conclude (5d), which is equivalent to the strengthened inference that the speaker believes that John read exactly three books.

(5) a. *Basic inference of (3):*
B_S(that Jack read at least 3 books)
b. *Scalar inference of (3):*
$\neg B_S$(that Jack read at least 4 books)
c. *Strengthened scalar inference of (3):*
$B_S \neg$(that Jack read at least 4 books)
d. *Strengthened inference of (3):*
B_S(that Jack read at least 3 books)$\wedge B_S \neg$(that Jack read at least 4 books)

It can now be seen why Horn-alternatives are necessary. Without such stipulated alternatives one would derive for (3) also an alternative proposition stating that Jack read three books but not four. This alternative is also strictly stronger than the basic interpretation of (3), and therefore by the same reasoning process as discussed above, the inference that the speaker does not believe that Jack read three books but not four would be drawn, i.e., the speaker does not believe that Jack read exactly

[2]This step does not follow from Gricean reasoning as such. In order for it to go through, it is necessary to assume that a speaker is opinionated with respect to the truth of the alternatives of a given sentence (cf. the discussion by Sauerland (2004) and Fox (2007a), but also Gazdar (1979)).

three books. But if that inference were derived, the strengthened scalar inference in (5c) could not be drawn. Together with the basic inference it would contradict the inference that the speaker does not believe that Jack read exactly three books. In fact, this latter inference could also not be strengthened to the inference that the speaker believes that Jack did not read exactly three books. Together with the basic inference it would entail that Jack read more than three books. But this contradicts the basic scalar inference in (5b). In other words, no strengthened inference could be drawn at all. For this reason the problem that would arise without Horn-alternatives is termed the symmetry problem. No strengthened scalar inference as in (5c) could ever be drawn.[3]

Evidence for the claim that numerals have the at-least-interpretation as their denotation and moreover form a Horn-set can be attained when embedding them in downward entailing (DE-) contexts. It is known that implicatures tend to disappear in such environments (cf. Gazdar (1979), Chierchia (2004)). This makes the prediction that the exact-interpretation of *three* in (3) should disappear or be weakened when the sentence is embedded in a DE-context, if the exact-interpretation is derived via an implicature. If *three*, however, had the exact-interpretation as its denotation, DE-contexts should not affect the availability of the exact-interpretation. Embedding (3) in DE-contexts such as clausal negation (6a), the antecedent of a conditional (6b), or the restrictor of a universal quantifier (6c) suggests that the former option is correct because in these contexts the exact-interpretation of the numeral *three* is systematically suspended. If the exact-interpretation were the denotation of *three*, the negative statement in (6a), for instance, should only be saying that Jack didn't read exactly three books, which would be compatible with Jack having read four books. The sentence, however, strongly implies that Jack didn't read four or more books either.[4] Similarly, (6b) should imply that only if Jack read exactly three books, he can participate. But, again, it is strongly implied that if Jack read four books, he can as well participate. Lastly, (6c) should imply that only people who read exactly three books can participate. But clearly it is suggested that people who read four books or more can also participate.

(6) a. Jack didn't read three books.
$\rightsquigarrow$ Jack didn't read four books
b. If Jack read three books, he can participate in the course.

[3]For discussion of the symmetry problem see (Fox 2007a) (also cf. Kroch (1972)). The symmetry problem arises whenever one inference prohibits the strengthening of another inference, and vice versa.

[4]This means that the implicature must have been suspended. If this were not the case, a contradiction would ensue. The sentence would convey that Jack didn't read three books, but that he read four books.

$\rightsquigarrow$ If Jack read four books, he can participate

c. Everyone who read three books can participate.
$\rightsquigarrow$ Everyone who read four books can participate

2.2 The absence of implicatures with modified numerals

(Krifka 1999:258) notes that sentences embedding *at least n* – that is, numerals modified by the superlative *at least* – do not lead to an implicature that *not at least n+1*. If (7) licensed such an inference, it would be taken to imply that Jack read exactly three books.

(7) Jack read at least three books.
$\not\rightsquigarrow$ Jack did not read at least four books

(Fox and Hackl 2006:540) note that numerals n modified by comparative *more than* similarly do not give rise to an implicature that *not more than n+1*. Otherwise (8) would imply that Jack read exactly four books.

(8) Jack read more than three books.
$\not\rightsquigarrow$ Jack did not read more than four books

This behavior is unexpected. If numerals form a Horn-set, as was argued in the preceding section for bare numerals, then they should also do so in (7) and (8) thereby yielding the unobserved interpretations. Two types of approaches have been suggested to deal with these data to my knowledge.[5] On the one hand, *at least* has been claimed by Krifka (1999) and Nouwen (2008) to behave similar to *only* in that it consumes the scalar alternatives in its scope (Rooth 1992) and leaves no alternatives to be evaluated for higher operators, i.e., the implicature generating mechanism has no alternatives to operate on because *at least* uses up the alternatives of the numeral. On the other hand, Fox and Hackl (2006) propose a detailed theory of why implicatures are missing for *more than n* making use of the assumption that all measurement scales in natural language are dense. Their argument does not carry over to *at least n*. Therefore a theory like the first one mentioned is needed on top of Fox and Hackl's 2006 one. I will now show that this position is untenable: first, implicatures appear under certain conditions for both *more than n* and *at least n*. And crucially these conditions are the same for both types of modified numerals. This does not follow from Fox and Hackl's 2006 account, and is also unexpected if *at least* consumes the alternatives of the numeral. Second, I point out a false empirical prediction, and suggest that a unified theory

[5] In section 6 we will discuss a further account argued for in particular by Spector (2005).

is needed that accounts for both types of modified numerals. This conjecture will be further strengthened in section 4.[6]

3 Further characterization of the problem

3.1 Density

Fox and Hackl (2006) argue for the following generalization (also cf. Fox (2007b), Nouwen (2008)):

(9) The Universal Density of Measurements (UDM): measurement scales needed for natural language semantics are *always* dense.
(Fox and Hackl 2006:542)

That is, scales in natural language are set up in such a way that for any two degrees there are infinitely many degrees between the former two. In other words, for any n and $n + \epsilon$ there is a degree $n + \delta$ such that $n < n + \delta < n + \epsilon$. This assumption has the consequence that the Horn-set for numerals is not the scale of cardinal numbers $\{1, 2, 3, \dots\}$ but rather a dense scale. What does this mean for the data at hand? Assume with Hackl (2000) that comparative quantifiers contain a silent *many*. (10) on its basic interpretation then states that the comparative quantifier *more than two girls* applies to a set of degrees d such that there is an individual x that John kissed and the cardinality of x is larger than d. This is equivalent to (11a). It is therefore required that John kissed more than two girls, i.e., he kissed $2 + \epsilon$ girls. But since the number scale is dense, it follows that he also kissed $2 + \epsilon/2$ girls. The implicature generating mechanism now negates all stronger alternatives to (11a).[7] The potential implicature states that for any degree larger than 2 it is not the case John kissed more than d-many girls. So he did not kiss more than $n + \epsilon/2$ girls. But this is contradictory because by the basic interpretation of the assertion in (11a) it is required that John kissed $2 + \epsilon$ girls. Contradictory implicatures are not generated. Thus (10) is predicted to not have an exact-interpretation.

(10) John kissed more than two girls.

(11) a. $[\![(10)]\!] = \exists x[|x| > 2$ and John kissed x and x is a girl$]$
b. $\forall d[d > 2 \rightarrow \neg($John kissed more than d-many girls$)]$

[6]There are in fact more cases of modified numerals (for instance, *no more than n*, *between n_i and n_j*), classified by Nouwen (2010) into two subtypes.

[7]Together with Chierchia (2006), Chierchia et al. (2008), Fox (2007a) a.o., Fox and Hackl (2006) assume the so-called grammatical theory of scalar implicatures. See subsection 5.3 below for discussion how the negation of stronger alternatives works technically.

This type of explanation does not carry over to examples with modification by *at least*. The basic interpretation of (12) states that there is an individual of at least three girls such that John kissed that individual. This is equivalent to saying that John kissed 3 girls or $3 + \epsilon$ girls. The potential implicature for (12) states that for all degrees d larger than 3 it is not the case that John kissed at least d-many girls. In other words, it cannot be the case that John kissed (at least) $3 + \epsilon$ girls. But this implicature is consistent with the assertion in (13a), and we get the strengthened interpretation $[\![\ ]\!]^S$ in (13c), where the negated stronger alternatives have been factored into the basic meaning, saying that John kissed exactly three girls.

(12) John kissed at least three girls.

(13) a. $[\![(12)]\!] = \exists x[|x| \geq 3$ and John kissed x and x is a girl]
b. $\forall d[d > 3 \rightarrow \neg$(John kissed at least d-many girls)]
c. $[\![(12)]\!]^S = \exists x[|x| = 3$ and John kissed x and x is a girl]

Thus clearly density only predicts the absence of any scalar implicature for sentences with *more than n* in them. (Fox and Hackl 2006:fn.4) suggest that *at least n* is different from *more than n* and the unavailability of the implicature with the former should be accounted for independently.[8] Nouwen (2008) for related reasons suggests to follow Krifka (1999) in the assumption that *at least* consumes the alternatives of the numeral, similar to the effects of the ~-operator argued for by Rooth (1992) (also cf. Beck (2006)), so that the implicature generating mechanism has no alternatives to work on anymore. Is such an assumption realistic?

3.2 Similarities between *at least n* and *more than n*

(Fox and Hackl 2006:544) note that universal modals reintroduce the implicature of *more than n*, (14), whereas existential modals do not do so, (15). In other words, in the former situation an exact interpretation of the numeral seems to become available.

(14) Jack is required to read more than three books.
$\rightsquigarrow$ There is no degree d larger than 3 such that Jack is required to read more than d-many books

(15) Jack is allowed to read more than three books.
$\not\rightsquigarrow$ There is no degree d larger than 3 such that Jack is allowed to read more than d-many books

[8]For arguments that *at least n* and *more than n* are inherently different see Geurts and Nouwen (2007) and Nouwen (2008, 2010). See the discussion in subsection 6.4.

The difference between embedding *more than n* under a universal modal and under an existential one follows naturally under Fox and Hackl's 2006 proposal. Consider first (14). The basic meaning in (16a) states that in all worlds Jack reads more than three books. This means that in all worlds w there is a degree ϵ such that Jack reads $3 + \epsilon$ books in w and that therefore Jack reads $3 + {}^{\epsilon}\!/{2}$ books in w. The potential implicature in (16b) states that for each degree d greater than 3 there is a world w such that Jack does not read more than d-many books in w. If we assume that the modal base over which the modals quantify are the set of worlds corresponding to the dense degrees d greater than 3, then it follows that for each d there is a world where Jack reads at most d-many books. But the basic meaning in (16a) and the implicature in (16b) are consistent. It is possible that Jack reads more than 3 books in each world and moreover that for each degree d larger than 3 there is a world where Jack reads at most d-many books. The strengthened interpretation in (16c) is derived: there is a world where Jack does not read more than 4 books.

(16) a. $[\![(14)]\!] = \forall w \exists x[|x| > 3$ and Jack reads x and x is a book in $w]$
b. $\forall d[d > 3 \rightarrow \neg\forall w[$Jack reads more than d-many books in $w]$
$= \forall d[d > 3 \rightarrow \exists w[\neg($Jack reads more than d-many books in $w)]$
c. $[\![(14)]\!]^S = \forall w \exists x[|x| > 3$ and Jack reads x and x is a book in $w]$ and $\exists w \neg\exists x[|x| > 4$ and Jack reads x and x is a book in $w]$

The exact interpretation for (15), however, is not allowed. The basic interpretation in (17a) states that in some world Jack read more than 3 books, i.e., there is a world w where Jack reads $3+\epsilon$ books, and thus Jack reads $3+{}^{\epsilon}\!/{2}$ books in w. The potential implicature requires that for all degrees d greater than 3 there is no world where Jack read more than d-many books, (17b). In particular, it requires that there is no world where Jack read more than $3 + {}^{\epsilon}\!/{2}$ books. But this contradicts the basic meaning stating that in w Jack read $3 + \epsilon$ books, and therefore strengthening does not apply.

(17) a. $[\![(15)]\!] = \exists w \exists x[|x| > 3$ and Jack read x in w and x is a book in $w]$
b. $\forall d[d > 3 \rightarrow \neg\exists w[$Jack reads more than d-many books in $w]$

But Fox and Hackl (2006) do not note that *at least n* shows a behavior parallel to the one of *more than n*. Only in a sentence where *at least n* is embedded under a universal modal does the exact interpretation become possible:

(18) Jack is required to read at least three books.
$\rightsquigarrow$ There is no degree d greater than 3 such that Jack is required to read at least d-many books

(19) Jack is allowed to read at least three books.

$\nrightarrow$ There is no degree d greater than 3 such that Jack is allowed to read at least d-many books

We said that Fox and Hackl (2006) do not intend their proposal to capture numerals modified by *at least n*. Nevertheless it is interesting to note for further discussion what their account would predict for the data in (18) and (19). Consider first (18). The basic meaning says that in all worlds w there is a degree ϵ such that Jack reads 3 books or 3 + ϵ books in w, (20a). The potential implicature states that for each degree d greater than 3 there is a world w such that Jack reads fewer than d-many books in w. If the modal base is again the set of worlds corresponding to the dense degrees greater than or equal to 3, the basic meaning and the implicature are consistent: it is possible that in each world Jack reads three books or more while there still being for each degree greater than 3 a world where he reads fewer than d-many books, as long as he reads at least 3 books in each of these worlds. But then it follows that there is no degree greater than 3 such that Jack must read at least d-many books. Strengthening can apply, (20c).

(20) a. $[\![(18)]\!] = \forall w \exists x[|x| \geq 3$ and Jack reads x in w and x is a book in $w]$
b. $\forall d[d > 3 \rightarrow \neg\forall w[$Jack reads at least d-many books in $w]$
$= \forall d[d > 3 \rightarrow \exists w[\neg($Jack reads at least d-many books in $w)]$
c. $[\![(18)]\!]^S = \forall w \exists x[|x| \geq 3$ and Jack reads x and x is a book in $w]$ and $\exists w \neg\exists x[|x| > 3$ and Jack reads x and x is a book in $w]$

What about (19)? Its basic meaning states that there is a world w such that Jack reads 3 books or 3 + ϵ books in w. But the implicature would state that for all degrees d greater than 3 there is no world such that Jack reads at least d-many books in w. It follows that Jack cannot read 3 + ϵ books in w. Nevertheless the implicature is consistent with the basic interpretation, as the strengthened meaning in (21c) would imply that Jack is only allowed to read exactly three books, contrary to fact.

(21) a. $[\![(19)]\!] = \exists w \exists x[x \geq 3$ and Jack reads x in w and x is a book in $w]$
b. $\forall d[d > 3 \rightarrow \neg\exists w[$Jack reads at least d-many books in $w]$
c. $[\![(19)]\!]^S = \exists w \exists x[|x| = 3$ and Jack reads x and x is a book in $w]$

The fact that the density-based approach predicts an implicature for (19) is not in itself problematic. We already know that the account is not meant to be applied to data with numerals modified by *at least*. But it must be noted that the parallel behavior of *at least n* and *more than n* would suggest that the density-based account misses a generalization. In other words, we have cast some initial doubts on the density-based approach to missing implicatures for *more than n*. Moreover, the observation that *at least n* does sometimes have a scalar implicature as in (18) is at odds with Krifka's 1999 and Nouwen's 2008 assumptions that *at least* consumes the alternatives of the numeral. If the latter were the case, implicatures should also be unavailable for (18). But then it follows that this analysis cannot be the independent theory needed by Fox and Hackl (2006) in order to make the correct predictions with respect to *at least n*.

3.3 Embedding under negation

Let us now consider modified numerals embedded under negation, having in mind the surface scope interpretation of the sentences discussed. First note that negative environments like other DE-contexts change the entailment patterns. Therefore the stronger alternatives to, say, 3 are now all the degrees smaller than 3. Consider (22a) under this aspect first. It does not seem to imply that Jack read exactly three books, which would be expected if the implicature noted were available.[9] Notice moreover that *not more than n* is equivalent to *at most n*, where *at most* is DE. In other words, (22a) is equivalent to (22b), and it is clear that (22b) does not have the exact-interpretation either.

(22) a. Jack didn't read more than three books.
$\not\rightsquigarrow$ Jack read more than two books
b. Jack read at most three books.
$\not\rightsquigarrow$ Jack didn't read at most two books

Similarly, (23a) does not have the exact-interpretation, i.e., it does not have the interpretation that Jack read exactly two books. But (23a) is slightly degraded, presumably due to the fact that *at least n* is a positive polarity item and can therefore not occur under negation.[10] The situation is, however, clearer with (23b). Note that *not at least n* is equivalent to *fewer than n*, where *fewer than* is of course DE. (23b) clearly does not have the exact-interpretation.

(23) a. ?Jack didn't read at least three books.
$\not\rightsquigarrow$ Jack read at least two books

[9] Nouwen (2008) discusses the complex numeral *no more than n*, which seems to be different from *more than n* embedded under clausal negation. In particular, he shows that *no more than n* appears to have the exact-interpretation, and he suggests that the very reason for this is density. But (22a) nevertheless does not have the exact-interpretation, and the problem discussed in the text remains.

[10] (23a) is acceptable under the inverse scope interpretation saying that at least three books are such that Jack did not read them. This is as expected from positive polarity items (cf. Szabolcsi (2004) a.o.) but tangential to the present discussion. Geurts and Nouwen (2007), however, suggest that numerals modified by superlatives are not positive polarity items but rather modal expressions. Modal expressions, they argue, are more restricted in distribution than non-modal ones.

b. Jack read fewer than three books.
$\not\rightsquigarrow$ Jack didn't read fewer than two books

In other words, the situation observed is parallel to the one discussed for positive environments. What does the density-based approach predict for the data above? Let us start by considering (22a). The basic meaning states that Jack read 3 or fewer books, (24a), i.e., Jack read 3 books or Jack read $3 - \epsilon$ books. The implicature says that for all degrees d smaller than 3 it is not the case that Jack didn't read more than d-many books, i.e., Jack read more than d-many books. Therefore Jack read more than $3 - \epsilon$ books. But the implicature is consistent: it follows that Jack read exactly three books, (24c), contrary to fact.

(24) a. $[\![(22a)]\!] = \neg\exists x[|x| > 3$ and Jack read x and x is a book$]$
b. $\forall d[d < 3 \rightarrow$ Jack read more than d-many books$]$
c. $[\![(22a)]\!]^S = \exists x[|x| = 3$ and Jack read x and x is a book$]$

The reasoning for (22b) is, of course, almost parallel. The basic meaning states that Jack read 3 books or Jack read $3 - \epsilon$ books, (25a), while the implicature says that for all degrees d smaller than 3 it is the case that Jack read more than d-many books, (25b). Therefore Jack read more than $3 - \epsilon$ books. So Jack read exactly three books, (25c).

(25) a. $[\![(22b)]\!] = \exists x[|x| \leq 3$ and Jack read x and x is a book$]$
b. $\forall d[d < 3 \rightarrow \neg$(Jack read at most d-many books)$]$
c. $[\![(22b)]\!]^S = \exists x[|x| = 3$ and Jack read x and x is a book$]$

Consider now (23a). Its basic interpretation is that Jack read fewer than 3 books, (26a). I.e., Jack read $3 - \epsilon$ books and therefore it is not the case that he read $3 - \epsilon/2$ books. But the potential implicature states that for all degrees d smaller than 3 Jack read at least d-many books (26b), from which it follows that Jack read at least $3 - \epsilon/2$ books. This is a contradiction and the implicature is not generated.

(26) a. $[\![(23a)]\!] = \neg\exists x[|x| \geq 3$ and Jack read x and x is a book$]$
b. $\forall d[d < 3 \rightarrow$ Jack read at least d-many books$]$

Again, the reasoning for (23b) is parallel to the one just given. The basic meaning says that Jack read $3 - \epsilon$ books and therefore he did not read $3 - \epsilon/2$ books, (27a). The implicature would require that he read at least $3 - \epsilon/2$ books (27b), which would be contradictory.[11]

[11]Two things should be noted for (22b) and (23b). The basic meanings given for them in (25a) and (27a), respectively, entail that Jack read at least one book. However, this does not seem to be required by either of them, as shown by the non-contradictory sentences in (i). Barwise and Cooper's 1981

(27) a. $[\![(23b)]\!] = \exists x[|x| < 3$ and Jack read x and x is a book$]$
b. $\forall d[d < 3 \rightarrow \neg$(Jack read fewer than d-many books)$]$

The situation observed is problematic for Fox and Hackl's 2006 account. Recall that according to this approach, the exact-interpretation for numerals modified by comparative *more than* is absent due to the hypothesized density of measurement scales. The absence of such an interpretation for numerals modified by superlative *at least* is in need of an independent explanation, as density alone would predict precisely that interpretation. As shown in the present subsection, however, the pattern switches when the modified numerals are embedded under negation: on the one hand, the absence of an exact-interpretation for *more than n* now does not follow from density anymore but would rather be predicted by it. The absence of such an interpretation for *at least n*, on the other hand, does now follow from density. This is unexpected. It seems that a generalization is missed: in the same environments where numerals with comparative modifiers do not have an exact-interpretation, numerals modified with superlative modifiers do not do so either. And the same holds for the environments where an exact-interpretation is available, as seen in the preceding subsection. In a way this is to be expected given that comparative *not more than n* can be replaced by superlative *at most n* and superlative *not at least n* can be replaced by *fewer than n* (but cf. Geurts and Nouwen (2007)). This immediately predicts that we should observe a parallel behavior for *fewer than n* and *at most n* when embedded under negation. The former is equivalent to *at least n*, whereas the latter can be expressed by *more than n*. Neither of them appears to have the exact-interpretation: (28) does not imply that Jack read exactly four books. Again, (28) is not perfect given the positive polarity status of superlative *at most*. And (29) does not imply that Jack read exactly three books. Crucially, Fox and Hackl (2006) would predict the latter.

(28) ?Jack didn't read at most three books.

Generalized Quantifier theory does not make this prediction (cf. the discussion by Krifka (1999)).

(i) a. Jack read at most three books, in fact he read none.
b. Jack read fewer than three books, in fact he read none.

This fact is, however, not problematic for the argument given in the text. For (25) observe that even if *at most 3* means includes the possibility of zero books, it would still follow that Jack read $3 - \epsilon$ books, which is compatible with the implicature that he read more than $3 - \epsilon$ books. I.e., the exact-interpretation is predicted. Similarly, for (27): even if it were possible that Jack read zero books, it is true that he read $3 - \epsilon$ books and therefore that he did not read $3 - \epsilon/2$ books. The latter is at odds with the implicature, and it would thus not be generated. Second, the interpretations given do not guarantee that Jack did not read more than three books because of cumulativity, as Krifka (1999) shows.

$\not\rightsquigarrow$ Jack read at most four books

(29) Jack didn't read fewer than three books.
$\not\rightsquigarrow$ Jack read fewer than four books

Finally, to complete the empirical picture, observe that scalar implicatures reappear under necessity modals for both *at most n* and *fewer than n*, whereas such inferences are absent under existential modals:

(30) a. Jack is required to read at most three books.
$\rightsquigarrow$ There is no degree d smaller than 3 such that Jack is required to read at most d-many books

b. Jack is allowed to read at most three books.
$\not\rightsquigarrow$ There is no degree d smaller than 3 such that Jack is allowed to read at most d-many books

(31) a. Jack is required to read fewer than three books.
$\rightsquigarrow$ There is no degree d smaller than 3 such that Jack is required to read fewer than d-many books

b. Jack is allowed to read fewer than three books.
$\not\rightsquigarrow$ There is no degree d smaller than 3 such that Jack is allowed to read fewer than d-many books

At present it is not clear to me how Fox and Hackl's 2006 approach could be modified in order to account for the parallelism between numerals modified by superlatives and those modified by comparatives. I therefore conclude that a new approach to the varying absence and presence of scalar implicatures for modified numerals is called for. In the following section, I will try to further strengthen the similarity between *at least n* and *more than n*.

4 Towards a generalization

In the present section I point out an almost obvious difference between examples with modified numerals showing a scalar implicature and examples not doing so. I submit that this fact cannot be a coincidence and propose that it should be the backbone of the descriptive generalization that we wish to account for.

4.1 When are scalar implicatures generated?

Recall that universal modals allow for the appearance of scalar implicatures with both more *more than n* and *at least n*:

(32) a. Jack is required to read more than three books.
$\rightsquigarrow$ Jack is not required to read more than four books
b. Jack is required to read at least three books.
$\rightsquigarrow$ Jack is not required to read four books

This leads us to expect that conditionals should also license scalar implicatures with modified numerals under Stalnaker's 1975 analysis and the approach advocated by Kratzer (1979, 1986) where conditionals are analyzed with implicit universal quantification. This is indeed the case. (33a) implies that there is a world where John is well prepared and he read exactly four books, whereas (33b) implies that there is a world where John is well prepared and he read exactly three books.[12]

(33) a. If John is well prepared, he read more than three books.
$\rightsquigarrow$ Not in all worlds where John is well prepared he read more than four books
b. If John is well prepared, he read at least three books.
$\rightsquigarrow$ Not in all worlds where John is well prepared he read four books

Embedding the modified numeral in the restrictor of the universal quantifier does not affect the availability of the implicature. Only the entailment pattern is reversed. (34a) suggests that there is a world where John read exactly three books and he is not well prepared. Similarly, in (34b) there is a world where John read exactly two books and he is not well prepared.

(34) a. If John read more than three books, he is well prepared.
$\rightsquigarrow$ Not in all worlds where John read more than two books, he is well prepared
b. If John read at least three books, he is well prepared.
$\rightsquigarrow$ Not in all worlds where John read at least two books he is well prepared

(Fox and Hackl 2006:576) note that not only do universal modals allow for the appearance of scalar implicatures with *more than n*, universal quantifiers over individuals do so as well, (35a). I.e., (35a) does have the inference that at least one person wrote exactly four books.[13] I add that the same holds for *at least n*, (35b).

[12]The inverse scope reading does not seem to be particularly salient in examples with conditionals where the modified numeral is in the consequent and should thus not interfere with the judgements. This is possibly due to a prohibition against inverse scope readings that would be stronger than the respective surface scope reading (Mayr and Spector 2011).

[13]Fox and Hackl (2006) have to resort to an infinite domain of individuals over which the universal quantifier ranges, so that no contradiction arises under the density-based approach for the example in (35a) when the scalar implicature is factored in. The infinite set of dense degrees is distributed

It suggests that someone wrote exactly three books.[14]

(35) a. Everyone wrote more than three books.
⇝ Not everyone wrote more than four books
b. Everyone wrote at least three books.
⇝ Not everyone wrote four books

Parallel behavior can be observed by embedding the modified numerals in the restrictor of the universal quantifier. (35a) and (49b) suggest that there is someone who read exactly three books and someone who read exactly two books, respectively, who did not participate.

(36) a. Everyone who read more than three books participated.
⇝ Not everyone who read more than two books participated
b. Everyone who read at least three books participated.
⇝ Not everyone who read at least two books participated

Furthermore, a negative quantifier like *no* has the same effect, with the caveat that *at least* is not perfect in the scope of negation, as already noted above. It seems, however, that *at least* is more acceptable in the restrictor of the negative quantifier, which would suggest that it is a local positive polarity item. (37a) implies that someone wrote exactly three books, and (37b) that someone wrote exactly two books. (38a) implies that someone who read exactly three books participated, and (38b) suggests that someone who read exactly two books participated.

(37) a. No one wrote read more than three books.
⇝ Someone wrote more than two books
b. ?No one wrote at least three books.
⇝ Someone wrote two books

(38) a. No one who read more than three books participated.
⇝ Someone who read more than two books participated
b. No one who read at least thee books participated.
⇝ Someone who read at least two books participated.

Comparative quantifiers like *more than half of the NPs* also allow for the scalar implicature associated with modified numerals to be generated. (39a) implies that

over the infinite set of individuals. This is parallel to *more than n* embedded under necessity modals, where they assume a modal base consisting of an infinite set of worlds over which the dense degrees are distributed.

[14]The inverse scope interpretation is non-sensical for (35a) and (35b) in normal contexts, because usually not everyone writes the same books. Similar considerations apply to examples below.

some students wrote exactly four books, while (39b) implies that some of the students wrote exactly three books.

(39) a. More than half of the students wrote more than three books.
$\leadsto$ At most half of the students wrote more than four books
b. More than half of the students wrote at least three books.
$\leadsto$ At most half of the students wrote four books

Moreover, we observe that distributive conjunctions of individuals show a parallel behavior. The examples in (40) suggest that one of John and Mary wrote exactly four books or exactly three books.[15]

(40) a. John and Mary both wrote more than three books.
$\leadsto$ Not both of John and Mary wrote more than four books
b. John and Mary both wrote at least three books.
$\leadsto$ Not both of John and Mary wrote at least four books

4.2 When are scalar implicatures not generated?

Recall that an exact-interpretation is not generated when modified numerals occur unembedded as in (41), when they occur embedded under negation as in (42), and when they are embedded under existential modals as in (43).

(41) a. Jack read more than three books
$\not\leadsto$ Jack read exactly four books
b. Jack read at least three books
$\not\leadsto$ Jack read exactly three books

(42) a. Jack didn't read more than three books.
$\not\leadsto$ Jack read more than two books
b. ?Jack didn't read at least three books.
$\not\leadsto$ Jack read at least two books

(43) a. Jack is allowed to read more than three books.
$\not\leadsto$ Jack is not allowed to read more than four books
b. Jack is allowed to read at least three books.
$\not\leadsto$ Jack is not allowed to read at least four books

[15] Note that it is not clear how Fox and Hackl (2006) would deal with (40a). Even though the conjunction can be analyzed as involving a universal quantifier of some sort, it appears to be impossible to make use of an infinite domain of individuals over which the quantifier ranges given that the domain of individuals is made explicit. But as noted in footnote 13 it is necessary for them to have infinite domains so that the infinite dense degrees can be distributed over the individuals.

Because of (43) we expect that no implicature is generated when the modified numeral is embedded in the antecedent of a conditional with an existential modal. This is the case, as (44) shows:

(44) a. If Jack read more than three books, he is allowed to participate.
$\not\rightsquigarrow$ There is no world where Jack read more than four books and he participates
b. If Jack read at least three books, he is allowed to participate.
$\not\rightsquigarrow$ There is no world where Jack read at least four books and he participates

(Fox and Hackl 2006:576) again note that existential quantifiers ranging over individuals do not license a scalar implicature for *more than n*, (45a). We note that it also does not do so for *at least n*, (45b). Moreover, we add that the same is true for modified numerals embedded in the restrictor of an existential quantifier, as in (46).

(45) a. Someone wrote more than three books.
$\not\rightsquigarrow$ No one wrote more than four books
b. Someone wrote at least three books.
$\not\rightsquigarrow$ No one wrote (at least) four books

(46) a. Someone who read more than three books participated.
$\not\rightsquigarrow$ No one who read more than four books participated
b. Someone who read at least three books participated.
$\not\rightsquigarrow$ No one who read (at least) four books participated

Finally, scalar implicatures for modified numerals are not generated when they are embedded under a disjunction of individuals:

(47) a. John or Mary wrote more than three books.
$\not\rightsquigarrow$ Neither John nor Mary wrote more than four books
b. John or Mary wrote at least thee books.
$\not\rightsquigarrow$ Neither John nor Mary wrote at least four books

4.3 A generalization

There is a feature that all the examples in subsection 4.1 share, and that is absent from all the examples surveyed in subsection 4.2: notice that in the cases where no scalar implicature is generated in a sentence $Op\phi$ – with Op being a possibly null operator –, the interpretation that had resulted if an implicature had been generated would be entailed by the *basic* interpretation of $Op\psi$, which is just like $Op\phi$ except that the modified numeral has been replaced by an exact numeral expression. This is obvious for the cases not involving any quantification. But for the examples involving existential quantifiers this is also the case. Consider (48a) and (48b) again, repeated from the preceding subsection. Factoring in the implicature would have the consequence that (48a) has as interpretation the proposition that someone wrote more than three books and no one wrote more than four books. (48b) would have the interpretation that someone wrote three books and no one wrote four books. In other words, in each case the strengthened interpretation of the sentence is entailed by the basic interpretation of a minimally differing sentence with an exact numeral expression.

(48) a. Someone wrote more than three books.
$\not\rightsquigarrow$ No one wrote more than four books
b. Someone wrote at least three books.
$\not\rightsquigarrow$ No one wrote (at least) four books

The cases in (48) differ from the ones in (49), repeated from (35a) and (35b) above. The strengthened interpretation in (49a) is not entailed by the proposition that everyone wrote exactly four books. And the strengthened interpretation of (49b) is not entailed by the proposition that everyone wrote exactly three books. Both of these propositions would be the basic interpretations of sentences differing from the ones in (49) only in the fact that the modified numeral has been replaced by an exact numeral expression.

(49) a. Everyone wrote more than three books.
$\rightsquigarrow$ Not everyone wrote more than four books
b. Everyone wrote at least three books.
$\rightsquigarrow$ Not everyone wrote four books

Parallel facts can be shown to hold for the other operators embedding modified numerals. It is thus natural to conclude that a scalar implicature for a modified numeral like *more than n* or *at least n* is generated whenever this does not partially undo the truth conditional contribution of *more than* or *at least* and reduces it to essentially the contribution that *exactly* would make for all worlds or individuals quantified over by the embedding operator if such an operator exists. In other words, the scalar implicature is generated as long as a world or individual can remain after strengthening the basic meaning of the sentence such that *more than* or *at least* makes a truth conditional difference for that world or individual when compared to *exactly*. Simply put, the strengthened interpretation of a sentence with a modified numeral in it should not be entailed by the basic interpretation of an alternative sentence with an *exactly n* expression in it:

(50) *Generalization*
For any ϕ with a modified numeral α *n* in it, a scalar implicature for α *n* is generated iff $[\![\psi]\!]$ does not entail $[\![\phi]\!]^S$, where ψ is just like ϕ except that α *n* is replaced by *exactly n+1*, *exactly n-1*, or *exactly n*.

Given this generalization it is furthermore natural to expect that the exact-interpretation of numerals should play a role in the account of the absence of scalar implicatures for modified numerals.

5 A new account

5.1 The basic idea

Consider the sentences in (51a) and (51b). When are such sentences typically uttered by a speaker?

(51) a. At least three boys left.
b. More than three boys left.

One particularly salient type of context where such sentences can be uttered felicitously is the one where the speaker does not know how many boys exactly left. Assume otherwise. In particular, assume that the speaker knows that exactly four boys left. In such a situation the basic interpretation of both (51a) and (51b) would be felicitous. Nevertheless one would probably judge the speaker to be not very cooperative because she has not made the strongest possible utterance. In other words, upon hearing one utter (51a), the hearer draws the ignorance inference that the speaker does not believe that exactly three boys left. In fact, it seems that (51a) has the ignorance inference that for any number n larger than three the speaker does not believe that n-many boys left. I will argue that for (51a) the alternatives are of the form in (52).

(52) {at least 3 boys left, at least 4 boys left, ..., at most 3 boys left, at most 4 boys left}

The basic ignorance inference is derived by employing the maxim of quantity. That is, we derive the inference that the speaker does not believe that at most three boys left and that at least four boys left, etc.

In essence I will argue that none of the basic ignorance inferences can be strengthened, although I will actually suggest that the grammatical view of scalar implicatures should be adopted. The basic interpretation of (51a) neither entails the alternatives *at least 4 boys left* nor *at most 3 boys left*. The implicature generating process negates all alternatives not entailed by the basic interpretation. But in the present situation this is not possible. The negations of *at least 4 boys left* and *at most 3 boys left* cannot both be true at the same time, i.e., it cannot be that both fewer than four boys left and more than three boys left. Because of this fact, neither of the two negations is factored into the meaning. The same holds for the remaining alternatives. In essence, this account relies on the symmetry problem discussed in subsection 2.1. A parallel account can be given for (51b). Let me now spell out the details.

5.2 Ingredient 1: the alternatives of modified numerals

I propose that the modifiers *at least* and *more than* themselves come with lexical alternatives. In particular, the following Horn-sets are proposed: {*at least*, *at most*} and {*more than*, *fewer than*}. It is important to see that the alternatives in these sets are not ordered by monotonicity.[16] This means that for the sentences in (53), the alternatives in (54a) and (54b) are derived, respectively.[17]

(53) a. At least three boys left.
b. More than three boys left.

(54) a. Alt([[At least three boys left]]) = {at least 3 boys left, at least 4 boys left, ..., at most 3 boys left, at most 4 boys left, ...}
b. Alt([[More than three boys left]]) = {more than 4 boys left, more than 5 boys left, ..., fewer than 4 boys left, fewer than 5 boys left, ...}

For concreteness let us make the rather uncontroversial assumption that the the interpretations of (53a) and (53b) involve existential quantification as in (55a) and (55b), respectively.[18]

[16]This contradicts Matsumoto's 1995 assumptions that the fundamental condition on Horn-sets is that the alternatives are ordered by monotonicity.

[17]Assume that the alternatives are constructed as in Rooth's 1985 theory of focus alternatives. That is, each constituent has two values: the ordinary semantic value and a set of alternative values as its secondary value. In the case of non-scalar items (and unfocused elements) the secondary value is the singleton set containing just the ordinary value. Otherwise, the secondary value is the set of Horn-alternatives. The secondary values of constituents are combined by point-wise functional application. The implicature generating mechanism factors the alternatives into the ordinary meaning, as discussed in subsection 5.3.

[18]What is more controversial is how these interpretations are compositionally derived. The generalized quantifier tradition, on the one hand, would argue that the modifier and the numeral form a constituent taking the NP as a restrictor (cf. Barwise and Cooper (1981), Keenan and Stavi (1986)). Krifka (1999) and Geurts and Nouwen (2007), on the other hand, claim that the NP and the numeral form a constituent. The modifier only supplies the modification of the numeral. The existential im-

(55) a. $[\![(53a)]\!]^w = \exists[|x| \geq 3 \wedge boy_w(x) \wedge left_w(x)]$
b. $[\![(53b)]\!]^w = \exists[|x| > 3 \wedge boy_w(x) \wedge left_w(x)]$

The goal is to create a symmetry problem by employing the alternatives given so that no scalar implicature is derived. Given the (Neo-)Gricean tradition this would mean for, say, (53a) that the hearer would have to reason why the speaker did not choose one of the alternatives in (54a). Recall the Neo-Gricean maxim of quantity, repeated from (56) above:

(56) *Maxim of quantity*
In world w of evaluation if ϕ and ψ are both relevant in the conversation, $\phi \subset \psi$, $\phi \in Alt(\psi)$, and the speaker believes both $\phi(w) = 1$ and $\psi(w) = 1$, the speaker should choose ϕ over ψ.

Notice that because of the informativeness condition in (56) only alternatives strictly stronger than the basic interpretation of (53a) will be negated. Crucially though, the alternatives *at most 4 boys left* , *at most 5 boys left* and so on will not be negated because they do not entail that at least three boys left. In fact, they are logically unrelated. Thus the maxim of quantity cannot be used to generate the symmetry problem that we set out to create. One could adapt (56) in such a way that the informativeness requirement that ϕ is strictly stronger than ψ is replaced by the requirement that ϕ is not weaker than ψ. But I am not sure whether the maxim of quantity should be tampered with. It seems to me that the Gricean spirit might be lost if this adaption is made. Be that as it may. As I will show below the grammatical theory of scalar implicatures has a way of dealing with the examples at hand and their proposed alternatives.

5.3 Ingredient 2: the exhaustivity operator and innocent exclusion

The grammatical view of scalar implicatures (cf. Chierchia (2006), Chierchia et al. (2008), Fox (2007a)) derives the strengthened interpretation of a sentence with a Horn-alternative in it by employing an exhaustivity operator O. That is, they assume that the strengthened interpretation of a sentence like *Jack read three books*, i.e., its exact-interpretation, is derived by applying an operator O such as (57) similar to *only* to the proposition denoted by the sentence, the prejacent (cf. Krifka (1995). This operator states that the prejacent is true and that all alternatives to the prejacent in the set C are either entailed by it or false. In other words, all alternatives to the proposition that Jack read three books not entailed by it must be false.

port is derived from existential closure. This latter option needs some additional assumptions. I do not want to take a side in this debate.

As a consequence it must be false that Jack read four books or any larger number of books.

(57) $[\![\mathrm{O}]\!](C_{\langle\langle s,t\rangle,t\rangle})(p_{\langle s,t\rangle})(w_{\langle s\rangle}) = p(w) \wedge \forall q \in C[q(w) \rightarrow p \subseteq q]$

Following Fox (2007a), who in turn follows Groenendijk and Stokhof (1984) and Sauerland (2004), I adopt a version of the exhaustivity operator O that only negates those alternatives in C whose negation does not automatically require the truth of some other alternative in C. These alternatives are called the *innocently excludable* ones. What this does is to spare those alternatives from negation that would otherwise contradict each other. That is, the negation of these alternatives is not factored into the strengthened interpretation. The definition of O is as in (58) (cf. (Fox 2007a:(61))).

(58) $[\![\mathrm{O}]\!](C_{\langle\langle s,t\rangle,t\rangle})(p_{\langle s,t\rangle})(w_{\langle s\rangle}) = p(w) \wedge \forall q \in C$
$[q$ is innocently excludable given $C \wedge p \nsubseteq q \rightarrow \neg q(w)]$
(where q is innocently excludable given C
if $\neg\exists q' \in C[p(w) \wedge \neg q(w) \rightarrow q'(w)]$)

Consider what this does for the example in (59a). The LF assumed is the one in (59b). As argued in the preceding subsection, the alternatives are as in (59c).

(59) a. At least three boys left.
b. O [C [at least three boys left]]
c. Alt([[At least three boys left]]) = {at least 3 boys left, at least 4 boys left, ..., at most 3 boys left, at most 4 boys left, ...}

O asserts that the prejacent is true – that is, it is true that at least three boys left. Which of the alternatives in (59c) are innocently excludable? Consider first the alternative *at most 3 boys left*. Negation of that alternative would require that at least 4 boys left. That is, the alternative *at least 4 boys left* would be automatically true, i.e., included in the strengthened meaning. Therefore *at most 3 boys* is not innocently excludable. Negation of the alternative *at most 4 boys left* would require the truth of the alternative *at least 5 boys left*. Again, *at most 4 boys left* is not innocently excludable. For completely parallel reasons *at-most*-alternatives with numerals larger than 4 are not innocently excludable either.

Consider next the *at-least*-alternatives. Of course *at least 3 boys left* cannot be negated. This would contradict the prejacent, which is required to be true. The negation of *at least 4 boys left* would entail together with the truth of the prejacent that exactly three boys left. But this would require the truth of the alternative *at most 3 boys left*. And therefore *at least 4 boys left* is not innocently excludable. Similarly, the negation of *at least five boys left* would require the truth of the alternative *at most 4 boys left*. Therefore it is not innocently excludable. And the same holds for *at-least*-alternatives with numerals larger than 5. None of them is innocently excludable.[19]

None of the alternatives in (59c) is innocently excludable. To see clearer what O defined as in (58) achieves, notice that the negations of these alternatives would contradict each other. For instance, the negation of *at least three 4 boys left* – that is, the proposition that fewer than four boys left – contradicts the negation of the alternative *at most 3 boys left* – that is, the proposition that more than three boys left. The negation of both alternatives cannot be true at the same time. O therefore does not negate them. In other words, they are not innocently excludable. But from this it follows that the strengthened interpretation of (59a) is equivalent to its basic interpretation. It does not have a scalar implicature.

For parallel reasons no implicature is generated for (60a) with comparative *more than n*, where the alternatives are as in (60c).

(60) a. More than three boys left.
b. O [C [more than three boys left]]
c. Alt([[More than three boys left]]) = {more than 4 boys left, more than 5 boys left, ..., fewer than 4 boys left, fewer than 5 boys left, ...}

The alternative *fewer than 4 boys left* can be negated given the basic interpretation of (60a). The negation says that at least four boys left, which is just the basic interpretation of (60a). That is, the negation forces the inclusion of *more than 3 boys left*, which is innocent. But *fewer than 5 boys left* cannot be negated. It would automatically make the alternative *more than 4 boys left* true. It is not innocently excludable. And the same holds for *fewer-than*-alternatives with larger numerals. They are all not innocently excludable.

The alternative *more than 4 boys left* cannot be negated. Doing so would automatically include the alternative *fewer than 5 boys left*. Similarly the alternative *more than 5 boys left* cannot be negated either. It would force the inclusion of the alternative *fewer than 6 boys left*. Again, the same holds for *more-than*-alternatives with numerals larger than 6. None of them is innocently excludable. Thus no scalar implicature is generated for (60a), and its strengthened interpretation is just its basic interpretation.

[19] Note that there are in fact also alternatives with numerals smaller than 3. Consider the alternative *at least 2 boys left*. It is entailed by the basic interpretation of (59a). By (58) it therefore cannot be negated. The same holds of course for *at least 1 boy left*. Consider next the negation of the alternative *at most 2 boys left*. It forces the inclusion of the alternative *at least 3 boys left*, which is just the basic meaning. of the sentence. Thus it is innocently excludable. The same holds for *at most 1 boy left*. Thus these alternatives do not affect the overall result obtained for (59a).

It should be noted that the negated versions of *more than n* and *at least n*, as well as the modified numerals *fewer than n* and *at most n* discussed in subsection 3.3 work in parallel way to the cases discussed here. They will not exhibit any scalar implicatures either.

5.4 Intermediate conclusion

We have made use of the alternatives proposed for the modifiers. In particular, we have used the exhaustivity operator argued for by adherents of the grammatical theory of scalar implicatures. This operator allowed us to essentially create a symmetry problem for alternatives that are not ordered by monotonicity, something that would be hard (although not impossible) for a Neo-Gricean theory. I showed that for each alternative which is not already entailed by the basic interpretation or whose negation is not entailed by the basic interpretation, we can find an alternative that prohibits innocent exclusion for it. In other words, for each such alternative we can find another alternative such that the negation of both would result in a contradiction. Because of this fact, none of the alternatives is excluded. Note also that the account given somewhat incorporates the insights from the generalization (50) argued for in subsection 4.3. This generalization said that scalar implicatures are not generated for modified numerals where the strengthened interpretation would entailed by the basic interpretation of a minimally differing sentence where the modified numeral is replaced by an exact numeral expression. Notice that in the account proposed exact numeral expressions play an indirect role: for instance, the conjunction of the alternatives *at least 3 boys left* and *at most 3 boys left* is equivalent to the non-monotonic proposition that exactly three boys left.

6 Further considerations

I now discuss some consequences of the present account and briefly contrast it with another alternative account.

6.1 Reappearance of scalar implicatures

Recall that scalar implicatures of modified numerals appear when embedded under universal quantifiers, but not when embedded under existential ones, as first noted by Fox and Hackl (2006). In terms of the descriptive generalization this means that a scalar implicature appears whenever the embedding operator has the effect that the strengthened interpretation is not entailed by the basic interpretation of a minimally differing sentence with an exact numeral expression. Consider the examples with *at least n* from above.

(61) Jack is required to read at least three books.
$\rightsquigarrow$ Jack is not required to read four books

(62) Jack is allowed to read at least three books.
$\not\rightsquigarrow$ Jack is not allowed to read four books

The symmetry problem does not arise for (61). The relevant alternatives are as in (63).

(63) $Alt([\![$Jack is required to read at least three books$]\!])$ = $\{$Jack is required to read at least 3 books, Jack is required to read at least 4 books, ..., Jack is required to read at most 3 books, Jack is required to read at most 4 books, ...$\}$

Can the *at-most*-alternatives be innocently excluded? The negation of the alternative *Jack is required to read at most 3 books* says that there is a world where Jack reads more than three books. In the example (59a) discussed above negation of the *at most 3* alternative automatically led to the inclusion of the *at least 4 alternative*. Not so in this case: if there is a world where Jack reads more than three books, it does not follow that in all worlds Jack reads more than three books, as would be required by the alternative *Jack is required to read at least 4 books*. Similarly, negation of *Jack is required to read at most 4 books* does not lead to the inclusion of the alternative *Jack read at least 5 books*. Parallel considerations apply for the remaining *at-most*-alternatives.

Consider next the *at-least*-alternatives: Negation of *Jack is required to read at least 3 books* is prohibited by the basic meaning of (61). *Jack is required to read at least 4 books*, however, can be innocently excluded. In particular its negation does not lead to the automatic inclusion of *Jack is required to read at most 3 books*. If there is a world where Jack reads fewer than 4 books – as the former requires –, it does not follow that in all worlds Jack reads fewer than 4 books – as the latter requires. The same holds for alternatives with larger numerals. We thus get the strengthened interpretation for (61) in (64), which is the desired outcome.

(64) $[\![$Jack is required to read at least three books$]\!]^S$ = Jack is required to read at least 3 books, for any $n > 3$ he is not required to read at least n-many books, for any $m \geq 3$ he is not required to read at most m-many books

Consider now (62), which has the alternatives in (65).

(65) $Alt([\![$Jack is allowed to read at least three books$]\!])$ = $\{$Jack is allowed to read at least 3 books, Jack is allowed to read at least 4 books, ..., Jack is allowed to read at most 3 books, Jack is allowed to read at most 4 books,

$\ldots\}$

Negating the alternative *Jack is allowed to read at most 3 books* automatically includes the alternative *Jack is allowed to read at least 4 books*. The negation of the former states that there is no world where Jack reads at most three books, i.e., in all worlds he reads at least four books. Thus the latter must be true. Similar considerations hold for the other *at-most*-alternatives.

Negating the alternative *Jack is allowed to read at least 4 books* would mean that in all worlds Jack reads fewer than 4 books. Therefore the alternative *Jack is allowed to read at most 3 books* would have to be true. In sum, none of the alternatives is innocently excludable, and the strengthened interpretation of (62) is equivalent to is basic meaning. In other words, the present theory correctly distinguishes between (61) and (62).

It is fairly easy to see that a parallel account for numerals modified by comparative *more than* embedded under universal or existential quantifiers can be given. As a way of illustration briefly consider again the example with distributive conjunction of individuals, (66) repeated from (40a), and the one with disjunction of individuals, (67) repeated from (47a).

(66) John and Mary both wrote more than thee books.
$\rightsquigarrow$ Not both of John and Mary wrote more than four books

(67) John or Mary wrote more than three books.
$\not\rightsquigarrow$ Neither John nor Mary wrote more than four books

The former has the relevant alternatives in (68).

(68) Alt([[John and Mary both wrote more than three books]]) = {John and Mary both wrote more than 4 books, John and Mary both wrote more than 5 books, $\ldots$, John and Mary both wrote fewer than 4 books, John and Mary both wrote fewer than 5 books, $\ldots\}$

Negation of the alternative *John and Mary both wrote fewer than 5 books* states that one of John and Mary wrote at least five books. This does not automatically include the alternative *John and Mary both wrote more than 4 books*. It is innocently excludable. But *John and Mary both wrote more than 4 books* can also be negated. Doing so says that one of John and Mary wrote at most 4 books. This does not lead to the inclusion of the alternative *John and Mary both wrote fewer than 5 books*. We get the strengthened interpretation in (69) for (66).

(69) [[John and Mary both wrote more than three books]]^S = John and Mary both wrote more than 3 books, and one of them wrote exactly 4 books.

The relevant alternatives for (67) are in (70).

(70) $Alt([\![$John or Mary wrote more than three books$]\!]) = \{$John or Mary wrote more than 4 books, John or Mary wrote more than 5 books, ..., John or Mary wrote fewer than 4 books, John or Mary wrote fewer than 5 books, ...$\}$

Negating *John or Mary wrote fewer than 5 books* means that both wrote at least five books. But by this the alternative *John or Mary wrote more than 4 books* is automatically included. Moreover negating *John or Mary wrote more than 4 books* means that both wrote at most 4 books. Thus the alternative *John or Mary wrote fewer than 5 books* is automatically included, as well. Therefore none of the alternatives is innocently excludable, and the strengthened interpretation of (67) is equivalent to its basic interpretation.

The distinction drawn by the present account between the examples (66) and (67) is crucial, because it is not predicted by Fox and Hackl's 2006 account, as far as I can see (cf. footnote 15 above).

6.2 An alternative account

Given the descriptive generalization argued for, one might suspect that another plausible account should be preferred to the one presented in the preceding section. In particular, one might want to argue that it is literally *exactly-n*-alternatives that create a problem for the generation of scalar implicatures. In this section I show that such an account faces at least one big problem. For reasons of space, I will only show how such an account would work for numerals modified by superlative *at least*. The problem shows up for comparative modifiers as well, though.

Assume that *at least n* has two types of alternatives. It has stronger alternatives of the form $\{$*exactly n, exactly n+1, exactly n+2, ...*$\}$, Moreover it has scalar alternatives where only the numeral is replaced by another, stronger numeral. The alternatives for the sentence in (71) are then as in (72). In other words, no *at-most*-alternatives would be needed.

(71) At least three boys left.

(72) $Alt([\![$At least three boys left$]\!]) = \{$exactly 3 boys left, exactly 4 boys left, ..., at least 3 boys left, at least 4 boys left, ...$\}$

Note that the alternatives in (72) are partially ordered by entailment. All the alternatives entail the basic meaning of (71) – that is, all entail that at least three boys left. Therefore the (Neo-)Gricean maxim of quantity appears to be more straightforwardly applicable than with the alternatives argued for in the preceding section.

In other words, the exhaustivity operator might not be needed, one might argue.

Let us see how the Neo-Gricean reasoning described in subsection 2.1 for the strengthening of bare numerals would handle the case of (71). A hearer of (71) draws the basic inference in (73a). That is, given the maxim of quality the hearer concludes that the speaker believes the plain meaning of (71), which says that at least three boys left. The alternatives to the plain meaning of (71) are as given in (72). Employing the maxim of quantity, the hearer reasons about the strictly stronger alternatives, which would have been relevant for the discussion: if the speaker believed that exactly three boys left or that exactly four boys left (and similarly for any higher numeral), the speaker would have said so. Since she did not say so, she does not believe these alternative propositions to be true. This derives the ignorance inference in (73b). In a completely parallel fashion, the hearer reasons about the derived scalar alternatives in (72). If the speaker had evidence that at least four boys left, she would have said so. She did not do so. Therefore she does not believe that the stronger scalar alternatives are true, (73c).

(73) a. *Basic inference of (71):*
B_S(that at least 3 boys left)

b. *Ignorance inference of (71):*
$\neg B_S$(that exactly 3 boys left) $\wedge \neg B_S$(that exactly 4 boys left)$\wedge \ldots$

c. *Scalar inference of (71):*
$\neg B_S$(that at least 4 boys left)$\wedge\neg B_S$(that at least 5 boys left)$\wedge \ldots$

The next question is whether any of the inferences in (73) can be strengthened. This is only possible if no contradiction arises. First, the hearer will not derive the stronger inference that the speaker believes it to be false that exactly three boys left. Otherwise it would entail together with the basic inference that the speaker believes it to be true that at least four boys left. But this contradicts the scalar inference in (73c), which says that the speaker does not believe that at least four boys left.

How about the scalar inference in (73c), can it be strengthened? If the speaker believed that it is false that at least four boys left, she would have to believe that exactly three boys left, given the basic inference. But this contradicts the ignorance inference in (73b). It can be seen that we are essentially facing the symmetry problem discussed in subsection 2.1 and moreover employed in the account offered in the preceding section. The hearer can neither conclude that the speaker believes it to be false that exactly three boys left nor that she believes it to be false that at least four boys left. But this also means that we have (almost) solved our initial problem. Sentences with non-embedded *at least n* do not give rise to scalar implicatures. The strengthened interpretation derived so far is then as in (74).

(74) *Strengthened interpretation of (71):*
B_S(that at least 3 boys left])$\wedge$
$\neg B_S$(that exactly 3 boys left)$\wedge\neg B_S$(that at least 4 boys left)

But what about the alternative propositions with numerals larger than 3 or 4? Notice that both the non-strengthened ignorance inference and the scalar inference in (73) contain further conjuncts that are said to not be believed by the speaker. The question arises whether the hearer can conclude for any numeral n larger than 3 that the speaker has the belief that it is false that exactly n-many boys left.

The hearer cannot conclude that for all numerals n larger than three that the speaker believes that not exactly n-many boys left. Together with the basic inference that the speaker believes that at least three boys left, this would entail that she also believes it to be the case that exactly three boys left. But this contradicts the ignorance inference in (73b). But the hearer could conclude that for any numeral m larger than 4 the speaker believes it to be false that exactly m-many boys left. Note that this does not clash with the ignorance inference: the speaker can still not believe the propositions that exactly three boys left and that exactly four boys left to be true. Moreover, the hearer could then also conclude that the speaker believes the scalar alternatives with numerals larger than 4 to be false. In other words, it would follow that the speaker believes that fewer than five boys left. Together with the basic inference a strengthened interpretation would follow that says that the speaker believes that either exactly three or exactly four boys, but not more left, given in (75) with the problematic part in boldface. Clearly, this interpretation is not attested for (71).[20]

(75) *Strengthened interpretation of (71):*
B_S(that at least 3 boys left])$\wedge$
$\neg B_S$(that exactly 3 boys left)$\wedge$
$\neg B_S$(that at least 4 boys left)$\wedge$
$B_S\neg$(that at least 5 boys left)$\wedge B_S\neg$(that at least 6 boys left)$\wedge\ldots$

A fully parallel problem would arise for a sentence with a numeral modified by comparative *more than*. This means that a scalar implicature is after all derived for modified numerals *at least n* and *more than n*, namely one stating that *not at least n+2* and one stating *not more than n+2* is the case, respectively. I.e., it appears that the initial problem has just been shifted one level up on the number scale, and we have only partially accounted for the problem. It is important to see the nature of

[20] For readability, the strengthened ignorance inferences for numerals larger than 4 have been left out. That is, the inference that the speaker believes it to be false that exactly five boys left has been left out, and the same for larger numerals.

the problem a little clearer. The result that no scalar implicature for the numeral $n + 1$ is generated was obtained because the basic inference of the sentence (71) sets a lower bound on how many boys left, namely three. Because of this and the respective ignorance inference the scalar inference where *three* is replaced by the next larger numeral cannot be strengthened. But for scalar alternatives where *three* is replaced with a numeral at least as large as five, this strategy does not work: the basic inference does not set a sufficiently high lower bound on how many boys left. I therefore conclude that an account such as the one proposed in the present paper is needed after all. That is, an account in terms of innocent exclusion and the exhaustivity operator is called for.

Spector (2005) offers an account an account to the present problem along the lines discussed just discussed. He proposes that *more than n* actually has the same formal alternatives as the disjunction *n+1 or more*. As is well known the symmetry problem arises in disjunction independently. In particular, Spector proposes that both sentences in (76) have the alternatives in (77).

(76) a. More than three boys left.
b. Four boys or more left.

(77) Alt([[More than three boys left]]/[[Four boys or more left]]) = {exactly 4 boys left, more than 4 boys left}

These alternatives are symmetric, and no scalar implicature is generated.

Fox (2007b) criticizes Spector's approach on two grounds, one of which might also apply to the present proposal. First, it is not clear why the sentences in (76) have the same alternatives. This necessary ingredient of Spector's proposal appears to be entirely stipulated. Fox notes that with the correct alternatives it is easy to derive the correct meaning. This criticism carries over to the present approach. However, there might be one slight difference: on an intuitive level *at least n* and *at most n* seem to be fairly natural alternatives. Be that as it may. We already know that the account based on the alternatives in (77) also faces independent empirical difficulties: scalar implicatures are derived for numerals at least as high as $n + 2$. This sets the present approach apart from Spector's.

Fox also notes another difference. Given that the sentences in (76) share one set of alternatives, he argues that we expect (76a) to behave similar to (76b) also in other respects. In particular, he suggests that (76a) should show free choice effects, which are common with disjunctions like (76b). Following Fox (2007a) and Kratzer and Shimoyama (2002) a.o. such effects are essentially also scalar implicatures. Therefore if the free choice effect in (76b) is dependent on the alternatives in (77), such an effect should also appear with (76a). But this does not seem to be the case, as the difference in (78) and (79) suggests. If there were a free choice effect associated with (78), the second sentence would license the inference that you are allowed to smoke seven cigarettes and that you are allowed to smoke more than seven cigarettes. This should contradict the third sentence. But intuitively no contradiction arises.

(78) You are allowed to smoke 6 or more cigarettes. I am luckier, I am allowed to smoke 7 or more. #More specifically, I am allowed to smoke 7 cigarettes but not more than 7.
(Fox 2007b:103)

(79) You are allowed to smoke more than 5 cigarettes. I am luckier, I am allowed to smoke more than 6. More specifically, I am allowed to smoke 7 cigarettes but not more than 7.
(Fox 2007b:103)

The present approach, however, has no difficulty with the different status of (78) and (79). It does not claim that (76a) and (76b) share a set of alternatives. Therefore no free choice effect is to be expected in (79).

6.3 Geurts & Nouwen 2007

Geurts and Nouwen (2007) argue that numerals modified by superlatives are inherently different from those modified by comparatives. Whereas the latter have an interpretation similar to the one assumed so far – that is, they are simple existential statements – the latter are modal statements. In particular, the interpretation of (80a) states that it must be the case that three boys left and that it is possible that more than three boys left, where the modals are preferably interpreted as epistemic. The authors present interesting evidence for this claim.

(80) a. At least three boys left.
b. $[\![(80a)]\!]^w = \forall w \exists x[x \text{ is an individual of 3 boys in } w \text{ and } x \text{ left in } w] \wedge \exists w \exists x[x \text{ is an individual of more than 3 boys in } w \text{ and } x \text{ left in } w]$

Given that the basic meaning of (80a) explicitly states that it is a possibility that more than three boys left, a scalar implicature might not be expected to begin with. It is, however, unclear to me how such a theory would account for the parallelism between *more than n* and *at least n* discussed in the present paper. In particular, it is not obvious how the exact-interpretation comes about when *at least n* is embedded under, say, a universal quantifier. In fact, this interpretation would already be part of the basic one. On an independent note it is unclear how such a theory would account for the wide scope interpretation of the universal quantifier in (81a). The interpretation in (81b) would say that every girl must have been met by three boys and for every girl it is possible that she was met by more than three boys.

(81) a. At least three boys met every girl.

b. $[\![(81a)]\!]^w = \forall y[y \text{ is a girl in } w \rightarrow$
$\forall w \exists x[x \text{ is an individual of 3 boys in } w \text{ and } x \text{ met } y \text{ in } w] \wedge$
$\forall y[y \text{ is a girl in } w \rightarrow$
$\exists w \exists x[x \text{ is an individual of 3 boys in } w \text{ and } x \text{ met } y \text{ in } w]$

But the second conjunct of the truth conditions does not appear to conform to our intuitions. Assume a situation with five girls, one of them being Mary. Assume furthermore that the speaker knows that Mary was met by exactly three boys. In such a situation the worlds conforming to the epistemic state of the speaker are such that Mary was met by exactly three boys in each of them. That is, it not the case that for every girl the speaker considers it possible that she was met by more than three boys. Thus (81a) should not be utterable in such a situation. But this appears to be a false prediction.

7 Conclusion

In the present paper I have offered a new account for the absence of scalar implicatures of modified numerals. I argued that both a density-based account[21] and a focus-operator-based one make certain unwelcome predictions. Moreover, I suggested that comparative and superlative modifiers behave on a par and therefore should be accounted for by a related mechanism. The particular account offered makes use of alternatives for the modifiers themselves. That is, I suggested that *at least* and *at most*, on the one hand, and *more than* and *fewer than*, on the other hand, form Horn-sets. Since the resulting alternatives are not completely ordered by entailment I adopted the grammatical view of scalar implicatures. In particular, Chierchia et al.'s 2008 and Fox's 2007a exhaustivity operator, which avoids strengthening that would lead to contradictions, has been put to use. This allowed us to treat comparative and superlative modifiers in a parallel way. I showed that this account makes certain interesting empirical predictions that are not made by competing accounts.

[21] This raises the question whether the universal density of measurement is needed at all for natural language semantics. Fox and Hackl (2006) argue that it is, for instance, also needed to account for weak island phenomena. If Abrusán and Spector (2011), however, are on the right track, then there might be a density-independent way to analyze weak islands.

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